9　M. 80

No.

Date 56.5.5

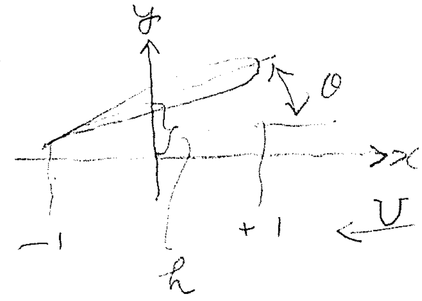
無限流体中の振動翼

M.B.

内容

1. 加速度ポテンシャルと速度ポテンシャル

加速度ポテンシャル Φ, 圧力を p とすると



$$\Phi(x,y) = \frac{1}{\rho} p(x,y), \quad \cdots (1)$$

(Pressure)

速度ポテンシャルとの関係は

$$\Phi(x,y) = \left(i\omega - U\frac{\partial}{\partial x}\right)\phi(x,y), \quad (2)$$

それゆえ

$$\phi(x,y) = \frac{1}{U} e^{i\alpha x}\int_x^\infty \Phi(x,y)\, e^{-i\alpha x}\, dx, \quad (3), \quad \alpha = \frac{\omega}{U}$$

翼面の上下で

$$\Phi_- - \Phi_+ = \frac{p}{\rho}, \quad \cdots (4)$$

であるとすると

$$\frac{1}{2\pi}\int_{-1}^{1} \frac{p(x')\, y\, dx'}{(x-x')^2+y^2} = \begin{cases} +\dfrac{p}{2} & \text{for } y=+0 \\ -\dfrac{p}{2} & \text{for } y=-0 \end{cases} \quad (5)$$

であるから

$$\Phi(x,y) = \frac{-1}{2\pi\rho}\int_{-1}^{1} \frac{p(x')\, y\, dx'}{(x-x')^2+y^2}, \quad (6)$$

それゆえ

$$S(x-x';y) = \frac{-e^{i\alpha x}}{2\pi U}\int_x^\infty \frac{y\, e^{-i\alpha\xi}\, d\xi}{(\xi-x')^2+y^2} \qquad \xi - x' = X$$

$$= +\frac{e^{i\alpha(x-x')}}{2\pi U}\int_\infty^{x-x'} \frac{y\, e^{-i\alpha X}}{X^2+y^2}\, dX, \quad \cdots (7)$$

とおくと (3) から

$$\phi(x,y) = \frac{1}{\rho U}\int_{-1}^{1} p(x')\, S(x-x';y)\, dx', \quad (8)$$

[illegible]

ここで

$$\frac{y}{x^2+y^2} = \frac{1}{2}\int_0^\infty \left(e^{ikx}+e^{-ikx}\right)e^{-ky}\,dk,\quad \text{for } y>0$$

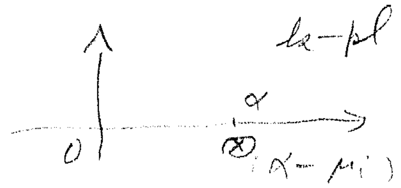
$$= \frac{1}{2}\int_0^\infty \left(e^{ikx}+e^{-ikx}\right)e^{ky}\,dk,\quad \text{for } y<0 \qquad (9)$$

それ故(7)は次のように書き表わされる。

$$S(x,y) = \frac{1}{4\pi i}\int_0^\infty \left(\frac{e^{ikx}}{k-\alpha+\mu i} - \frac{e^{-ikx}}{k+\alpha}\right)e^{-ky}\,dk,\quad \text{for } y>0$$

$$= \frac{1}{4\pi i}\int_0^\infty \left(\frac{e^{ikx}}{k-\alpha+\mu i} - \frac{e^{-ikx}}{k+\alpha}\right)e^{ky}\,dk,\quad \text{for } y<0 \qquad (10)$$

充分遠方では



$$S(x,y) = \frac{1}{4\pi i}\int_0^\infty \left(\frac{e^{-mx-imy}}{m+i\alpha} - \frac{e^{-mx+imy}}{m-i\alpha}\right)dm,$$

$$\longrightarrow 0 \quad \text{for } x>0, \qquad (11)$$

$$S(x,y) \xrightarrow[x\ll -1]{} \begin{cases} -\frac{1}{2}e^{i\alpha x-\alpha y} & \text{for } y>0 \\ \frac{1}{2}e^{i\alpha x+\alpha y} & \text{for } y<0 \end{cases} \qquad (12)$$

従って

$$\phi(x,y) \xrightarrow[(x\ll -1)]{} -\frac{U}{2}e^{i\alpha x-\alpha y}\bar{H}(\alpha),\quad \text{for } y>0$$

$$+\frac{U}{2}e^{i\alpha x+\alpha y}\bar{H}(\alpha),\quad \text{for } y<0 \qquad (13)$$

$$\bar{H}(\alpha) = \frac{1}{\rho U^2}\int_{-1}^{1} p(x)\,e^{-i\alpha x}\,dx, \qquad (14)$$

$$\frac{\partial}{\partial y} S(x,y) = -\frac{e^{i\alpha x}}{2\pi}\int_\infty^x e^{-i\alpha x}\frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right)dx$$

$$= -\frac{x}{2\pi(x^2+y^2)} + \frac{e^{i\alpha x}}{2\pi}(-i\alpha)\int_\infty^x e^{-i\alpha x}\frac{x\,dx}{x^2+y^2}, \quad (15)$$

ここで

$$g(x,y) = \frac{1}{4\pi\rho U^2}\int_{-1}^{1}\frac{p(x')(x-x')}{(x-x')^2+y^2}dx', \quad \cdots (16)$$

とおくと (8) から

$$\frac{1}{U}\frac{\partial}{\partial y}\phi(x,y) = -\frac{g(x,y)}{2} - \frac{i\alpha}{2}e^{i\alpha x}\int_\infty^x g(x,y)e^{-i\alpha x}dx, \quad (17)$$

と書ける。

ここで新しく、

$$\left(i\omega - U\frac{\partial}{\partial x}\right)\eta(x) = -\frac{\partial\phi}{\partial y}(x,0), \quad (18)$$

のような関数 η を導入すれば η は翼面の変位である。

$$\eta(x) = \frac{e^{i\alpha x}}{U}\int_\infty^x \phi_y(x,0)e^{-i\alpha x}dx, \quad \cdots (19)$$

(17)を代入すれば

$$\eta(x) = -\frac{e^{+i\alpha x}}{2}\int_\infty^x g(\xi)\{1+i\alpha(x-\xi)\}e^{-i\alpha\xi}d\xi, \quad (20)$$

$$\left[= -\frac{1}{2}\left[1+\alpha\frac{\partial}{\partial\alpha}\right]e^{i\alpha x}\int_\infty^x g\,e^{-i\alpha\xi}d\xi\right]$$

あるいは (10) を用いると

$$\eta(x) = \frac{1}{\rho U^2}\int p(x')S_H(x-x';y)\,dx', \quad \cdots (21)$$

$$\bar{S}_H(x,y) = +\frac{1}{4\pi}\int_0^\infty\left[\frac{-e^{ikx}}{(k-\alpha+i\mu)^2}+\frac{e^{-ikx}}{(k+\alpha)^2}\right]e^{-k|y|}dk \quad \cdots (22)$$

=

後流側では

$$\eta(x) \xrightarrow[x \ll -1]{} \frac{1}{2i} \frac{\partial}{\partial k}\left[k e^{ikx} \bar{H}(k)\right]\Big|_{k=\alpha} \quad \cdots\cdots (23)$$

境界条件は

$$\frac{\partial \phi}{\partial y}\Big|_{y=0} = -\left(i\omega - U\frac{\partial}{\partial x}\right)\eta(x) \quad , \quad \cdots\cdots (24)$$

と与えられるので

heaving に対し　（h : heaving 振巾, θ : pitching 振巾）

$$\left.\begin{aligned} \frac{\partial \phi}{\partial y} &= -i\omega h \quad , \\ \frac{\partial \phi}{\partial y} &= -(i\omega x + U)\,\theta \ , \end{aligned}\right\} \cdots (25)$$

$$\eta(x) e^{-i\alpha x}\Big|_{x=-X} \longrightarrow -\frac{\alpha}{2} X \bar{H}(\alpha) + \frac{1}{2i}\left[\bar{H}(\alpha) + \alpha \bar{H}'(\alpha)\right].$$

$$\phi_y \Big|_{x=-X} \longrightarrow \frac{\quad}{2i}$$

2. 解.

$$p = \frac{2i\rho\omega U}{\sin\theta} \sum_{n=0}^{\infty} A_n \cos n\theta \quad , \quad x = \cos\theta , \qquad (1)$$

とおこう。

$$q = \frac{1}{\pi\rho U^2} \int_{-1}^{1} \frac{p(x')dx'}{x-x'} = \frac{2i\alpha}{\pi} \int_0^{\pi} \frac{p \sin\theta d\theta}{\operatorname{ch}u - \cos\theta} ,$$

$$= \frac{2i\alpha}{\pi} \sum_n A_n \int_0^{\pi} \frac{\cos n\theta d\theta}{\operatorname{ch}u - \cos\theta} = \frac{2i\alpha}{\operatorname{sh}u} \sum_{n=0}^{\infty} A_n e^{-nu} , \qquad (2)$$

$$F(\alpha) = \frac{1}{\rho U^2} \int_{-1}^{1} p e^{-i\alpha x} dx = 2i\alpha \sum A_n \int_0^{\pi} e^{-i\alpha\cos\theta} \cos n\theta d\theta$$

$$= 2\pi i\alpha \sum_n (-i)^n A_n J_n(\alpha) \quad , \quad \cdots \quad (3).$$

$$q(x,0) = q_1 + q_2 \qquad \text{for } |x|<1 .$$

$$q_1 = \begin{cases} 0 & \text{for } |x|<1 . \\ \dfrac{2i\alpha}{\operatorname{sh}u} \sum A_n \operatorname{ch} nu & \text{for } x>1 \end{cases}$$

$$q_2 = \begin{cases} -\dfrac{2i\alpha}{\sin\theta} \sum A_n \sin n\theta , & \text{for } |x|<1 \\ -\dfrac{2i\alpha}{\operatorname{sh}u} \sum A_n \operatorname{sh} nu & \text{for } x>1 \end{cases} \qquad (4)$$

$$I_1 = \int_{\infty}^{x} q_1 e^{-i\alpha x} dx$$

$$I_2 = \int_{\infty}^{x} q_2 e^{-i\alpha x} dx \qquad (5)$$

とおくと

$x<1$ に対して

$$I_1 = -2i\alpha \sum A_n \int_0^\infty e^{-ik\,ch\,u}\, ch\,nu\,du = -\pi\alpha \sum (-i)^n A_n H_n^{(2)}(k)$$

$(-i)^n \frac{\pi}{2i}$

$$= \frac{i}{2} G^{(2)}(k)$$

$$G^{(2)}(k) = 2\pi i\alpha \sum_n (-i)^n A_n H_n^{(2)}(k).$$

$$I_2 = +2i\alpha \sum A_n \left[\int_0^{\theta_1} e^{-ik\cos\theta} \sin n\theta\, d\theta + \int_{\theta}^{\infty} e^{-ik\,ch\,u}\, sh\,nu\,du \right],$$

$$\phi_y(x,0) = -\frac{q}{2} - \frac{i\alpha}{2} e^{i\alpha x}(I_1 + I_2)$$

$$\eta(x,0) = \frac{-1}{2}\left[1 + \alpha\frac{\partial}{\partial k}\right] e^{ikx}(I_1 + I_2)\Big|_{k=\alpha}$$

$e^{i\alpha x} I_2$ は x の中組数となるし, 又 $|x|<1$ で ϕ_y は $e^{i\alpha x}$ なる因子を含まぬ故.

$$G^{(2)}(k) = 0,$$

$$\phi_y(x,0) = -\frac{q}{2} - \frac{i\alpha}{2} e^{i\alpha x} I_2,$$

$$\eta(x,0) = \eta^*(x,0) - \frac{\alpha}{2}\left(\frac{\partial}{\partial k} I_1\right) e^{i\alpha x}$$

$$= \eta^* - \frac{i\alpha}{4} e^{i\alpha x} \frac{\partial}{\partial k} G^{(2)}(k)$$

$$\eta^*(x,0) = -\frac{1}{2}\left(1 + \alpha\frac{\partial}{\partial k}\right)\{e^{ikx} I_2\}$$

$$e^{ikx} I_2 = 2i\alpha \cdot \left[A_1 \left(\frac{1}{ik} \right) + A_2 \left(\frac{2x}{ik} - \frac{2}{k^2} \right) \right.$$
$$\left. + A_3 \left(\frac{i}{k} + \frac{8i}{k^3} - \frac{8}{k^2} x - \frac{4i x^2}{k} \right) + \cdots \right],$$

$$\eta^{k}(x, 0) = -i\alpha \left[A_2 \left\{ \frac{2x}{i\alpha} - \frac{2}{\alpha^2} - \frac{2\alpha x}{i\alpha^2} + \frac{4}{\alpha^2} \right\} \right.$$
$$+ A_3 \left\{ \frac{i}{\alpha} + \frac{8i}{\alpha^3} - \frac{8x}{\alpha^2} - \frac{4i}{\alpha} x^2 \right.$$
$$\left. - \frac{i}{\alpha} - \frac{24i}{\alpha^3} + \frac{16}{\alpha^2} x + \frac{4i}{\alpha} x^2 \right\}$$
$$= -i\alpha \left[\frac{2}{\alpha^2} A_2 + A_3 \left(\frac{8}{\alpha^2} x - \frac{16i}{\alpha^3} \right) \right]$$
$$= -\frac{2i}{\alpha} \left[A_2 + 4\left(x - \frac{2i}{\alpha}\right) A_3 \right]$$

$$\phi_y = -2A_2 - 8\left(x - \frac{i}{\alpha}\right) A_3 + \cdots$$
$$+ \frac{\pi i \alpha^2}{2} \left[\sum_n A_n (-i)^n H_n^{(1)}(\alpha) \right] e^{i\alpha x}$$

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$$\frac{1}{U}\phi_y = -\frac{\delta}{2} - \frac{i\alpha}{2} e^{i\alpha x}\int_\infty^x \delta e^{-i\alpha x} dx, \quad \eta = -\frac{1}{2}\frac{\partial}{\partial\alpha}\left[\alpha e^{i\alpha x}\int_\infty^x \delta e^{-i\alpha\xi} d\xi\right]$$

$$\delta^{(2)} = -2i\alpha \sum A_n \frac{\sin n\theta}{\sin\theta} \qquad \Big| \qquad \delta^{(1)} = 2i\alpha\sum A_n \frac{\mathrm{ch}\, nu}{\mathrm{sh}\, u}$$

$$= -2i\alpha \sum A_n \frac{\mathrm{sh}\, nu}{\mathrm{sh}\, u}$$

$$\frac{1}{U}\phi_y = i\alpha\sum A_n \frac{\sin n\theta}{\sin\theta} + \alpha^2 \sum A_n e^{i\alpha x}\int_\infty^x e^{-i\alpha x' - nu}\frac{dx}{\mathrm{sh}\,u}$$

$$\frac{1}{U}\phi_y^{(1)} = -\alpha^2 e^{i\alpha x}\sum_n A_n \int_0^\infty e^{-i\alpha\,\mathrm{ch}\, u}\,\mathrm{ch}\, nu\, du = -\frac{\pi\alpha^2}{2} i e^{i\alpha x}\sum A_n (-i)^n H_n^{(2)} = \frac{\alpha}{4} e^{i\alpha x} G^{(2)}$$

$$\frac{1}{U}\phi_y^{(2)} = \sum A_n\left[i\alpha\frac{\sin n\theta}{\sin\theta} - \alpha^2 e^{i\alpha x}\int_\infty^x e^{-i\alpha\,\mathrm{ch}\, u}\,\mathrm{sh}\, nu\, du\right]$$

$$\eta^{(1)} = +i\frac{\partial}{\partial\alpha}\left[\alpha^2 e^{i\alpha x}\sum_n A_n\int_0^\infty e^{-i\alpha\,\mathrm{ch}\,u}\,\mathrm{ch}\,nu\,du\right]$$

$$\eta^{(2)} = +i\frac{\partial}{\partial\alpha}\left[\alpha^2 e^{i\alpha x}\sum A_n \int_\infty^x e^{-i\alpha\,\mathrm{ch}\,u}\,\mathrm{sh}\,nu\,du\right]$$

$$\eta^{(1)} = -\frac{i}{4}\frac{\partial}{\partial\alpha}\left[\alpha e^{i\alpha x} G^{(2)}\right] \quad //$$

$$\frac{1}{U}\phi_y^{(2)} = \sum A_n L_n, \quad L_n = i\alpha\frac{\sin n\theta}{\sin\theta} + M_n$$

$$\eta^{(2)} = -i\alpha\frac{\partial}{\partial\alpha}\left(\frac{M_n}{\alpha}\right) = i\alpha \frac{\partial}{\partial\alpha}\left(\frac{L_n}{\alpha}\right)$$

M_n 注意

$M_n = \alpha N_n$

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$\frac{\sin n\theta}{\sin\theta}dx$ $\frac{\sinh nu}{\sinh u}dx$ Date

$$M_n = -\alpha^2 e^{i\alpha x}\int_\infty^x e^{-i\alpha \cosh u}\sinh nu\,du = \alpha^2 e^{i\alpha x}\left[-\int_0^\infty e^{-i\alpha \cosh u}\sinh nu\,du - \int_1^x e^{i\alpha x}dx + \int_0^\theta e^{-i\alpha\cos\theta}\sin n\theta\,d\theta.\right]$$

$$= +\alpha^2 e^{i\alpha x}\left[\int_0^\theta e^{-i\alpha x\cos\theta}\sin n\theta\,d\theta + \int_0^u e^{-i\alpha x\cosh u}\sinh nu\,du\right.$$

$$M_0 = 0,\quad M_1 = +\alpha^2 e^{i\alpha x}\left[+\frac{e^{-i\alpha x}}{i\alpha}\Big|_1^x + \frac{e^{-i\alpha x}}{-i\alpha}\Big|_1^\infty\right] = -i\alpha$$

$$M_{n+1} - M_{n-1} = 2\alpha^2 e^{i\alpha x}\left[\int e^{-i\alpha\cos\theta}\cos n\theta\sin\theta\,d\theta + \int_0 e^{-i\alpha\cosh u}\cosh nu\sinh u\,du\right]$$

$$= +\frac{1}{i\alpha}\left[\cos n\theta\, e^{-i\alpha\cos\theta}\right]_0^\theta + \frac{1}{-i\alpha}\left[e^{-i\alpha\cosh u}\cosh nu\right]_0^\infty$$
$$+\frac{n}{i\alpha}\int\sin n\theta\, e^{-i\alpha}d\theta + \frac{n}{i\alpha}\int e^{-i\alpha\cosh u}\sinh nu\,du$$

$$= -2i\alpha\cos n\theta - 2i\alpha n\left[\int_0^\theta \sin n\theta\, e^{-i\alpha\cos}d\theta + \int_0^\infty e^{-i\alpha\cosh u}\sinh nu\,du\right]$$

$$= -2i\alpha\cos n\theta - \frac{2in}{\alpha}M_n$$

$$L_{n+1} - L_{n-1} = 2i\alpha\cos n\theta + M_{n+1} - M_{n-1} = -\frac{2in}{\alpha}M_n$$

$$= -\frac{2in}{\alpha}L_n - 2n\frac{\sin n\theta}{\sin\theta},$$

$$L_0 = 0,\quad L_1 = i\alpha - i\alpha = 0,$$

$$M_2 = -2i\alpha\cos\theta - \frac{2i}{\alpha}(-i\alpha) = -2 - 2i\alpha\cos\theta$$

$$L_2 = -\frac{2i}{\alpha}M_1 = -2,\quad M_3 = -i\alpha - 2i\alpha\cos 2\theta - \frac{4i}{\alpha}M_2$$

$$= -i\alpha - 2i\alpha\cos 2\theta + \frac{8i}{\alpha}(1 + i\alpha\cos\theta)$$

$$L_3 = -\frac{4i}{\alpha}M_2 \qquad M_3 = -i\alpha + \frac{8i}{\alpha} - 8\cos\theta - 2i\alpha\cos 2\theta.$$

$$= \frac{8i}{\alpha}(1 + i\alpha\cos\theta) = \frac{8i}{\alpha} - 8\cos\theta$$

$$i\alpha\left(\frac{M_1}{\alpha}\right)' = 0,\quad i\alpha\left(\frac{M_2}{\alpha}\right)' = i\alpha\left[+\frac{2}{\alpha^2}\right] = \frac{2i}{\alpha}$$

$$i\alpha\left(\frac{M_3}{\alpha}\right)' = \ldots \frac{\cos\theta}{} \ldots 2i \ldots (2\ldots i\cos\theta)$$

3. 逆流れとの関係　可逆性

§1の所論で一様流れの方向を逆にした量に(~)印を附す事にする。

加速度ポテンシャルは　$\tilde{\Phi} = \left(i\omega + U\frac{\partial}{\partial x}\right)\tilde{\varphi}$,　(1)

$$\therefore\ \tilde{\varphi} = \frac{1}{U} e^{-i\alpha x}\int_{-\infty}^{x} \tilde{\Phi} e^{i\alpha x} dx , \qquad (2)$$

$$\left[\tilde{\Phi}\right]_{y=+0}^{y=-0} = \tilde{p}/\rho , \qquad (3)$$

とおくと

$$\tilde{\Phi}(x,y) = -\frac{1}{2\pi\rho}\int_{-1}^{1}\frac{\tilde{p}\, y\, dx'}{(x-x')^2+y^2} , \qquad (4)$$

今

$$\tilde{S}(x,y) = -\frac{e^{-i\alpha x}}{2\pi}\int_{-\infty}^{x}\frac{y e^{i\alpha\xi} d\xi}{\xi^2+y^2} , \qquad (5)$$

とおくと

$$\tilde{\varphi}(x,y) = +\frac{1}{\rho U}\int_{-1}^{1}\tilde{p}(x')\,\tilde{S}(x-x';y)\,dx' , \qquad (6)$$

となる。

所で§1(7)のSと$\tilde{S}$の関係は、

$$-\tilde{S} - S = \frac{e^{-i\alpha x}}{2\pi}\int_{-\infty}^{\infty}\frac{y e^{i\alpha\xi} d\xi}{\xi^2+y^2} = \frac{\operatorname{sgn}(y)}{2} e^{-i\alpha x-\alpha|y|} , \qquad (7)$$

境界条件は次のように定義しよう。

$$\left.\begin{aligned} \tilde{\eta}(x) &= \overline{\eta(x)} \\ \tilde{\varphi}_y &= \left(i\omega + U\frac{\partial}{\partial x}\right)\tilde{\eta} = \left(i\omega + U\frac{\partial}{\partial x}\right)\bar{\eta} = -\bar{\varphi}_y , \end{aligned}\right\} \qquad (8)$$

なお$\tilde{\varphi}=\bar{\varphi}$の関係が成立てば$\tilde{\varphi}_y=-\bar{\varphi}_y$は成り立つが逆は成立たない。

また副条件としては クッタの条件

$$\bar{\tilde p}(1) = 0 \quad , \qquad (9)$$

で $\bar{\tilde\varphi}$ は $x\to-\infty$ で正則，$x\to+\infty$ に渦層をもつ。

さて (6) に (7) を代入すると、

$$\bar{\tilde\varphi}(x,y) = -\frac{1}{\rho U}\int_{-1}^{1}\bar{\tilde p}\left[S + \frac{\mathrm{sgn}(y)}{2}e^{i\alpha(x-x')-\alpha|y|}\right]dx'$$

となるから 今

$$\frac{1}{\rho U^2}\int \bar{\tilde p}\, e^{i\alpha x}dx = \bar{\tilde H}(\alpha) \quad , \qquad (10)$$

とおけば

$$\bar{\tilde\varphi}(x,y) = -\frac{1}{\rho U}\int_{-1}^{1}\bar{\tilde p}\, S\, dx' - \frac{U\,\mathrm{sgn}(y)}{2}\bar{\tilde H}(\alpha)e^{i\alpha x-\alpha|y|} \quad , \qquad (11)$$

を得る。

境界条件は (8) 第2式を適用すると

$$\bar{\tilde\varphi}_y(x,0) = -\phi_y(x,0) = -\frac{1}{\rho U}\int \bar{\tilde p}\, S_y\Big|_{y=0}dx' + \frac{\omega}{2}\bar{\tilde H}\, e^{i\alpha x},$$

ここで §1(8)より

$$\phi_y = \frac{1}{\rho U}\int_{-1}^{1} p\, S_y\Big|_{y=0}dx'$$

であるから 結局

$$\frac{1}{\rho U}\int_{-1}^{1}(p-\bar{\tilde p})\, S_y\Big|_{y=0}dx' + \frac{\omega}{2}\bar{\tilde H}(\alpha)e^{i\alpha x} = 0$$

$$\text{for } |x|<1 \,, \qquad (12)$$

を得、これに (9) なる条件がつく。

今

$$\phi_s = \frac{1}{\rho U}\int_{-1}^{1} p_s\, S\, dx, \quad \cdots \quad (13)$$

$$p_s = \frac{2i\rho\omega U}{\alpha_\cdot \theta}\left(A_0^s + A_1^s \cos\theta\right),$$

なるポテンシャルを考えると、§2 により

$$\frac{\partial}{\partial y}\phi_s\Big|_{y=0} = \frac{\pi i \alpha^2}{2}\left(A_0^s H_0^{(2)}(\alpha) - i A_1^s H_1^{(2)}(\alpha)\right)e^{i\alpha x},$$

$$\text{for } |x|<1, \quad (14)$$

であるから

$$\tilde{p}(x) = p(x) + p_s(x), \quad \cdots \quad (15)$$

とおくと、(12) は

$$A_0^s H_0^{(2)}(\alpha) - i H_1^{(2)}(\alpha) A_1^s = \frac{U}{\pi i \alpha}\tilde{\bar{H}}(\alpha), \quad \cdots (16)$$

とおけば満たされる。

一方 $\tilde{p}$ のクッタの条件は次式によって満たされる。

$$A_0^s + A_1^s = -\sigma = -\sum_{n=0}^{\infty} A_n, \quad (17)$$

ここで A_n は §2 (1) の展開係数である。

それ故

$$A_0^s = \frac{-i\sigma H_1^{(2)} + \frac{U}{\pi i \alpha}\tilde{\bar{H}}}{H_0^{(2)} + i H_1^{(2)}}$$

$$A_1^s = \frac{-\sigma H_0^{(2)} - \frac{U}{\pi i \alpha}\tilde{\bar{H}}}{H_0^{(2)} + i H_1^{(2)}} \quad \cdots (18)$$

となる。

今 $\tilde{p}$ を (1) に対応して、

$$\tilde{p} = \frac{2\rho\omega U}{i\alpha\sin\theta}\sum_{n=0}\tilde{A}_n\cos n\theta \quad , \quad \cdots\cdots (19)$$

と展開出来るとすれば (15), (18) により

$$\left.\begin{aligned} A_n &= \tilde{A}_n \quad \text{for } n \geq 2, \\ \tilde{A}_0 &= A_0 + A_0^s, \\ \tilde{A}_1 &= A_1 + A_1^s, \end{aligned}\right\} (20)$$

$Y_1 J_0 - J$

$$\bar{F}_s(\alpha) = 2\pi i\alpha\left(A_0^s J_0 - iA_1^s J_1\right) = \frac{2\pi i\alpha}{H_0 + iH_1}\left[-\sigma\left(iH_1 J_0 - iH_0 J_1\right) + \frac{U}{\pi i\alpha}\tilde{F}\left(J_0 + iJ_1\right)\right]$$

$-\frac{2}{\pi\alpha}$

$$= \frac{1}{H_0 + iH_1}\left[+4\sigma i + 2U\tilde{F}\left(J_0 + iJ_1\right)\right], \quad \cdots (21)$$

となるから (15) より

$$\tilde{F}(\alpha) = \bar{F}(\alpha) + \bar{F}_s(\alpha) = \bar{F}(\alpha) + \frac{4\sigma i}{H_0 + iH_1} + \frac{2U(J_0 + iJ_1)\tilde{F}}{H_0 + iH_1}$$

$$\tilde{F}(\alpha) = (-)\frac{H_0^{(2)} + iH_1^{(2)}}{H_0^{(1)} + iH_1^{(1)}}\left[+\bar{F}(\alpha) + \frac{4\sigma i}{H_0^{(2)} + iH_1^{(2)}}\right], \quad (22)$$

それ故 (18) は又

$$\left.\begin{aligned} A_0^s &= \frac{i}{H_1^{(1)} - iH_0^{(1)}}\left[\frac{\bar{F}(\alpha)}{\pi\alpha} - \sigma H_1^{(1)}(\alpha)\right], \\ A_1^s &= \frac{1}{H_1^{(1)} - iH_0^{(1)}}\left[-\frac{\bar{F}(\alpha)}{\pi\alpha} + i\sigma H_0^{(1)}(\alpha)\right], \end{aligned}\right\} (18')$$

可逆定理は

$$\int_{-1}^{1} p_n \frac{\partial \tilde{\phi}_m}{\partial y}\, dx = \int_{-1}^{1} \tilde{p}_m \frac{\partial \phi_n}{\partial y}\, dx, \qquad (23)$$

$$\int_{-1}^{1} p_n \tilde{\eta}_m\, dx = \int_{-1}^{1} \tilde{p}_m \eta_n\, dx, \qquad (24)$$

の2形式あるが η は一般に $e^{i\alpha x}$ の成分を含むので(24)は複雑な形となり不便である。

(23)からはMunkの定理が得られる。

平板のheaving ($\eta=1,\ \phi_y=-i\omega$) の圧力を p_0, ポテンシャルを ϕ_0 と書く

$$L = \int_{-1}^{1} p\, dx = \frac{i}{\omega}\int_{-1}^{1} \tilde{p}_0(x) \frac{\partial \phi}{\partial y}\, dx \qquad , \quad (25)$$

ピッチングに対し ($\eta = x,\ \phi_y = -i\omega x + U,\ \tilde{\phi}_y = -i\omega x - U$)

$$\int p(-i\omega x - U)\, dx = \int \tilde{p}_1 \frac{\partial \phi}{\partial y}\, dx = -i\omega M - UL,$$

$$\therefore\ M = \frac{i}{\omega}\int \tilde{p}_1 \frac{\partial \phi}{\partial y} dx + \frac{U}{\omega^2}\int_{-1}^{1} \tilde{p}_0 \frac{\partial \phi}{\partial y}\, dx$$

$$= \frac{i}{\omega}\int_{-1}^{1} \left(\tilde{p}_1 + \frac{iU}{\omega}\tilde{p}_0\right) \frac{\partial \phi}{\partial y}\, dx, \qquad (26)$$

また(13)の形のポテンシャルでクッタの条件を満たすものを考えると

$$\overline{p}_H = \frac{2\rho\omega U}{i\lambda \sin\theta}\left(\overline{A}_0^H + \overline{A}_1^H \cos\theta\right)$$

$$A_0^H = A_1^H = \frac{2}{\pi i \alpha^2 (H_0^{(2)} - iH_1^{(2)})} = \frac{2}{\pi\alpha^2 (H_1^{(2)} + iH_0^{(2)})} \qquad (27)$$

とおくと

$$\frac{\partial}{\partial y}\varphi_H = e^{i\alpha x}, \qquad (28)$$

これは (23) より

$$\tilde{H}(\alpha) = \frac{1}{\rho U^2}\int_{-1}^{1} \tilde{p} e^{i\alpha x} dx = \frac{1}{\rho U^2}\int_{-1}^{1} p_H \frac{\partial \tilde{\Phi}}{\partial y} dx$$

$$= -\frac{1}{\rho U^2}\int_{-1}^{1} p_H \overline{\frac{\partial \Phi}{\partial y}} dx, \qquad (29)$$

この φ_H の逆流れを考えると

$$\frac{\partial}{\partial y}\tilde{\varphi}_H = -\overline{\frac{\partial \varphi_H}{\partial y}} = -e^{-i\alpha x}$$

であるから

$$\overline{H}(\alpha) = \frac{1}{\rho U^2}\int_{-1}^{1} p e^{-i\alpha x} dx = \frac{-1}{\rho U^2}\int_{-1}^{1} \tilde{p}_H \frac{\partial \Phi}{\partial y} dx, \qquad (30)$$

$$\left.\begin{aligned} \tilde{p}_H &= \frac{2\rho\omega U}{i\lambda \sin\theta}\left[\tilde{A}_0^H + \tilde{A}_1^H \cos\theta\right] \\ \tilde{A}_0^H &= \frac{2}{\pi\alpha^2 (H_1^{(2)} + iH_0^{(2)})} = -\tilde{A}_1^H = A_0^H = A_1^H \end{aligned}\right\} \qquad (31)$$

(29) (30) と (22) を比較すると σ の表示が得られて

$$\sigma = \frac{2i}{\pi\alpha\{H_1^{(2)} + iH_0^{(2)}\}}\left[H_1^{(2)}\int_0^{\pi} \varphi_1 d\theta + iH_0 \int_0^{\pi} \varphi_1 \cos\theta d\theta\right], \qquad (32)$$

$$\hat{H}_H = \frac{2\pi\alpha}{i}\left[\hat{A}_0^H J_0 + i\hat{A}_1^H J_1\right] \qquad (p.9\,n?)$$

$$= \frac{4\pi}{i}(J_0 - iJ_1)$$

又 $(p\overline{\Phi_y}) = -(p\overline{\Phi'_y}) = -(\overline{p'}\Phi_y) = -(\overline{p}\Phi_y) - (\overline{p_s}\Phi_y)$

$\therefore (p\overline{\Phi_y} + \overline{p}\Phi_y) = -(\overline{p_s}\Phi_y)$

この形では先端吸引力の項は現われ~~ない~~ている。(p_s の[illegible]の項)

$\int_C p\Phi\,\Phi\frac{\partial}{\partial n}$ [illegible]

$$\zeta = \frac{1}{\rho U^2}\int p\, S_H\, dx'$$
$$\tilde{\zeta} = \frac{1}{\rho U^2}\int \tilde{p}\, \tilde{S}_H\, dx'$$

$$\tilde{S}_H(x,0) = S_H(x,0) + \frac{i}{2}\frac{\partial}{\partial k}\left[k e^{ikx}\right].$$

$$S_H = \frac{1}{4\pi}\int_0^\infty \left[\frac{e^{ikx}}{(k-\alpha+\mu i)^2} + \frac{e^{-ikx}}{(k+\alpha)^2}\right] k\,dk,$$
$$\tilde{S}_H = \frac{1}{4\pi}\int_0^\infty \left[\frac{e^{-ikx}}{(k-\alpha+\mu i)^2} + \frac{e^{ikx}}{(-k+\alpha)^2}\right] k\,dk,$$

$$\zeta = \zeta^* - \frac{i}{4}\frac{\partial}{\partial k}\left[k e^{ikx} G_T^{(2)}(k)\right],$$
$$\tilde{\zeta} = \tilde{\zeta}^* - \frac{i}{4}\frac{\partial}{\partial k}\left[k e^{-ikx} \tilde{G}_T^{(2)}(k)\right],$$

$$\tilde{H}(k) = -2\pi i\alpha \sum \hat{A}_n i^n J_n(\alpha) = \frac{1}{2}\left[\tilde{G}^{(1)}(k) + \tilde{G}_T^{(2)}(k)\right],$$
$$\tilde{G}_T^{(2)}(k) = -2\pi i\alpha \sum_n \hat{A}_n i^n H_n^{(2)}(k)$$

それ故 $\tilde{p} = \bar{p}$ とおくと $\hat{A}_n = \bar{A}_n$, $\tilde{H} = \bar{H}$, $\tilde{G}_T^{(2)} = G^{(1)}$
$\tilde{\zeta}^* = \bar{\zeta}^*$ 互に等しいとなり、

$$\tilde{\zeta} - \zeta = \frac{i}{4}\frac{\partial}{\partial k}\left[k e^{ikx}\left\{\tilde{G}_T^{(2)}(k) - G_T^{(2)}(k)\right\}\right] //$$

$$= \frac{i}{4}\frac{\partial}{\partial k}\left[k e^{ikx}\left\{G_T^{(1)}(k) - G^{(2)}(k)\right\}\right]$$
($2G^{(1)} = 2\bar{H}$)

$$= \frac{i}{4}(1+i\alpha x)e^{i\alpha x} G_T^{(1)}(\alpha) + \frac{i}{2}\alpha e^{i\alpha x}\left\{G^{(1)\prime} - \bar{H}'\right\}.$$

$\because G_T^{(2)}(\alpha) = 0$

$= \quad \bar{H}(\alpha) = \frac{1}{2} G(\alpha)$

$$p_I = \frac{2\rho i\omega U}{\lambda\Theta}\left[A_0^I + A_1^I \cos\theta\right]$$

とおくと

$$\eta_I = -\frac{i}{4}\frac{\partial}{\partial\alpha}\left[\alpha e^{i\alpha x} G_I^{(2)}(\alpha)\right]$$

$$= -\frac{i}{4}\left[(1+i\alpha x) G_I^{(2)}(\alpha) + \alpha G_I^{(2)\prime}(\alpha)\right]e^{i\alpha x},$$

$\eta_I = e^{i\alpha x}$ とおくと.

$$\left.\begin{aligned} G_I^{(2)}(\alpha) &= 2\pi i\alpha\left[A_0^I H_0^{(2)} - iA_1^I H_1^{(2)}\right] = 0 \\ G_I^{(2)\prime}(\alpha) &= 2\pi i\alpha\left[A_0^I H_0^{(2)\prime} - iA_1^I H_1^{(2)\prime}\right] = \frac{4i}{\alpha} \end{aligned}\right\}$$

$$\therefore \quad A_0^I = \frac{\frac{2}{\pi\alpha^2}H_1^{(2)}}{H_0^{\prime}H_1^{(2)} - H_0 H_1^{\prime}}, \quad A_1^I = \frac{\frac{2}{\pi i\alpha^2}H_0^{(2)}}{\text{〃}},$$

$$\tilde{p}_I = -\frac{2\rho i\omega U}{\lambda\Theta}\left[\hat{A}_0^I + \hat{A}_1^I \cos\theta\right]$$

$$\tilde{\eta}_I = -\frac{i}{4}\left[(1+i\alpha x)\tilde{G}_I^{(2)}(\alpha) + \alpha\tilde{G}_I^{(2)\prime}\right]e^{-i\alpha x}$$

$\tilde{\eta}_I = e^{-i\alpha x}$ とすると

$$\left.\begin{aligned} \tilde{G}_I^{(2)}(\alpha) &= -2\pi i\alpha\left(\hat{A}_0^I H_0^{(2)} + i\hat{A}_1^I H_1^{(2)}\right) = 0 \\ \tilde{G}_I^{(2)\prime}(\alpha) &= -2\pi i\alpha\left(\hat{A}_0^I H_0^{(2)\prime} + i\hat{A}_1^I H_1^{(2)\prime}\right) = \frac{4i}{\alpha} \end{aligned}\right\}$$

$$\tilde{A}_0^I = -A_0^I, \quad \tilde{A}_1^I = A_1^I \;//$$

故に

$$\overline{H}_I(\alpha) = 2\pi i\alpha\left(A_0^I J_0 - iA_1^I J_1\right) = -\frac{\frac{8}{\pi\alpha^2}}{H_0^{\prime}H_1^{(2)} - H_0 H_1^{(2)\prime}},$$

$$\tilde{\overline{H}}_I(\alpha) = -2\pi i\alpha\left(\tilde{A}_0^I J_0 + i\hat{A}_1^I J_1\right) = \overline{H}_I(\alpha)$$

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$$\tilde{\varphi}_J = \left(i\omega + U\frac{\partial}{\partial x}\right)\tilde{\eta}_J$$

$$= i\omega(-i\alpha x e^{-i\alpha x}) - i\alpha U e^{-i\alpha x} - \alpha^2 U x e^{-i\alpha x}$$

$$= -i\omega e^{-i\alpha x}$$

$$\tilde{p}_J = -\frac{2\rho i\omega}{\pi i\sigma}\left[\tilde{A}_0^J + \tilde{A}_1^J\cos\theta\right] = -\rho\omega(\bar{H}_1 + i\bar{H}_0\cos\theta)$$

$$\tilde{\eta}_J = (1 - i\alpha x)e^{-i\alpha x} - \frac{i\alpha}{4}\tilde{G}_J^{(2)\prime}e^{-i\alpha x}.$$

$$\tilde{H}_J = 2i.$$

$$\frac{1}{4i}(-2\pi i\alpha)\left[\tilde{A}_0^J H_0^{(2)} + i\tilde{A}_1^J H_1^{(2)}\right] = 1$$

$$(-2\pi i\alpha)\left[\tilde{A}_0^J J_0 + i\tilde{A}_1^J J_1\right] = 2i$$

$$\tilde{A}_0^J = \frac{\frac{1}{\pi\alpha}(H_1 - 2J_1)}{J_1 H_0 - J_0 H_1} = \frac{1}{2i}H_1^{(1)}, \quad \tilde{A}_1^J = \frac{\frac{1}{\pi\alpha}(-H_0 + 2J_0)}{i(J_1 H_0 - J_0 H_1)} = +\frac{H_0^{(1)}}{2}$$

$$\Delta = -\frac{i\alpha}{4}\tilde{G}_J^{(2)\prime} = -\frac{\pi\alpha^2}{2}\left(\tilde{A}_0^J H_0^{(2)\prime} + i\tilde{A}_1^J H_1^{(2)\prime}\right) = \frac{\pi i\alpha^2}{4}(\bar{H}_1 H_0' - \bar{H}_0 H_1') -$$

$$\tilde{G}_J^{(1)} = 0$$

$$\begin{cases} p_J = \frac{2\rho i\omega}{\pi i\sigma}\left[A_0^J + A_1^J\cos\theta\right] \\ \eta_J = (1 + i\alpha x)e^{+i\alpha x} - \frac{i\alpha}{4}G_J^{(2)\prime}e^{i\alpha x} \end{cases}$$

$$\left.\begin{aligned} A_0^J H_0^{(2)} - i A_1^J H_1^{(2)} &= \frac{2}{\pi\alpha} \\ A_0^J \bar{H}_0 - i A_1^J \bar{H}_1 &= 0 \end{aligned}\right\} \quad \begin{aligned} &(-\tfrac{i}{4}G^{(1)} = 1) \\ &(G_J^{(1)} = 0)\ (H_J = 2i) \end{aligned}$$

$$A_0^J = \frac{\frac{2}{\pi\alpha}\bar{H}_1}{H_0\bar{H}_1 - \bar{H}_0 H_1} = \frac{i\bar{H}_1}{2}, \quad A_1^J = \frac{\frac{2}{\pi\alpha}\bar{H}_0}{-i(H_1\bar{H}_0 - H_0\bar{H}_1)} = +\frac{\bar{H}_0}{2}$$

$$-\frac{i\alpha}{4}G_J^{(2)\prime} = \frac{\pi\alpha^2}{2}\left(A_0^J H_0' - i A_1^J H_1'\right) = \frac{\pi i\alpha^2}{4}(\bar{H}_1 H_0' - \bar{H}_0 H_1') =$$

$$= \frac{-i\alpha}{4}\tilde{G}_J^{(2)\prime}$$

$$H_J = 2i = \tilde{H}_J$$

$$\Delta = \frac{\pi i\alpha^2}{4}\left[-H_1\bar{H}_1 - \bar{H}_0\left(H_0 - \frac{H_1}{\alpha}\right)\right] = \frac{\pi\alpha^2}{4i}(H_0\bar{H}_0 + H_1\bar{H}_1) + \frac{\alpha\pi i}{4}\bar{H}_0 H_1$$

$$= \frac{\pi\alpha^2}{4i}(H_0\bar{H}_0 + H_1\bar{H}_1) + \frac{\alpha\pi i}{4}(J_0 J_1 + Y_0 Y_1) \to \frac{1}{2}$$

$$\bar{H}(\alpha)=\frac{1}{\rho U^2}\int_{-1}^{1} p\, e^{-i\alpha x}dx=\frac{1}{\rho U^2}\int p\,\tilde{\eta}_I\, dx=\frac{1}{\rho U^2}\int \tilde{p}_I\, \eta\, dx$$

$$=\frac{1}{\rho U^2}\int \tilde{p}_I\, \eta^{*} dx+\frac{1}{\rho U^2}\int \tilde{p}_I\, dx\cdot\left[-\frac{i}{4}\alpha G^{(2)\prime} e^{i\alpha x}\right]$$

$$=\frac{1}{\rho U^2}\int \tilde{p}_I\, \eta^{*} dx+\frac{\alpha}{4i}\tilde{H}_I\; G^{(2)\prime}(\alpha)$$ //

p_J 不要

$\bar{H}$ は一方では (30) で与えられるから これから $G^{(2)\prime}$ が求められる

$$\tilde{\bar{H}}(\alpha)=\frac{1}{\rho U^2}\int \tilde{p}\, e^{i\alpha x}dx=\frac{1}{\rho U^2}\int \tilde{p}\,\eta\, dx=\frac{1}{\rho U^2}\int p_I\,\tilde{\eta}\, dx$$

$$=\frac{1}{\rho U^2}\int p_J\,\tilde{\eta}^{*} dx+\frac{\alpha}{4i}H_I\tilde{G}_2^{(2)\prime}$$ //

$$+\frac{\Delta}{\rho U^2}\int p\, e^{-i\alpha x}dx,$$

$$(1+\Delta)\bar{H}(\alpha)+\alpha\bar{H}'(\alpha)=\frac{1}{\rho U^2}\int(1-i\alpha x)\,p\, e^{-i\alpha x}dx=\frac{1}{\rho U^2}\int p\,\tilde{\eta}_J\, dx=\frac{1}{\rho U^2}\int \tilde{p}_J\,\eta\, dx$$

$$=\frac{1}{\rho U^2}\int \tilde{p}_J\,\eta^{*} dx-\frac{i\alpha}{4}\tilde{H}_J\; G_J^{(2)\prime}(\alpha)=\frac{1}{\rho U^2}\int \tilde{p}_J\,\eta^{*}dx+\frac{\alpha}{2}G^{(2)\prime},$$

$$(1+\Delta)\bar{H}(\alpha)+\frac{\alpha}{2}G_J^{(1)\prime}(\alpha)=\frac{1}{\rho U^2}\int \tilde{p}_J\,\eta^{*}dx$$ //

$$\Delta=\frac{i\alpha}{4i}\tilde{G}_J^{(2)\prime}$$ //

$A_1 = \int p_0\,\tilde{\phi}_{1y}\, dx$

$A_0' = \int \tilde{p}_1\,\phi_{0y}\, dx$

$$\eta = \sum_{n=0} C_n \cos n\theta \qquad , \frac{\partial \eta}{\partial x} = + \sum_n n C_n \frac{\sin n\theta}{\sin\theta}$$

$$\phi_y = \left(i\omega - U\frac{\partial}{\partial x}\right)\eta$$

$$\frac{\sin 2n\theta}{\sin\theta} = 2\left[\cos\theta + \cos 3\theta + \cdots + \cos \overline{2n-1}\,\theta\right]$$

$$\frac{\sin(2n+1)\theta}{\sin\theta} = 1 + 2\left[\cos 2\theta + \cdots + \cos 2n\theta\right]$$

$$\therefore \sum n C_n \frac{\sin n\theta}{\sin\theta} = \sum \overline{2n+1}\, C_{2n+1} \frac{\sin(2n+1)\theta}{\sin\theta} + \sum_{n=1} 2n C_{2n} \frac{\sin 2n\theta}{\sin\theta}$$

$$= \sum_n (2n+1) C_{2n+1} + 2\sum_{n=2}(2n+1) C_{2n+1}(\cos 2\theta + \cdots + \cos 2n\theta)$$

$$+ 2\sum_{n=1} 2n C_{2n} + 2\sum_n 2n C_{2n}(\cos 3\theta + \cdots + \cos \overline{2n-1}\,\theta)$$

$$\frac{1}{\pi}\int_0^\pi \phi_y \, d\theta = i\omega C_0 - U\sum_{n=0}(2n+1)C_{2n+1} = i\omega C_0 - UA_m$$

$$\frac{1}{\pi}\int_0^\pi \phi_y \cos\theta \, d\theta = \frac{i\omega}{2} C_1 - U\sum_{n=1} 2n C_{2n} = \frac{i\omega}{2} C_1 - UC_m$$

$$\left.\begin{aligned} \sum_{n=0}(2n+1)C_{2n+1} &= \frac{1}{\pi}\int_0^\pi \frac{\partial \eta}{\partial x} d\theta = A_m \\ \sum_{n=1}^{\infty} 2n C_{2n} &= \frac{1}{\pi}\int^\pi \frac{\partial \eta}{\partial x}\cos\theta \, d\theta = C_m \end{aligned}\right\}$$

$$\phi_y = (i\omega C_0 - UA_m) + (i\omega C_1 - 2UC_m)\cos\theta$$

$$\frac{\partial \eta}{\partial x} = A_m + 2C_m \cos\theta + \quad \cos 2\theta +$$

$$\frac{1}{\pi}\int_0^\pi (1-\cos\theta)\phi_y \, d\theta = i\omega\left(C_0 - \frac{C_1}{2}\right) - U(A_m - C_m)$$

$$\bar{H}(\alpha) = 2\pi i\alpha \tilde{A}_0^H \left[i\alpha\left(C_0 - \frac{C_1}{2}\right) + (A_m - C_m) \right]$$

$$A_m - C_m = C_1 + B_m .$$

$$\bar{H}(\alpha) = 2\pi i\alpha \tilde{A}_0^H \left[i\alpha C_0 - \left(1 + \frac{i\alpha}{2}\right) C_1 - B_m \right].$$

$$(1+\Delta)\bar{H} + \frac{\alpha}{2} G^{(1)\prime} = -\pi\alpha \left[C_0 H_1 + \frac{i}{2} C_1 H_0 \right]$$

$$F = a C_0 + b C_1 + c B_m$$

$$P' = -\left(\frac{\alpha}{2} G^{(1)\prime} + \bar{H}\right) = \Delta F + (A C_0 + B C_1), \quad P = A C_0 + B C_1)$$

$$\frac{E}{\rho} = \frac{\alpha}{8} \bar{H}\bar{\bar{H}}$$

$$\frac{TU}{\rho} = \frac{\alpha}{16}\left[P'\bar{\bar{H}} + \bar{P}'\bar{H} \right] = \frac{\alpha}{16}(\Delta + \bar{\Delta})\bar{H}\bar{\bar{H}} + \frac{\alpha}{16}(P\bar{\bar{H}} + \bar{P}\bar{H})$$

$$\eta = \frac{TU}{E + TU} = \frac{\frac{1}{2}(\Delta+\bar{\Delta})\bar{H}\bar{\bar{H}} + \frac{1}{2}(P\bar{\bar{H}} + \bar{P}\bar{H})}{\left(1 + \frac{\Delta+\bar{\Delta}}{2}\right)\bar{H}\bar{\bar{H}} + \frac{1}{2}(P\bar{\bar{H}} + \bar{P}\bar{H})}$$

$$= \frac{\Delta + \bar{\Delta} + \frac{P}{\bar{H}} + \frac{\bar{P}}{\bar{\bar{H}}}}{2 + \Delta + \bar{\Delta} + \frac{P}{\bar{H}} + \frac{\bar{P}}{\bar{\bar{H}}}}$$

$$W = -\frac{\rho\omega U^2}{8} \left[\frac{\alpha}{2}\bar{H}\,\bar{G}' + \frac{\alpha}{2}\bar{\bar{H}}\,G' + 4\bar{H}\bar{\bar{H}} \right]$$

$$\alpha\bar{H}\left(\frac{\bar{G}'}{2} + \bar{H}\right) + \bar{H}\left(\frac{1}{2} \ldots\right)$$

$$+ 2\bar{H}\bar{\bar{H}}$$

$$W = -\frac{\rho\omega^2}{8}\left[\bar{H}\,\frac{\alpha}{2}\bar{G}' \ldots \bar{H} + \bar{H}\,G' \ldots \right]$$

$$\bar{H}\left(\bar{\bar{H}} - P \ldots - \frac{\alpha}{2}(\bar{H} + \ldots)\right)$$

$$-P - \Delta\bar{H} \qquad \ldots \bar{H}\bar{\bar{H}}$$

$$-(P\bar{\bar{H}} + \bar{P}\bar{H}) - (2 + \Delta + \bar{\Delta})$$

極値問題

$$I' = \frac{W}{\rho} - \lambda' \frac{TU}{\rho} = \frac{E}{\rho} + (1-\lambda')\frac{TU}{\rho} = \frac{\alpha}{8}\Pi\overline{\Pi} + (1-\lambda')\frac{\alpha}{16}\left[(\Delta+\overline{\Delta})\Pi\overline{\Pi} + P\overline{\Pi} + \overline{P}\Pi\right]$$

新しく

$$I = 2\Pi\overline{\Pi} - \lambda(P\overline{\Pi} + \overline{P}\Pi) \qquad \Delta + \overline{\Delta} = -1$$

$$\frac{\partial I}{\partial C_1} = \frac{\partial I}{\partial \overline{C_1}} = 0, \quad \frac{\partial I}{\partial B_m} = \frac{\partial I}{\partial \overline{B_m}} = 0$$

$$F = aC_0 + bC_1 + cB_m, \qquad F_{C_1} = b, \quad F_{B_m} = c$$
$$P = AC_0 + BC_1, \qquad P_{C_1} = B, \quad P_{B_m} = 0$$

$$2b\overline{\Pi} = \lambda[B\overline{\Pi} + \overline{P}b]$$
$$2\overline{b}\Pi = \lambda[P\overline{b} + \Pi\overline{B}]$$
$$2c\overline{\Pi} = \lambda[c\overline{P}]$$
$$2\overline{c}F = \lambda[\overline{c}P]$$

$$\longrightarrow \quad 2\overline{\Pi} = \lambda\overline{P}, \quad 2F = \lambda P$$

$$\alpha C_T = \frac{TU}{\rho} = \frac{\alpha}{16}\left[1 - \frac{\lambda^2}{4} + \lambda\right]P\overline{P}$$

$$\frac{E}{\rho} = \frac{\alpha}{8} \times \frac{\lambda^2}{4}P\overline{P}$$

$$\frac{TU}{E} = \left(\frac{\Delta+\overline{\Delta}}{2} + \frac{4}{\lambda}\right)$$

$$\frac{W}{\rho} = \frac{\alpha}{16}\left[\frac{\lambda^2}{4} + \lambda\right]P\overline{P} \qquad \Pi\overline{\Pi} =$$

$$\therefore \eta = \frac{TU}{W} = \frac{1 - \frac{\lambda}{4}}{1 + \frac{\lambda}{4}}$$

これから λ を決めると η が決まり thrust も決まる.

$\lambda = 4$ で $T = 0$, $\eta = 0$ → $\lambda > 4$ で ($C_T < 0$), $\eta < 0$)

$\mathrm{Max}\, \frac{TU}{\rho} = \frac{\alpha}{16}P\overline{P}$, $\eta = \frac{1}{3}$

T を大きくするには $C_1 = C_0 \frac{H_1}{iH_0} p$ (p: real 任意) とし p を大きくすればよい。

$B_m = 0$.

$$\overline{H} = aC_0 + bC_1$$
$$P = AC_0 + BC_1$$

$$2b\overline{H} = \lambda\left[\beta\overline{H} + b\overline{P}\right)$$
$$2\bar{b}H = \lambda(P\bar{b} + H\overline{B})$$

$$\frac{b}{\bar b}\frac{\overline{H}}{H} = \frac{B\overline{H} + b\overline{P}}{\overline{B}H + \bar{b}P} \quad \frac{\bar b \overline{P}}{b\overline{H}}$$

$$\overline{H}(2b - \lambda B) = \lambda b\overline{P}$$
$$H(2\bar b - \lambda\overline{B}) = \lambda\bar b P$$

$$(b\overline{B} - \bar{b}B)H\overline{H} = b\bar{b}(\overline{P}H - P\overline{H})$$

$$\frac{P}{H} =$$

$$\frac{E}{P} = \frac{\alpha}{8}H\overline{H} = \frac{\alpha}{8}\frac{\lambda^2 b\bar b P\overline{P}}{(2b - \lambda B)(2\bar b - \lambda\overline{B})}$$

$$\frac{W}{P},\ \frac{TU}{P} = \frac{\alpha}{16}\left[\pm H\overline{H} + P\overline{H} + \overline{P}H\right] = \frac{\alpha}{16}\times\frac{\lambda^2 b\bar b}{(2b-\lambda B)(2\bar b - \lambda\overline{B})}\left[\pm 4 + \frac{1}{\lambda}\left(\frac{2\bar b - \lambda\overline{B}}{\bar b} + \frac{2b - \lambda B}{b}\right)\right]P\overline{P}$$

$$\therefore \eta = \frac{-\lambda b\bar b + b(2\bar b - \lambda\overline{B}) + \bar b(2b - \lambda B)}{\lambda b\bar b + b(2\bar b - \lambda\overline{B}) + \bar b(2b - \lambda B)}$$

$$= \frac{4 - \lambda\left(1 + \frac{\overline{B}}{\bar b} + \frac{B}{b}\right)}{4 + \lambda\left(1 - \frac{\overline{B}}{\bar b} - \frac{B}{b}\right)} = \frac{\frac{4}{\lambda} - 2\left(\frac{B}{b} + \frac{\overline{B}}{\bar b}\right) - 1 + \frac{B}{b} + \frac{\overline{B}}{\bar b}}{\frac{4}{\lambda} - 2\left(\frac{B}{b} + \frac{\overline{B}}{\bar b}\right) + 1 + \frac{B}{b} + \frac{\overline{B}}{\bar b}}$$

$$\frac{TU}{P} = \frac{\alpha}{16}H\overline{H}\left[-1 - \frac{\overline{B}}{\bar b} - \frac{B}{b} + \frac{4}{\lambda}\right]$$

$$(\bar{a}\overline{C}_0 + \bar{b}\overline{C}_1)\left(\frac{2}{\lambda} - \frac{\overline{B}}{\bar b}\right) = AC_0 + BC_1$$

$$\therefore C_1 = \frac{a\left(\frac{2}{\lambda} - \frac{\overline{B}}{\bar b}\right) - A}{B - b\left(\frac{2}{\lambda} - \frac{\overline{B}}{\bar b}\right)}C_0 = \frac{A - a\left(\frac{2}{\lambda} - \frac{\overline{B}}{\bar b}\right)}{b\left[\frac{2}{\lambda} - \frac{B}{b} - \frac{\overline{B}}{\bar b}\right]}C_0$$

$$\overline{H} = aC_0 + bC_1 = \frac{\left(A - \frac{a}{b}B\right)C_0}{\frac{2}{\lambda} - \frac{B}{b} - \frac{\overline{B}}{\bar b}}$$

$$\frac{B}{a} + \frac{\overline{B}}{c} = -P$$

$$\eta = \frac{2 - \frac{1+P}{\frac{2}{\lambda}+P}}{2 + \frac{1-P}{\frac{2}{\lambda}+P}}$$

$$\frac{1}{\frac{2}{\lambda}+P} = \mu$$

$$\frac{TU}{P} = \frac{\alpha}{16} \frac{|A - \frac{a}{c}B|^2 |C_0|^2}{(\frac{2}{\lambda}+P)^2}$$

$$\eta = \frac{2-(1+P)\mu}{2+(1-P)\mu}, \quad \frac{TU}{P} = \frac{\alpha}{16} \frac{|A - \frac{a}{c}B|^2 |C_0|^2 \mu^2 (\frac{2}{\mu} - 1 - P)}{c}$$

$$\frac{TU}{P} = \frac{\alpha}{16} |A - \frac{a}{c}B|^2 |C_0|^2 \mu[2-(1+P)\mu]$$

$$2(aC_0 + bC_1 + cB_M) = \lambda(AC_0 + BC_1)$$

$$(2a - \lambda A)C_0 + (2b - \lambda B)C_1 + 2cB_M = 0$$

$$B_M = \left(\frac{\lambda A}{2c} - \frac{a}{c}\right)C_0 + \left(\frac{\lambda B}{2c} - \frac{b}{c}\right)C_1$$

$$\overline{B} = \left(\frac{\lambda \overline{A}}{2c} - \frac{a}{c}\right)\overline{C_0} + \left(\frac{\lambda \overline{B}}{2c} - \frac{b}{c}\right)\overline{C_1}$$

$$2a\overline{A}C_0\overline{C_0} + 2b\overline{A}C_1\overline{C_0} + 2b\overline{B}C_1\overline{C_1} + 2c(\overline{A}\overline{C_0} + \overline{B}\overline{C_1})B_M + 2a\overline{B}C_0\overline{C_1}$$

$$\frac{B}{r_c} = \frac{\pi\alpha \frac{i}{2}\overline{H_0}}{-2\pi i\alpha A_0'' (1+\frac{i\alpha}{2})} = \frac{\frac{\pi\alpha}{2}\overline{H_0}(H_1 + iH_0)}{-2\pi\alpha(1+\frac{i\alpha}{2})\cdot\frac{2}{\pi\alpha}}$$

$$= \frac{-\pi\alpha^2}{8(1+\frac{i\alpha}{2})}(iH_0\overline{H_0} + \overline{H_0}H_1)$$

$$\frac{B}{r_c} + \frac{\overline{B}}{r_c} = \frac{-\pi\alpha^2}{8}\left[\frac{iH_0\overline{H_0} + \overline{H_0}H_1}{1+\frac{i\alpha}{2}} + \frac{-iH_0\overline{H_0} + H_0\overline{H_1}}{1-\frac{i\alpha}{2}}\right]$$

$$= -\frac{\pi\alpha^2}{8(1+\frac{\alpha^2}{4})}\left[\alpha H_0\overline{H_0} + \overline{H_0}H_1 + H_0\overline{H_1} - \frac{i\alpha}{2}(\overline{H_0}H_1 - \overline{H_1}H_0)\right]$$

$$2(J_0J_1 + Y_0Y_1) \qquad (Y_0J_1 - J_1Y_0) \qquad + iY_0J_1 \qquad \frac{2}{\pi\alpha}$$

$$= -\frac{\pi\alpha^2}{8(1+\frac{\alpha^2}{4})}\left[\frac{1}{\pi} + \alpha H_0\overline{H_0} + 2(J_0J_1 + Y_0Y_1)\right] \quad \frac{1}{\pi}$$

$$1 + \frac{B}{r_c} + \frac{\overline{B}}{r_c} = 1 - \frac{\alpha^2}{8(1+\frac{\alpha^2}{4})} -$$

$$A - \frac{a}{r_c}B = \pi\alpha\overline{H_1} + \frac{i}{2}\pi\alpha\overline{H_0} * \frac{i\alpha}{1+\frac{i\alpha}{2}}$$

$$= \pi\alpha\left[\overline{H_1} - \frac{\frac{\alpha}{2}\overline{H_0}}{1+\frac{i\alpha}{2}}\right]$$

$$\frac{1}{2\pi} \qquad \frac{i\alpha}{2}\,2(Y_1J_0 - J_1Y_0)$$

$$\left|A - \frac{a}{r_c}B\right|^2 = \pi^2\alpha^2\left[H_1\overline{H_1} + \frac{\frac{\alpha^2}{4}H_0\overline{H_0}}{1+\frac{\alpha^2}{4}} - \frac{\alpha}{2}\left\{\frac{H_1\overline{H_0}}{1+\frac{i\alpha}{2}} + \frac{\overline{H_1}H_0}{1-\frac{i\alpha}{2}}\right\}\right]$$

$$= \frac{\pi^2\alpha^2}{1+\frac{\alpha^2}{4}}\left[H_1\overline{H_1}(1+\frac{\alpha^2}{4}) + \frac{\alpha^2}{4}H_0\overline{H_0} - \frac{\alpha}{2}\left(J_0Y_1 + J_1Y_0 + \frac{1}{2\pi}\right)\right]$$

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$$2F = \lambda P.$$

$$(2a - \lambda A)C_0 + (2b - \lambda B)C_1 + 2c\,B_m = 0$$

$$B_m = \left(-\frac{a}{c} + \frac{\lambda A}{2c}\right)C_0 + \left(\frac{\lambda B}{2c} - \frac{b}{c}\right)C_1$$

$$= \left(-1 + \frac{\lambda A}{2a}\right)\frac{a}{c}C_0 + \left(\frac{\lambda B}{2b} - 1\right)\frac{b}{c}C_1$$

$$\frac{a}{c} = -i\alpha, \quad \frac{b}{c} = \left(1 + \frac{i\alpha}{2}\right)$$

$$\frac{A}{a} = \frac{\pi\alpha\overline{H_1}}{2\pi(i\alpha)^2 A_0^H} = \frac{\pi\alpha\overline{H_1}(H_1 + iH_0)}{-4}$$

$$\frac{B}{b} = \frac{\pi\alpha\frac{\alpha}{2}\overline{H_0}}{-2\pi i\alpha(1+\frac{i}{2}\alpha)A_0^H} = \frac{\pi\alpha^2\overline{H_0}(H_1 + iH_0)}{-8(1+\frac{i}{2}\alpha)}$$

$$C_1 = \frac{\mu\left[A - \frac{aB}{b} - a\left(\frac{2}{\lambda} - \frac{B}{b} - \frac{B}{b}\right)\right]}{b}C_0 = \frac{1}{b}\left[\mu\left(A - \frac{aB}{b}\right) - a\right]C_0.$$

$$= \left[\mu\left(\frac{A}{b} - \frac{aB}{b^2}\right) - \frac{a}{b}\right]C_0 = \frac{a}{b}C_0\left[\mu\left(\frac{A}{a} - \frac{B}{b}\right) - 1\right]$$

$$\frac{1}{b}\left(A - \frac{a}{b}B\right) = \frac{\pi\alpha(H_1 + iH_0)}{-4\pi i\alpha(1+\frac{i\alpha}{2})}\Big/\pi\alpha^2\left[\overline{H_1} - \frac{\frac{\alpha}{2}\overline{H_0}}{1+\frac{i}{2}\alpha}\right]$$

$$= \frac{i\pi\alpha^2}{4(1+\frac{i\alpha}{2})^2}(H_1 + iH_0)\left[\left(1+\frac{i\alpha}{2}\right)\overline{H_1} - \frac{\alpha}{2}\overline{H_0}\right]$$

$$\overline{H_1} + \frac{i\alpha}{2}(\overline{H_1} + i\overline{H_0})$$

$$= \frac{i\pi\alpha^2}{4(1+\frac{i\alpha}{2})^2}\left[H_1\overline{H_1} + iH_0\overline{H_1} + \frac{i\alpha}{2}\left\{H_1\overline{H_1} - H_0\overline{H_0} + iH_0\overline{H_1} + i\overline{H_0}H_1\right\}\right]$$

$$(\overline{H_1} - i\overline{H_0}) \qquad 2(J_0J_1 + Y_0Y_1)$$

$$\frac{A}{a} - \frac{B}{b} = -\frac{\pi\alpha(H_1 + iH_0)}{8(1+\frac{i}{2}\alpha)}\left[2\left(1+\frac{i\alpha}{2}\right)\overline{H_1} - \alpha\overline{H_0}\right]$$

$$= -\frac{\pi\alpha}{8(1+\frac{i}{2}\alpha)}\left[2\overline{H_1}(H_1 + iH_0) + \alpha i\left\{H_1\overline{H_1} + H_0\overline{H_0} + i(H_0\overline{H_1} - \overline{H_0}H_1)\right\}\right]$$

$$\frac{4}{\pi\alpha}$$

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W の 極値条件

$$W = \frac{\alpha}{16}\left[\bar{P}\bar{H} + P\bar{\bar{H}} + \bar{P}\bar{H}\right]$$

$$\frac{\partial P}{\partial B_m} = 0$$

$$\frac{\partial W}{\partial B_m} = 0 \qquad c(\bar{\bar{H}} + \bar{P}) = 0$$

$$\bar{c}(\bar{H} + P) = 0$$

前の $\lambda = -2$ に相当している

$$l = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$$

$$W = \frac{\alpha}{16}\left[-\bar{H}\bar{\bar{H}}\right]$$

$$(\alpha + 1)^2 - 1$$

$$\frac{\partial W}{\partial c_1} = 0 \qquad b\bar{\bar{H}} + B\bar{\bar{H}} + \bar{P}b = 0$$

$$(\bar{b} + \bar{B})\bar{H} + \bar{b}\bar{P} = 0$$

$$\bar{H} = \frac{-\bar{b}}{\bar{b} + \bar{B}} P.$$

$$\left\{ \begin{array}{l} \lambda = -2 \\ \mu = \frac{1}{P-1} \end{array} \right\}$$

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$$\eta = C_0 + C_1 x + f(x)\delta$$

$$f(x) = \frac{1}{2}|x| - a = \frac{|x|}{2} - \frac{1}{\pi}$$

$$\frac{1}{\pi}\int_0^{\pi} \eta \, d\theta = C_0 = h_0, \quad \frac{1}{\pi}\int \eta \, x \, d\theta = \frac{C_1}{2} = \frac{\theta}{2}$$

$$\int_0^{\pi} f(x)\, d\theta = 0 = \int_0^{\pi/2} (\cos\theta - 2a)\, d\theta = \sin\theta \Big|_0^{\frac{\pi}{2}} - 2a \times \frac{\pi}{2}$$

$$\therefore \quad a = \frac{1}{\pi}$$

$$\eta = h_0 + \theta x + \left(\frac{x}{2} - \frac{1}{\pi}\right)\delta$$

$$\frac{1}{\pi}\int_0^{\pi} f(x) \cos 2n\theta \, d\theta = \frac{4}{\pi}\int_0^{\pi/2} \left(\cos\theta - \frac{2}{\pi}\right) \cos 2n\theta \, d\theta \qquad n \geq 1$$

$$= \frac{1}{2\pi}\int_0^{\frac{\pi}{2}} \left[\cos \overline{2n-1}\,\theta + \cos \overline{2n+1}\,\theta\right] d\theta$$

$$= \frac{1}{2\pi}\left[\frac{\sin(n-\frac{1}{2})\pi}{2n-1} + \frac{\sin(n+\frac{1}{2})\pi}{2n+1}\right] = \frac{(-)^n}{2\pi}\left(\frac{1}{2n+1} - \frac{1}{2n-1}\right)$$

$$\frac{1}{\pi}\int_0^{\pi} p_y (1-\cos\theta)\, d\theta = \frac{i\omega}{\pi}\int \eta (1-\cos\theta)\, d\theta - \frac{U}{\pi}\int_0^{\pi} \left[\theta + \frac{\delta}{2}\,\mathrm{sgn}(x)\right](1-\cos\theta)\, d\theta$$

$$= i\omega\left(h - \frac{\theta}{2}\right) - U\left[\theta - \frac{\delta}{\pi}\right] \quad //$$

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \cos\theta \, d\theta$$

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$$\overline{H}(\alpha) = \frac{1}{\rho U^2}\int_{-1}^{1} p e^{-i\alpha x} dx = \frac{-1}{\rho U^2}\int_{-1}^{1} p \,\tilde{\phi}_{Hy} dx = -\frac{1}{\rho U^2}\int_{-1}^{1} \tilde{p}_H \phi_y dx$$

$$\overline{F} = \frac{1}{\rho U^2}\int e^{i\alpha x}\left(\tilde{p} + \overline{\sigma}\,\tilde{p}_S \mp \frac{\omega}{2}\overline{H}\,\tilde{p}_H\right)dx$$

$$= \tilde{H} + \overline{\sigma}\,\tilde{H}_S \mp \frac{\omega}{2}\overline{H}\,\tilde{H}_H$$

$$\tilde{F} = \frac{1}{\rho U^2}\int \tilde{p} e^{i\alpha x} dx = \frac{1}{\rho U^2}\int \tilde{p}\,\phi_{Hy} dx = \frac{1}{\rho U^2}\int p_H \tilde{\phi}_y dx.$$

$$\tilde{\overline{H}} = \frac{-1}{\rho U^2}\int \overline{p}_H \phi_y dx$$

$$\frac{1}{\rho U^2}\int(\overline{p}_H - \tilde{p}_H)\phi_y dx = \overline{\sigma}\,\tilde{\overline{H}}_S \mp \frac{\omega}{2}\overline{H}\,\tilde{\overline{p}}_H$$

$$\sigma\tilde{\overline{H}}_S = \frac{1}{\rho U^2}\int\left[\overline{p}_H - \left(1 \mp \frac{\omega}{2}\tilde{\overline{H}}_H\right)\tilde{p}_H\right]\phi_y dx$$

($\tilde{p}_H \mp \frac{\omega}{2}$; $\frac{\overline{H}_S}{\pi\alpha^2}\,\tilde{p}_S$)

$$\frac{4\sigma}{H_1^{(1)} - iH_0^{(1)}} = \frac{2\alpha}{\pi} \cdot \frac{4}{\pi\alpha^2 [H_1^{(2)} - iH_0^{(2)}]^2}\int_0^{\pi}\left(H_1^{(2)} + iH_0^{(2)}\cos\theta\right)\phi_y d\theta$$

$$\sigma = \frac{2}{\pi i\alpha(H_1^{(2)} + iH_0^{(2)})}\int_0^{\pi}\left(H_1^{(2)} + iH_0^{(2)}\cos\theta\right)\phi_y d\theta$$

O.K.

$$\therefore \sigma = \frac{1}{\pi\alpha^2} \times \frac{1}{\rho U^2}\int_{-1}^{1} \tilde{p}_S \phi_y dx.$$

$$\bar{\Phi}_y(x,0) = \frac{1}{\rho U}\int_{-1}^{1} \bar{p}\,\bar{S}_y\,dx' = -\tilde{\Phi}_y$$

$$\bar{S}_y\big|_{y=0} = \frac{\alpha}{2}e^{-i\alpha x} - \tilde{S}_y\big|_{y=0}$$

$$\therefore\ \bar{\Phi}_y = \frac{-1}{\rho U}\int_{-1}^{1} \bar{p}\,\tilde{S}_y\big|_{y=0}dx' + \frac{\alpha}{2}\left[\int_{-1}^{1}\bar{p}\,e^{+i\alpha x}dx\right]e^{-i\alpha x}$$

$$\tilde{\Phi}_y = \frac{1}{\rho U}\int \bar{p}\,\tilde{S}_y\big|_{y=0}dx' - \frac{\omega}{2}\overline{\tilde{H}(\alpha)}\,e^{-i\alpha x}$$

$$= \frac{1}{\rho U}\int \bar{p}\,\tilde{S}_y\big|_{y=0}dx'$$

$$\therefore\ \bar{p} + \frac{\omega}{2}\bar{H}\,\tilde{p}_H = \tilde{p} + \bar{\sigma}\tilde{p}_s$$

$$\frac{1}{\rho U}\int \tilde{p}_H\,\tilde{S}_y\,dx' = -e^{-i\alpha x}$$

$$\frac{1}{\rho U}\int \tilde{p}_s\,\tilde{S}_y\,dx' = 0$$

すると

$$\bar{p}_H - \tilde{p}_H = \frac{2\rho\omega U}{i\,\lambda\ldots\theta}\left[\bar{A}^H_0 - \tilde{A}^H_0 + (\bar{A}^H_1 - \tilde{A}^H_1)\cos\theta\right]$$

$$= \frac{2\rho\omega U}{i\,\lambda\ldots\theta}\left[\tilde{\bar{A}}^H_0 - \tilde{A}^H_0 + (\tilde{\bar{A}}^H_0 + \tilde{A}^H_0)\cos\theta\right]$$

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$$1 + \frac{\alpha}{2}\overline{\tilde{H}_{1}}$$

$$1 + \frac{\alpha}{2}\tilde{H}_{1} = 1 + \frac{2i(J_0 + iJ_1)}{\overline{H}_1 - i\overline{H}_0} = \frac{\overline{H}_1 - 2J_1 - i(\overline{H}_0 - 2J_0)}{\overline{H}_1 - i\overline{H}_0}$$

$$= -\frac{H_1 - iH_0}{\overline{H}_1 - i\overline{H}_0}$$

$$\overline{A}^H_0 - \left(1 + \frac{\alpha}{2}\overline{\tilde{H}}_\mu\right)\tilde{A}^H_0 = \frac{2}{\pi\alpha^2(\overline{H}_1 - i\overline{H}_0)} + \frac{2(H_1 - iH_0)}{\pi\alpha^2(H_1 + iH_0)\times(\overline{H}_1 - i\overline{H}_0)}$$

$$= \frac{4H_1}{\pi\alpha^2(H_1 + iH_0)^2}$$

$$\overline{A}^H_1 - \left(1 + \frac{\alpha}{2}\overline{\tilde{H}}_\mu\right)\tilde{A}^H_1 = \overline{A}^H_0 + \left(1 + \frac{\alpha}{2}\overline{\tilde{H}}_\mu\right)\tilde{A}^H_0 = \frac{4iH_0}{\pi\alpha^2(H_1 + iH_0)^2}$$

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$$\tilde{P}_s = \frac{2\rho\omega U}{i\kappa\theta}\left[\tilde{A}_0^s + \tilde{A}_1^s \cos\theta\right] \xrightarrow[\theta\to 0]{} \frac{2\rho\omega U(\tilde{A}_0^s + \tilde{A}_1^s)}{i\kappa\theta}$$

$$\tilde{A}_0^s + \tilde{A}_1^s = 1$$

$$\tilde{A}_0^s H_1^{(2)} + i\tilde{A}_1^s H_1^{(2)} = 0$$

$$\frac{2i}{\pi\alpha}$$

$$\tilde{A}_0^s = \frac{iH_1^{(2)}}{iH_1^{(2)} - H_0}, \quad \tilde{A}_1^s = \frac{-H_0^{(2)}}{iH_1 - H_0} \qquad -iY_1J_0 + iJ_1Y_0$$

$$\tilde{F}_s = \frac{2\pi\alpha}{i}\left[\tilde{A}_0^s J_0 + i\tilde{A}_1^s J_1\right] = \frac{2\pi\alpha\{H_1J_0 - H_0J_1\}}{i\,(H_1^{(2)} + iH_0)}$$

$$= \frac{4}{H_1^{(2)} + iH_0^{(2)}}$$

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$$\overline{P_H} + \frac{\alpha}{2}\overline{F_H}\,\tilde{P_H} = \tilde{P_H} + \sigma_H \tilde{P_S}$$

$$\overline{P_H} - (1 - \frac{\alpha}{2}\overline{F_H})\tilde{P_H} = \sigma_H \tilde{P_S} = \frac{\tilde{\overline{F_S}}}{\pi\alpha^2}\tilde{P_S}$$

$$\overline{F_H} = 2\pi i\alpha (A_0^H J_0 - i A_1^H J_1) = 2\pi i \alpha (\tilde{A}_0^H J_0 + i \tilde{A}_1^H J_1) = -\tilde{\overline{F_H}}$$

?

$$\sigma_H = 2A_0^H = \frac{\alpha}{\pi\alpha^2 (H_1^{(2)} + i H_0^{(2)})} = \frac{\tilde{\overline{F_S}}}{\pi\alpha^2}$$

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$$(p\,\bar{\Phi}_y) = -(p\,\tilde{\Phi}_y) = -(\tilde{p}\,\Phi_y) = -(\bar{p} + \tfrac{\alpha}{2}\bar{F}\,\tilde{p}_H - \bar{\sigma}\,\tilde{p}_S,\ \Phi_y)$$

$$(p\,\bar{\Phi}_y) + (\bar{p}\,\Phi_y) = \bar{\sigma}(\tilde{p}_S\,\Phi_y) - \tfrac{\alpha}{2}\bar{F}(\tilde{p}_H\,\Phi_y)$$

$$= 4\,E^{*} \quad = \rho U^2\left[\bar{\pi}\alpha^2\sigma\bar{\sigma} + \tfrac{\alpha}{2}\bar{F}\bar{F}\right]$$

S''

前縁吸引力

$$E + SU = E^{*}$$

$$W = TU + E$$

$$W - E^{*} = (TU - S)U$$

$$(p\,\bar{\eta}^{*}) = (p\,\tilde{\eta}_x^{*}) = (p\,\eta) + \tfrac{i}{4}\int p\,dx\left[\tfrac{\partial}{\partial x}Re^{i\alpha x}\,\overline{G''}\right](x)$$

$=$

$$(p\,\bar{\eta}) = (\bar{p}\,\eta) = (\bar{p},\eta) = \bar{p}\,\eta^{*}) - \tfrac{i}{4}\alpha\,G''' \int \bar{p}\,e^{i\alpha x}dx$$

$$\bar{F} = \overline{G''}$$

$$\frac{1}{\rho U^2}\left[(p\,\bar{\eta}^{*}) - (\bar{p}\,\eta^{*})\right] = -\tfrac{i\alpha}{4}G'''\,\bar{F}(\alpha) + \tfrac{i}{4}\overline{G''}\left(\bar{F} + \alpha\bar{F}'\right) + \tfrac{i}{4}\alpha^2\bar{F}\,\overline{G'''}$$

$$= \tfrac{i}{2}F\bar{F} + \tfrac{i\alpha}{2}\bar{F}F' - \tfrac{i\alpha}{4}\bar{F}G''' + \tfrac{i\alpha}{4}F\,\overline{G'''}$$

$$= \tfrac{i}{2}F\bar{F} + \tfrac{i\alpha}{4}\left(\bar{F}G''' + F\,\overline{G'''}\right)$$

$$= \tfrac{i}{8}G''\overline{G''} + \tfrac{i\alpha}{8}\left(\overline{G''}G''' + G''\overline{G'''}\right)$$

$$= \tfrac{i}{8}\tfrac{\partial}{\partial\alpha}\left[\alpha|G''|^2\right]$$

4. 仕事, 減衰, 推力

外力のなす仕事率つまり減衰 W は

$$W = \frac{i\omega}{4}\int_{-1}^{1}\left[p(x)\overline{\zeta^*(x)} - \overline{p(x)}\,\zeta^*(x)\right]dx, \qquad (1)$$

一方 翼が水に対してなす仕事率 E は

$$E = \frac{1}{4}\int_{-1}^{1}\left[p(x)\overline{\Phi_y(x,0)} + \overline{p(x)}\,\Phi_y(x,0)\right]dx, \qquad (2)$$

エネルギー不滅則から推力を T とすると

$$W = E + TU, \qquad (3)$$

でなければならない。

§1 の討論より

$$\Phi_y(x,0) = \frac{iU}{4\pi}\int_0^{\infty}\left[\frac{\overline{H}(k)e^{ikx}}{k-\alpha+\mu i} - \frac{\overline{H}(-k)e^{-ikx}}{k+\alpha}\right]k\,dk, \qquad (4)$$

$$\zeta(x) = \frac{1}{4\pi}\int_0^{\infty}\left[\frac{\overline{H}(k)e^{ikx}}{(k-\alpha+\mu i)^2} + \frac{\overline{H}(-k)e^{-ikx}}{(k+\alpha)^2}\right]k\,dk, \qquad (5)$$

であるから (1), (2) の積分を実行すると留数分だけ残って

まず $$E = \frac{\rho\omega U^2}{8}\left|\overline{H}(\alpha)\right|^2, \qquad (6)$$

次に §2 の討論より

$$\zeta(x) = \zeta^*(x) - \frac{i\alpha}{4}e^{i\alpha x}\frac{\partial}{\partial k}G^{(2)}(k)\Big|_{k=\alpha}$$

であるから

$$W = \frac{\rho\alpha\omega U^2}{16}\left[\overline{H}(\alpha)\,\overline{G^{(2)\prime}(\alpha)} + \overline{H}(\alpha)\,G^{(2)\prime}(\alpha)\right]$$

$$+ W', \qquad (7)$$

と両方が とれ[illegible]

$$W' = \frac{i\omega}{4}\int_{-1}^{1}(p\bar{\eta} - \bar{p}\eta)\,dx = -\frac{\rho\omega U^2}{8}\frac{\partial}{\partial k}\left(k\,H(k)\overline{H(k)}\right) \quad , \cdots (8)$$

∴

$$W = -\frac{\rho\omega}{8}U^2\,H(\alpha)\overline{H(\alpha)} - \frac{\rho\omega}{8}U^2\alpha\left[H(\alpha)\overline{H'(\alpha)} + H'(\alpha)\overline{H(\alpha)}\right]$$

$$+ \frac{\rho\alpha\omega}{16}U^2\left[H(\alpha)\overline{G^{(2)\prime}(\alpha)} + \overline{H(\alpha)}\,G^{(2)\prime}(\alpha)\right],$$

を得るが

$$G^{(1)}(\alpha) + G^{(2)}(\alpha) = 2H(\alpha), \cdots \quad (9)$$

$$G^{(1)}(k) = 2\pi i\alpha\sum_n (-i)^n A_n H_n^{(1)}(k)$$

であるから

$$W = -\frac{\rho\omega U^2}{8}H(\alpha)\overline{H(\alpha)} - \frac{\rho\alpha\omega}{16}U^2\left[H(\alpha)\overline{G^{(1)\prime}(\alpha)} + \overline{H}\,G^{(1)\prime}(\alpha)\right],$$

$$= -\frac{\rho\omega U^2}{32}\frac{\partial}{\partial k}\left[k\,G^{(1)}\overline{G^{(1)}}\right]_{k=\alpha} \quad // \qquad \cdots (10)$$

これ[illegible](5)より

$$T = -\frac{\rho\omega U}{4}H(\alpha)\overline{H(\alpha)} - \frac{\rho\omega^2}{16}\left[H\,\overline{G^{(1)\prime}} + \overline{H}\,G^{(1)\prime}\right],$$

$$= -\frac{\rho\omega^2}{16}\left[H\left(\overline{G^{(1)\prime} + \frac{2H}{\alpha}}\right) + \overline{H}\left(G^{(1)\prime} + \frac{2H}{\alpha}\right)\right], \qquad (11)$$

を得 前端吸引力の助けをかりなくても推力が

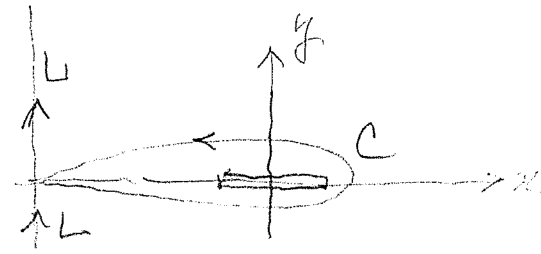
出て来る。 $H = \frac{1}{2}G^{(1)}$

$$T = -\frac{\rho\omega^2}{16}\left[[illegible] + \frac{\alpha}{2}\left(G^{(1)}\overline{G^{(1)\prime}} + \overline{G^{(1)}}\,G^{(1)\prime}\right)\right] //$$

[illegible]

これらの式は遠場の展開からも得られるがこの時(2)については注意が必要である。

(11),(8)の積分は右図C上の積分としてもよいから



$$E = \frac{\rho}{4}\int_C \left[(i\omega\phi - U\phi_x)\bar{\phi}_y - (i\omega\bar{\phi} + U\bar{\phi}_x)\phi_y \right] dx, \quad (12)$$

$$W = \frac{\rho i\omega}{4}\int_C \left[(i\omega\phi - U\phi_x)\bar{\zeta} + (i\omega\bar{\phi} + U\bar{\phi}_x)\zeta \right] dx \quad (13)$$

となるが E の積分中 $\phi_x\phi_y$ の部分をグリーンの定理で変形する際に前縁特異性の部分があるのでこれを

$$\int_C (\phi_x\bar{\phi}_y + \bar{\phi}_x\phi_y)\, dx = \int_C \left[\phi_x\bar{\phi}_y + \bar{\phi}_x\phi_y - |\nabla\phi|^2 \frac{\partial x}{\partial n} \right] dx$$

$$= \int_L \left[\!\left[|\phi_x|^2 - |\phi_y|^2 \right]\!\right] dx \neq 0, \cdots \quad (14)$$

と理解すべきである。

この2番目の式の第3項が前縁特異性の項であるから、これを含めておく訳である。

こうすれば

$$E = \frac{\rho i\omega}{4}\int_L (\phi\bar{\phi}_y - \bar{\phi}\phi_y)\, dy, \cdots (15)$$

は正しく(6)に一致する。

次に W' は一度部分積分して

$$W' = -\frac{\rho i\omega U}{4}\left[\phi\bar{\zeta} - \bar{\phi}\zeta\right]_{-\infty-i\varepsilon}^{+\infty-i\varepsilon} + E, \quad \cdots (16)$$

となり，ϕ と ζ の無限後方の値を代入すると(8)に一致する。

また(1)から(2)を差引くと，

$$W - E = T = \frac{1}{4}\int_{-\infty}^{\infty}\left(p\bar{\zeta}_x^{*} + \bar{p}\zeta_x^{*}\right)dx, \quad \cdots (17)$$

をうるが，この式では前端吸引力が含まれていないので具合が悪い事はよく知られている通りである。

$$F_0(\alpha) = 2\pi i\alpha\left[A_0^0 J_0 - iA_1^0 J_1 - A_2^0 J_2\right] = \frac{4}{H_1^{(2)} + iH_0^{(2)}}$$

$$G_0^{(1)\prime}(\alpha) = 2\pi i\alpha\left[A_0^0 \overline{H_0} - iA_1^0 \overline{H_1} - A_2^0 \overline{H_2}\right].$$

$$\overline{H_n} = H_n^{(1)}.$$

$$\frac{G_0^{(1)\prime}}{2\pi i\alpha} = \left(-\frac{i\alpha}{2} - C\right)\overline{H_0'} + iC\overline{H_1'} - \frac{i\alpha}{2}\overline{H_2'}$$

$$= -\frac{i\alpha}{2}\left(\frac{2}{\alpha}\overline{H_1'} - \frac{2}{\alpha^2}\overline{H_1}\right) + C\overline{H_1} + iC\overline{H_1'}$$

$$= i(C-1)\left(\overline{H_0} - \frac{1}{\alpha}\overline{H_1}\right) + \left(C + \frac{i}{\alpha}\right)\overline{H_1}$$

$$= i\overline{H_0}(C-1) + \overline{H_1}\left(\frac{2i}{\alpha} + C - \frac{i}{\alpha}C\right)$$

$$= \frac{H_0\overline{H_0}}{H_1 + iH_0} + \frac{\overline{H_1}}{H_1 + iH_0}\left\{\left(1 + \frac{i}{\alpha}\right)H_1 - \frac{2}{\alpha}H_0\right\}$$

$$H_0\overline{H_1} = J_0J_1 + Y_0Y_1 - i\underbrace{(J_1Y_0 - J_0Y_1)}_{\frac{2}{\pi\alpha}}$$

$$G_0^{(1)\prime} = \frac{1}{H_1 + iH_0}\left[2\pi i\alpha\begin{pmatrix}H_0\overline{H_0} + H_1\overline{H_1} \\ -\frac{2}{\alpha}(J_0J_1 + Y_0Y_1)\end{pmatrix} - 2\pi\left(H_1\overline{H_1} - \frac{8}{\alpha}\right)\right]$$

$$\mathrm{Re}\left[\overline{F_0}\left(G_0^{(1)\prime} + \frac{2}{\alpha}F_0\right)\right] = -2\pi\frac{H_1\overline{H_1}}{|H_1 + iH_0|^2} \quad //$$

$$F_1 = 2\pi i\alpha \left[A_0' J_0 - i A_1' J_1 - A_2' J_2 + i A_3' J_3 \right] = \frac{4\left(i\frac{2}{\alpha} - \frac{1}{2}\right)}{H_1 + iH_0}$$

$$\frac{G^{(1)'}}{2\pi i\alpha} = \left\{ \frac{1}{2} + \left(\frac{1}{2} - \frac{i}{\alpha}\right)C \right\} \overline{H_0'} - i\left\{ -\frac{1}{2} - \frac{i\alpha}{8} + \left(\frac{1}{2} - \frac{i}{\alpha}\right)C \right\} \overline{H_1'}$$

$$+ \overline{H_2'} - \frac{\alpha}{8}\overline{H_3'} \qquad \frac{1}{2\alpha}\left(\overline{H_2} - \frac{2}{\alpha}H_1\right)$$

$$= \left(\frac{1}{\alpha} + \frac{i}{2}\right)\overline{H_1'} - \frac{\overline{H_1}}{\alpha^2} + \frac{1}{2\alpha}\overline{H_2} + \left(\frac{1}{2} - \frac{i}{\alpha}\right)C\left(\overline{H_1} - i\overline{H_1'}\right)$$

$$= \left(\frac{1}{2} - \frac{i}{\alpha}\right) i\overline{H_1'}(1 - C) - \frac{1}{2\alpha}\overline{H_0} \qquad - \overline{H_1}\left(\frac{1}{2} - \frac{i}{\alpha}\right)C$$

$$= \frac{\left(\frac{1}{2} - \frac{i}{\alpha}\right)\left[i\overline{H_1'}H_0 - H_1\overline{H_1}\right]}{H_1 + iH_0} - \frac{\overline{H_0}}{2\alpha}, \qquad H_1' = H_0 - \frac{H_1}{\alpha}$$

$$= \frac{1}{H_1 + iH_0}\left[\left(\frac{1}{2} - \frac{i}{\alpha}\right)\left(\frac{H_0\overline{H_1}}{\alpha} - H_0\overline{H_0} - H_1\overline{H_1}\right) - \right.$$

$$\left. - \frac{1}{2\alpha}\left(H_1\overline{H_0} + iH_0\overline{H_0}\right) \right].$$

$$H_0\overline{H_1} = J_0J_1 + Y_0Y_1 - i(J_1Y_0 - J_0Y_1) = -\frac{2i}{\pi\alpha} + J_0J_1 + Y_0Y_1.$$

$$G^{(1)'} = \frac{1}{H_1 + iH_0}\left[\left(\frac{1}{2} - \frac{i}{\alpha}\right)\frac{4}{\alpha} - 2\pi i\alpha\left(\frac{1}{2} - \frac{i}{\alpha}\right)\left\{H_0\overline{H_0} + H_1\overline{H_1} - \frac{1}{\alpha}(J_0J_1 + Y_0Y_1)\right\} \right.$$

$$\left. + \frac{\pi}{\alpha}H_0\overline{H_0} + \frac{2}{\alpha} - \frac{i\pi}{\alpha}(J_0J_1 + Y_0Y_1) \right].$$

$$\frac{4i}{\alpha}\left(\frac{i}{\alpha} - \frac{1}{2}\right)$$

$$G_1^{(1)'} + \frac{2F_1}{\alpha} = \frac{1}{H_1 + iH_0}\left[\frac{4i}{\alpha^2} - \frac{i\pi}{\alpha}(J_0J_1 + Y_0Y_1) + \frac{\pi}{\alpha}H_0\overline{H_0} \right.$$

$$\left. - 2\pi i\alpha\left(\frac{1}{2} - \frac{i}{\alpha}\right)\left\{H_0\overline{H_0} + H_1\overline{H_1} - \frac{1}{\alpha}(J_0J_1 + Y_0Y_1)\right\} \right]$$

$$\mathrm{Re}\,\overline{F_1}\left(G_1^{(1)'} + \frac{2F_1}{\alpha}\right) = \frac{-4}{|H_1 + iH_0|^2}\left[\left(\frac{1}{2} + \frac{i}{\alpha}\right)\left\{\frac{\pi}{\alpha}H_0\overline{H_0} + \frac{4i}{\alpha^2} - \frac{i\pi}{\alpha}(J_0J_1 + Y_0Y_1)\right\} \right.$$

$$= \frac{-4}{|\quad|^2}\left[\frac{\pi}{2\alpha}H_0\overline{H_0} + \frac{\pi}{\alpha^2}\left\{J_0J_1 + Y_0Y_1 - \frac{4}{\pi\alpha}\right\} \right]$$

$$= \frac{-2\pi\alpha}{\quad}\left[-\frac{8}{\quad} + J_0^2 + Y_0^2 + \frac{2}{\alpha}(J_0J_1 + Y_0Y_1) \right] //$$

4. 解

heaving については　$z=\cos\theta$ とおいて

$$\frac{p_0}{i\alpha}=\frac{p}{i\rho\omega U}=\frac{2}{\sin\theta}\left[A_0^0+A_1^0\cos\theta+A_2^0\cos2\theta\right], \qquad (1)$$

$$\left.\begin{aligned} A_0^0&=-\frac{i\alpha}{2}-C(\alpha)\\ A_1^0&=-C(\alpha)\\ A_2^0&=+\frac{i\alpha}{2}\end{aligned}\right\} \qquad (2)$$

$$\left.\begin{aligned} C(\alpha)&=\frac{H_1^{(2)}(\alpha)}{H_1^{(2)}(\alpha)+iH_0^{(2)}(\alpha)},\\ C(\alpha)&=C_r(\alpha)-iC_i(\alpha)\\ C_r&=\frac{\frac{2}{\pi\alpha}+J_1^2(\alpha)+Y_1^2(\alpha)}{\frac{4}{\pi\alpha}+J_0^2(\alpha)+J_1^2(\alpha)+Y_0^2(\alpha)+Y_1^2(\alpha)}\\ C_i&=\frac{J_0(\alpha)J_1(\alpha)+Y_0(\alpha)Y_1(\alpha)}{\text{〃}},\end{aligned}\right\} \qquad (3)$$

変位は

$$\eta(x)=1+B_0e^{i\alpha x},\ \text{for } |x|<1. \qquad (4')$$

pitching に対しては.

$$\frac{p_1}{i\alpha}=\frac{p}{i\omega\rho U}=\frac{2}{\sin\theta}\left[A_0^1+A_1^1\cos\theta+A_2^1\cos2\theta+A_3^1\cos3\theta\right], \qquad (5)$$

$$\left.\begin{aligned} A_0^1&=\frac{1}{2}+\left(\frac{1}{2}+\frac{1}{i\alpha}\right)C(\alpha),\\ A_1^1&=-\frac{1}{2}-\frac{i\alpha}{8}+\left(\frac{1}{2}+\frac{1}{i\alpha}\right)C(\alpha),\\ A_2^1&=-1,\quad A_3^1=\frac{i\alpha}{8},\end{aligned}\right\} \qquad (6)$$

$$\eta(x)=x+B_1e^{i\alpha x},\ \text{for } |x|<1, \qquad (7)$$

$$\left.\begin{aligned} F_{0,1}(\alpha)&=\int_{-1}^{1}p_{0,1}\,e^{-i\alpha x}dx,\\ F_0(\alpha)&=4/\left[H_1^{(2)}(\alpha)+iH_0^{(2)}(\alpha)\right],\\ F_1(\alpha)&=\left(-\frac{1}{2}+\frac{i}{\alpha}\right)F_0(\alpha),\end{aligned}\right\} \qquad (8)$$

$$L = \int_{-1}^{1} p\,dx, \quad M = \int_{-1}^{1} p x\,dx.$$

$$C_L = \frac{L}{i\rho\omega U} = 2\pi\left[-h A_0^0 + \theta A_0^1\right]$$

$$C_M = \frac{M}{i\rho\omega U} = \pi\left[-h A_1^0 + \theta A_1^1\right] \qquad (9)$$

$$W = \frac{-i\omega}{4}\left[\bar{L} h - L\bar{h} + \bar{M}\theta - M\bar{\theta}\right]$$

$$= \frac{-\rho\omega^2 U}{4}\left[C_L\bar{h} + \bar{C}_L h + C_M\bar{\theta} + \bar{C}_M\theta\right]$$

$$C_W = \frac{W}{\rho\omega^2 U} = \frac{-\pi}{4}\left[2\bar{h}(-h A_0^0 + \theta A_0^1) + 2h(-\bar{h}\bar{A}_0^0 + \bar{\theta}\bar{A}_0^1) + \bar{\theta}(hA_1^0 + \theta A_1^1) + \theta(\bar{h}\bar{A}_1^0 + \bar{\theta}\bar{A}_1^1)\right]$$

$$= \frac{\pi}{4}\left[2h\bar{h}(A_0^0 + \bar{A}_0^0) + \theta\bar{\theta}(A_1^1 + \bar{A}_1^1) + \bar{h}\theta(2A_0^1 + \bar{A}_1^0) + h\bar{\theta}(2\bar{A}_0^1 + A_1^0)\right], \qquad (10)$$

$$\bar{E} = \frac{\rho\omega U^2}{8}|F|^2 =$$

$$C_E = \frac{E}{\rho\omega^2 U} = \frac{|F|^2}{8\alpha} = \frac{|F_0(\alpha)|^2}{8\alpha}\left|h + \left(\frac{i}{\alpha} - \frac{1}{2}\right)\theta\right|^2$$

$$= \frac{|F_0(\alpha)|^2}{8\alpha}\left[h\bar{h} + \bar{h}\theta\left(\frac{i}{\alpha} - \frac{1}{2}\right) + h\bar{\theta}\left(-\frac{i}{\alpha} - \frac{1}{2}\right) + \left(\frac{1}{4} + \frac{1}{\alpha^2}\right)\theta\bar{\theta}\right], \qquad (11)$$

$$C_W - C_E = C_T = \frac{TU}{\rho\omega^2 U} = \frac{T}{\rho\omega^2}, \qquad (12)$$

i) heaving Osc.

$$C_L = 2\pi h A_0^0 \quad , \quad C_L/h = 2\pi A_0^0$$

$$C_M = \pi h A_1^0 \quad , \quad C_M/h = \pi A_1^0$$

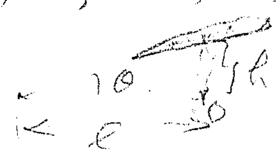
$$C_W / |h|^2 = -\frac{\pi}{2}(A_0^0 + \bar{A}_0^0) \;, = \pi C_Y = \frac{\frac{2}{\pi\alpha} + \pi(J_1^2 + Y_1^2)}{|\quad|^2}$$

$$C_E / h\bar{h} = \frac{1}{8\alpha}|F_0(\alpha)|^2 \;, = \frac{2/\alpha}{|H_1 + iH_0|^2}$$

$$C_T = C_W - C_E \;, = \frac{\pi\,|H_1|^2}{|H_1 + iH_0|^2}$$

$$\eta = C_T / C_W \;, = \frac{[illegible]}{\frac{2}{\pi\alpha} + J_1^2 + Y_1^2}$$

ii) $h = l\theta$ （lは実，正定） [$l = 0, \pm\frac{1}{2}, \pm 1, \pm 1.5, \pm 2$]



$$C_L/\theta = 2\pi\,[\,lA_0^0 + A_0^1\,],$$

$$C_M/\theta = \pi\,[\,lA_1^0 + A_1^1\,],$$

$$C_W/\theta\bar{\theta} = -\frac{\pi}{4}\Big[\,2l^2(A_0^0 + \bar{A}_0^0) + l\{\theta(2A_0^1 + \bar{A}_1^0) + \bar{\theta}(2\bar{A}_0^1 + A_1^0)\} + A_1^1 + \bar{A}_1^1\,\Big],$$

$$C_E/\theta\bar{\theta} = \frac{|F_0(\alpha)|^2}{8\alpha}\Big[\,l^2 + l\{\theta(\tfrac{i}{\alpha} - \tfrac{1}{2}) - \bar{\theta}(\tfrac{1}{2} + \tfrac{i}{\alpha})\} + \frac{1}{4} + \frac{1}{\alpha^2}\,\Big],$$

$$C_T/\theta\bar{\theta} = \frac{C_W}{\theta\bar{\theta}} - \frac{C_E}{\theta\bar{\theta}} \;, \quad [\,l > 0 \text{ならば } C_T < 0\ ?\,]$$

$$\eta = \frac{C_T}{C_W}$$

iii) $h = (\frac{1}{2} - \frac{i}{\alpha})\theta$, (C_E への場合)

$$\frac{C_L}{\theta} = 2\pi\left[(\frac{1}{2} - \frac{i}{\alpha})A_0^0 + A_0^1 \right]$$

$$C_M/\theta = \pi\left[(\frac{1}{2} - \frac{i}{\alpha})A_1^0 + A_1^1 \right].$$

$$\frac{C_W}{\theta\bar{\theta}} = -\frac{\pi}{4}\Big[A_1^1 + \overline{A_1^1} + (\frac{1}{2} - \frac{i}{\alpha})(2\overline{A_0^0} + A_1^0) + (\frac{1}{2} + \frac{i}{\alpha})(2A_0^0 + \overline{A_1^0}) + 2(\frac{1}{4} + \frac{1}{\alpha^2})(A_0^0 + \overline{A_0^0}) \Big],$$

⟶ 0 の(?)

$$A_1^1 + \overline{A_1^1} = -1 + C_r - \frac{C_i}{\alpha}$$

$$\tfrac{1}{2}(1 - \tfrac{2i}{\alpha}) \quad 2\overline{A_0^0} + A_1^0 = 1 + (1 + \frac{2i}{\alpha})\overline{C} - C$$

$$\tfrac{1}{2}(1 + \tfrac{2i}{\alpha}) \quad 2A_0^0 + \overline{A_1^0} = 1 + (1 - \frac{2i}{\alpha})C - \overline{C}$$

$$= 1 + (1 + \frac{4}{\alpha^2})C_r - C_r + \frac{C_i}{\alpha} = 1 + \frac{C_r}{\alpha^2} + \frac{C_i}{\alpha}$$

$$\frac{1}{2}(~~1~~ + \frac{4}{\alpha^2})(A_0^0 + \overline{A_0^0}) = -(1 + \frac{4}{\alpha^2})C_r$$

$$\frac{C_W}{\theta\bar{\theta}} = 0$$

$$\frac{C_L}{2\pi\theta} = (\frac{1}{2} - \frac{i}{\alpha})(-\frac{i}{2}\alpha - C) + \frac{1}{2} + (\frac{1}{2} - \frac{i}{\alpha})C$$

$$= -\frac{i}{4}\alpha \quad //$$

$$\frac{C_M}{\pi\theta} = -(\frac{1}{2} - \frac{i}{\alpha})C + \frac{1}{2} - \frac{i\alpha}{8} + (\frac{1}{2} - \frac{i}{\alpha})C$$

$$= -\frac{1}{2} - \frac{i\alpha}{8} \quad //$$

iv) 推進効率最大の場合

$$\theta = (x e^{i\varepsilon}) h \quad , \quad x = \frac{c}{2l} \quad , \quad c : \text{cord. length}$$
$$= (p + iq) h \qquad (\varepsilon = -\delta)$$

$$C_{LH} = \frac{L}{i\rho\omega h U \times \frac{c}{2}} = 2\pi [A_0^0 + (p+iq) A_0^1] ,$$

$$C_{MH} = \frac{M}{i\rho\omega h U \times (\frac{c}{2})^2} = \pi [A_1^0 + (p+iq) A_1^1] ,$$

$$C_{WH} = B_0 + B_1 p - B_2 q + B_3 (p^2 + q^2) ,$$
$$C_{EH} = C_0 + C_1 p - C_2 q + C_3 (p^2 + q^2) ,$$
$$C_{TH} = D_0 + D_1 p - D_2 q + D_3 (p^2 + q^2) ,$$

$C_{TH} = C_{WH} - C_{EH}$ が与えられた時 C_W を極値とするには

$$I = C_{WH} - \lambda C_{TH} \quad (\lambda : \text{ラグランジュの定数})$$

を極値とすればよいから

$$I = (1-\lambda) C_{WH} + \lambda C_{EH}$$

$$\frac{\partial I}{\partial p} = \frac{\partial I}{\partial q} = 0 \quad \text{から}$$

$$\frac{\lambda - 1}{\lambda} = \frac{C_1 + 2C_3 p}{B_1 + 2B_3 p} = \frac{-C_2 + 2C_3 q}{-B_2 + 2B_3 q} ,$$

$$\therefore -C_1 B_2 + 2 C_1 B_3 q - 2 B_2 C_3 p = -B_1 C_2 + 2 B_1 C_3 q - 2 B_3 C_2 p .$$

$$(B_1 C_2 - C_1 B_2) + 2(C_1 B_3 - B_1 C_3) q + 2(B_3 C_2 - B_2 C_3) p = 0 \;/\!/$$

これを $mp + nq + l = 0$ と書こう。

本来ならばこの関係を C_{TH} に代入すれば与えられた C_{TH} に対して p, q が定まるのであるが C_{TH} がどの程度の値かよくわからないので以下陶解的に求めよう。

上式を代入すると

$$C_{WH} = B_0 + B_1 p + \frac{B_2}{n}(mp+l) + B_3 p^2 + \frac{B_3}{n^2}(mp+l)^2$$

$$= B_0 + \frac{l}{n}B_2 + \frac{B_3}{n^2}l^2 + \left(B_1 + \frac{B_2}{n}m + \frac{2B_3}{n^2}ml\right)p + B_3\left(1+\frac{m^2}{n^2}\right)p^2,$$

$$C_{TH} = D_0 + \frac{l}{n}D_2 + \frac{D_3}{n^2}l^2 + \left(D_1 + \frac{m}{n}D_2 + \frac{2ml}{n^2}D_3\right)p + D_3\left(1+\frac{m^2}{n^2}\right)p^2,$$

あらためて

$$\left.\begin{aligned} C_{WH} &= B_0' + B_1' p + B_2' p^2 \\ C_{TH} &= D_0' + D_1' p + D_2' p^2 \end{aligned}\right\}$$

と書き

$$\eta = \frac{C_{TH}}{C_W}$$

が極値になる条件を求めると $\left(\frac{\partial \eta}{\partial p} = 0\right)$

$$\frac{B_1' + 2B_2' p}{B_0' + B_1' p + B_2' p^2} = \frac{D_1' + 2D_2' p}{D_0' + D_1' p + D_2' p^2}$$

$$B_1'D_0' + p(B_1'D_1' + 2D_0'B_2') + p^2(B_1'D_2' + 2B_2'D_1')$$

$$= B_0'D_1' + p(B_1'D_1' + 2B_0'D_2') + p^2(B_2'D_1' + 2B_1'D_2')$$

$$p^2(B_2'D_1' - B_1'D_2') + 2p(D_0'B_2' - B_0'D_2') + B_1'D_0' - B_0'D_1' = 0$$

これから p の2根が求まる。一つは

一つの根は $C_{WH}, C_{TH} > 0$，$\eta < 1$ で今の問題の解

他は $C_{WH}, C_{TH} < 0, \eta > 1$ で W_H の論ずる一様流からのエネルギー抽出つまり水車である。

p が求まると $q = -\frac{mp+l}{n}$ で q が求まり

これを $xe^{i\varepsilon} = (p+iq)$ で x と ε が求まる。

v) 水車の場合 (一様流れからのエネルギー抽出)

Wu のように与えられた C_W に対して C_T (今はどちらの前節と符号が違うとして) を極値とすると考えると前節で得たもう一つの解がそれである。

しかしこの場合の効率は一様流れから如何に多くのエネルギーを取り出すかであるから、一様流れの持つエネルギーに対して考えなければならない (Wuも最終的にはそうしている)

上下動振幅 $2h$ の間に流入するエネルギーは単位時間に $2h\times\frac{\rho}{2}U^3$ であるから

$$\eta_T = \frac{-W}{\rho h U^3} = \frac{\omega^2 h}{U^2}\times\frac{c}{2}(-C_{WH}) = \frac{2h}{c}\alpha^2(-C_{WH})$$

それ故その最大値は単純に p と q で微分して得られる。
(尚 C_{WH} は前節と符号が異なっている事に注意)

$$C_{WH} = +B_0 + B_1 p \mp B_2 q + B_3(p^2+q^2)$$

$$p = -\frac{B_1}{2B_3},\quad q = \frac{B_2}{2B_3},\quad \left(\tan\varepsilon = -\frac{B_2}{B_1},\ \tau^2 = p^2+q^2\right)$$

$$\mathrm{Max}[-C_{WH}] = \frac{B_1^2+B_2^2}{4B_3} - B_0 = B_3\tau^2 - B_0$$

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2. 力，仕事，減衰，推力

揚力 L，モーメント M（原点まわり）は

$$L = \int_{-1}^{1} p(x)\,dx$$

$$M = \int_{-1}^{1} p(x)\,x\,dx \qquad (1)$$

外力のなす仕事 W は

$$W = \frac{\omega}{4i}\int_{-1}^{1}\left[p(x)\bar{z}(x) - \bar{p}\,z(x)\right]dx \qquad (2)$$

これはまた $W = \frac{\omega}{4i}\left[\bar{h}L - h\bar{L} + \bar{\theta}M - \theta\bar{M}\right]$，

翼が水に対してなす仕事 E は

$$E = \frac{+1}{4}\int_{-1}^{1}\left(p\bar{\phi}_y + \bar{p}\phi_y\right)dx \qquad (3)$$

[illegible]

$$W = E + \bar{T}U \qquad (4)$$

[illegible]

$$D = \frac{[illegible]}{4}\int^{\infty} [illegible]$$

[illegible]

$$\bar{T} = [illegible]$$

[illegible]

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$$\bar{H}(k) = \frac{1}{\rho U^2} \int_{-1}^{1} p\, e^{-ikx} dx$$

$$\int_{+\infty}^{-\infty} g(x) e^{-ikx} dx = \frac{1}{\pi \rho U^2} \int_{-\infty}^{\infty} \left[\int_{-1}^{1} \frac{p(x')(x-x')}{(x-x')^2 + y^2} dx' \right] e^{-ikx} dx.$$

$y>0.$

$$= \frac{-i}{\rho U^2} \int_{-1}^{1} p\, e^{-ikx'} dx' = -i\bar{H}(k).$$

$g(x) = g_1(x) + g_2(x)$, g_1 analytic

$g_2(x) = 0$ for $|x|<1$

$$\int_{+\infty}^{-\infty} g_1(x) e^{-ikx} dx = 0.$$

$$\int_{+\infty}^{1} g_2(x) e^{-ikx} dx = -\frac{i}{2} G_2(k)$$

$$\int_{-1}^{-\infty} g_2(x) e^{-ikx} dx = -\frac{i}{2} G_1(k)$$

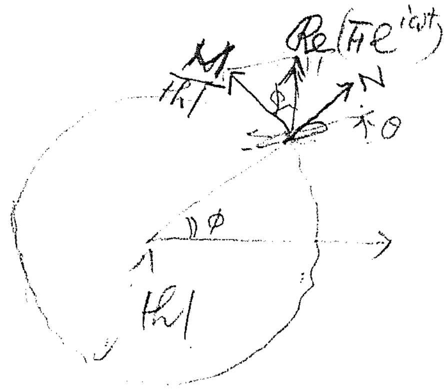
$$\bar{H}(k) = \frac{1}{2}(G_1 + G_2).$$

5. 考察

最初に気付く所は $\alpha\left(\frac{\omega c}{2U}\right)$ の小さい所では ピッチングでは抗力を生じ、推力を生ずる為にはヒービングでなければならず又それはかなりの高効率である。

この事から従来のアンチ・ピッチング・フィンのように迎角を制御する方法では抗力を生ずる可能性が多く望ましいのは鳥のように羽ばたく方法となろう。

又、この理論では前後方向の運動による力は無視しているので フォイト・シュナイダー・プロペラのように翼が回転していると考えてもよい。



そこで羽搏運動の方程式を考えて見よう。

起振力をFとすると

$$\left.\begin{aligned} \dot{h} Z_{00} + \dot{\theta} Z_{10} &= F \\ \dot{h} Z_{01} + \dot{\theta} Z_{11} &= 0 \end{aligned}\right\}$$

即ち回転運動に自由にして外力は加えないものとしよう。

以下 ピッチ(W) を(E)に 変更

$$\left\{\begin{aligned} Z_{00} &= Z_{00}^{W} + i\left(\omega M - \frac{S_h}{\omega}\right), \quad Z_{00}^{W} = -\pi \rho U c A_0^0, \\ Z_{10} &= Z_{10}^{W} = -\pi \rho U c A_0^1, \\ Z_{01} &= Z_{01}^{W} = -\frac{\pi}{4} \rho U c^2 A_1^0, \\ Z_{11} &= Z_{11}^{W} + i\left(\omega I - \frac{S_\theta}{\omega}\right), \quad Z_{11}^{W} = -\frac{\pi}{4} \rho U c^2 A_1^1, \end{aligned}\right\}$$

θ が Optimum な値になる為には前節のように計算される

$$\theta = h X e^{i\varepsilon}$$

とならねばならないので上の第2式と等置すると

$$Z_{01} + Z_{11} X e^{i\varepsilon} = 0.$$

つまり $$i\left(\omega I - \frac{S_\theta}{\omega}\right) = -\left(Z_{11}^{W} + \frac{Z_{01}^{W}}{X} e^{-i\varepsilon}\right)$$

$$= \frac{\pi}{4}\rho U c^2 \left(A_1' + \frac{A_1^0}{X} e^{-i\varepsilon}\right)$$

であればよいが左辺は純虚数で，右辺は複素数と考えられるので，バネだけでなくダムパー（負も含めて）を入れねばなるまい。

（↑ hinge 点を変えれば不要）

この時 $$h = \frac{F}{Z_{00} + X Z_{10} e^{i\varepsilon}},$$

フォイト・シュナイダー式の場合も同じでその時は

$S_h = 0$ とし，トルクを M とすると図から

$$\frac{M}{|h|}\cos\phi + \hat{N}\sin\phi = Re\{F e^{i\omega t}\} = Re[F e^{-i\delta} e^{i\phi}]$$

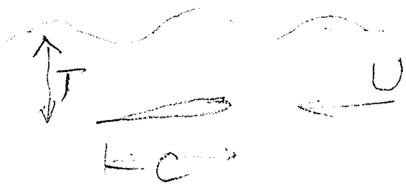
であるから（$h = |h| e^{i\phi}$, $\phi = \omega t + \delta$），

$$Re\{F e^{-i\delta}\} = \frac{M}{|h|}$$

とおけばよい。

6. 波からのエネルギー抽出

波長が充分長いものとし、又没水深度も弦長程度以上とすると水中翼性能は無限流体中のものと同じと見なす事が出来よう。



また(ωの小さい部分で考えるので)推力発生にはピッチングはあまり貢献しないし、複雑にもなるので以下ヒーピングのみ考える。

波強制力 V_W は近似的に

$$V_W = I_W Z_{00}^W \quad , \quad I_W = i\omega_0 a e^{-KT} \quad , \quad ?$$

$$\omega = \omega_0 + KU \quad , \quad K = \frac{\omega_0^2}{g} = \frac{2\pi}{\lambda} \quad , \quad a \text{: 波振巾}$$

p.17の運動方程式($\theta=0$とおいて)に代入すると

$$h = \frac{V_W}{Z_{00}} = \frac{I_W Z_{00}^E}{Z_{00}} ,$$

波のなす仕事 W は

$$W = Re\{h \overline{V_W}\} = Re\left[\frac{|I_W Z_{00}^E|^2}{Z_{00}}\right]$$

$$Z_{00} = R_{00}^E + i(X_{00}^E + X_0) \quad , \quad X_0 = \omega M - \frac{S_h}{\omega} ,$$

$$\therefore \quad W = \frac{|I_W Z_{00}^E|^2}{|Z_{00}|^2} \cdot R_{00}^E ,$$

振巾 a の入射波から単位時間に流れ込むエネルギー E_w は

$$E = \rho g a^2 \left(U + \frac{c_p}{2}\right) \quad , \quad c_p ; 位相速度$$

であるから，Wの最大値（共役整合）をそれで割ると

$$\mathrm{Max}.B_e = \frac{\mathrm{Max}.W}{E} = \frac{\omega_0^2 \, e^{-2KT}}{\rho g \left(U + \frac{c_p}{2}\right)} \times \frac{|Z_{00}^E|^2}{R_{00}^E}$$

αが充分小さいとすると

$$Z_{00}^E \doteqdot \pi \rho U c \left(1 + \frac{i\alpha}{2}\right)$$

$$R_{00}^E \doteqdot \pi \rho U c$$

$$\mathrm{Max}.[B_e] \doteqdot \frac{K \cdot \pi \rho U c}{\rho \left(U + \frac{c_p}{2}\right)} \left(1 + \frac{\alpha^2}{4}\right) e^{-2KT} = \frac{\pi K c \left(1 + \frac{\alpha^2}{4}\right)}{1 + \frac{c_p}{2U}} e^{-2KT}$$

$$\doteqdot \frac{\pi K c}{1 + \sqrt{\frac{g}{KU^2}}} e^{-2KT}$$

一方この仕事は後続渦列のエネルギーとして消費されるから　それによる推力Tは推進効率ηとして

$$TU = \eta W$$

あるいは

$$\frac{TU}{E} = \eta \frac{W}{E} = \eta B_e$$

$$\therefore \mathrm{Max}\left(\frac{TU}{E}\right) = \eta \cdot \mathrm{max}(B_e) \doteqdot \frac{\pi K c \eta}{1 + \sqrt{\frac{g}{KU^2}}} e^{-2KT}$$

今　$\lambda = 150\,m$,　$U = 10\,m/s$,　$c_p = \frac{g}{\omega} \doteqdot 16\,m/s$
と$a \doteqdot 1/\sqrt{2}\,m$，とすると

$$E = 102 \times 9.8 \times \frac{1}{2} \times (10 + 8) \doteqdot 13000 \quad kg \cdot m/sec \cdot m$$

$$= 90\,(KW/m)$$

$C = 2m$ とすると．

$$\frac{\pi K c}{1+\frac{C_p}{2U}} = \left(\frac{19.74}{2.6}\right)\frac{2}{150} = .10$$

それ故 翼の全幅を 十数m とすると 得られる推力は 100KW. 程度となろう。

いづれにしても 吃水を充分小さくして大きい翼をもつ必要がある。

今は 造波減衰を無視しているが $Kc = \frac{2\pi c}{\lambda}$ が大きくなると無視出来ない。

ほか推出には 負荷減衰抵抗(発電機)による方法も考えられるが今の場合は 偏減衰が 大変大きいので それをやると 益々 利得が小さくなって不利である。

もう一例)

$\lambda = 1m$, $a = \frac{.01}{\sqrt{2}} m$, 全巾 B_A 0.5m, $c = 0.1m$.
$C_p = 1.25 m/s$, $U = .625 m/s$.

今 $\frac{\pi K c B_A}{1+\frac{c}{2U}} = \frac{2\pi^2}{2}\cdot\frac{c}{\lambda}B_A = \pi^2 \times 0.1 \times 0.5 \doteqdot 0.5$ (m)

$\frac{E}{U} = 1000 \times 10^{-4} = 0.1$ kg/m

$\therefore$ Max T = .05 (kg)

(なお $B_A < \frac{1}{2}$ と考えられるので この例はかなり 過大な見積もりとおもわれる)

§1, (8), (10), (14) より

$$\phi_1(x,y)\Big|_{y=0} = \frac{i}{4\pi}U\int_0^\infty\left[\frac{e^{ikx}F(k)}{k-\alpha+\mu i} - \frac{e^{-ikx}F(-k)}{k+\alpha}\right]k\,dk, \quad (8)$$

∴

$$\frac{E}{\rho U^2} = \frac{i}{16\pi}U\int_0^\infty\left[\frac{F(k)\overline{F(k)}}{k-\alpha+\mu i} - \frac{F(-k)\overline{F(-k)}}{k+\alpha}\right]k\,dk$$

$$+\frac{-i}{16\pi}U\int_0^\infty\left[\frac{|F(k)|^2}{k-\alpha-\mu i} - \frac{|F(-k)|^2}{k+\alpha}\right]k\,dk,$$

$$= \frac{\alpha U}{8}|F(\alpha)|^2 = \ldots$$

となって … leading edge suction S' を

考えねばならない。

それは §1 (17) 右辺第1項から出て来るものである

$$SU = \frac{U}{4}\int_{-1}^{1}(p\bar{q} + \bar{p}q)\,dx$$

$$S = \frac{1}{4}\left[\int_{-1}^{1}p(x)\,dx\int_{-1}^{1}\frac{\overline{p(x')}}{x-x'}dx' + \int_{-1}^{1}\overline{p(x)}\,dx\int_{-1}^{1}\frac{p(x')}{x-x'}dx'\right]$$

今 $x = \cos\theta$

$$\frac{p}{\rho U^2} = \frac{1}{\sin\theta}\left[\sigma(1+\cos\theta) + \sum b_n \sin n\theta\right]$$

とおくと

$$S' = +\frac{\pi^2}{8}|\sigma|^2\rho^2U^4 \quad \ldots\ldots (9)$$

一方 §1 (21), (22) により (2) を求めると

$$\eta(x) = \frac{1}{4\pi}\int_0^\infty \left[\frac{e^{ikx}\overline{F(k)}}{(k-\alpha+\mu i)^2} + \frac{e^{-ikx}\overline{F(-k)}}{(k+\alpha)^2}\right] k\,dk, \quad (10)$$

$$\frac{1}{\rho U^2}\int_{-1}^{1} \overline{p}\,\eta\,dx = \frac{1}{4\pi}\int_0^\infty \left[\frac{|\overline{F}(k)|^2}{(k-\alpha+\mu i)^2} + \frac{|\overline{F}(-k)|^2}{(k+\alpha)^2}\right] k\,dk$$

$$= \frac{1}{\rho U^2}\int_{-1}^{1} \overline{p}\,\eta_0^{*}\,dx - \frac{i\alpha G'}{4}\int \frac{\overline{p}\,e^{i\alpha x}}{\rho U^2}dx$$

$$= \int_{-1}^{1}\frac{\overline{p}\,\eta_0}{\rho U^2}dx - \frac{i\alpha}{4}\overline{F}\,G' \quad (11)$$

$$\therefore \eta = \eta^{*} - \frac{i\alpha}{4}e^{i\alpha x}G^{(2)}(\alpha)$$

$\therefore \tilde{W} =$

$$\frac{W}{\rho U^2} = +\frac{\omega}{4}\left[\mathrm{Im}\left[+B\overline{F(\alpha)}\right] + \frac{1}{4}\frac{\partial}{\partial k}\left\{k|\overline{F}(k)|^2\right\}\Big|_{k=\alpha}\right], \quad (12)$$

(上式 験証不能かま.)

$$G^{(2)} + G^{(1)} = 2\overline{F}$$

$$\frac{\tilde{W}}{\rho U^2} = \frac{\omega}{4}\int_{-1}^{1}(\overline{p}\eta - p\overline{\eta})\,dx = \frac{\omega}{8}\frac{\partial}{\partial k}\{k|F|^2\}$$

$$\frac{\tilde{W}}{\rho U^2} = \frac{W}{\rho U^2} + \frac{\omega\alpha}{4\times4}(\overline{F}G' + F\overline{G'})$$

$$\frac{W}{\rho U^2} = \frac{\alpha}{8}|F|^2 + \frac{\omega\alpha}{16}\left[2\overline{F}'F + 2F'\overline{F} - G'\overline{F} - \overline{G'}F\right]$$

$$= \frac{\alpha}{8}|F|^2 + \frac{\omega\alpha}{16}\left[\frac{d}{dk}G^{(1)}\overline{F} + F\frac{d}{dk}\overline{G^{(1)}}\right]$$

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$$C_W = -\frac{\pi}{2}\left(A_0^0 + \bar{A}_0^0\right) = \pi C_r = \frac{\pi\left(\frac{2}{\pi r} + J_1 \cdots\right)}{\Delta}$$

$$C_E = \frac{|H_0^0|^2}{8\alpha} = \frac{2}{\alpha}\,\frac{|H|}{|H_1 + i H_0|^2} = \frac{2}{\alpha}\,\frac{1}{(J_1+Y_0)^2 + (J_0 - Y_1)^2}$$

$$\equiv \frac{2}{\alpha}\,\frac{1}{J_0^2 + J_1^2 + Y_0^2 + Y_1^2 + 2(J_1 Y_0 - J_0 Y_1)}$$

$$C_T = C_W - C_E = \frac{\pi}{\Delta}\left[\frac{2}{\alpha} J_1^2 + Y_1^2\right] - \pi\,|C(\alpha)|^2$$

$$\frac{1}{2}(H_0 - H_2 \cdots \qquad \left(\frac{2}{\alpha} H_0 H_1 - H_0 H_2 - H_1 \cdots\right)$$

$$B_0 = \frac{i\pi\alpha^2}{2(H_1 + iH_0)}\left[H_1' H_1 + \underbrace{i H_0 H_1}_{-iH_1^2} + \underbrace{\frac{2}{\alpha} H_1^2 - H_0 H_1}_{H_1 H_2} + \frac{2i}{\alpha} H_0 H_1 - i H_0^2\right]$$

$$\left[\frac{H_1^2}{\alpha}\right.$$

$$\overline{B_0 H_0} - \bar{B}_0 \bar{F}_0 = \frac{2 i\pi\alpha^2}{|H_1 + iH_0|^2}\left[\frac{H_1^2}{\alpha} \cdots H_0 + \frac{2}{\alpha} \cdots\right.$$

$$\left. + H_1' H_1 + \frac{2}{\alpha} H_1^2 - \cdots\right]$$

$$H_1' H_1 = J_1' J_1 - Y_1' Y_1$$

$$\frac{1}{\alpha} H_1^2 = \frac{1}{\alpha}\left(J_1^2 - Y_1^2\right)$$

$$H_0 H_2 - H_1^2 = (J_0 - iY_0)(J_2 - iY_2) - (J_1 - iY_1)^2$$

$$\Rightarrow -i\left[Y_0 J_2 + J_0 Y_2 - 2 J_1 Y_1\right]$$

$$2\left[\frac{1}{\alpha}\left(J_1^2 - Y_1^2\right) + \left(J_0 Y_2 + Y_0 J_2 - 2 J_1 Y_1\right)\right]$$

$$J_0\left(\frac{2}{\alpha} Y_1 - Y_0\right) + Y_0\left(\frac{2}{\alpha} J_1 - J_0\right),$$

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$$\overline{H}_{\gamma} = \sum_n 2\int_0^{\pi} d\theta \left[A_n^{\gamma} \cos n\theta \right] e^{-i\alpha\cos\theta} = 2\pi \sum (-i)^n A_n J_n(\alpha)$$

$$\int_0^{\pi} e^{-i\alpha\cos\theta} \cos n\theta \, d\theta = \pi (-i)^n J_n(\alpha)$$

$$G_{\gamma}^{(1)(2)} = 2\pi \sum (-i)^n A_n H_n^{(1)(2)}(\alpha) \qquad \frac{G^{(2)}}{i\alpha}$$

$$F_0 = 2\pi \left[A_0^0 J_0 + (-i) A_1^0 J_1 - A_2^0 J_2 \right] = \frac{F_1}{i\alpha}$$

$$= 2\pi \left[\left(-\frac{i\alpha}{2} - C\right) J_0 - i(-C) J_1 - \frac{i\alpha}{2} J_2 \right]$$

$$= 2\pi \left[-iJ_1 + iCJ_1 - CJ_0 \right] = \begin{cases} 2\pi i J_1 (C-1) \\ -2\pi C J_0 \end{cases} = 2\pi i J_1 \frac{-iH_0}{H_1 + iH_0} - 2\pi J_0 \frac{H_1}{H_1 + iH_0}$$

$$= \frac{-2\pi}{H_1 + iH_0} \left(J_0 H_1 - J_1 H_0 \right) = \frac{2\pi (+i)}{H_1 + iH_0} \left(J_0 Y_1 - J_1 Y_0 \right) = \frac{-4i/\alpha}{H_1 + iH_0}$$

$$G_0' = 2\pi \left[\left(-\frac{i\alpha}{2} - C\right) \overline{H_0'} + iC\overline{H_1'} - \frac{i\alpha}{2} \overline{H_2'} \right]$$

$$= 2\pi \left[-\frac{i\alpha}{2} \left(\overline{H_1'} \frac{2}{\alpha} - \frac{2}{\alpha^2} \overline{H_1} \right) - C \left(\overline{H_0'} - i\overline{H_1'} \right) \right]$$

$$\frac{G_0'}{2\pi} = +i\overline{H_1'}(C-1) + \frac{i}{\alpha}\overline{H_1} + C\overline{H_1}, \qquad H_1' = H_0 - \frac{1}{\alpha} H_1$$

$$= i\left(\overline{H_0} - \frac{1}{\alpha}\overline{H_1}\right)(C-1) + \left(\frac{i}{\alpha} + C\right)\overline{H_1}$$

$$= i\overline{H_0}(C-1) - \frac{i}{\alpha}\overline{H_1}(C-1) + \left(\frac{i}{\alpha} + C\right)\overline{H_1}$$

$$\overline{H_1} \left\{ \frac{2i}{\alpha} + C - \frac{i}{\alpha} C \right\}$$

$$= \frac{H_0 \overline{H_0}}{H_1 + iH_0} + \frac{\overline{H_1}}{H_1 + iH_0} \left\{ \frac{2i}{\alpha}(H_1 + iH_0) + \left(1 - \frac{i}{\alpha}\right) H_1 \right\}$$

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$$\frac{G_0'}{2\pi} = \frac{1}{H_1 + iH_0}\left[H_0\overline{H_0} + \left(1+\frac{i}{\alpha}\right) H_1\overline{H_1} - \frac{2}{\alpha} H_0\overline{H_1} \right]$$

$$H_0\overline{H_1} = J_0J_1 + Y_0Y_1 + i(\underbrace{J_0Y_1 - Y_0J_1}_{-\frac{2}{\pi\alpha}})$$

$$= \frac{1}{H_1 + iH_0}\left[H_0\overline{H_0} + H_1\overline{H_1} + \frac{i}{\alpha} H_1\overline{H_1} + \frac{4i}{\pi\alpha^2} \right]$$

$$\overline{F_0} G_0' = \frac{-4i}{|H_1 + iH_0|^2}\left[H_0\overline{H_0} + H_1\overline{H_1} + \frac{i}{\alpha}\left(H_1\overline{H_1} + \frac{4}{\pi\alpha}\right) \right]$$

$$\mathcal{R}e\{\overline{F_0} G_0'\} = \frac{4}{\alpha}\left[|C|^2 + \frac{4}{\pi\alpha}\frac{1}{|\quad|^2} \right] = \frac{\pi G''}{\alpha^2}$$

$$\frac{4}{\alpha}|C|^2 + \frac{8}{\pi\alpha} C_E = \frac{4}{\pi\alpha} C_T + \frac{8}{\pi\alpha} C_E$$

$$\frac{\pi\alpha}{4} \mathcal{R}e\{\overline{F_0} G_0'\} = C_T + 2 C_E$$

$$\frac{|H_1|^2}{4\alpha}, \frac{\pi}{4\alpha}$$

$$C_W = C_T + C_E$$

$$C_W = -\frac{\pi}{4}\left(A_1' + \bar{A}_1'\right)$$

$$C_E = \frac{|F_1|^2}{8\alpha} = \frac{\frac{2}{\pi\alpha}\left(\frac{1}{4}+\frac{1}{\alpha^2}\right)}{|H_1+iH_0|^2} = \frac{\pi}{4}\frac{\left(\frac{2}{\pi\alpha}+\frac{8}{\pi\alpha^2}\right)}{|\ \ |}$$

$$C_W = -\frac{\pi}{4}(A_1' + \bar{A}_1') = -\frac{\pi}{4}\left[-1 + C_r - \frac{2C_i}{\alpha}\right] = \frac{\pi}{4}\left[1 - C_r + \frac{2}{\alpha}C_i\right]$$

$$= \frac{\pi}{4|H_1+iH_0|^2}\left[\frac{2}{\pi\alpha} + J_0^2 + Y_0^2 + \frac{2}{\alpha}(J_0J_1 + Y_0Y_1)\right]$$

$$C_T = C_W - C_E = \frac{\pi}{4|\ \ |^2}\left[-\frac{8}{\pi\alpha^2} + \qquad\right]$$

$$F_1 = 2\pi\left[A_0' J_0 - iA_1' J_1 - A_2' J_2 + iA_3' J_3\right]$$

$$\frac{F_1}{2\pi} = \left\{\frac{1}{2} + \left(\frac{1}{2} - \frac{i}{\alpha}\right)C\right\}J_0 - i\left\{-\frac{i}{2} - \frac{i\alpha}{8} + \left(\frac{1}{2} - \frac{i}{\alpha}\right)C\right\}J_1$$

$$+ J_2 - \frac{\alpha}{8}J_3$$

$$= \frac{2}{\alpha}J_1 - \frac{\alpha}{8}\times\frac{4}{\alpha}J_2 - \frac{J_0}{2} + \frac{i}{2}J_1 + \left(\frac{1}{2} - \frac{i}{\alpha}\right)C\,(J_0 + iJ_1)$$

$$= \left(\frac{1}{\alpha} + \frac{i}{2}\right)J_1$$

$$= \left(\frac{1}{2} - \frac{i}{\alpha}\right)\left[iJ_1 + C(J_0 + iJ_1)\right]$$

$$= \frac{\left(\frac{1}{2} - \frac{i}{\alpha}\right)}{H_1 + iH_0}\left[iJ_1(H_1 + iH_0) + H_1(J_0 + iJ_1)\right]$$

$$J_0 H_1 - J_1 H_0$$

$$-i(J_0Y_1 - J_1Y_0)$$

$$= \frac{\left(\frac{1}{2} - \frac{i}{\alpha}\right)}{H_1 + iH_0}\times(+i)\frac{2}{\pi\alpha}$$

$$\frac{G_1'}{2\pi} = \left\{\frac{1}{2} + \left(\frac{1}{2} - \frac{i}{\alpha}\right)C\right\}\overline{H_0'} - i\left\{-\frac{1}{2} - \frac{i\alpha}{8} + \left(\frac{1}{2} - \frac{i}{\alpha}\right)C\right\}\overline{H_1'}$$

$$+ \overline{H_2'} - \frac{\alpha}{8}\overline{H_3'} \qquad \left(\frac{4}{\alpha}\overline{H_2'} - \frac{4}{\alpha^2}\overline{H_2}\right)$$

$$= \frac{1}{2}\overline{H_0'} + \overline{H_2'} + \frac{i}{2}\overline{H_1'} - \frac{\alpha}{8}\left(\overline{H_1'} + \overline{H_3'}\right)$$

$$+ \left(\frac{1}{2} - \frac{i}{\alpha}\right)C\left\{\overline{H_0'} - i\overline{H_1'}\right\}$$

$$= \frac{1}{2}\left(\overline{H_0'} + \overline{H_2'}\right) + \frac{i}{2}\overline{H_1'} + \frac{1}{2\alpha}\overline{H_2} +$$

$$\frac{2}{\alpha}H_1' - \frac{2}{\alpha^2}H_1$$

$$= \left(\frac{1}{\alpha} + \frac{i}{2}\right)\overline{H_1'} - \frac{\overline{H_1}}{\alpha^2} + \frac{1}{2\alpha}\overline{H_2} +$$

$$= \frac{\left(\frac{1}{2} - \frac{i}{\alpha}\right)}{H_1 + iH_0}\left[(H_1 + iH_0)\,i\overline{H_1'} + H_1\left(\overline{H_0'} - i\overline{H_1'}\right)\right]$$

$$+ \frac{1}{2\alpha}\overline{H_2} - \frac{1}{\alpha^2}\overline{H_1}$$

$$-H_0\left(\overline{H_0} - \frac{1}{\alpha}\overline{H_1}\right) \qquad \frac{1}{2\alpha}\left(\overline{H_2} - \frac{2}{\alpha}\overline{H_1}\right) = -\overline{H_0}$$

$$= \frac{1}{H_1 + iH_0}\left[\left(\frac{1}{2} - \frac{i}{\alpha}\right)\left(-H_0\overline{H_1'} - H_1\overline{H_1}\right)\right.$$

$$\left. - \frac{1}{2\alpha}\overline{H_0}\,(H_1 + iH_0)\right]$$

$$\left[\left(\frac{1}{2} - \frac{i}{\alpha}\right)\left(\frac{1}{\alpha}H_0\overline{H_1} - H_0\overline{H_0} - H_1\overline{H_1}\right) - \frac{1}{2\alpha}\overline{H_0}(H_1 + iH_0)\right]$$

$$\left[\frac{1}{2\alpha}\left(H_0\overline{H_1} - \overline{H_0}H_1\right) - \frac{i}{\alpha^2}H_0\overline{H_1} - \frac{i}{2\alpha}H_0\overline{H_0} + \left(\frac{1}{2} - \frac{i}{\alpha}\right)\left(|H_0|^2 + |H_1|^2\right)\right]$$

$$2i\left(J_0Y_1 - Y_0J_1\right)$$

$$-\frac{2}{\pi\alpha} \qquad -\frac{2i}{\pi\alpha^2} \qquad \frac{-i2}{\pi\alpha} + J_0J_1 + Y_0Y_1$$

$$= \frac{1}{H_1 + iH_0}\left[-\frac{2i}{\pi\alpha^2} - \frac{2}{\pi\alpha^3} - \frac{i}{\alpha^2}\left(J_0J_1 + Y_1Y_0\right) - \frac{i}{2\alpha}H_0\overline{H_0}\right.$$

$$\left. - \left(\frac{1}{2} - \frac{i}{\alpha}\right)\left(|H_0|^2 + |H_1|^2\right)\right]$$

$$\alpha \frac{\overline{H}\,G_1}{2\pi} = \frac{4}{i\,|H_1 + iH_0|^2}\Big[-\left(\frac{1}{4}+\frac{1}{\alpha^2}\right)(|H_0|^2+|H_1|^2) - \frac{2}{\pi\alpha^3}\left(\frac{1}{2}+\frac{2}{\alpha}\right)$$

$$+ i\left(\frac{1}{2}+\frac{2}{\alpha}\right)\left\{\frac{-2}{\pi\alpha^2} - \frac{1}{\alpha^2}(J_0J_1+Y_0Y_1) - \frac{1}{2\alpha}|H_0|^2\right\}\Big]$$

$$= \frac{-4i}{|\quad|^2}\Big[-\left(\frac{1}{4}+\frac{1}{\alpha^2}\right)(|H_0|^2+|H_1|^2) - \frac{1}{\pi\alpha^3} + \frac{1}{\alpha}\left\{\frac{2}{\pi\alpha^2}+\frac{1}{\alpha^2}(J_0J_1+Y_0Y_1) + \frac{1}{2\alpha}|H_0|^2\right\}$$

$$+ i\left\{-\frac{2}{\pi\alpha^2} + \frac{1}{2}\left\{\frac{-2}{\pi\alpha^2} - \frac{1}{\alpha^2}(J_0J_1+Y_0Y_1) - \frac{1}{2\alpha}|H_0|^2\right\}\right\}$$

$$-\frac{i}{2}\left[\frac{+2}{\pi\alpha^2} + \frac{1}{\alpha^2}(J_0J_1+Y_0Y_1) + \frac{1}{2\alpha}(J_0^2+Y_0^2)\right]$$

$$+\frac{4}{\pi\alpha^4} \quad + \frac{2}{\pi\alpha^2}$$

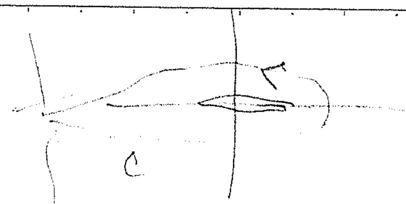
$$-\frac{1}{2\pi\alpha^2} + \frac{3}{2\pi\alpha^2} = \frac{1}{\pi\alpha^2}$$

$$-\frac{i}{4\alpha}\Big[J_0^2+Y_0^2 + \frac{2}{\alpha}(J_0J_1+Y_0Y_1) + \frac{8}{\pi\alpha^3} + \frac{4}{\pi\alpha}$$

$$+\frac{2}{\pi\alpha} \qquad \frac{2}{\pi\alpha}\left(\frac{4}{\alpha^2}+1\right)\Big]$$

$$Re \simeq \frac{-1}{\alpha\,|H_1+iH_0|^2}\Big[\left\{\frac{2}{\pi\alpha} + J_0^2+Y_0^2 + \frac{2}{\alpha}(J_0J_1+Y_0Y_1)\right\} + \frac{8}{\pi\alpha}\left(\frac{1}{4}+\frac{1}{\alpha^2}\right)\Big]$$

$$E = \frac{1}{4}\int_{-1}^{1}(p\bar{\phi}_y + \bar{p}\phi_y)\,dx ,$$



$$(i\omega - U\tfrac{\partial}{\partial x})\eta = -\phi_y , \quad \phi_y = U\eta_x - i\omega\eta .$$

$$\frac{p}{\rho} = (i\omega - U\tfrac{\partial}{\partial x})[\phi]^{-}_{+} \qquad \bar{\phi}_y = U\bar{\eta}_x + i\omega\bar{\eta}$$

$$W^{*} = \frac{i\omega}{4}\int_{-1}^{1}(p\bar{\eta}^{*} - \bar{p}\eta^{*})\,dx ,$$

$$W^{*} - E = \frac{U}{4}\int_{-1}^{1}(p\bar{\eta}_x + \bar{p}\eta_x)\,dx = TU$$

$$\frac{E}{\rho} = \frac{1}{4}\oint\left[(i\omega\phi - U\phi_x)\bar{\phi}_y + (-i\omega\bar{\phi} - U\bar{\phi}_x)\phi_y\right]dx$$

$$i\omega(\phi\bar{\phi}_y - \bar{\phi}\phi_y) - U(\phi_x\bar{\phi}_y + \bar{\phi}_x\phi_y)$$

$$\frac{W}{\rho} = \frac{i\omega}{4}\oint\left[(i\omega\phi - U\phi_x)\bar{\eta}^{*} - (-i\omega\bar{\phi} - U\bar{\phi}_x)\eta^{*}\right]dx$$

$$i\omega(\phi\bar{\eta}^{*} + \bar{\phi}\eta^{*}) - U(\phi_x\bar{\eta}^{*} - \bar{\phi}_x\eta^{*})$$

$$\overset{\#}{=} -\frac{i}{4}\omega U\left[\phi\bar{\eta}^{*} - \bar{\phi}\eta^{*}\right]_{-\infty-i\varepsilon}^{-\infty+i\varepsilon} + \frac{i\omega}{4}\oint(\phi\bar{\phi}_y - \bar{\phi}\phi_y)\,dx$$

$$E^{*} = \frac{1}{4}\int_{-1}^{1}(p\bar{\phi}_y + \bar{p}\phi_y)\,dx + \frac{U}{4}\oint|\nabla\phi|^2\frac{\partial x}{\partial n}\,ds$$

$$\oint\left[\phi_x\bar{\phi}_y + \bar{\phi}_x\phi_y - |\nabla\phi|^2\frac{\partial x}{\partial n}\right]ds = \int_{-\infty}^{+\infty}\left[|\phi_x|^2 - |\phi_y|^2\right]dy\Big|_{x=-\infty} = 0$$

$$\therefore\ E^{*} = \frac{i\omega}{4}\oint(\phi\bar{\phi}_y - \bar{\phi}\phi_y)\,dx .$$

$$W = E^{*} - \frac{i\omega U}{4}\Big[-U\,\overline{H(\alpha)}\,\{ -\tfrac{\alpha}{2}X\overline{H} + \tfrac{i}{2}(\overline{H} + \alpha\overline{H'})\} + U\overline{H}\,\{ -\tfrac{\alpha}{2}XH - \tfrac{i}{2}(H + \alpha H')\}\Big].$$

$$W = E^{*} - \frac{\omega U^2}{8}\{ H(\overline{H} + \alpha\overline{H'}) + \overline{H}(H + \alpha H')\}.$$

$$E^{*} = \frac{i\omega}{4}\int_{-\infty}^{\infty}(\phi\overline{\phi}_x - \overline{\phi}\phi_x)\,dy\Big|_{x=-\infty} = \frac{\omega}{8}U^2 H\overline{H}$$

$$\phi\overline{\phi}_x \doteq \frac{U^2}{4}(He^{-\alpha y + i\alpha x})\times(\overline{i\alpha H e^{-\alpha y + i\alpha x}})$$

$$= -\frac{i\alpha}{4}U^2 H\overline{H}e^{-2\alpha y}. \qquad -\tfrac{i}{2}\alpha \times \tfrac{1}{\alpha}$$

$$\therefore\ W = -\frac{\omega U^2}{8}H\overline{H} - \frac{\alpha\omega}{8}U^2(H\overline{H'} + H'\overline{H})$$

$$\eta = \eta^{*} - \frac{i\alpha}{4}e^{i\alpha x}G_1^{(2)\prime}$$

$$W = \frac{i\omega}{4}\int(p\overline{\eta} - \overline{p}\eta)\,dx = W^{*}$$

$$= W^{*} - \frac{\alpha\omega}{16}\int\big[\overline{G_1^{(2)\prime}}\,p\,e^{-i\alpha x} + G_1^{(2)\prime}\,\overline{p}\,e^{i\alpha x}\big]\,dx$$

$$= W^{*} - \frac{\alpha\omega}{16}(H\overline{G_1^{(2)\prime}} + \overline{H}G_1^{(2)\prime})$$

$$\therefore\ W^{*} = -\frac{\omega U^2}{8}H\overline{H} + \frac{\alpha\omega}{16}(H\overline{G_1^{(2)\prime}} + \overline{H}G_1^{(2)\prime})$$
$$- \frac{\alpha\omega}{8}U^2(H\overline{H'} + H'\overline{H})$$
$$= -\frac{\omega U^2}{8}H\overline{H} - \frac{\alpha\omega}{16}(H\overline{G_1^{(1)\prime}} + \overline{H}G_1^{(1)\prime})$$

$$W^{*} - E^{*} = DU - \frac{\alpha\omega}{}\big[\ldots \big]$$

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$$\hat{W} = \frac{1}{4}\int_{-1}^{1} (p\bar{\Phi}_y + \bar{p}\Phi_y)\,dx$$

$$p = \Phi_- - \Phi_+ = (i\omega - U\frac{\partial}{\partial x})[\phi]_+^-$$

$$\hat{W} = \frac{1}{4}\oint [i\omega(\phi\bar{\Phi}_y - \bar{\phi}\Phi_y) - U(\phi_x\bar{\Phi}_y + \bar{\phi}_x\Phi_y)]\,dx$$

$$E = \tilde{W} = D\,\bar{\omega}\,TU$$

$$D = \frac{i\omega}{4}\oint(\phi\bar{\Phi}_y - \bar{\phi}\Phi_y)\,dx$$

$$TU = -\frac{U}{4}\oint(\phi_x\bar{\Phi}_y + \bar{\Phi}_x\phi_y)\,dx$$

$$T = \frac{1}{4}\oint\left[\phi_x\frac{\partial\bar{\Phi}}{\partial n} + \bar{\Phi}_x\frac{\partial\phi}{\partial n} - \{\ldots\}\frac{\partial x}{\partial n}\right]dx$$

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$$q = \frac{1}{\pi}\int_{-1}^{1} \frac{p \, d\xi}{x-\xi}$$

$$-\frac{\sin\theta}{\sin\theta} + i\frac{\cos\theta}{\sin\theta} \quad \frac{e^{i\theta}}{\sin\theta}$$

$$\chi(z) = \frac{1}{\pi}\int_{-1}^{1} \frac{p \, d\xi}{z-\xi} = q + ip, \quad \quad y<0$$

$$x = ch\,u, \quad q = \frac{e^{-u}}{sh\,u} \xrightarrow[u=i\theta]{} \frac{e^{-i\theta}}{-i\sin\theta} = \frac{i\cos\theta - \sin\theta}{\sin\theta}$$

$$\int_{-i\varepsilon-\infty}^{-i\varepsilon+\infty} \chi(z)\, e^{-i\alpha z} dz = 0 = \int_{-\infty}^{\infty} q e^{-i\alpha x} dx + i\int_{-1}^{1} p e^{-i\alpha x} dx$$

$$\int_{-1}^{1} p e^{-i\alpha x} dx = \overline{H}(\alpha)$$

$$\int_{\infty}^{x} q e^{-i\alpha x} dx = \int_{\infty}^{-\infty} q e^{-i\alpha x} dx - \int_{x}^{-\infty} q e^{-i\alpha x} dx$$

$$= i\overline{H}(\alpha) + \int_{-\infty}^{x} q e^{-i\alpha x} dx$$

$$\int_{-\infty}^{-1} q e^{-i\alpha x} dx = -i\overline{H} + \int_{\infty}^{-1} q e^{-i\alpha x} dx$$

$$\zeta(-1)e^{+i\alpha} = -\frac{1}{2}\int_{\infty}^{-1} q(\xi)\{1 - i\alpha(1+\xi)\} e^{-i\alpha\xi} d\xi$$

$$= -\frac{1}{2}\left[(1-i\alpha) + \alpha\frac{\partial}{\partial\alpha}\right\} \int_{\infty}^{-1} q(\xi) e^{-i\alpha\xi} d\xi$$

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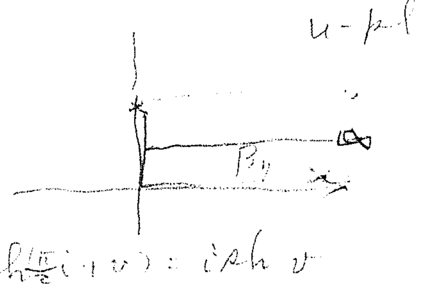
$$A_n = \int_{-\infty}^{-1} g_n e^{-i\alpha x} dx$$

$$x = -ch\,u \quad , \quad g_n = \frac{e^{-nu+in\pi}}{-sh\,u} \quad , \quad dx = -sh\,u\,du$$

$$A_n = (-)^{n+1} \int_0^\infty e^{i\alpha ch\,u - nu} du .$$



$$B_n = \int_1^\infty g_n e^{-i\alpha x} dx = +\int_0^\infty e^{-i\alpha ch\,u - nu} du$$

$$x = ch\,u$$

$$C_n = \int_{-1}^{1} g_n e^{-i\alpha x} dx = \int_\pi^0 \sin n\theta\, e^{-i\alpha\cos\theta} d\theta$$

$$x = \cos\theta .$$

$$A_n + B_n + C_n = -i\,\overline{H}(\alpha)$$

$$B_n = \frac{\pi}{2}(-i)^{n+1} H_n^{(2)}(\alpha) - b_n , \quad -$$

$$b_n = \int_0^\infty e^{-i\alpha ch\,u} sh\,nu\,du = \overline{a_n}$$

$$\overline{A_n} = (-)^{n+1} B_n$$

$$A_n = (-)^n a_n + (-)^{n+1}\left(\frac{\pi}{2}(+i)^{n+1}\right) H_n^{(1)}(\alpha) .$$

$$A_n + B_n = \frac{\pi}{2}(-i)^{n+1}\left\{ H_n^{(1)}(\alpha) + H_n^{(2)}(\alpha) \right\} + (-)^n a_n - b_n$$

$$\overline{H}_n(\alpha) = \int_0^\pi \cos n\theta\, e^{-i\alpha\cos\theta} d\theta = \pi$$

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$$I = \int_{-\infty}^{-1} \phi_y e^{-i\alpha x} dx = \frac{-U}{2}\int_{-\infty}^{-1} g e^{-i\alpha x} dx - \frac{i\alpha U}{2}\int_{-\infty}^{-1} dx \int_{-\infty}^{x} g e^{-i\alpha x} dx$$

$$\int_{-\infty}^{-1} dx \int_{\infty}^{x} g e^{-i\alpha x} dx = i \bar{F}(\alpha)[-1+X] + \int_{-\infty}^{-1}dx\int_{-\infty}^{x} g e^{-i\alpha x}dx$$

$$= i(X-1)\bar{F}(\alpha) + x\int_{\infty}^{x} g e^{-i\alpha x}dx \Big|_{\infty}^{-1} - \int_{-\infty}^{-1} x g e^{-i\alpha x} dx$$

$$= i(X-1)\bar F(\alpha) - \int_{-\infty}^{-1} g e^{-i\alpha x}dx - i\frac{\partial}{\partial\alpha}\int_{-\infty}^{-1} g e^{-i\alpha x} dx$$

$$\int_{\infty}^{-1} g e^{-i\alpha x} dx = A \quad , -\frac{1}{2}\left\{1 - i\alpha + \alpha\frac{\partial}{\partial\alpha}\right\}(A) = \left\{\eta(-1)e^{i\alpha} + i\bar F\right\}$$

$$I = -\frac{U}{2}A - \frac{i\alpha U}{2}\left[i(X-1)\bar F(\alpha) - A - i\frac{\partial A}{\partial\alpha}\right]$$

$$= \frac{\alpha U}{2}(X-1)\bar F(\alpha) - \frac{U}{2}(1-i\alpha)A - \frac{\alpha U}{2}\frac{\partial A}{\partial \alpha}$$

$$- \frac{U}{2}\left[(1 \mp i\alpha) + \alpha\frac{\partial}{\partial\alpha}\right]A$$

~~I e~~ $U\eta(x)e^{-i\alpha x} = \int_{\infty}^{x}\phi_y e^{-i\alpha x}dx$.

$$I = \int_{-\infty}^{-1}\phi_y e^{-i\alpha x}dx = \left[U\int_{-\infty}^{\infty} - \int_{\infty}^{-1}\right]\phi_y e^{-i\alpha x} dx$$

$$= -U\left[\eta(-X)e^{+i\alpha X} + \eta(-1)e^{+i\alpha}\right]$$

$$\bar F I - \bar{\bar F} \bar I = U\left[\bar F\left\{\overline{\eta(-X)e^{i\alpha X}} + \overline{\eta(-1)}e^{i\alpha}\right\} - \bar F\left\{\eta(-X)e^{+i\alpha X} + \eta(-1)e^{i\alpha}\right\}\right]$$

$$= U\left[\frac{i\bar F}{2}\{\bar F + \alpha \bar F'\} - \frac{\bar F}{2i}\{\bar F + \alpha\bar F'\} + \right.$$

$$\left. + \bar F\eta(-1)e^{i\alpha} - \bar F\eta(-1)e^{i\alpha}\right]$$

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$$\Delta\phi_x = \phi_x\Big|_{y=-\varepsilon}^{y=+\varepsilon} = -\frac{i\alpha U}{\ } e^{i\alpha x} \overline{A(\alpha)}$$

$$I = \int_{-\infty}^{-1} \left[\overline{\Delta\phi_x}\, \phi_y + (\Delta\phi_x)\, \overline{\phi_y} \right] dx$$

$$= i\alpha U\, \overline{A(\alpha)}\, I - i\alpha U\, A(\alpha)\, \overline{I}$$

$$= i\alpha U \left[\overline{A(\alpha)}\, I - A\overline{I} \right]$$

$$= i\alpha U \Big[\overline{A} \left\{ \frac{\alpha U}{2}(x-1)\overline{A} - \frac{U}{2}(1-i\alpha)A - \frac{\alpha U}{2}A' \right\}$$
$$- A\left\{ \frac{\alpha U}{2}(x-1)\overline{A} - \frac{U}{2}(1+i\alpha)\overline{A} - \frac{\alpha U}{2}\overline{A'} \right\} \Big]$$

$$= i\alpha U \Big[-\frac{U}{2}(1-i\alpha)A\overline{A} + \frac{U}{2}(1+i\alpha)\overline{A}A$$
$$- \frac{\alpha U}{2}\left(\overline{A}A' - A\overline{A'}\right) \Big]$$

$$= i\alpha U \Big[\frac{U}{2}\left(\overline{A}A - A\overline{A}\right) + \frac{i\alpha U}{2}\left(\overline{A}A + A\overline{A}\right)$$
$$- \frac{\alpha U}{2}\left(\overline{A}A' - A\overline{A'}\right) \Big].$$

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$$\eta \doteqdot \frac{1}{2i}\left[e^{i\alpha x}\bar{H}(\alpha) + ix\alpha e^{i\alpha x}\bar{H}(\alpha) + \alpha e^{i\alpha x}\bar{H}'(\alpha)\right].$$

$$\eta e^{-i\alpha x} \doteqdot \frac{1}{2i}\left[\bar{H}(\alpha) + \alpha\bar{H}'(\alpha) - i\alpha X\bar{H}(\alpha)\right]$$

$x = -X$

$$\eta(x) = \frac{e^{i\alpha x}}{U}\int_{\infty}^{x}\phi_y e^{-i\alpha x}dx = \frac{e^{i\alpha x}}{U}\left[\int_{\infty}^{-\infty} + \int_{-\infty}^{x}\right]\phi_y e^{-i\alpha x}dx$$

$$U\eta(x)e^{-i\alpha x} = \int_{\infty}^{-\infty}\phi_y e^{-i\alpha x}dx + \int_{-\infty}^{x}\phi_y e^{-i\alpha x}dx$$

$$\xrightarrow[x\to -X]{} \int_{\infty}^{-\infty}\phi_y e^{-i\alpha x}dx + \frac{\alpha U}{2}\bar{H}(\alpha)(x+X)$$

$$\phi_y \longrightarrow \frac{\alpha U}{2}e^{i\alpha x}\bar{H}(\alpha)$$

$$\int_{-\infty}^{x}\phi_y e^{-i\alpha x}dx = \frac{\alpha U}{2}\bar{H}(\alpha)\int_{-\infty}^{x}dx = \frac{\alpha U}{2}\bar{H}\,(x+X)$$

$x - X$

$$\int_{1}^{\infty}\phi_y e^{-i\alpha x}dx = -\int_{1}^{\infty}g e^{-i\alpha x}dx - \frac{i\alpha}{2}\int_{1}^{\infty}dx\int_{\infty}^{x}g e^{-i\alpha x}dx$$

$$x\int_{\infty}^{x}g e^{-i\alpha x}dx\Big|_{1}^{\infty} - \int_{1}^{\infty}x g e^{-i\alpha x}dx$$

$$= -\frac{1}{2}\int_{1}^{\infty}g e^{-i\alpha x}dx - \frac{i\alpha}{2}\int_{1}^{\infty}g e^{-i\alpha x}dx + \frac{i\alpha}{2}\int_{1}^{\infty}x g e^{-i\alpha x}dx$$

$$-\int_{\infty}^{1}g e^{-i\alpha x}dx$$

$$-\frac{\alpha}{2}\frac{\partial}{\partial\alpha}\int_{1}^{\infty}g e^{-i\alpha x}dx$$

$$= \left(-\frac{1}{2} - \frac{i\alpha}{2} - \frac{\alpha}{2}\frac{\partial}{\partial\alpha}\right)\int_{1}^{\infty}g e^{-i\alpha x}$$

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freezing

$$\eta(x) = 1 + B_0 e^{i\alpha x}, \quad = \frac{e^{i\alpha x}}{U}\int_{\infty}^{x} \phi_y e^{-i\alpha x} dx$$

$$= \frac{e^{i\alpha x}}{U}\int_{\infty}^{1} \phi_y e^{-i\alpha x} dx - \frac{i\omega}{U} e^{i\alpha x}\int_{1}^{x} e^{-i\alpha x} dx$$

$$= \frac{e^{i\alpha x}}{U}\int_{\infty}^{1} \phi_y e^{-i\alpha x} dx - i\alpha \frac{(1 - e^{i\alpha(x-1)})}{-i\alpha}$$

$$= 1 - e^{i\alpha(x-1)} - \frac{e^{i\alpha x}}{U}\int_{1}^{\infty} \phi_y e^{-i\alpha x} dx$$

$$B_0 = -e^{-i\alpha} - \frac{1}{U}\int_{1}^{\infty} \phi_y e^{-i\alpha x} dx.$$

$$\eta(x) = \frac{e^{i\alpha x}}{U}\left[\int_{\infty}^{-1} \phi_y e^{-i\alpha x} dx + \int_{-1}^{x} \phi_y e^{-i\alpha x} dx\right]$$

$$\int_{-1}^{x} \phi_y e^{-i\alpha x} dx = -i\omega\int_{-1}^{x} e^{-i\alpha x} dx = \frac{-i\omega}{-i\alpha}\left[e^{-i\alpha x} - e^{i\alpha}\right].$$

$$\eta = (1 - e^{i\alpha + i\alpha x}) + \frac{e^{i\alpha x}}{U}\int_{\infty}^{-1} \phi_y e^{-i\alpha x} dx.$$

$$\eta(-1)e^{i\alpha} = e^{i\alpha} + B_0 \quad \bar{A}$$

$$\overline{\eta(-1)e^{i\alpha}} = e^{-i\alpha} + \overline{B_0} \quad \bar{A} \quad (-)$$

$$= B_0\bar{A} - \overline{B_0}A + \bar{A}e^{i\alpha} - Ae^{-i\alpha}.$$

$$\overline{\bar{A}e^{i\alpha}} - \bar{A}e^{-i\alpha}$$

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$$(i\omega - U\tfrac{\partial}{\partial x})\eta^* = -\Phi_y$$

$$\eta^*(x) = \eta^*(1) + \frac{e^{i\alpha x}}{U}\int_1^x \Phi_y e^{-i\alpha x} dx$$

$$\eta(x) = \frac{e^{i\alpha x}}{U}\int_\infty^x \Phi_y e^{-i\alpha x} dx$$

$$\eta^*(x) - \eta^*(1) = \eta(x) - \frac{e^{i\alpha x}}{U}\int_\infty^1 \Phi_y e^{-i\alpha x} dx$$

$$= \eta(x) - \eta(1) e^{i\alpha(x-1)}$$

$$\eta^*(x) = \eta(x) + \eta^*(1) - \eta(1) e^{i\alpha(x-1)}$$

$$\eta^* = \eta(x) + \eta^*(-1) - \eta(-1) e^{i\alpha(x+1)}$$

$$\eta^*(-1) = \eta(-1) + \eta^*(-1) - \eta(-1) e^{i\alpha}$$

$$e^{i\alpha x}\int_1^x e^{-i\alpha x} dx = \frac{e^{i\alpha x}}{-i\alpha}\left[e^{-i\alpha x}\right]_1^x = \frac{1 - e^{i\alpha(x-1)}}{-i\alpha}$$

$$-\frac{i}{2}\left(1 - i\alpha + \alpha\frac{\partial}{\partial\alpha}\right)\bar{H} \qquad \bar{\bar{H}}$$

$$\frac{i}{2}\left(1 + i\alpha + \alpha\frac{\partial}{\partial\alpha}\right)\bar{\bar{H}} \qquad \bar{H}$$

$$-i\bar{H}\bar{\bar{H}} + \frac{\alpha}{2i}\left(\bar{H}'\bar{\bar{H}} + \bar{H}\bar{\bar{H}}'\right)$$

$$-i\left(1 + \alpha\frac{\partial}{\partial\alpha}\right)\bar{H}\bar{\bar{H}}$$