円孔の解

$$\varphi(z) = Az - \frac{2\bar{B}b^2}{z},$$

$$\psi(z) = Bz^2 - 2Ab^2\log z + \frac{\bar{B}b^4}{z^2},$$

$$A = \frac{\sigma_x+\sigma_y}{4}, \quad B = \frac{1}{2}\left(\frac{\sigma_y-\sigma_x}{2} + i\tau\right)$$

$$u - iv = \frac{1}{2G}\left[\frac{3-\nu}{1+\nu}\bar{\varphi} - \{\bar{z}\varphi' + \psi'\}\right]$$

$$-g - ip = \bar{\varphi} + \{\bar{z}\varphi' + \psi'\}$$

$$\chi = \mathrm{Re}\{\bar{z}\varphi + \psi\}$$

$$\bar{z}\varphi + \psi = Az\bar{z} - 2\bar{B}b^2\frac{\bar{z}}{z} + B z^2 - 2Ab^2\log z + \frac{\bar{B}b^4}{z^2}$$

$$= \underline{Ar^2 + Br^2e^{2i\theta}} - 2Ab^2(\log r + i\theta) + \bar{B}b^2\left(\frac{b^2}{r^2} - 2\right)e^{-2i\theta}$$

$1 + \cos 2\theta = 2\cos^2\theta$

$$Ar^2 + Br^2e^{2i\theta} = \left(\frac{\sigma_x+\sigma_y}{4} + \frac{\sigma_y-\sigma_x}{4}e^{2i\theta}\right)r^2 + \frac{i}{2}\tau \cdot r^2 e^{2i\theta}$$

$$\mathrm{Re}\{\quad\} = \frac{\sigma_y}{2}x^2 + \frac{\sigma_x}{2}y^2 - \tau(xy) \quad ///$$

$$\chi^{(p)} = -2Ab^2\log r - b^2\left(2 - \frac{b^2}{r^2}\right)\{B_r\cos 2\theta + B_i\sin 2\theta\} \quad ///$$

$$u - iv = \frac{1}{2G}\left[\frac{3-\nu}{1+\nu}\left(A\bar{z} - \frac{2Bb^2}{\bar{z}}\right) - \left\{\bar{z}\left(A + \frac{2\bar{B}b^2}{z^2}\right) + 2Bz - \frac{2Ab^2}{z} - \frac{2\bar{B}b^4}{z^3}\right\}\right]$$

$$= \frac{1}{2G}\left[\left(\frac{3-\nu}{1+\nu}A\bar{z} - \bar{z}A - 2Bz\right) - \frac{3-\nu}{1+\nu}\frac{2Bb^2}{\bar{z}} - \frac{\bar{z}}{z^2}2\bar{B}b^2 + \frac{2Ab^2}{z} + \frac{2\bar{B}b^4}{z^3}\right]$$

$$= \frac{1}{2G}\Big[\frac{2(1-\nu)}{1+\nu}A(x-iy) - 2B(x+iy)$$

$$- \frac{3-\nu}{1+\nu}\frac{2b^2}{r}(B_r\cos\theta + B_i\sin\theta + iB_r\sin\theta + iB_i\cos\theta) + \cdots$$

$$\frac{2(1-\nu)}{1+\nu}A - 2B = -i\tau + \frac{1-\nu}{2(1+\nu)}(\sigma_x+\sigma_y) - \frac{\sigma_y-\sigma_x}{2} = -i\tau + \frac{\sigma_x}{(1+\nu)} - \frac{\nu\sigma_y}{1+\nu}$$

$$\frac{2(1-\nu)}{1+\nu}A + 2B = +i\tau + \quad '' \quad + \quad '' = i\tau + \frac{\sigma_y - \nu\sigma_x}{1+\nu}$$

$$\frac{2A\cdot b^2}{z} - \frac{3-\nu}{1+\nu}\frac{2B\cdot b^2}{\bar z} = 2\frac{A b^2}{r}(\cos\theta - i\sin\theta) - \frac{2(3-\nu)b^2}{(1+\nu)r}\{(B_r+iB_i)(\cos\theta+i\sin\theta)\}$$

$$= \frac{2b^2\cos\theta}{r}\left[A - \frac{3-\nu}{1+\nu}(B_r+iB_i)\right]$$

$$- \frac{2ib^2}{r}\sin\theta\left[A + \frac{3-\nu}{1+\nu}(B_r+iB_i)\right]$$

$$+\frac{2\bar B}{z^3}b^4 - \frac{\bar z}{z^2}2\bar B b^2 = \frac{2\bar B b^2}{r}\left[\frac{b^2}{r^2}-1\right]e^{-3i\theta}$$

$$A \mp \frac{3-\nu}{1+\nu}B_r = \frac{\sigma_y+\sigma_x}{4} \mp \frac{3-\nu}{1+\nu}\frac{(\sigma_y-\sigma_x)}{4} = \begin{cases} -\frac{\sigma_y(1-\nu)}{2(1+\nu)} + \frac{\sigma_x}{(1+\nu)} \\ \frac{\sigma_y}{1+\nu} - \frac{\sigma_x(1-\nu)}{2(1+\nu)} \end{cases}$$

$$u - iv = \frac{1}{2G}\Big[x\left\{\frac{\sigma_x-\nu\sigma_y}{1+\nu} - i\tau\right\} - iy\left\{\frac{\sigma_y-\nu\sigma_x}{1+\nu}+i\tau\right\}$$

$$+ \frac{2b^2\cos\theta}{2r(1+\nu)}\left\{2\sigma_x - \sigma_y(1-\nu) - (3-\nu)i\frac{\tau}{2}\right\}$$

$$- \frac{ib^2\sin\theta}{r(1+\nu)}\left\{2\sigma_y - (1-\nu)\sigma_x + \frac{i\tau}{2}(3-\nu)\right\}$$

$$+ \frac{2b^2}{r}\left(\frac{b^2}{r^2}-1\right)\left\{B_r\cos3\theta - B_i\sin3\theta + i(-B_r\sin3\theta - B_i\cos3\theta)\right\}\Big]$$

$$\frac{1}{\pi}\int_0^{2\pi}(u-iv)\cos\theta\,d\theta = \frac{b}{2G}\left[\left\{\frac{\sigma_x-\nu\sigma_y}{1+\nu} - i\tau\right\} + \frac{1}{1+\nu}\left\{2\sigma_x - \sigma_y(1-\nu) - (3-\nu)i\frac{\tau}{2}\right\}\right]$$

$$= \frac{b}{E}\left[3\sigma_x - \sigma_y - \frac{(4-\nu)}{2}i\tau\right]$$

$$\frac{i}{\pi}\int_0^{2\pi}(u-iv)\sin\theta\,d\theta = -\frac{i\,b}{2G}\left[\left\{\frac{\sigma_y-\nu\sigma_x}{1+\nu}+i\tau\right\} - \frac{1}{1+\nu}\left\{2\sigma_y - (1-\nu)\sigma_x + \frac{3-\nu}{2}i\tau\right\}\right]$$

$$= -\frac{i\,b}{E}\left\{-\sigma_y + (1-2\nu)\sigma_x - \ldots\right\}$$

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楕円孔の解.

$$\overline{\Phi}(\zeta) = \frac{c}{2}A\zeta - \left(\frac{c}{2R^2}A + \overline{B}c\right)\frac{R^2}{\zeta}$$

$$X(\zeta) = \left(A\frac{c}{2R^2} + Bc\right)\zeta - \frac{c}{2}A\frac{R^2}{\zeta} = \overline{Z}\,\overline{\Phi}^{I} + \Psi^{I}$$

$z = Z = \frac{c}{2}\left(\zeta + \frac{1}{\zeta}\right)$, $\zeta = Re^{i\theta}$ が楕円

$\therefore\ a = \frac{c}{2}\left(R + \frac{1}{R}\right)$, $b = \frac{c}{2}\left(R - \frac{1}{R}\right)$

$$\overline{\Phi}^{I} = \overline{\Phi}'\frac{d\zeta}{dz} = \left\{\frac{c}{2}A + \left(\frac{c}{2R^2}A + \overline{B}c\right)\frac{R^2}{\zeta^2}\right\}\frac{1}{\frac{c}{2}\left(1 - \frac{1}{\zeta^2}\right)}$$

$$\Psi^{I} = X - \overline{Z}\,\overline{\Phi}^{I} = X - \left(\frac{\frac{R^2}{\zeta} + \frac{\zeta}{R^2}}{1 - \frac{1}{\zeta^2}}\right)\left\{\frac{c}{2}A + \left(\frac{c}{2R^2}A + \overline{B}c\right)\frac{R^2}{\zeta^2}\right\}$$

$z \to \infty$ $\quad Z - \frac{c}{2}\zeta + \frac{c}{2\zeta}$, $\quad \frac{c}{2}\zeta = z - \frac{c}{2\zeta} \fallingdotseq z - \frac{c^2}{4z}$

$$\overline{\Phi} \longrightarrow A\left(z - \frac{c^2}{4z}\right) - \left(\frac{c}{2R^2}A + \overline{B}c\right)\frac{cR^2}{2z}$$

$$= Az - \left(A\frac{c^2}{2} + \frac{c^2}{2}R^2\overline{B}\right)\frac{1}{z}$$

$$X \longrightarrow \left(\frac{A}{R^2} + 2B\right)\left(z - \frac{c^2}{4z}\right) - \frac{c^2}{4}\frac{AR^2}{z}$$

$$= \left(\frac{A}{R^2} + 2B\right)z - \frac{c^2}{4z}\left(AR^2 + \frac{A}{R^2} + 2B\right)$$

$$\overline{Z}\left(\frac{R^2}{\zeta}\right)\overline{\Phi}^{I} \fallingdotseq \frac{c}{2}\left(\frac{R^2}{\zeta} + \frac{\zeta}{R^2}\right)\left\{A + \frac{c^2}{z^2}(A + \overline{B}R^2)\right\}$$

$$\left\{\frac{z - \frac{c^2}{4z}}{R^2} + \frac{c^2R^2}{4z}\right\}$$

$$\left\{\frac{z}{R^2} + \frac{c^2}{4z}\left(R^2 - \frac{1}{R^2}\right)\right\}\left\{A + \frac{c^2}{z^2}(A + \overline{B}R^2)\right\}$$

$$= \frac{A}{R^2}z + \frac{1}{z}\left\{\frac{c^2}{4}\left(R^2 - \frac{1}{R^2}\right)A + \frac{c^2}{R^2}(A + \overline{B}R^2)\right\} + \cdots$$

$$\Psi^{I} = 2Bz$$

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$x^2+y^2 \to \frac{c^2}{4} e^{2\alpha}$, $\alpha \gg 1$ ... $\frac{4r^2}{c^2}$ $\alpha = \log r$

$x = c\,\mathrm{ch}\,\alpha \cos\beta$, $y = c\,\mathrm{sh}\,\alpha \sin\beta$. $x^2 \to \frac{c^2 e^{2\alpha}}{4} \cos^2\beta$.

$a = c\,\mathrm{ch}\,\alpha_0$, $b = c\,\mathrm{sh}\,\alpha_0$, $a^2 - b^2 = c^2$

$a+b = c\,e^{\alpha_0}$, $\left(\frac{a+b}{c}\right)^2 = e^{2\alpha_0}$.

$1 + 2e^{2\alpha_0} = 1 + 2\frac{(a+b)^2}{c^2} =$, $\mathrm{ch}\,2\alpha_0 = \mathrm{ch}^2\alpha_0 + \mathrm{sh}^2\alpha_0$
$= \frac{a^2+b^2}{c^2}$

$\mathrm{ch}\,2\alpha_0 + 1 = \frac{a^2+b^2+a^2-b^2}{c^2} = \frac{2a^2}{c^2}$ //

$1 + 2e^{2\alpha_0} = \frac{1}{c^2}\left[a^2 - b^2 + 2(a+b)^2\right] = \frac{3a^2+b^2+4ab}{c^2}$

$F = \frac{Tc^2}{16}\left[F_1 - (1+2e^{2\alpha_0})F_2 + e^{2\alpha_0} F_3 - 4(1+\mathrm{ch}\,2\alpha_0)F_4 + F_5\right]$

$e^{2\alpha} \cdots$

$F_1 = e^{2\alpha} + \cos 2\beta$ ⟶
$F_5 = e^{2\alpha} \cos 2\beta$.

$F_1 + F_5 = e^{2\alpha}\, 2\cos^2\beta + \cos 2\beta$.
$\longrightarrow \frac{8x^2}{c^2} + \cos 2\beta$,

$F_2 = e^{-2\alpha} + \cos 2\beta \longrightarrow \cos 2\beta$.

$F_3 = e^{-2\alpha} \cos 2\beta \longrightarrow 0$.

$$F = \frac{Tc^2}{16}\left[\frac{8x^2}{c^2} + \underbrace{\cos 2\beta - (1+2e^{2\alpha_0})\cos 2\beta}_{-2e^{2\alpha_0}\cos 2\beta} - 4\alpha(1+\mathrm{ch}\,2\alpha_0)\right]$$

$\simeq \frac{Tx^2}{2} - \frac{e^{2\alpha_0}}{8} Tc^2 \cos 2\beta - \frac{(1+\mathrm{ch}\,2\alpha_0)Tc^2}{4}[\alpha]$

$\left(\frac{1+\mathrm{ch}\,2\alpha_0}{4}\right) = \frac{a^2}{2c^2}$, $\frac{e^{2\alpha_0}}{8} = \frac{(a+b)^2}{8c^2} \xrightarrow{a-b} \frac{a^2}{2c^2}$

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$$U(p,Q) = \frac{1}{2\pi}\left[y\theta - \frac{1-\nu}{2} x \log r \right]$$

$$V(p,Q) = \frac{-1}{2\pi}\left[x\theta + \frac{1-\nu}{2} y \log r \right]$$

$$\frac{\partial U}{\partial x} = \frac{1}{2\pi}\left[y\theta_x - \frac{1-\nu}{2}\log r - \frac{1-\nu}{2}\frac{x^2}{r^2} \right]$$

$$\frac{\partial U}{\partial y} = \frac{1}{2\pi}\left[\theta + y\theta_y - \frac{1-\nu}{2}\frac{xy}{r^2} \right] \qquad \theta_y = \frac{x}{r^2}$$

$$V_x = -\frac{1}{2\pi}\left[\theta + x\theta_x + \frac{1+\nu}{2}\frac{xy}{r^2} \right] \qquad \theta_x = -\frac{y}{r^2}$$

$$V_y = \frac{-1}{2\pi}\left[x\theta_y + \frac{1+\nu}{2}\log r + \frac{1+\nu}{2}\frac{y^2}{r^2} \right]$$

$$U_x = \frac{1}{2\pi}\left[-\frac{y^2}{r^2} - \frac{1-\nu}{2}\frac{x^2}{r^2} - \frac{1-\nu}{2}\log r \right]$$

$$V_y = -\frac{1}{2\pi}\left[\frac{x^2}{r^2} + \frac{1+\nu}{2}\frac{y^2}{r^2} + \frac{1+\nu}{2}\log r \right]$$

$$U_y = \frac{1}{2\pi}\left[\theta + \frac{1+\nu}{2}\frac{xy}{r^2} \right] \qquad \frac{1-\nu}{2} - 1 = \frac{-(1+\nu)}{2}$$

$$V_x = \frac{1}{2\pi}\left[-\theta + \frac{1+\nu}{2}\frac{xy}{r^2} \right]$$

$U_0 x_1 + V_0 y_1$

$$U_x = -\frac{1}{2\pi}\left[\frac{y^2}{r^2} + \frac{1-\nu}{2}\frac{x^2}{r^2} + \frac{1-\nu}{2}\log r \right]$$

$$V_y = -\frac{1}{2\pi}\left[\frac{x^2}{r^2} + \frac{1-\nu}{2}\frac{y^2}{r^2} + \frac{1-\nu}{2}\log r \right]$$

$$\int \left\{ p(x U_x + y V_y) + q(x V_x + y V_y) \right\} ds \qquad \int p x + q y$$

$$\theta \int (py - qx)\, ds = \theta M = 0$$

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$$U_x = -\frac{1}{2\pi}\left[\frac{1}{2}\left(1+\frac{1-\nu}{2}\right) + \frac{1}{4}\left(\frac{1-\nu}{2}-1\right)\cos 2\theta + \frac{1-\nu}{2}\log r\right]$$

$$V_y = -\frac{1}{2\pi}\left[\frac{3-\nu}{4} + \frac{1}{2}\left(1-\frac{1-\nu}{2}\right)\cos 2\theta + \frac{1-\nu}{2}\log r\right]$$

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$$I = \int_c U y_s \, ds = \int_c (u_0 + u) y_s \, ds \qquad u_0 = \frac{-\nu X}{E}$$

$$y_s = -p_1 = -\frac{\partial^2 \left(\frac{y^2}{2}\right)}{\partial s \partial y}, \quad q_1 = 0, \quad u_{10} = \frac{X}{E}, \quad v_{10} = -\frac{\nu y}{E}$$

$q =$

$$I = \int_c \left(u y_s + \frac{\nu}{E} x x_n \right) ds = \int_c \left(u y_s + v q_{10} - u_{10} p - u_{10} q \right) ds$$

$y \, y_n$

$u_{10} p = 0$; $u_{10} q = 0 + v_{10} q_{30} = -\frac{\nu}{E} y x_s = \frac{\nu}{E} y y_n$

$$= -\int_c \left(y u_s + \frac{x f_{ys} + \nu y f_{xs}}{E} \right) ds$$

$$u_s = u_x x_s + u_y y_s = \xi_x x_s + \xi_y y_s - \frac{1}{2G}(f_{xx} x_s + f_{xy} y_s)$$

$$= \xi_s - \frac{1}{2G} f_{xs}$$

$$\int y u_s \, ds = \int \left(y \xi_s - \frac{y}{2G} f_{xs} \right) ds$$

$$I = -\int \left[y \xi_s + \frac{x f_{ys} - y f_{xs}}{E} \right] ds = -\int \left[y \xi_s + \frac{y_s f_x - x_s f_y}{E} \right] ds$$

$$= -\int \left[y \xi_s + \frac{f_r}{E} \right] ds = \frac{A}{2} \qquad ?$$

$$-\int V x_s \, ds = \frac{A}{2}$$

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$$2\pi\,\xi_x = -B\frac{x^2-y^2}{r^4} + C\frac{2xy}{r^4} = \frac{\partial}{\partial x}\left(B\frac{x}{r^2} + C\frac{y}{r^2}\right)$$

$$2\pi\,\eta_x = +B\frac{2xy}{r^4} + \frac{C(x^2-y^2)}{r^4} = \frac{\partial}{\partial x}\left(B\frac{y}{r^2} - C\frac{x}{r^2}\right)$$

$$\frac{\partial}{\partial x}\frac{x}{r^2} = \frac{1}{r^2} - \frac{2x^2}{r^4} = \frac{y^2-x^2}{r^4}$$

$$\frac{\partial}{\partial x}\frac{y}{r^2} = -\frac{2xy}{r^4}$$

$-y + ix =$

$-i(x-iy)$

$$2\pi\xi = \frac{Bx + Cy}{r^2}, \quad 2\pi\eta = -\frac{By + Cx}{r^2}$$

$$2\pi(\xi + i\eta) = \frac{B - iC}{z}$$

$C\,(y + ix)$

$$\xi = \frac{1}{2\pi r}(B\cos\theta + C\sin\theta)$$

$$\frac{y^2-x^2}{r^2} = -\frac{x^2-y^2}{r^2}$$

$$\xi_s = \frac{1}{2\pi r^2}(-B\sin\theta + C\cos\theta)$$

$$2\pi\,\xi_y = -\frac{2Bxy}{r^4} + C\frac{x^2-y^2}{r^4}$$

$$\eta = \frac{-1}{2\pi r}(B\sin\theta + C\cos\theta)$$

$C_1 = -\pi a^2;\ C_3 = 3\pi a^2,\ C_2 = 0$

円 $\frac{A}{4\pi} = \frac{a^2}{2}$, $\frac{B}{8\pi} = \frac{a^2}{2}$, $A = 2\pi a^2;\ B = 4\pi a^2;\ C = 0$

楕円. $\frac{A}{4\pi} = \frac{a^2}{2}$, $\frac{B}{8\pi} = \frac{(a+b)^2}{8}$, $\begin{cases} \frac{C_3}{4\pi} = \frac{a^2}{2} + \frac{(a+b)^2}{4} = \frac{3a^2+b^2+2ab}{4} \\ \frac{C_1}{4\pi} = \frac{a^2}{2} - \frac{(a+b)^2}{4}, \quad C_2 = 0. \end{cases}$

$$S = \frac{r^2}{8\pi} \lg r$$

$$S_x = \frac{1}{8\pi}\left[2x \lg r + x\right]$$

$$S_y = \frac{1}{8\pi}\left[2y \lg r + y\right]$$

$$S_{xx} = \frac{1}{4\pi}\left[\lg r + \frac{1}{2} + \frac{x^2}{r^2}\right] = \frac{1}{4\pi}\left[\lg r + \frac{1+\cos 2\theta}{2} + \frac{1}{2}\right]$$

$$S_{xy} = \frac{xy}{4\pi r^2} = \frac{\sin 2\theta}{8\pi}$$

$$S_{yy} = \frac{1}{4\pi}\left[\lg r + \frac{1}{2} + \frac{y^2}{r^2}\right] = \frac{1}{4\pi}\left[\lg r - \frac{\cos 2\theta}{2} + 1\right].$$

$$f \Rightarrow -C_1 S_{yy} - C_2 S_{xy} - C_3 S_{xx}$$

$C_1 = \int U y_s\, ds$ | $A = \int (U y_s - V x_s)\, ds$

$C_2 = \int_C (U x_s - V y_s)\, ds$ | $B = -\int (U y_s + V x_s)\, ds,$

$C_3 = -\int V x_s\, ds.$ | $C_2 = C$

$$f \to -\frac{C_1}{4\pi}\left(\lg r - \frac{\cos 2\theta}{2}\right) - \frac{C_2}{8\pi}\sin 2\theta - \frac{C_3}{4\pi}\left(\lg r + \frac{\cos 2\theta}{2}\right)$$

$$= -\frac{(C_1+C_3)}{4\pi}\lg r + \frac{\cos 2\theta}{8\pi}(C_1 - C_3) - \frac{C_2}{8\pi}\sin 2\theta.$$

$$A = (C_1 + C_3), \quad B = C_3 - C_1, \quad \frac{C_2}{2} = C.$$

$$= -\frac{1}{4\pi}\left[A \lg r + \frac{B}{2}\cos 2\theta + \frac{C}{2}\sin 2\theta\right]$$

$C_3 = (A+B)/2, \quad C_1 = A - B$

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$$I = V(F,F)(x x_n + y y_n) - \{ F_{ys}(x U_x + y U_y) - F_{xs}(x V_x + y V_y) \}$$

$$I = \frac{1}{2}\{ F_{yy} U_x + F_{xx} V_y - F_{xy}(U_y + V_x) \} - \{ \quad '' \quad \}$$

$$F = \frac{x^2}{2} + f, \quad U = -\frac{\nu x}{E} + u, \quad V = \frac{y}{E} + v$$

$$I = \frac{1}{2}\left[f_{yy}\left(-\frac{\nu}{E} + u_x\right) + (1 + f_{xx})\left(\frac{1}{E} + v_y\right) - f_{xy}(u_y + v_x) \right]$$

$$- \left[f_{ys}\left\{ x\left(-\frac{\nu}{E} + u_x\right) + y u_y \right\} - (x_s + f_{xs})\left\{ x v_x + y\left(\frac{1}{E} + v_y\right) \right\} \right]$$

$$= \frac{1}{2}\left[-\frac{\nu}{E} f_{yy} + \frac{1}{E} + \frac{f_{xx}}{E} + v_y \right](x x_n + y y_n) \quad O(\varepsilon^2)$$

$$- \left[-\frac{\nu x}{E} f_{ys} - x x_s v_x - \frac{x_s y}{E} - x_s y v_y - \frac{y}{E} f_{xs} \right]$$

$$= \frac{(x y_s + y x_s)}{2E} + \frac{1}{2}\left(v_y + \frac{f_{xx} - \nu f_{yy}}{E} \right)(x y_s - y x_s)$$

$$+ \frac{\nu x}{E} f_{ys} + \frac{y f_{xs}}{E} + x_s (x v_x + y v_y)$$

$v_y = \eta_y - \frac{f_{yy}}{2G}$

$v_x = \eta_x - \frac{f_{xy}}{2G}$

$$= \frac{x y_s + y x_s}{2E} + \frac{1}{2E}(f_{xx} - \nu f_{yy})(x y_s - y x_s) + \frac{1}{E}(\nu x f_{ys} + y f_{xs})$$

$$+ x x_s v_x + \frac{v_y}{2}(x y_s + y x_s)$$

$$\nu x f_{ys} + y f_{xs} = \nu x (f_{yy} y_s + f_{yx} x_s) + y (f_{xx} x_s + f_{xy} y_s)$$

$$x x_s v_x + \frac{v_y}{2}(x y_s + y x_s) = \overset{r\cos^2\theta}{x x_s} \eta_x + \overset{r\cos 2\theta}{\frac{(x y_s + y x_s)}{2}} \eta_y$$

$$- \frac{1}{2G}\left[x x_s f_{xy} + \frac{(x y_s + y x_s)}{2} f_{yy} \right]$$

~~$= \frac{r}{2}\cos 2\theta (\eta_x + \eta_y) + \frac{r}{2}\eta_x - \frac{r}{2G}\left[f_{xy}\cos^2\theta + \frac{\cos 2\theta}{2} f_{yy} \right]$~~

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$$I = I_0 + I_1 + I_2 + I_3$$

$$I_1 = \frac{r}{2E}(f_{xx} - \nu f_{yy})$$

$$I_2 = \frac{1}{E}(\nu x f_{ys} + y f_{xs}) \rightarrow -\frac{1}{E}(\nu f_y x_s + f_x y_s)$$

$$I_3 = x x_s v_x + \frac{v_y}{2} r \cos 2\theta = r\left[-v_x \sin\theta\cos\theta + \frac{v_y}{2}\cos 2\theta\right]$$

$$= I_4 + I_5$$

$$I_4 = \frac{r}{2}\left[-\eta_x \sin 2\theta + \eta_y \cos 2\theta\right]$$

$$I_5 = -\frac{r}{2G}\left[f_{xy}\sin\theta\cos\theta + \frac{\cos 2\theta}{2} f_{yy}\right]$$

$$\int_R I_1\, ds = 0 = \int I_0\, ds$$

$$\int I_2\, ds = -\frac{1}{E}\int (f_x y_s + \nu f_y x_s)\, ds = \frac{1}{4\pi}\left[\pi A + \frac{B}{2}\pi - \nu\pi A + \frac{\nu}{2}\pi B\right]$$

$$= \frac{1}{4}\left[(1-\nu)A + \left(\frac{1+\nu}{2}\right)B\right]$$

$$\int I_4\, ds = \frac{1}{2}[B] \quad -\frac{1+\nu}{2E}\int f_{xy}\sin 2\theta\, ds \quad \frac{1+\nu}{8}A +$$

$$\int I_5\, ds = -\frac{1+\nu}{2E}\int f_{yy}\cos 2\theta\, ds = -\frac{(1+\nu)}{8}(-A + B)$$

$$\int I\, ds = \frac{A}{2} + \frac{B}{2}$$

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$$\frac{\partial}{\partial x} = \frac{\partial r}{\partial x}\frac{\partial}{\partial r} + \frac{\partial \theta}{\partial x}\frac{\partial}{\partial \theta} = \cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta} \qquad \frac{\partial x}{\partial r} = \cos\theta$$

$$\frac{\partial}{\partial y} = \sin\theta \frac{\partial}{\partial r} + \frac{\cos\theta}{r}\frac{\partial}{\partial \theta} \qquad \frac{\partial y}{\partial r} = \sin\theta$$

$$f_x = f_r \cos\theta - \frac{\sin\theta}{r} f_\theta$$

$$f_y = f_r \sin\theta + \frac{\cos\theta}{r} f_\theta$$

$$f_{xx} = \left(\cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\right)\left(f_r \cos\theta - \frac{\sin\theta}{r} f_\theta\right)$$

$$= \cos^2\theta f_{rr} + \frac{\sin^2\theta}{r} f_r - \frac{2\sin\theta\cos\theta}{r} f_{r\theta} + \frac{2\cos\theta\sin\theta}{r^2} f_\theta + \frac{\sin^2\theta}{r^2} f_{\theta\theta}$$

$$f_{xy} = \sin\theta\left(f_{rr}\cos\theta + \frac{\sin\theta}{r^2} f_\theta - \frac{\sin\theta}{r} f_{\theta r}\right) + \frac{\cos\theta}{r}\left(f_{r\theta}\cos\theta +\right.$$

$$+ \frac{\cos\theta}{r}\left(f_{r\theta}\cos\theta - f_r \sin\theta - \frac{\cos\theta}{r} f_\theta - \frac{\sin\theta}{r} f_{\theta\theta}\right)$$

$$= f_{rr}\sin\theta\cos\theta + \frac{\cos 2\theta}{r} f_{\theta r} - \frac{\sin\theta\cos\theta}{r^2} f_{\theta\theta}$$

$$- \frac{\sin\theta\cos\theta}{r} f_r - \frac{\cos 2\theta}{r^2} f_\theta$$

$$f_{yy} = \sin\theta\left(f_{rr}\sin\theta - \frac{\cos\theta}{r^2} f_\theta + \frac{\cos\theta}{r} f_{\theta r}\right)$$

$$+ \frac{\cos\theta}{r}\left(f_r\cos\theta + f_{r\theta}\sin\theta - \frac{\sin\theta}{r} f_\theta + \frac{\cos\theta}{r} f_{\theta\theta}\right)$$

$$= f_{rr}\sin^2\theta + \frac{\sin 2\theta}{r} f_{\theta r} + \frac{\cos^2\theta}{r^2} f_{\theta\theta}$$

$$+ \frac{\cos^2\theta}{r} f_r - \frac{\sin 2\theta}{r} f_\theta$$

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$$\frac{4\pi}{E} f_{xx} = \frac{A}{r^2}\cos^2\theta + \frac{\sin^2\theta}{r^2}(2B\cos2\theta + 2C\sin2\theta)$$

$$+ \frac{\sin^2\theta}{r}\left(-\frac{A}{r}\right) + \frac{\sin2\theta}{r^2}(B\sin2\theta - C\cos2\theta)$$

$$(\cos2\theta - \cos4\theta)$$

$$= \frac{A}{r^2}\cos2\theta + \frac{B}{r^2}\left(\cos2\theta(1-\cos2\theta) + \sin^2 2\theta\right)$$

$$+ \frac{C}{r^2}\left\{\sin2\theta(1-\cos2\theta) - \sin2\theta\cos2\theta\right\}$$

$$\frac{4\pi}{E} f_{yy} = \frac{A}{r^2}\sin^2\theta + \frac{\cos^2\theta}{r^2}(2B\cos2\theta + C\sin2\theta) - \frac{A}{r^2}\cos^2\theta$$

$$- \frac{\sin2\theta}{r}(B\sin2\theta - C\cos2\theta)$$

$$= \frac{-A}{r^2}\cos2\theta + \frac{B}{r^2}\left(\cos2\theta(1+\cos2\theta) - \sin^2 2\theta\right)$$

$$+ \frac{C}{r^2}\left(\sin2\theta(1+\cos2\theta) + \sin2\theta\cos2\theta\right)$$

$$\frac{4\pi}{E}\Delta f = \frac{2B}{r^2}\cos2\theta + \frac{2C}{r^2}\sin2\theta.$$

$$\frac{4\pi}{E} f_{xy} = \frac{A}{r^2}\sin\theta\cos\theta - \frac{\sin2\theta}{r^2}(B\cos2\theta + C\sin2\theta)$$

$$+ \frac{A}{r^2}\sin\theta\cos\theta + \frac{\cos2\theta}{r^2}(-B\sin2\theta + C\cos2\theta)$$

$$= \frac{A}{r^2}\sin2\theta - \frac{B}{r^2}\sin4\theta + \frac{C}{r^2}\cos4\theta$$

$$+ \frac{?}{?}(\cos3\theta - \cos\theta)$$

$$\frac{4\pi}{E} f_x = -\frac{A}{r}\cos\theta - \frac{\sin\theta}{r}(B\sin2\theta - C\cos2\theta) \quad -\cos\theta$$

$$\sin\theta\cos2\theta = \sin3\theta - \sin\theta$$

$$\frac{4\pi}{E} f_y = -\frac{A}{r}\sin\theta + \frac{\cos\theta}{r}(B\sin2\theta - C\cos2\theta) \quad +\sin\theta$$

$$\downarrow$$

$$\frac{B}{2r}(\sin3\theta + \sin\theta)$$