# 水面変位計算用核関数について

圧力分布 $p(x,y)$ によって水面変位 $\zeta(x,y)$ は

$$\zeta(x,y) = \frac{1}{\rho U^2} \iint P(\xi,\eta)\, \Omega_2(x-\xi, y-\eta, 0)\, d\xi\, d\eta \qquad (1)$$

$$p(x,y) = \rho g T P(x,y) \qquad (2)$$

とおくと

$$\frac{1}{T}\zeta(x,y) = K \iint P(\xi,\eta)\, \Omega_2(x-\xi, y-\eta, 0)\, d\xi\, d\eta \qquad (1')$$

$$K = g/U^2$$

$$\Omega_2(x,y,0) = -\frac{\partial T(x,y,0)}{\partial z} = \lim_{\mu \to +0} \frac{1}{4\pi^2} \int_{-\pi}^{\pi}\int_0^{\infty} \frac{e^{ik(x\cos u + y\sin u)}\, dk\, du}{\cos^2 u - \frac{K}{k} \pm i\mu\cos u} \qquad (3)$$

十 前

さて $h = K/m^2$ とおくと

$$\frac{1}{\cos^2 u - \frac{K}{h} + 2i\mu\cos u} = \frac{1}{\cos^2 u + 2i\mu\cos u - m^2} = \frac{1}{(\cos u + i\mu)^2 - m^2 + \mu^2}$$

$$= \frac{1}{(\cos u + i\mu + \sqrt{m^2-\mu^2})(\cos u + i\mu - \sqrt{m^2-\mu^2})} =$$

$$= \frac{1}{2m}\left(\frac{1}{\cos u + i\mu - \sqrt{m^2-\mu^2}} - \frac{1}{\cos u + i\mu + \sqrt{m^2-\mu^2}}\right) \quad ,\ \cdots (4)$$

さらに $z = e^{iu}$ とおくと，$\frac{dz}{iz} = du$

$$\frac{1}{\cos u + i\mu - m} = \frac{2z}{z^2 + 1 - 2z(m - i\mu)} =$$

分母の根は

$m > 1$ 場合 $z = m - i\mu \pm \sqrt{m^2 - \mu^2 - 1 - 2\mu m i} \doteqdot m - i\mu \pm (\sqrt{m^2-1} + m\mu i)$

$= m \pm \sqrt{m^2-1} \doteqdot e^{\pm v}$, $\cosh v = m$, $\sinh v = \sqrt{m^2-1}$

$m < 1$ 場合 $z = e^{\pm i(\theta \mp i\varepsilon)}$, $\varepsilon > 0$, $\cos\theta = m$, $\sin\theta = \sqrt{1-m^2}$.

$[\ \cos u = m - i\mu \doteqdot \cos\theta - i\mu,\ \cos(\theta + i\varepsilon) = \cos\theta\cos\varepsilon - i\varepsilon\sin\theta\ ]$

$$\therefore\ \frac{1}{\cos u - m + i\mu} = \frac{z}{\sinh v}\left(\frac{1}{(z - e^{v})} - \frac{1}{z - e^{-v}}\right) \quad \text{for } m > 1$$

$$= \frac{z}{i\sin\theta}\left(\frac{1}{z - e^{i(\theta - i\varepsilon)}} - \frac{1}{z - e^{-i(\theta - i\varepsilon)}}\right), \quad \text{for } m < 1$$

$\cdots (5)$

$$\frac{1}{\cos u + m + i\mu} = \frac{2z}{z^2+1+2z(m+\mu i)}$$

分母の零点は

$m>1$ ならば $\quad z = -m \pm\sqrt{m^2-1} = -e^{\pm v}, \quad m = ch\, v, \quad m>1$

$m<1$ ならば $\quad z = -e^{\pm i(\theta - i\varepsilon)}, \quad \cos\theta = m$

$u = \pi + \theta + i\varepsilon$

$\left[ \cos u = -\cos\theta - i\mu, \quad -\cos(\theta + i\varepsilon) = -[\cos\theta \cos i\varepsilon + \sin\theta\, \varepsilon] = -\cos\theta - i\mu \right.$

$$\frac{1}{\cos u + m + i\mu} = \frac{z}{sh\, v}\left(\frac{-1}{z+e^{v}} + \frac{1}{z+e^{-v}}\right) \quad \text{for } m>1$$

$$\frac{z}{i\sin\theta}\left(\frac{1}{z+e^{-i\theta}} - \frac{1}{z+e^{+i\theta}}\right), \quad m<1 \qquad (6)$$

一方 (5) 式の場合

$$E = -k(x\cos u + y\sin u) = \frac{K}{ch^2 v}(x\, ch\, v + iy\, sh\, v) \quad \text{for } m>1$$

$$\frac{K}{\cos^2\theta}(x\cos\theta + y\sin\theta) \quad \text{for } m<1, \quad (7)$$

(6) の場合は

$$E = \frac{-K}{ch^2 v}(+x\, ch\, v + iy\, sh\, v) \quad \text{for } m>1$$

$$\frac{-K}{\cos^2\theta}(+x\cos\theta + y\sin\theta) \quad \text{for } m<1 \qquad (8)$$

(3)の $u$ に関する積分は $z$-面の単位円上の積分となり

$$I = \int_{-\pi}^{\pi} \frac{e^{iE(u)}\,du}{\cos^2 u - m^2 + 2i\mu\cos u}$$

$$= \frac{1}{2im}\oint \frac{dz}{sh\,v}\left[\left(\frac{1}{z-e^{v}} - \frac{1}{z-e^{-v}}\right) + \left(\frac{1}{z+e^{v}} - \frac{1}{z-e^{-v}}\right)\right] e^{iE(u)}$$

for $m<1$

$$= -\frac{\pi}{m\,sh\,v}\left(e^{i\frac{K}{ch^2 v}(x\,ch\,v - iy\,sh\,v)} + e^{-i\frac{K}{ch^2 v}(x\,ch\,v - iy\,sh\,v)}\right), \quad (9)$$

$$= \frac{-1}{2im\sin\theta}\oint e^{iE(u)}\,dz\,\Big[\frac{1}{z-e^{i\alpha+i\theta}} - \frac{1}{z-e^{i\alpha-i\theta}}$$

$$+ \frac{1}{z+e^{i\alpha+i\theta}} - \frac{1}{z+e^{i\alpha-i\theta}}\Big]$$

for

$$= \frac{\pi i}{m\sin\theta}\left[e^{i\frac{K}{\cos^2\theta}(x\cos\theta + y\sin\theta)} - e^{-i\frac{K}{\cos^2\theta}(x\cos\theta + y\sin\theta)}\right], \quad (10)$$

for $m>1$

$$k = \frac{K}{m^2} = \frac{K}{ch^2 v}, \quad dk = -2K\frac{sh\,v}{ch^3 v}\,dv$$

$$k_2 = \frac{K}{m^2} = K\sec^2\theta, \quad dk = 2K\frac{\sin\theta}{\cos^3\theta}\,d\theta \quad (11)$$

従って(3)は

$$S_2 = -\frac{K}{\pi}\int_0^{\infty}\frac{dv}{ch^2 v}\cos\left[K\left\{\frac{x}{ch\,v} - iy\frac{sh\,v}{ch^2 v}\right\}\right]$$

$$-\frac{K}{\pi}\int_0^{\frac{\pi}{2}}\sin\left[\frac{K}{\cos^2\theta}(x\cos\theta + y\sin\theta)\right]\frac{d\theta}{\cos^2\theta}, \quad (12)$$

右辺第1項で変数変換すると

$\frac{1}{\cosh v} = \cos\phi$ , $\frac{\sinh v}{\cosh^2 v}dv = \sin\phi\, d\phi$ , $\sinh v = \tan\phi$

∴ $dv = \frac{d\phi}{\cos\phi}$ , となるから $\left( d\phi = \frac{dv}{\cosh v} \right)$

$$-\frac{K}{\pi}\int_0^\infty \frac{dv}{\cosh^3 v}\cos\left[\frac{K}{\cosh^2 v}(x\cosh v - iy\sinh v)\right]$$

$$= -\frac{K}{\pi}\int_0^{\frac{\pi}{2}} \frac{d\phi}{\cos^3\phi}\cos\left[K\cos^2\phi\,(x\sec\phi - iy\tan\phi)\right] , \quad (13)$$

これは $\phi = \frac{\pi}{2}$ の近くで積分が収束しない！

同じ事は第2項についても言える。

次に 第2項は $\sec\theta = \cosh u$ , $d\theta = \frac{du}{\cosh u}$ , $\tan\theta = \sinh u$

$$-\frac{K}{\pi}\int_0^{\frac{\pi}{2}} \sin\left[\frac{K}{\cos^2\theta}(x\cos\theta + y\sin\theta)\right]\frac{d\theta}{\cos^4\theta} = -\frac{K}{\pi}\int_0^\infty \sin\left[K\cosh^2 u\left(\frac{x}{\cosh u} + y\tanh u\right)\right]\times \cosh^3 u\, du$$

(14)