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Stirring of a Viscous Fluid

別所

参照文献

1) 池田隆穂, 第12回 運動生理委員会 昭62年2月
2) 別所, 昭62春. 造学 講演会
3) N. Riley, Z.A.M.P. vol.22, 1971
4) M.S. Longuet-Higgins, J.F.M. vol.32, 1970
5) J.R. Chaplin, J.F.M. vol.147, 1984

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Stirring of a Viscous Fluid

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1) 池田隆, 第12回 [illegible] 1986年2月

2) 別府, 昭62春, 造船 講演会

3) N. Riley, Z.A.M.P. vol.22, 1971

4) M.S. Longuet-Higgins, J.F.M. vol.42, 1970

5) J.R. Chaplin, J.F.M. vol.147, 1984

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概要

円筒が粘性流体中で回転する事なく円運動している時に円筒に働く力をもとめるのが主題である。

これはまた水平没水円筒が波からうける力の問題でもあり、力は排水容積の慣性力分だけ異る。[3]

ここでは表題と少し異なるがこの振動流れ中の力を論じよう。

さてLonguet-Higgins, Rileyによって解かれているようにこの場合周期運動により、サーキュレーションを持つ定常流が誘起されるがそのサーキュレーションによる力については言及されていない。

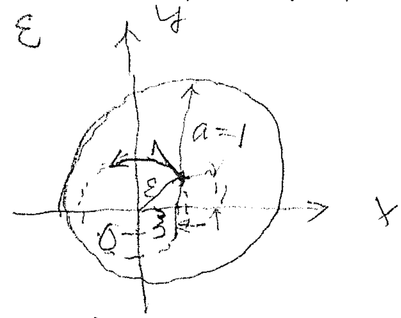
しかし明らかにそれと振動流れが干渉して流れの方向に直角の揚力を生ずる訳でそれは復元力として働く事がわかる。つまり慣性力を減ずる役割を果し実験[1,5]の傾向に一致する。

本報はナビアストークス方程式を近似して、先ず振動分の線型解をもとめ、次にそれを使って定常流れの近似解をもとめ文献[3,4]の解と一致する事を確かめ、それらを使って振動流れの第2近似をもとめようとするものであるが、その解は大変複雑でまたかなり悪い近似のように見える。

しかしGalerkin近似に相当する積分定理[2]を使えばかなり良い近似を与える事が示される。

1. 第1近似

半径 $a=1$ の円筒が原点のまわりの半径 ε の円周上を半時計まわりに回転する事なく円運動をしているものとする。



円筒中心の座標を (ξ,η) とおくと流れ関数は円筒座標で

$$\Psi = i\omega(\xi\sin\theta - \eta\cos\theta)\Psi_1(r), \qquad (1.1)$$

のように書けるが、ξ, η の位相差を考えて（振巾 ε の）円運動な場合

$$\xi = \varepsilon, \quad \eta = -i\varepsilon, \quad (\varepsilon\ \text{real}) \qquad (1.2)$$

とおけるから (1.1) は

$$\Psi = +\varepsilon\omega e^{-i\theta}\Psi_1(r), \qquad (1.3)$$

の形に書ける。（大文字で攪乱分を示す事にする）

また円筒が静止して振動流れ（あるいは波の中）の中にある場合はそれを加えて小文字で示す事とすると

$$\psi(r,\theta) = +\varepsilon\omega e^{-i\theta}\psi_1(r), \qquad (1.4)$$

〃 $$\zeta(r,\theta) = \varepsilon\omega e^{-i\theta}\zeta_1(r), \qquad (1.5)$$

B.C. は $\psi_1(1) = 0$, $\psi_{1r}(1) = 0$, (1.6)

となる。

以下の計算はすべて振動流れの中で行う。

$|k| \gg 1$ では近似的に

$$\psi_1(r) = +r - \frac{1}{r}\left(1+\frac{2}{k}+\frac{1}{k^2}\right) + \frac{\zeta_1(r)}{k^2},$$

$$\zeta_1(r) = \frac{2k e^{-k(r-1)}}{\sqrt{r}}\left[1 + \frac{1+\frac{3}{r}}{8k} + \frac{7+\frac{6}{r}+\frac{15}{r^2}}{128k^2}\right], \quad (1.7)$$

$$p = -\rho i\omega\Phi, \quad \Phi = -i\varepsilon\omega e^{-it}\phi_1(r)$$

$$\phi_1(r) = r + \frac{1}{r}\left(1+\frac{2}{k}+\frac{1}{k^2}\right),$$

$$p = -\rho\omega^2\varepsilon e^{-it}\phi_1(r), \quad \begin{matrix}|k| \gg 1 \\ (1.7)\end{matrix}$$

それ故 X−方向の力は 第1近似として（振動流れで）

$$X_1 = -\int_0^{2\pi}\left(p\cos\theta + p_{r\theta}\sin\theta\right)d\theta = +\pi\rho\omega^2\varepsilon\left(2+\frac{4}{k}\right), \quad (1.8)$$

円筒が動く場合は右辺括弧の中の2を1とすればよい。
（静止流体中で）

またY方向の力は90°位相が遅れるだけであるから以後考えない。

Sarpkaya等の記号に合せて無次元係数をとると、

$$C_{M1} = \frac{[X]_R}{\pi\rho\varepsilon\omega^2} = 2 + \left[\frac{4}{k}\right]_R = 2 + \frac{4}{\sqrt{\pi\beta}}, \quad (1.9)$$

$$C_{D1} = \frac{[-X]_I}{\frac{8}{3\pi}\rho\omega^2\varepsilon^2} = \frac{3\pi^3}{2K\sqrt{\pi\beta}}, \quad (1.10)$$

但し $\beta = \frac{2\omega}{\pi\nu}a^2$, $K = \pi\frac{\varepsilon}{a}$.

(Notice; 今は $u = -i\omega\varepsilon$, $\dot{u} = \omega^2\varepsilon$ となっている！)

2、第2近似定常流れ

前節の解を(29)第1式に代入し $O(\epsilon^3)$ の項も無視すれば

$$+\nabla^4\Psi = -\frac{1}{\nu}\left[(u\hat{\zeta}+\hat{u}\zeta)_x+(v\hat{\zeta}+\hat{v}\zeta)_y\right]$$

$$=\frac{-1}{2\nu r}\left[\zeta_r\psi_\theta-\zeta_\theta\psi_r\right]_R=\frac{\epsilon^2\omega^2}{2\nu r}\left[-i\zeta_{1r}\hat{\psi}_1-i\zeta_1\hat{\psi}_{1r}\right]_R$$

$$=\omega\epsilon^2\left[k^4F(r)e^{-k(r-1)}+(k+\hat{k})^4G(r)e^{-(k+\hat{k})(r-1)}\right]_R \quad (2\cdot1)$$

$$\hat{\zeta}_1\psi_{1r}+\hat{\zeta}_{1r}\psi_1=\hat{\zeta}_1\psi_{1pr}+\hat{\zeta}_{1r}\psi_{1p}+\frac{1}{k^2}(\hat{\zeta}_1\zeta_{1r}+\hat{\zeta}_{1r}\zeta_1) \qquad \left[\Psi=\omega\epsilon^2\Psi_2\right.$$

$$\psi_1=\psi_{1p}+\frac{\zeta_1}{k^2},\quad \psi_{1p}=r-\frac{1}{r}\left(1+\frac{2}{k}+\frac{1}{k^2}\right)$$

$$\psi_{1r}=\psi_{1pr}+\frac{\zeta_{1r}}{k^2},\quad \psi_{1pr}=1+\frac{1}{r^2}\left(1+\frac{2}{k}+\frac{1}{k^2}\right)$$

$$\zeta_1=\frac{2k}{\sqrt{r}}e^{-k(r-1)}\left(1+\frac{1+\frac{3}{r}}{8k}+\frac{7+\frac{6}{r}+\frac{15}{r^2}}{128k^2}\right)$$

$$\zeta_{1r}=\frac{-2k^2}{\sqrt{r}}e\left(1+\frac{1+\frac{3}{r}}{8k}+\frac{7+\frac{6}{r}+\frac{15}{r^2}}{128k^2}+\frac{1}{k}\left\{\frac{1}{2r}\left(1+\frac{1}{8k}\right)+\frac{45}{8kr^2}\right\}\right)$$

$$=-\frac{2k^2}{\sqrt{r}}e^{-k(r-1)}\left[1+\frac{1}{8k}+\frac{7}{8kr}+\frac{7+\frac{14}{r}+\frac{87}{r^2}}{128k^2}\right]$$

$$\hat{\zeta}_1\psi_{1pr}=\frac{2\hat{k}}{\sqrt{r}}e^{-\hat{k}(r-1)}\left(1+\frac{1+\frac{3}{r}}{8\hat{k}}+\frac{7+\frac{6}{r}+\frac{15}{r^2}}{128\hat{k}^2}\right)\left\{1+\frac{1}{r^2}\left(1+\frac{2}{k}+\frac{1}{k^2}\right)\right\}$$

$$=\frac{2\hat{k}}{\sqrt{r}}e^{-\hat{k}(r-1)}\left(1+\frac{1}{r^2}+\frac{2}{kr^2}+\frac{1+\frac{3}{r}}{8\hat{k}}\right)$$

$$\hat{\zeta}_{1r}\psi_{1p}=-\frac{2\hat{k}^2}{\sqrt{r}}e^{-\hat{k}(r-1)}\left(1+\frac{1+\frac{7}{r}}{8\hat{k}}+O(\epsilon^2)\right)\left\{r-\frac{1}{r}\left(1+\frac{2}{k}+\frac{1}{k^2}\right)\right\}$$

$$\left(r-\frac{1}{r}+\frac{(r-\frac{1}{r})(1+\frac{7}{r})}{8\hat{k}}-\frac{2}{kr}-\frac{1}{k^2r}\right.$$

$$\frac{1}{k^2}\left(\hat{\zeta}_{1r}\zeta_1+\hat{\zeta}_1\zeta_{1r}\right)=\frac{-4}{k^2r}e^{-(k+\hat{k})(r-1)}\left[k^2\hat{k}\left(1+\frac{1+\frac{7}{r}}{8k}\right)\left(1+\frac{1+\frac{3}{r}}{8\hat{k}}\right)+k\hat{k}^2\left(1+\frac{1+\frac{7}{r}}{8\hat{k}}\right)\left(\frac{1+\frac{3}{r}}{8k}\right)\right]$$

$$S_{1r}\hat{\Psi}_1 + S_1\hat{\Psi}_{1r} = S_{1r}\hat{\Psi}_{1p} + S_1\hat{\Psi}_{1pr} + (S_{1r}S_1 + S_1S_{1r})\frac{1}{\hat{k}^2}$$

$$= -\frac{2k^2}{\sqrt{r}}e^{-k(r-1)}\left(1+\frac{1+\frac{7}{r}}{8k}+\frac{7+\frac{14}{r}+\frac{87}{r^2}}{128k^2}\right)\left(r+\frac{1}{r}-\frac{2}{\hat{k}r}-\frac{1}{\hat{k}^2 r}\right)$$

$$+\frac{2k}{\sqrt{r}}e^{-k(r-1)}\left(1+\frac{1+\frac{3}{r}}{8k}+\frac{7+\frac{6}{r}+\frac{15}{r^2}}{128k^2}\right)\left(1+\frac{1}{r^2}+\frac{2}{\hat{k}r^2}+\frac{1}{\hat{k}^2r^2}\right)$$

$$-\frac{4\cancel{k}}{r\hat{k}^2}e^{-(k+\hat{k})(r-1)}\left[k^2\hat{k}\left(1+\frac{1+\frac{7}{r}}{8k}\right)\left(1+\frac{1+\frac{3}{r}}{8\hat{k}}\right)+\hat{k}^2k\left(1+\frac{1+\frac{7}{r}}{8\hat{k}}\right)\left(1+\frac{1+\frac{3}{r}}{8k}\right)\right.$$

$$= -\frac{2k^2}{\sqrt{r}}e^{-k(r-1)}\left[\left(r-\frac{1}{r}\right)\left(1+\frac{1+\frac{7}{r}}{8k}+\frac{7+\frac{14}{r}+\frac{87}{r^2}}{128k^2}\right)-\frac{2}{\hat{k}r}\left(1+\frac{1+\frac{7}{r}}{8k}\right)-\frac{1}{\hat{k}^2r}\right.$$

$$\left.-\frac{1}{k}\left\{1+\frac{1}{r^2}+\frac{2}{\hat{k}r^2}+\frac{1+\frac{3}{r}}{8k}\left(1+\frac{1}{r^2}\right)\right\}\right]$$

$$-\frac{4\cancel{k}}{\hat{k}r}e^{-(k+\hat{k})(r-1)}\left[k\left(1+\frac{1+\frac{7}{r}}{8k}+\frac{1+\frac{3}{r}}{8\hat{k}}\right)+\hat{k}\left(1+\frac{1+\frac{7}{r}}{8\hat{k}}+\frac{1+\frac{3}{r}}{8k}\right)\right]$$

$$F(r) = +\frac{2}{2r^{1.5}}\left[\left(r-\frac{1}{r}\right)\left(1+\frac{1+\frac{7}{r}}{8k}+\frac{7+\cdots}{128k^2}\right)-\frac{2}{\hat{k}r}-\frac{1}{k}\left(1+\frac{1}{r^2}\right)\right.$$

$$\left.-\frac{1}{\hat{k}^2r}-\frac{1+\frac{7}{r}}{4k\hat{k}r}-\frac{2}{k\hat{k}r^2}-\frac{\left(1+\frac{3}{r}\right)\left(1+\frac{1}{r^2}\right)}{8k^2}\right] \quad (2.2)$$

$$G(r) = +\frac{+k^2\cdot 4i}{2(k+\hat{k})^4r^2}\left[k+\hat{k}+\frac{1+\frac{7}{r}}{\cancel{8}}\right]$$

$$= \frac{-1}{2\alpha^2r^2}\left(\sqrt{2}\alpha+\frac{1+\frac{7}{r}}{4}\right) = \frac{-1}{\sqrt{2}\alpha r^2}\left(1+\frac{1+\frac{7}{r}}{4\sqrt{2}\alpha}\right),$$

$$F_r = \left(\frac{-0.5}{r^{1.5}}+\frac{2.5}{r^{3.5}}\right)\left(1+\frac{1+\frac{7}{r}}{8k}+\cdots\right)+\frac{5}{\hat{k}r^{3.5}}+\frac{1}{k}\left(\frac{1.5}{r^{2.5}}+\frac{3.5}{r^{4.5}}\right)+O\left(\frac{1}{k^2}\right)$$

$$G_r(1) = -2G(1), \quad G(1) = \frac{-1}{\sqrt{2}\alpha}\left(1+\frac{\sqrt{2}}{\alpha}\right), \quad F(1) = -2G(1) \quad (2.3)$$

$$H(1) = -\frac{2}{k}-\frac{2}{\hat{k}}-\frac{\cancel{1}}{\cancel{\hat{k}^2}}-\frac{4}{k\hat{k}}-\frac{1}{k^2} = -\frac{2(k+\hat{k}+2)}{k\hat{k}} \quad (2.4)$$

$$F_r(1) = 2+\frac{17}{k}+\frac{5}{\hat{k}} = 2+\frac{2}{\sqrt{2}}(6-i), \quad = -\frac{2\sqrt{2}(\alpha+\sqrt{2})}{\cdots}$$

補助定理により

$$\bar{\Psi}_{2s} = \left[C_1 e^{-k(r-1)}\right]_R + D_1 e^{-(k+\hat{k})(r-1)}$$

$$\bar{\Psi}_{2sr} = \left[C_2 e^{-k(r-1)}\right]_R + D_2 e^{-(k+\hat{k})(r-1)} \qquad (2.5)$$

$$\bar{\zeta}_{2} = \left[C_3 e^{-k(r-1)}\right]_R + D_3 e^{-(k+\hat{k})(r-1)}$$

$$\bar{\zeta}_{2r} = \left[C_4 e^{-k(r-1)}\right]_R + D_4 e^{-(k+\hat{k})(r-1)}$$

とおけば

$$C_1 = \left(1+\frac{2}{kr}\right)F + \frac{4}{k}F_r = -\frac{2}{k} \qquad C_{1r} = F_r + \frac{2}{kr}F_r - \frac{2}{kr^2}F + \frac{4}{k}F_{rr}$$

$$D_1 = \left(1+\frac{2}{k+\hat{k}}\right)G + \frac{4}{k+\hat{k}}G_r =$$

$$C_2 = -k\left(1+\frac{2}{kr}\right)F - 3F_r$$

$$D_2 = -(k+\hat{k})\left(1+\frac{2}{k+\hat{k}}\right)G - 3G_r \qquad (2.6)$$

$$C_3 = k^2\left(1+\frac{1}{kr}\right)F + 2kF_r$$

$$D_3 = (k+\hat{k})^2\left[\left(1+\frac{1}{(k+\hat{k})r}\right)G + \frac{2}{k+\hat{k}}G_r\right]$$

$$C_4 = -k^3\left[\left(1+\frac{1}{kr}\right)F + \frac{1}{k}F_r\right]$$

$$D_4 = -(k+\hat{k})^3\left[\left(1+\frac{1}{(k+\hat{k})r}\right)G + \frac{1}{k+\hat{k}}G_r\right]$$

$$1+\frac{2}{k} = 1+\frac{2\mp 2i}{\sqrt{2}\alpha} = 1+\frac{\sqrt{2}}{\alpha} - \frac{\sqrt{2}}{\alpha}i$$

$$F_r(1) = 2 + \frac{7-7i+5+5i}{\sqrt{2}\alpha} = 2 + \frac{12}{\sqrt{2}\alpha} - \frac{\sqrt{2}}{\alpha}i$$

r=1 とおく

$$C_1 = -\frac{2}{k}\left(1+i+\frac{2}{k^2}\right)\left(1+\frac{2}{k}\right) + \frac{4}{k}\left(2+\frac{7}{k}+\frac{5}{k^2}\right)$$

$$= \frac{2}{k}\left[-\left(1+i+\frac{2+2i}{k}+\frac{2}{k^2}\right)+4+\frac{14}{k}+\frac{10}{k^2}\right]$$

$$= \frac{2}{k}\left[3-i+\frac{12-2i}{k}+\frac{8}{k^2}\right] = \frac{2}{\sqrt{2}\alpha}\left[2-4i\right].$$

$$D_1 = -\left(1-\frac{6}{\sqrt{2}\alpha}\right)\frac{1}{\sqrt{2}\alpha}\left(1+\frac{\sqrt{2}}{\alpha}\right) = \frac{-1}{\sqrt{2}\alpha}\left(1-\frac{2\sqrt{2}}{\alpha}\right)$$

$$C_2 = +2\left(1+i+\frac{2}{k^2}\right)\left(1+\frac{2}{k}\right) - 3\left(2+\frac{7}{k}+\frac{5}{k^2}\right)$$

$$= -4+2i+\frac{4}{k}(1+i)+\frac{4}{k^2}-\frac{21}{k}-\frac{15}{k^2}$$

$$= -4+2i+\frac{-17+4i}{k}-\frac{11}{k^2} = -4+2i+\frac{-24+32i}{\sqrt{2}\alpha}$$

$$D_2 = +\sqrt{2}\alpha\left(1-\frac{4}{\sqrt{2}\alpha}\right)\frac{1}{\sqrt{2}\alpha}\left(1+\frac{\sqrt{2}}{\alpha}\right) = 4\left(1-\frac{\sqrt{2}}{\alpha}\right) \qquad (2.6')$$

$$C_3 = -2k\left(1+\frac{1}{k}\right)\left(1+i+\frac{2}{k^2}\right) + 2k\left(2+\frac{7}{k}+\frac{5}{k^2}\right)$$

$$= 2k\left[1-i+\frac{6-i}{k}+\frac{3}{k^2}\right]$$

$$D_3 = -\sqrt{2}\alpha\left[\left(1-\frac{3}{\sqrt{2}\alpha}\right)\left(1+\frac{\sqrt{2}}{\alpha}\right)\right] = -\sqrt{2}\alpha\left(1-\frac{1}{\sqrt{2}\alpha}\right)$$

$$C_4 = -k^3\left[\frac{-2}{k}\left(1+\frac{1}{k}\right)\left(1+i+\frac{2}{k^2}\right)+\frac{1}{k}\left(2+\frac{7}{k}+\frac{5}{k^2}\right)\right]$$

$$= k^2\left[+2i+\frac{-5+2i}{k}-\frac{1}{k^2}\right],$$

$$D_4 = +2\alpha^2\left(1-\frac{1}{\sqrt{2}\alpha}\right)\left(1+\frac{\sqrt{2}}{\alpha}\right) = +2\alpha^2\left(1+\frac{1}{\sqrt{2}\alpha}\right),$$

$$\left.\begin{aligned}
[C_1+D_1]_R &= \frac{4}{\sqrt{2}\alpha}-\frac{1}{\sqrt{2}\alpha} = \frac{3}{\sqrt{2}\alpha},\\
[C_2+D_2]_R &= -4-\frac{24}{\sqrt{2}\alpha}+1-\frac{\sqrt{2}}{\alpha} = -3-\frac{26}{\sqrt{2}\alpha},
\end{aligned}\right\} \quad (2.7)$$

$[C_3+D_3]_R = -\sqrt{2}\alpha \ldots$

$$\bar{\psi}_2(r) = \bar{\psi}_{2s}(r) + A\log r + B, \qquad (2.8)$$

とおくと

$$\left.\begin{aligned} A &= -[C_2+D_2]_R = 3+\frac{26}{\sqrt{2}\alpha}, \\ B &= -[C_1+D_1]_R = \frac{-3}{\sqrt{2}\alpha}, \end{aligned}\right\} \qquad (2.9)$$

$$\left.\begin{aligned} \bar{\psi} &= \omega\varepsilon^2\,\bar{\psi}_2(r), \\ \bar{\zeta} &= \omega\varepsilon^2\,\bar{\zeta}_2(r) \end{aligned}\right\} \qquad (2.10)$$

次に総圧は (2.9) より今の近似では

$$\frac{1}{p}\nabla^2\overline{p} = \frac{1}{2}\left[(\hat{\zeta}\hat{v})_x + (\hat{u}\hat{\zeta})_y\right]_R = \zeta\hat{\zeta} - \frac{1}{2}\left[\zeta_r\hat{\psi}_r + \frac{\zeta_\theta}{r^2}\hat{\psi}_\theta\right]_R, \quad (2.11)$$

$$\nabla^2\overline{g}_2 = \frac{\nabla^2\overline{G}}{p\varepsilon^2\omega^2} = \zeta_1(r)\hat{\zeta}_1^* + \frac{1}{2}\left[\zeta_{1r}\hat{\psi}_{1r} + \frac{1}{r^2}\zeta_1\hat{\psi}_1\right]_R, \quad (2.11')$$

$$= \frac{1}{2}\left[\zeta_{1r}\hat{\psi}_{1pr} + \frac{l}{r^2}\zeta_1\hat{\psi}_{1p}\right]_R + \zeta_1\hat{\zeta}_1 + \frac{1}{2}\left[\frac{\zeta_{1r}\hat{\zeta}_{1r}}{\hat{k}^2} + \frac{\zeta_1\hat{\zeta}_1}{\hat{k}^2r^2}\right]_R$$

$$\doteqdot \frac{1}{2}\left[\zeta_{1r}\hat{\psi}_{1pr} + \frac{\zeta_{1r}\hat{\zeta}_{1r}}{\hat{k}^2}\right]_R + \zeta_1(r)\hat{\zeta}_1(r)$$

$$\doteqdot \frac{4k\hat{k}}{r}e^{-(k+\hat{k})(r-1)} + \frac{1}{2}\left[-2\frac{k^2}{\sqrt{r}}e^{-k(r-1)}\left\{\left(-\frac{2e^{-\hat{k}(r-1)}}{\sqrt{r}}\right) + 1 + \frac{1}{r^2}\left(1-\frac{2}{r}\right)\right\}\right]_R$$

$$\doteqdot 4\frac{k\hat{k}}{r}e^{-(k+\hat{k})(r-1)} - \left[\frac{k^2e^{-k(r-1)}}{\sqrt{r}}\left\{1+\frac{1}{r^2}+\cdots\right\}\right]_R$$

$$\nabla^2\overline{g}_2(r) \doteqdot -\frac{k^2}{2}\frac{e^{-k(r-1)}}{\sqrt{r}}\left(1+\frac{1}{r^2}\right) - \frac{\hat{k}^2}{2\sqrt{r}}e^{-\hat{k}(r-1)}\left(1+\frac{1}{r^2}\right) + \frac{4k\hat{k}}{r}e^{-(k+\hat{k})(r-1)}, \quad (2.12)$$

$$\overline{g}_{2s}(r) = f_1e^{-k(r-1)} + f_2e^{-\hat{k}(r-1)} + f_3e^{-(k+\hat{k})(r-1)}, \quad (2.13)$$

$$\left.\begin{aligned} f_1 &= -\frac{1+\frac{1}{r^2}}{2\sqrt{r}} \underset{r=1}{=} -1 \\ f_2 &= -\frac{1+\frac{1}{r^2}}{2\sqrt{r}} = -1 \\ f_3 &= \frac{4k\hat{k}}{r(k+\hat{k})^2} = \frac{4\alpha^2}{2\alpha^2} = 2 \end{aligned}\right\} \quad (2.14)$$

$$\overline{g}_{2sr}(r) = -kf_1e^{-k(r-1)} - \hat{k}f_2e^{-\hat{k}(r-1)} - (k+\hat{k})e^{-(k+\hat{k})(r-1)}, \quad (2.15)$$

$$\overline{g}_{2s}(1) = 0, \quad \overline{g}_{2sr}(1) = k + \hat{k} - 2(k+\hat{k}) = -\sqrt{2}\alpha, \quad (2.16)$$

(34*) より

$$\left.\begin{array}{l}\frac{1}{\rho}G_r = -\frac{\nu}{r}\zeta_\theta = 0 \\ \frac{1}{\rho r}G_\theta - \nu\zeta_r = 0\end{array}\right\} \quad \left.\begin{array}{l}\overline{G_r}\big|_{r=1} = 0 \\ \nu\overline{\zeta_r}\big|_{r=1} = 0\end{array}\right\} \qquad (2.17)$$

この後の関係は (2.6′) より

$$\overline{\zeta_r}\big|_{r=1} = 0\left(\frac{1}{\alpha}\right), \qquad \cdots (2.18)$$

が満たされている。

g は (2.13) に斉次解を加えて

$$\overline{g_2}(r) = A\log r + \overline{g_{2s}}(r), \qquad \cdots (2.19)$$

となるから (2.17) 第1式より

$$A + \overline{g_{2sr}}(1) = 0, \quad A = \sqrt{2}\alpha, \qquad (2.20)$$

また $$\overline{g_2}(1) = 0, \qquad \cdots (2.21)$$

となり, $$\overline{g_2}(r) \xrightarrow{r \gg 1} A\log r, \qquad (2.22)$$

なお $$\overline{G} = \rho\varepsilon^2\omega^2\overline{g_2}(r), \qquad (2.23)$$

3. 力の第2近似

附録Bによって~~変動力~~力の第2近似を求めよう。

(B·22)より

$$\left.\begin{aligned} X &= X_1 - \pi\rho\varepsilon^3\omega^2 A + \rho\pi\omega^2\varepsilon^3(I^\psi + II^\psi), \\ X_1 &= \pi\rho\,\omega^2\varepsilon\left(2+\frac{4}{k}\right), \end{aligned}\right\} \quad (3.1)$$

$$\left.\begin{aligned} I^\psi &= 2\int_1^\infty \bar{\zeta}_2(r)\,\psi_1(r)\,\psi_{1r}\,dr, \\ II^\psi &= -\int_1^\infty \zeta_1(r)\,\psi_1(r)\,\bar{\psi}_{2r}(r)\,dr, \end{aligned}\right\} \quad (3.2)$$

これらの積分は $r=1$ の近傍の値にのみ関係し，かつ初項のみ考えればよいので各関数は次のように近似してよい

$$\left.\begin{aligned} \psi_1(r) &= -\frac{2}{k}\left(1-e^{-k(r-1)}\right) \\ \psi_{1r}(r) &= 2\left(1-e^{-k(r-1)}\right) \\ \zeta_1(r) &= 2k\,e^{-k(r-1)} \\ \bar{\psi}_{2r}(r) &= A + \frac{C_2}{2}e^{-k(r-1)} + \frac{\hat{C}_2}{2}e^{-\hat{k}(r-1)} + D_2\,e^{-(k+\hat{k})(r-1)} \\ \bar{\zeta}_2(r) &= \frac{C_3}{2}e^{-k(r-1)} + \frac{\hat{C}_3}{2}e^{-\hat{k}(r-1)} + D_3\,e^{-(k+\hat{k})(r-1)}, \\ C_2 &= -4+2i,\quad D_2 = 1,\quad A \fallingdotseq 3 \\ C_3 &= 2k(1-i),\quad D_3 = -\sqrt{2}\alpha \end{aligned}\right\} \quad (3.3)$$

$$\frac{C_3}{2k} = 1-i,\quad \frac{\hat{C}_3}{2\hat{k}} = \cancel{i(1+i)} = 1-i,\quad \frac{D_3}{k} = -\frac{\sqrt{2}\alpha(1-i)}{\sqrt{2}\alpha} = -(1-i)$$

$$\bar{\zeta}_2(r) = \underbrace{k}_{\fallingdotseq\sqrt{2}\alpha}(1-i)\left(e^{-k(r-1)} + e^{-\hat{k}(r-1)} - e^{-(k+\hat{k})(r-1)}\right),$$

$$I^* = -\frac{8}{\alpha}\int_1^\infty \left(1-2e^{-k(r-1)}+e^{-2k(r-1)}\right)\left(\frac{C_3}{2k}e^{-k(r-1)}+\frac{\hat{C}_3}{2\hat{k}}e^{-\hat{k}(r-1)}+\frac{D_3}{k}e^{-(k+\hat{k})(r-1)}\right)dr$$

$$= -8(1-i)\int_1^\infty \left(\quad '' \quad \right)\left(e^{-k(r-1)}+e^{-\hat{k}(r-1)}-e^{-(k+\hat{k})(r-1)}\right)dr$$

$$= -8(1-i)\left[\frac{1}{k}+\frac{1}{\hat{k}}-\frac{1}{k+\hat{k}}+\frac{1}{3k}+\frac{1}{2k+\hat{k}}-\frac{1}{3k+\hat{k}}\right.$$
$$\left. -2\left(\frac{1}{2k}+\frac{1}{k+\hat{k}}-\frac{1}{2k+\hat{k}}\right)\right]$$

$$= -\frac{8\sqrt{2}}{\alpha}\left[\frac{-i}{3}-\frac{3}{1+i}+\frac{3}{1+2i}-\frac{1}{1+3i}\right]=\frac{8\sqrt{2}}{15\alpha}(15-4i), \quad (3.4)$$

$$II^* = 4\int_1^\infty e^{-k(r-1)}\left(1-e^{-k(r-1)}\right)\left(3+(-2+i)e^{-k(r-1)}+(-2-i)e^{-\hat{k}(r-1)}+e^{-(k+\hat{k})(r-1)}\right)dr$$

$$= 4\left[\frac{3}{k}+\frac{-2+i}{2k}+\frac{-2-i}{k+\hat{k}}+\frac{1}{2k+\hat{k}}\right.$$
$$\left. -\left(\frac{3}{2k}+\frac{-2+i}{3k}-\frac{2+i}{2k+\hat{k}}+\frac{1}{3k+\hat{k}}\right)\right]$$

$$= \frac{4}{k}\left[\frac{7+i}{6}+\frac{3+i}{2-i}-\frac{1}{3-i}\right]=\frac{16}{15k}(7+4i), \quad (3.5)$$
$$= \frac{16}{15\sqrt{2}\alpha}(11-3i),$$

$$I^*+II^* = \frac{16}{15\sqrt{2}\alpha}(26-7i) \qquad (3.6)$$

$$\therefore \frac{X}{\pi\rho\omega^2\varepsilon l} = 2+\frac{4-4i}{\sqrt{2}\alpha} - \left(3-\frac{16}{15\sqrt{2}\alpha}(26-7i)\right)\varepsilon^2$$

$$= 2-\left(3-\frac{26}{15\sqrt{2}\alpha}\right)\varepsilon^2-\frac{4i}{\sqrt{2}\alpha}\left(1+\frac{28}{15}\varepsilon^2\right), \quad (3.7)$$

$$\therefore\ C_m = 2 - \frac{3}{\pi^2}K^2 + \frac{4}{\sqrt{\pi\beta}}\left(1 + \frac{13K^2}{30\pi^2}\right), \qquad (3.8)$$

$$C_D / C_{D1} = 1 + \frac{28}{15}\frac{K^2}{\pi^2}, \qquad (3.9)$$

一方 定常流れによる損失パワーは

$$W = \mu \iint \bar{\zeta}^2\,dx\,dy = 2\pi\mu\,\omega^3\varepsilon^4 \int_1^\infty \bar{\zeta}_2^{\,2}\, r\,dr$$

$$= 2\pi\rho\frac{\omega^2\varepsilon^4}{\alpha^2}\cancel{2}\alpha^2\int_1^\infty \left(e^{-k(r-1)} + e^{-\bar{k}(r-1)} - e^{-(k+\bar{k})(r-1)}\right)^2 dr$$

$$= \cancel{16}4\pi\rho\omega^2\varepsilon^4\left(\frac{1}{2k} + \frac{1}{2\bar{k}} + \frac{1}{2(k+\bar{k})} + \frac{2}{k+\bar{k}} - \frac{2}{2k+\bar{k}} - \frac{2}{k+2\bar{k}}\right)$$

$$= \pi\rho\omega^2\varepsilon^4\frac{\cancel{4}\times1.9}{\sqrt{2}\,\alpha} \qquad (3.10)$$

$$\frac{W}{2W_1} = 1.9\varepsilon^2 = 1.9\,\varepsilon^2, \qquad (3.11)$$

これは (3.9) の第2項の係数 1.87 とよく合っている。

4. 直接解法（振動分）

(4^b)より

$$\nabla^4\psi - k^2\nabla^2\psi = -\frac{1}{\nu}\left[(u\zeta+\zeta u)_x + (v\zeta+\bar{v}\zeta)_y\right]$$

$$= \frac{1}{\nu}\left[\zeta_\theta \bar{\psi}_r + \bar{\zeta}_\theta \psi_r - \zeta_r \bar{\psi}_\theta - \bar{\zeta}_r \psi_\theta\right]$$

$$= \frac{\omega^2\varepsilon^3}{\nu}\left[i\zeta_1(r)\bar{\psi}_{2r}(r) + i\bar{\zeta}_{2r}(r)\psi_1(r)\right]e^{-i\theta}$$

$$= \omega\varepsilon^3 k^2\left[\zeta_1(r)\bar{\psi}_{2r}(r) + \bar{\zeta}_{2r}(r)\psi_1(r)\right]e^{-i\theta},$$

$$\psi = \omega\varepsilon^3 e^{-i\theta}\psi_2(r), \tag{4.1}$$

$$(\nabla^4 - k^2\nabla^2)\psi_2(r) = k^2(\zeta_1\bar{\psi}_{2r} + \bar{\zeta}_{2r}\psi_1), \tag{4.2}$$

但し $\nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}$,

総圧については (5^b)より

$$\frac{1}{\rho}\nabla^2 G = 2\zeta\bar{\zeta} - \left(\bar{\zeta}_r\psi_r + \zeta_r\bar{\psi}_r + \frac{\bar{\zeta}_\theta}{r^2}\psi_\theta + \frac{\zeta_\theta}{r^2}\bar{\psi}_\theta\right)$$

$$= 2\zeta\bar{\zeta} - \bar{\zeta}_r\psi_r - \zeta_r\bar{\psi}_r$$

$$= \omega^2\varepsilon^3 e^{-i\theta}\left[-\zeta_1\bar{\zeta}_2 - \bar{\zeta}_{2r}\psi_{1r} + \zeta_{1r}\bar{\psi}_{2r}\right]$$

$$G = \rho\omega^2\varepsilon^3 e^{-i\theta} g_2(r) \tag{4.3}$$

とおくと

$$\nabla^2 g_2(r) = \zeta_{1r}\bar{\psi}_{2r} - \bar{\zeta}_{2r}\psi_{1r} - \zeta_1\bar{\zeta}_2, \tag{4.4}$$

境界では

$$\left.\begin{aligned} \frac{1}{\rho}G_r &= -\frac{\nu}{r}\zeta_\theta \Big|_{r=1} \\ \frac{1}{\rho r}G_\theta &= \nu\zeta_r \end{aligned}\right\} \qquad \left.\begin{aligned} g_{2r}(1) &= +i\frac{\nu}{\omega}\zeta_2(1) \\ g_2(1) &= \frac{\nu}{\omega}\zeta_{2r}(1) \end{aligned}\right\} \tag{4.5}$$

$$\zeta = \omega\varepsilon^3\zeta_2(r)e^{-i\theta},$$

$A=3$

$$k^2(\zeta_1 \bar{y}_{2r} + \bar{\zeta}_{2r} y_1) = k^2 h_1 e^{-k(r-1)} + \hat{k}^2 h_2 e^{-\hat{k}(r-1)} + 4k^2 h_3 e^{-2k(r-1)} + (k+\hat{k})^2 h_4 e^{-(k+\hat{k})(r-1)} + (2k+\hat{k})^2 h_5 e^{-(2k+\hat{k})(r-1)} \quad (4.6)$$

とおけば

$$= \frac{2k^3}{\sqrt{r}} e^{-k(r-1)} \left[\frac{A}{r} + \frac{C_2}{2} e^{-k(r-1)} + \frac{\hat{C}_2}{2} e^{-\hat{k}(r-1)} + D_2 e^{-(k+\hat{k})(r-1)} \right]$$

$$+ k^2 \left\{ \left(r - \frac{1}{r} - \frac{2}{kr}\right) + \frac{2 e^{-k(r-1)}}{k\sqrt{r}} \right\} \left\{ \frac{C_4}{2} e^{-k(r-1)} + \frac{\hat{C}_4}{2} e^{-\hat{k}(r-1)} + D_4 e^{-(k+\hat{k})(r-1)} \right\}$$

であるから

$$\left.\begin{aligned}
h_1 &= \frac{2k}{\sqrt{r}} \frac{A}{r} + \frac{C_4}{2}\left(r - \frac{1}{r} - \frac{2}{kr}\right), \\
h_2 &= \frac{k^2}{2\hat{k}^2} \hat{C}_4 \left(r - \frac{1}{r} - \frac{2}{kr}\right), \\
h_3 &= \frac{k}{4\sqrt{r}} C_2 + \frac{C_4}{4k\sqrt{r}}, \\
h_4 &= \frac{1}{(k+\hat{k})^2}\left[\frac{k^3}{\sqrt{r}} \hat{C}_2 + \frac{k \hat{C}_4}{\sqrt{r}} + k^2\left(r - \frac{1}{r} - \frac{2}{kr}\right) D_4 \right], \\
h_5 &= \frac{1}{(2k+\hat{k})^2}\left[\frac{2k^3 D_2}{\sqrt{r}} + \frac{2k}{\sqrt{r}} D_4 \right],
\end{aligned}\right\} \quad (4.7)$$

今特解を

$$y_{2s}(r) = f_1 e^{-k(r-1)} + f_2 e^{-\hat{k}(r-1)} + f_3 e^{-2k(r-1)} + f_4 e^{-(k+\hat{k})(r-1)} + f_5 e^{-(2k+\hat{k})(r-1)}, \quad (4.7)$$

とおけば一般解は

$$y_2(r) = \frac{A_2}{r} + \frac{B_2}{\sqrt{r}} e^{-k(r-1)} + y_{2s}(r); \quad (4.8)$$

となる。

また $$y_{2r}(r) = \sum_{j=1}^{5} f_j' e^{-p_j(r-1)} - \frac{A_2}{r^2} - \frac{k B_2}{\sqrt{r}} e^{-k(r-1)}, \quad (4.9)$$

$p_1 = k,\ p_2 = \hat{k},\ p_3 = 2k,\ p_4 = k+\hat{k},\ p_5 = 2k+\hat{k}$

境界条件は

$$\psi_2(1) = 0 = A_2 + B_2 + \psi_{2s}(1), \quad \left.\right\} (4\cdot10)$$
$$\psi_{2r}(1) = 0 = -A_2 - kB_2 + \psi_{2sr}(1),$$

$$\therefore \quad B_2 = \frac{\psi_{2sr}(1) + \psi_{2s}(1)}{k-1}, \quad A_2 = \frac{-(k\psi_{2s}(1) + \psi_{2sr})}{k-1}, \quad (4\cdot11)$$

$$\psi_{2s}(1) = \sum_j f_j(1), \quad \psi_{2sr}(1) = \sum_j f_j'(1),$$

補助定理により

$$f_1(r) = \frac{1}{2k\sqrt{r}}\left[\sqrt{r}\,h_1\right]_1^r, \quad f_1'(r) = \frac{h_1}{2k} - \frac{1}{2\sqrt{r}}\left[\sqrt{r}\,h_1\right]_1^r,$$

$$f_j(r) = \frac{1}{\beta_j p_j^2}\left[h_j(r) + \frac{2}{p_j}(1+\beta_j)h_{jr}\right], \quad \beta_j = 1 - \frac{k^2}{p_j^2} \quad (4\cdot1)$$

$$f_j'(r) = \frac{-1}{\beta_j p_j}\left[-h_j + \frac{2}{p_j}(1+\beta_j)h_{jr}\right] + \frac{1}{\beta_j p_j^2}h_{jr}$$

$$h_1(1) = 2k - \frac{1}{k}C_4 = 2k(1-i),$$
$$h_2(1) = +\frac{\hat{C}_4}{k} = \frac{-2i\hat{k}^2}{k} = -2\hat{k},$$
$$h_3(1) = \frac{k}{4}C_2 + \frac{C_4}{4k} = \frac{k}{2}(-2+i) + \frac{ik}{2} = k(-1+i), \quad (4\cdot1)$$
$$h_4(1) = \frac{k^3}{(k+\hat{k})^2}\left[\hat{C}_2 + \frac{\hat{C}_4}{k^2} - \frac{2}{k^2}D_4\right],$$
$$\left[-4-2i + 2i - \frac{4}{k^2}\alpha^2\right] = \frac{k^3}{(k+\hat{k})^2}(-4+4i),$$
$$h_5(1) = \frac{2k^3}{(2k+\hat{k})^2}\left[D_2 + \frac{D_4}{k^2}\right], \quad = \frac{i\alpha^2 k}{[\ ]^2}(-4+4i)$$

$$1 + \ldots$$

$$= \frac{2k}{\ }(1-2i) = \frac{2k(1-2i)(3+4i)}{\ } \quad 2k \ldots = k(-2-2i)$$

$$h_{1r} = -\frac{3kA}{r^{2.5}} + \frac{C_4}{2k}\left(1+\frac{1}{r^2}+\frac{2}{kr^2}\right) + \frac{1}{2}\left(r-\frac{1}{r}-\frac{2}{kr}\right)C_{4r}$$

$$C_{4r} = -k^3\left[\bar{F}_r + \frac{1}{k}\bar{F}_{rr} + \frac{\bar{F}_r}{kr} - \frac{\bar{F}}{kr^2}\right] \fallingdotseq -2k^3$$

$$C_4(1) = 2ik^2, \quad \bar{F}(1) = -\frac{2}{k}(1+i), \quad \bar{F}_r(1) = 2$$

$$\underline{h_{1r}(1) = ik^2 + 2k^2 = (2+i)k^2} \quad //$$

$$h_{3r} = -\frac{1}{2}\hat{C}_4\left(1+\frac{1}{r^2}\right) - \frac{1}{2}\hat{C}_{4r}\left(r-\frac{1}{r}-\frac{2}{kr}\right)$$

$$\underset{r=1}{=} -\hat{C}_4 + \frac{1}{k}\hat{C}_{4r} = 2i\hat{k}^2 - \frac{2}{k}\hat{k}^3 = 2\hat{k}^2(i+i) = 4i\hat{k}^2 \quad //$$

$$h_{7r} = -\frac{k}{8r^{1.5}}C_2 + \frac{k}{4\sqrt{r}}C_{2r} - \frac{C_4}{8kr^{1.5}} + \frac{C_{4r}}{4k\sqrt{r}} \underset{r=1}{\fallingdotseq} -k^2 \quad // \qquad (4.12)$$

$$C_2 = -4+2i, \quad C_{2r} \fallingdotseq -k\bar{F}_r \underset{r=1}{=} -2k$$

$$h_{4r} \fallingdotseq \frac{k^3}{(k+\hat{k})^2}\left[\frac{\hat{C}_{2r}}{\sqrt{r}} + \frac{\hat{C}_{4r}}{k^2\sqrt{r}} + \frac{1}{k}\left(1+\frac{1}{r^2}\right)D_4 + \frac{1}{k}\left(r-\frac{1}{r}-\frac{2}{kr}\right)D_{4r}\right]$$

$$D_4 = 2\alpha^2, \qquad D_{4r} = -(k+\hat{k})^3 G \underset{r=1}{\fallingdotseq} + \frac{2\sqrt{2}\alpha^3}{\sqrt{2}\alpha} = 2\alpha^2,$$

$$h_{4r} \fallingdotseq \frac{k^3}{(k+\hat{k})^2}\left[-2\hat{k} - 2\frac{\hat{k}^3}{k^2} + \frac{4}{k}\alpha^2\right] = \frac{-2\hat{k}k^3}{(k+\hat{k})^2}\left[\underbrace{1+\frac{\hat{k}^2}{k^2}}_{0} + 4\right]$$

$$= -4i\alpha^2 = -4k^2 \quad //$$

$$h_{5r} \fallingdotseq \frac{2k^3}{(2k+\hat{k})^2}\left[\frac{D_{2r}}{\sqrt{r}} - \frac{D_2}{2r^{1.5}} + \frac{1}{k\sqrt{r}}\left(D_{4r} - \frac{D_4}{2r}\right)\right]$$

$$D_2 = 1, \quad D_{2r} \fallingdotseq -(k+\hat{k})G_r = 2(k+\hat{k})\left(-\frac{1}{\sqrt{2}\alpha}\right) = -2 \quad //$$

$$h_{5r} = \frac{2k^3}{(2k+\hat{k})^2}\left[-2.5 + \frac{\alpha^2}{k^2}\right] = \frac{2k^3(-2.5-i)}{(2k+\hat{k})^2} \quad //$$

No. 17

$\beta_5 = \frac{2(1-3i)(3+4i)}{25} = \frac{2(11-5i)}{25}$

Date

$\beta_2 = 1 - \frac{k^2}{\hat{k}^2} = 2, \quad \beta_3 = 1 - \frac{1}{4} = \frac{3}{4}, \quad \beta_4 = 1 - \frac{-i}{(k+\hat{k})^2} = 1 - \frac{i}{2}, \quad \beta_5 = 1 - \frac{1}{(2-i)^2} = \frac{2-6i}{(2-i)^2}$

$$f_1(1) = -\frac{h_1(1)}{k^2} = -\frac{2}{k}(1-i) = -$$

$$f_2(1) = \frac{1}{\beta_2 \hat{k}^2}\left[h_2(1) + \frac{2}{\hat{k}}(1+\beta_2) h_{2r}\right] = \frac{1}{\beta_2 \hat{k}^2}\left[-2\hat{k} + \frac{2}{\hat{k}}(1+\beta_2) 4i \hat{k}^2\right]$$

$$= \frac{2}{\beta_2 \hat{k}}\left[4i(1+\beta_2) - 1\right] = \frac{1}{k}\left[-i - 12\right]$$

$$f_3(1) = \frac{1}{4\beta_3 k^2}\left[h_3(1) + \frac{1+\beta_3}{k} h_{3r}\right] = \frac{1}{4\beta_3 k^2}\left[k(i-1) - k(1+\beta_3)\right]$$

$$= \frac{1}{4\beta_3 k}(i - 1 - 1 - \beta_3) = \frac{1}{3k}(i - 2.75)$$

$$f_4(1) = \frac{1}{\beta_4 (k+\hat{k})^2}\left(h_4 + \frac{2(1+\beta_4)}{k+\hat{k}} h_{4r}\right) = \frac{1}{\beta_3 (k+\hat{k})^2}\left[\frac{4k^3(i-1)}{(k+\hat{k})^2} - \frac{8k^2(1+\beta_4)}{k+\hat{k}}\right]$$

$$= \frac{4k^4}{k\beta_3 (k+\hat{k})^4}\left[i - 1 - 2(1+\beta_2)\frac{k+\hat{k}}{k}\right] = \frac{(1-i)}{\beta_3 k}\left[1 + 2(1+\beta_4)\right],$$

$$= \frac{1}{k}\frac{4}{5}(7-4i) \qquad 1-i \qquad = \frac{2(1-i)}{k(2-i)}(3 + 2 - i)$$

$$f_5(1) = \frac{1}{\beta_4(2k+\hat{k})^2}\left[h_5 + \frac{2(1+\beta_5)}{2k+\hat{k}} h_{5r}\right] = \; \left[2k\frac{(1-2i)}{(2-i)^2} + \frac{4k^3(-2.5-i)(1+\beta_5)}{(2k+\hat{k})^3}\right]$$

$$= \frac{2k(1-2i)}{\beta_4(2k+\hat{k})^2 (2-i)^2} = \frac{2(1-2i)}{k(2-i)^2\, 2(1-3i)} = \frac{(4+2i)}{k(+9+13i)} = \frac{17+31i}{k\,250}$$

$(3-4i)$

26
−9
17

$(9-13i)$
15

169
81
250

$$k\sum_j f_j(1) = -2+i - i - 12 + \frac{i}{3} - \frac{2.75}{3} + \frac{28-16i}{5} + \frac{17+31i}{250} \simeq .068 - 2.743i$$

$$f_1'(1) = \frac{5h_1}{2k} - \frac{h_{1r}}{k^2} = 5(1-i) - (2+i) = 3-6i$$

$$f_2' = \frac{-1}{\beta_2 k}\left[h_2 + \frac{2}{k}(1+\beta_2)h_{2r}\right] = \frac{-1}{2k}\left[-2k + \frac{6}{k}\times 4ik^2\right] = 1-12i$$

$$f_3' = \frac{-1}{\beta_3 k}\left[h_3 + \frac{1+\beta_3}{k}h_{3r}\right] = \frac{-1}{3k}\left[k(-1+i) + \frac{1.75}{k}(-k^2)\right]$$

$$= \frac{1}{3}(1-i+1.75) = \frac{2.75-i}{3}$$

$$f_4' = \frac{-1}{\beta_4(k+k)}\left[h_4 + \frac{2(1+\beta_4)}{k+k}h_{4r}\right] = \frac{-1}{\beta_4(k+k)}\left[\frac{+4k^3(1-i)}{(k+k)^2} + \frac{8k^2(1+\beta_4)}{(k+k)}\right]$$

$$= \frac{4k^2}{(1-\frac{i}{2})(k+k)^2}\left[2(1+1-\frac{i}{2}) + \frac{1-i}{1-i}\right] = \frac{2i}{1-\frac{i}{2}}(5-i) = \frac{4(1+5i)(2+i)}{5} = \frac{4(-3+11i)}{5}$$

$$f_5' = \frac{-1}{\beta_5(2k+k)}\left[\frac{2k(1-2i)}{(2-i)} + \frac{4k^3}{(2k+k)^3}(1+\beta_5)(-2.5-i)\right]$$

$$= \frac{-2(1-2i)}{2-6i} = \frac{-(1-2i)(1+3i)}{1-3i} = \frac{-7-i}{10}$$

$$\psi_{2,5r}(1) = \sum f_j' = 3+1+\frac{2.75}{3} - \frac{12}{5} - \frac{7}{10} + i\left[-6-12-\frac{1}{3}+\frac{44}{5}-\frac{1}{10}\right] = 1.817 - i(9.633)$$

$$A_2 = \frac{-.068+2.743i - 1.817 + 9.633i}{-k-1} = \frac{-1.749+12.38i}{-k-1}$$

$$B_2 = \frac{1.817-9.633i}{-k}$$

$$\zeta = \omega \varepsilon^3 e^{-i\theta} \zeta_2(r), \quad = -\nabla^2 \psi_2 = -\left[\tilde{\nabla}^2 \psi_{2s} + k^2 B_2 \left(1 - \frac{2}{kr}\right) \frac{e^{-k(r-1)}}{\sqrt{r}}\right]$$

$$\nabla^2 \zeta_2 - k^2 \zeta_2 = -k^2 \left[\zeta_1 \bar{\psi}_{2r} + \bar{\zeta}_{2r} \psi_1\right]$$

$$= -\sum p_j^2 h_j e^{-p_j(r-1)},$$

$p_1 = k$, $p_2 = \hat{k}$, $p_3 = 2k$, $p_4 = k + \hat{k}$, $p_5 = 2k + \hat{k}$

$$\zeta_2(r) = \sum_{j=1}^{5} z_j e^{-p_j(r-1)} + z_0 \frac{e^{-k(r-1)}}{\sqrt{r}}, \qquad z_0 = k^2 B_2,$$

$$\left.\begin{aligned} z_1 &= +\frac{k}{2\sqrt{r}}\left[\sqrt{r} h_1\right]_r' + \frac{k}{4} h_1, \\ z_j &= \frac{1}{\beta_j}\left\{\left(1 + \frac{1}{\beta_j p_j r}\right) h_j + \frac{2}{\beta_j p_j} h_{jr}\right\}, \quad \text{for } j \geq 2 \end{aligned}\right\}$$

$$\zeta_{2r}(r) = \sum z_j' e^{-p_j(r-1)} - k z_0 \frac{e^{-k(r-1)}}{\sqrt{r}},$$

$$\left.\begin{aligned} z_j' &= +\frac{p_j}{\beta_j}\left\{\left(1 + \frac{1}{\beta_j p_j r}\right) h_j + \frac{2}{\beta_j p_j} h_{jr}\right\} - \frac{h_{jr}}{\beta_j} \\ &= \frac{p_j}{\beta_j}\left[\left(1 + \frac{1}{\beta_j p_j r}\right) h_j + \frac{1}{p_j}\left(\frac{2}{\beta_j} - 1\right) h_{jr}\right], \quad \text{for } j \geq 2 \\ z_1' &= -\frac{k^2}{2\sqrt{r}}\left[\sqrt{r} h_1\right]_r' - \frac{k^2}{4} h_1 + \frac{k}{2} h_1 \underset{r=1}{=} \frac{k}{4} h_1(1) \end{aligned}\right\}$$

$r=1$ $\alpha^{\prime}$ $\qquad \frac{k}{k+k} = \frac{\alpha(1+i)}{2\alpha} = \frac{1+i}{2}$

$$z_1 = \frac{1}{4} h_1(1) = \frac{k}{2}(1-i)$$

$$z_2 = -\frac{1}{2}\left[\left(1+\frac{1}{2k}\right)h_2(1) + \frac{1}{k}h_{2r}(1)\right] = -\frac{1}{2}\left[-2k\left(1+\frac{1}{2k}\right) + 4ik\right]$$

$$= k[1-2i] = k[-i-2]$$

$$z_3 = -\frac{1}{\frac{3}{4}}\left[\left(1+\frac{2}{3k}\right)h_3 + \frac{2}{\frac{3k}{2}}h_{3r}\right] = -\frac{4k}{3}\left[-1+i - \frac{2}{3}\right]$$

$$= k\left[-\frac{20}{9} - \frac{4}{3}i\right], \qquad \frac{16}{2\cdot i} \times \frac{1+i}{2} = 8\frac{(1+3i)}{5}$$

$$z_4 = \frac{-k}{\left(1-\frac{i}{2}\right)}\left[-2-2i + \frac{2(-4k)}{\left(1-\frac{i}{2}\right)(k+k)}\right] = \frac{4k}{2-i}\left[2+2i+\frac{8}{5}(1+3i)\right]$$

$$\frac{18}{5} + \frac{34}{5}i$$

$$= \frac{8k}{5}\frac{9+17i}{2-i} = \frac{8k}{25}(1+43i)$$

$$z_5 = -\frac{k}{\beta_5}\left[\frac{22-4i}{25} + \frac{2\times 2(-2.5-i)k^3}{\beta_5(2k+k)^3}\right] = -\frac{25k}{2(11-5i)} \times \frac{22-4i}{25}$$

$$= -\frac{k(11-2i)}{11-5i} = -k\left(\frac{131+33i}{146}\right).$$

$$\left.\begin{aligned} \frac{1}{k}\sum_j z_j = & \frac{1}{2} - 2 - \frac{20}{9} + \frac{8}{25} - \frac{131}{146} \\ & + i\left(-\frac{1}{2} - 1 - \frac{3}{4} + \frac{344}{25} - \frac{33}{146}\right) \end{aligned}\right\} = -4.30 + 11.28i$$

1.82

$$\frac{1}{}\; k^2 B_2 + k\sum z_j =$$
$$= k($$

$$\frac{1}{k^2}\zeta_1' = \frac{1}{4k}h_1(1) = \frac{1-i}{2}$$

$$\frac{1}{k^2}\zeta_2' = \frac{\hat{k}}{2k^2}\left[-2\hat{k} + \frac{1}{2}\times 0\right] = +1,$$

$-\frac{5}{6}$

$$\frac{1}{k^2}\zeta_3' = \frac{4\times 2k^2}{3k^2}\left[-1+i + \frac{k}{2k}\left(\frac{8}{3}-1\right)\right] = \frac{8}{3}\left[-\frac{11}{6}+i\right]$$

$$-\frac{(2+i)^2}{(2-i)^2}(1+i) = -2($$

$$\frac{1}{k^2}\zeta_4' = \frac{(k+\hat{k})k}{\beta_4 k^2}\left[-2-2i + \frac{(-4k)}{}\left(\frac{4}{2-i}-1\right)\right] = \frac{2(1-i)}{2-i}\left[-2-2i+\frac{2\pm 14i}{5}\right]$$

$$= \frac{2(3-i)}{5\times 5}(-8-24i) = \frac{-16}{25}(3-i)(1+3i) = -\frac{32}{25}(3+4i)$$

$$\frac{1}{k^2}\zeta_5' = \frac{(2k+\hat{k})k}{\beta_5 k^2}\left[\frac{2(11-2i)}{25}\right] = \frac{(11-2i)(2-i)}{(11-5i)} = \frac{20-15i}{11-5i} = \frac{295-65i}{146}$$

$$\frac{1}{k^2}\sum\zeta_j' = \begin{cases}\frac{1}{2}+1-\frac{44}{12}-\frac{96}{25}+\frac{295}{146} \\ +i\left(-\frac{1}{2}+\frac{8}{3}-\frac{128}{25}-\frac{65}{146}\right)\end{cases} = 3.35 - 3.40i$$

$(1.82 \quad -9.6)$

$$kB_2 \simeq 1.82 - 9.63i$$

$$g_2(1) = \frac{i}{k^2}S_{2r}(1) = i\left(-kB_2 + \frac{1}{k^2}\sum\zeta_j'\right) = (13.03 + 5.17i)$$

?

あまりうまくない結果である。

No. 5-0

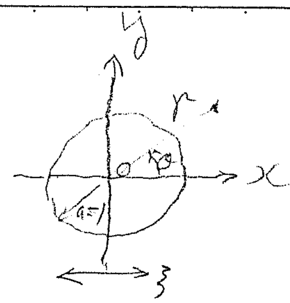
Date 62.5.15

5. 円筒の左右振動

頁

1. 第1近似解

単位半径の円筒が振巾ξで正弦運動を(円筒消数のの)しているとしよう。



擾乱流れ関数は

$$\psi(r,\theta) = i\omega\xi\,\psi_1(r)\sin\theta, \qquad (1.1)$$

$$\psi_1(r) = \frac{1}{r}\left[\frac{kK_1'(k)-K_1(k)}{kK_1'(k)+K_1(k)}\right] - \frac{1}{k^2}\zeta_1(r), \qquad (1.2)$$

$$\zeta(r,\theta) = i\omega\xi\sin\theta\,\zeta_1(r), \qquad (1.3)$$

$(\text{無}) = i\omega\xi\sin\theta \times 2k\left(1+\frac{1}{2k}\right)$

$$\zeta_1(r) = \frac{2k^2K_1(kr)}{kK_1'(k)+K_1(k)}, \qquad (1.4)$$

$\zeta_1(1)$ -

$|k| \gg 1$ では

$$K_1(k) \fallingdotseq \sqrt{\frac{\pi}{2k}}\,e^{-k}\left(1+\frac{3}{8k}+\frac{15}{128k^2}\right)$$

$$K_1'(k) \fallingdotseq -\sqrt{\frac{\pi}{2k}}\,e^{-k}\left(1+\frac{7}{8k}+\frac{57}{128k^2}\right) \qquad (1.5)$$

故

$$K_1'(k)+\frac{1}{k}K_1(k) \fallingdotseq -\sqrt{\frac{\pi}{2k}}\,e^{-k}\left[1-\frac{1}{8k}+\frac{9}{128k^2}\right],$$

$$K_1'(k)-\frac{1}{k}K_1(k) \fallingdotseq -\sqrt{\frac{\pi}{2k}}\,e^{-k}\left[1+\frac{15}{8k}+\frac{105}{128k^2}\right]$$

$$\frac{K_1'(k)-\frac{1}{k}K_1(k)}{K_1'(k)+\frac{1}{k}K_1} = 1+\frac{2}{k}+\frac{1}{k^2}$$

$$\frac{K_1(kr)}{kK_1'+K_1(k)} \fallingdotseq -\frac{e^{-k(r-1)}}{k\sqrt{r}}\left[1+\frac{1}{8k}+\frac{3}{8kr}+\frac{1}{128k^2}\left(7+\frac{6}{r}+\frac{15}{r^2}\right)\right]$$

$$\therefore \quad \psi_1(r) \doteqdot \frac{1}{r}\left(1+\frac{2}{k}+\frac{1}{k^2}\right) - \frac{\zeta_1(r)}{k^2}, \quad \cdots (1.3')$$

$$\zeta_1(r) \doteqdot 2k\frac{e^{-k(r-1)}}{\sqrt{r}}\left[1+\frac{1}{8k}\left(1+\frac{3}{r}\right)+\frac{1}{128k^2}\left(7+\frac{6}{r}+\frac{15}{r^2}\right)\right], \quad (1.4')$$

圧力 p は

$$\frac{p}{\rho} = -i\omega\xi\,\phi(r,\theta), \qquad (1.6)$$

$-\psi + \frac{\psi}{r}$

$$\phi(r,\theta) = \cos\theta\,\phi_1(r), \quad \phi_1(r) \doteqdot \frac{-1}{r}\left(1+\frac{2}{k}+\frac{1}{k^2}\right), \quad (1.7)$$

ϕ は ψ の調和部分の共役関数で速度ポテンシャルである。

また以下大文字で振動流れを加えたものとしよう。

$$\left.\begin{aligned} \Psi_1(r) &= -r + \psi_1(r), \\ \Phi_1(r) &= -r + \phi_1(r), \\ \frac{P_\infty}{\rho} &= -\omega^2\xi\cos\theta\,\Phi_1(r), \end{aligned}\right\} \quad (1.8)$$

$\frac{P(1)}{\rho} = \omega^2\xi\cos\theta\left(2+\frac{2}{k}+\frac{1}{k^2}\right)$

円筒に働く力は振動流れ（円筒は静止）については

$$X_1 = -\int_0^{2\pi}(P\cos\theta + \mu\zeta\sin\theta)\,d\theta = \pi\omega^2\xi\left[2+\frac{2}{k}+\frac{2}{k}\right]$$

$$= -\pi\rho\omega^2\xi\left[2+\frac{4}{k}\right], \quad \cdots (1.9)$$

$$\frac{4}{k} = \frac{4(1-i)}{\sqrt{2}\sqrt{\frac{\omega}{\nu}}} = \frac{4}{\sqrt{\pi\beta}}(1-i), \quad \beta = \frac{\omega}{\nu}(1)^2, \quad (1.10)$$

故に

$$C_{m1} = [-X_1]_R / \pi\rho\omega^2\xi = 2 + \frac{4}{\sqrt{\pi\beta}}, \quad (1.11)$$

抵抗係数は

$$C_{D1} = \frac{[-X]_I}{\frac{8}{3\pi}\rho\omega^2\zeta^2} = \frac{3\pi^3}{2K\sqrt{\pi\beta}}, \quad K = \pi\zeta, \qquad \cdots (1.12)$$

一方 粘性によって失われるパワーは

$$W_D = \frac{\mu}{2}\iint \zeta\hat{\zeta}\, dx\, dy = \frac{\pi}{2}\mu\omega^2\zeta^2 \int_1^\infty \zeta_1(r)\hat{\zeta}_1(r)\, r\, dr$$

$$= 2\pi\mu\omega^2\zeta^2 \cdot \frac{h\hat{h}}{h+\hat{h}} = 2\pi\rho\omega^3\frac{\zeta^2}{\sqrt{\pi\beta}}, \qquad \cdots (1.13)$$

一方 X_1 のなす単位時間の仕事は

$$W_P = \frac{\omega\zeta}{2}[-X]_I = \frac{\pi\rho\omega^3\zeta^2}{2}\times\frac{4}{\sqrt{\pi\beta}}, \qquad (1.14)$$

となって 確かに 等しい。

また C_D と W_P の関係は

$$C_D = \frac{3\pi}{4\rho\omega^3\zeta^3} W, \qquad (1.15)$$

2. 第2近似定常解

定常分の近似微分方程式は（振動流れとして）

$$\nabla^4\Psi = -\frac{1}{2\nu}\left[(\hat{U}\zeta)_x + (\hat{V}\zeta)_y\right]_R = -\frac{1}{2\nu r}\left[\zeta_r\hat{\Psi}_\theta - \zeta_\theta\hat{\Psi}_r\right]_R$$

$$= -\frac{\omega^2}{2\nu r}\xi^2\sin\theta\cos\theta\left[\zeta_{1r}(r)\hat{\Psi}_{1r}(r) - \zeta_1(r)\hat{\Psi}_{1r}\right]_R,$$

$$= \omega\xi^2\sin 2\theta\left[k^4 e^{-k(r-1)}M(r) + e^{-(k+\bar{k})(r-1)}N(r)\right]_R \quad (2.1)$$

$$M(r) = \frac{1}{2i\,r^{1.5}}\Big[-r + \frac{1}{r} + \frac{2}{\bar{k}r} - \frac{1}{8k}\left(r+15-\frac{1}{r}+\frac{1}{r^2}\right) + \frac{1}{k^2 r} - \frac{\left(1+\frac{7}{r}+\frac{6}{r^2}\right)}{4k\bar{k}} + \frac{1+\frac{3}{r}+\frac{1}{r^2}+\frac{3}{r^3}}{8k^2} + \frac{7+\frac{6}{r}+\frac{15}{r^2}}{128k^2}\left(r-\frac{1}{r}\right)\Big]$$

$$N(r) = -\frac{i(k+\bar{k})}{k^2 r^2}\left[1 + \frac{1+\frac{3}{r}}{4(k+\bar{k})}\right]\times\left(\frac{k}{k+\bar{k}}\right)^4 \qquad \left(\frac{k}{k+\bar{k}}\right)^4 = -\frac{1}{4} \quad (2.2)$$

$$M_r(r) = \frac{1}{2i}\left[\frac{0.5}{r^{1.5}} - \frac{2.5}{r^{3.5}} - \frac{5}{\bar{k}r^{3.5}} + \frac{1}{8k}\left(\frac{0.5}{r^{1.5}} + \frac{22.5}{r^{2.5}} - \frac{2.5}{r^{3.5}} + \frac{3.5}{r^{4.5}}\right)\right] + O\left(\frac{1}{k\bar{k}}\right)$$

$$N_r(r) = +\frac{2i(k+\bar{k})}{4k^2 r^3} + O\left(\frac{1}{k^2}\right), \quad (2.3)$$

$$M(1) = \frac{1}{2i}\left[\frac{2}{\bar{k}} - \frac{2}{4k} + \frac{1}{\bar{k}^2} - \frac{4}{k\bar{k}} + \frac{1}{k^2}\right]$$

$$= -i\left[\frac{1}{\bar{k}} - \frac{1}{k} - \frac{2}{k\bar{k}}\right],$$

$$N(1) = \frac{+i(k+\bar{k})}{4k^2}\left[1 + \frac{1}{k+\bar{k}}\right],$$

$$M_r(1) = \frac{1}{2i}\left[-2 - \frac{5}{\bar{k}} + \frac{3}{k}\right] = +i\left[1 + \frac{2.5}{\bar{k}} - \frac{1.5}{k}\right],$$

$$N_r(1) \fallingdotseq -2N(1) \quad (2.4)$$

$$\Psi(r,\theta) = \omega \xi^2 \sin 2\theta\, \Psi_2(r) \, ,$$
$$\zeta(r,\theta) = \omega \xi^2 \sin 2\theta\, \zeta_2(r) \, , \qquad (2.5)$$

とおいておくと (2.1)は

$$\nabla^4 \Psi_2(r) = \left[k^4 e^{-k(r-1)} M(r) + (k+\hat{k})^4 e^{-(k+\hat{k})(r-1)} N(r) \right]_R \qquad (2.6)$$

但しここでは $\nabla^2 = \dfrac{d^2}{dr^2} + \dfrac{1}{r}\dfrac{d}{dr} - \dfrac{4}{r^2}$, — — (2.7)

補助定理Iにより特解は（上式の最後の項は今の近似には関係ない）

$$\Psi_{2\xi}(r) = \left[C_1 e^{-k(r-1)} + D_1 e^{-(k+\hat{k})(r-1)} \right]_R ,$$
$$\Psi_{2\xi r}(r) = \left[C_2 e^{-k(r-1)} + D_2 e^{-(k+\hat{k})(r-1)} \right]_R ,$$
$$\zeta_{2r}(r) = \left[C_3 e^{-k(r-1)} + D_3 e^{-(k+\hat{k})(r-1)} \right]_R , \qquad (2.8)$$

とおくと

$$C_1 = \left(1 + \frac{2}{kr}\right) M(r) + \frac{4}{k} M_r(r)$$
$$D_1 = \left(1 + \frac{2}{(k+\hat{k})r}\right) N(r) + \frac{4}{k+\hat{k}} N_r(r) \qquad (2.9)$$

$$C_2 = -k\left(1 + \frac{2}{kr}\right) M(r) - 3 M_r(r)$$
$$D_2 = -(k+\hat{k})\left(1 + \frac{2}{(k+\hat{k})r}\right) N(r) - 3 N_r(r) \qquad (2.10)$$

$$C_3 = -k^2 \left[\left(1 + \frac{1}{kr}\right) M(r) + \frac{2}{k} M_r \right] ,$$
$$D_3 = -(k+\hat{k})^2 \left[\left(1 + \frac{1}{(k+\hat{k})r}\right) N(r) + \frac{2}{k+\hat{k}} N_r \right] \qquad (2.11)$$

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特に $r=1$ では

$1+i+\frac{2}{\hat{k}}$

$$C_1 = i\left(1+\frac{2}{k}\right)\left(\frac{1}{k}-\frac{1}{\hat{k}}+\frac{2}{k\hat{k}}\right)+\frac{4i}{k}\left(1+\frac{2.5}{\hat{k}}-\frac{3}{2k}\right)$$

$$= \frac{i}{k}\left[5-i+\frac{12}{\hat{k}}+\frac{-4-2i}{k}\right] = \frac{1}{ik}\left[1+5i+\frac{2-4i}{k}+\frac{12i}{\hat{k}}\right],$$ (2.12)

$$D_1 = \frac{-(k+\hat{k})}{4ik^2}\left(1-\frac{6}{k+\hat{k}}\right)\left(1+\frac{1}{k+\hat{k}}\right) = \frac{-(1-i)}{4ik}\left(1-\frac{5}{k+\hat{k}}\right),$$

$$= \frac{+(1+i)}{4k}\left(1-\frac{5}{k+\hat{k}}\right) \quad ;\text{real}$$

$+1.5i$

-2

$-1-i-\frac{2}{\hat{k}}i$

$$C_2 = +i\left(1+\frac{2}{k}\right)\left(i-1-\frac{2}{\hat{k}}\right)-3i\left(1+\frac{2.5}{\hat{k}}-\frac{3}{2k}\right)$$

$$= -1-4i+\frac{-2+2.5i}{k}+\frac{9.5i}{\hat{k}},$$ (2.13)

$$D_2 = \frac{-i(k+\hat{k})^2}{4k^2}\left(1-\frac{4}{k+\hat{k}}\right)\left(1+\frac{1}{k+\hat{k}}\right) = \frac{1}{2}\left(1-\frac{3}{k+\hat{k}}\right),$$

$(1-i-\frac{2}{\hat{k}})$

$$C_3 = -k^2\left[\left(1+\frac{1}{k}\right)(+i)\left(\frac{1}{k}-\frac{1}{\hat{k}}+\frac{2}{k\hat{k}}\right)+\frac{2i}{k}\left(1-\frac{3}{2k}+\frac{5}{2\hat{k}}\right)\right]$$

$$= -ik\left[3-i-\frac{i+2}{k}+\frac{7}{\hat{k}}\right] = k\left[-1-3i+\frac{2i-1}{k}-\frac{7i}{\hat{k}}\right],$$ (2.14)

$$D_3 = \frac{-i(k+\hat{k})^3}{4k^2}\left(1-\frac{2}{k+\hat{k}}\right) \quad ;\text{real}$$

$$= \frac{-(k+k)^3}{4\alpha^2} = -\frac{2\sqrt{2}\,\alpha}{\sqrt{2}}\left(1-\frac{2}{k+\hat{k}}\right)$$

$$C_3 = \sqrt{2}\,\alpha(1-2i), \qquad D_3 = -\frac{\alpha}{\sqrt{2}}$$

$$C_1+D_1 = \frac{1}{k}(1.25+5.25i) = \frac{1}{\sqrt{2}\,\alpha}(6.5+4i)$$

$$C_2+D_2 = -1.5-4i+\frac{3}{2\sqrt{2}\,\alpha}+\frac{10-5i}{\sqrt{2}\,\alpha} = -1.5+\frac{13-5i}{\sqrt{2}\,\alpha}$$

$-4i$

これに一般解をかえて解は

$$\overline{\Psi}_2(r) = A + \frac{B}{r^2} + \overline{\Psi}_{2S}(r) \quad , \qquad (2.15)$$

境界条件から

$$A + B = -\overline{\Psi}_{2S}(1) = -[C_1 + D_1]_R = -\frac{6.5}{\sqrt{2}\alpha} ,$$

$$2B = -\overline{\Psi}_{2S}'(1) = [C_2 + D_2]_R = 1.5 - \frac{13}{\sqrt{2}\alpha}$$

$$, \quad \alpha = \sqrt{R/2} ,$$

$$\therefore \quad A = +\frac{3}{4} - \frac{13}{\sqrt{2}\alpha} \qquad \qquad \qquad (2.16)^*$$

$$B = \frac{3}{4} - \frac{13}{2\sqrt{2}\alpha} \qquad \alpha = \sqrt{\frac{R}{2}} ,$$

また

$$\overline{\zeta}_2(r) = \frac{A}{r^2} + \overline{\zeta}_{2S}(r) \quad , \qquad (2.17)$$

($\overline{\Psi} = \cos 2\theta$ は 重調和)

*(Text では $A = \frac{3}{4} - \frac{17}{\sqrt{2}\alpha}$, $B = \frac{3}{4} - \frac{7.5}{\sqrt{2}\alpha}$ となっている)

3. 力の才2近似

才2近似での力は(B.8)より

$$X = X_1 + \rho \mathrm{II},$$

$$\mathrm{II} = \iint_D s(\bar{v}U^* - \bar{u}V^*)\,dx\,dy = \iint_D \bar{\Psi}\{(sU^*)_x + (sV^*)_y\}\,dx\,dy, \quad (3.1)$$

但し $\Psi^* = \sin\theta\, \Psi_1(r)$, - - - (3.2)

とする。

$$\mathrm{II} = \iint_D \bar{\Psi}(s_r \Psi_\theta^* - s_\theta \Psi_r^*)\,dr\,d\theta = \frac{i\pi}{2}\omega^2\zeta^3 \int_1^\infty \bar{\Psi}_2(r)(s_{1r}\Psi_1 - s_1\Psi_{1r})\,dr,$$

$$= \pi\varepsilon\omega^2\zeta^3\, \mathrm{II}^*, \quad \cdots\cdots (3.3)$$

$$s_{1r}\Psi_1 - s_1\Psi_{1r} = s_{1r}\bar{\Psi}_{1P} - s_1\Psi_{1Pr} = -\frac{2k^2}{\sqrt{r}} e^{-k(r-1)}\{-r + \frac{1}{r}(1+\frac{2}{k})\} - \frac{2k}{\sqrt{r}} e^{-k(r-1)}\{-1 - \frac{1}{r^2}(1+\frac{2}{k})\}$$

$$= +\frac{2k^2}{\sqrt{r}} e^{-k(r-1)}\left[r - \frac{1}{r} - \frac{1}{k}\left(\frac{2}{r} - 1 - \frac{1}{r^2}\right)\right]$$

$$= \frac{2k^2}{r^{1.5}} e^{-k(r-1)} (r-1)\left(r+1 + \frac{r-1}{kr}\right) \doteqdot \frac{2k^2}{r^{1.5}}(r^2-1)e^{-k(r-1)}, \quad (3.4)$$

$$\therefore\ \mathrm{II}^* = ik^2 \int_1^\infty \bar{\Psi}_2(r) e^{-k(r-1)} (r^2-1)\frac{dr}{r^{1.5}}, \quad (3.5)$$

所で

$$\int_1^\infty (r-1)e^{-p(r-1)} f(r)\,dr \doteqdot -\frac{f(1)}{p^2}, \quad |p| \gg 1 \quad (3.6)$$

であるから

(2.8), (2.15) より

$$\tilde{\gamma}_2(r) = A - \frac{B}{r^2} + \frac{C_1}{2} e^{-k(r-1)} + \frac{\hat{C}_1}{2} e^{-\hat{k}(r-1)} + D_1 e^{-(k+\hat{k})(r-1)}, \quad (D_1 : \text{real})$$

故に

$$I^{+} = +i(A-B) + \frac{iC_1}{8}\alpha + \frac{i\hat{C}_1 k^2}{2(k+\hat{k})^2} + \frac{ik^2 D_1}{(2k+\hat{k})^2}$$

$$= \frac{6.5i}{\sqrt{2}\alpha} + \frac{-2+3i}{4\sqrt{2}\alpha} + \frac{2i-3}{\sqrt{2}\alpha} + \frac{3i-4}{50\sqrt{2}\alpha} = \frac{-3.58+3.69i}{\sqrt{2}\alpha}$$

$$\doteq \frac{-4}{\sqrt{2}\alpha}(.895+.9225i), \qquad (3.8)$$

故に

$$X = X_1 - \pi\rho\omega^2\xi^3\left[\frac{4}{\sqrt{2}\alpha}(.895+.923i)\right], \quad (3.9)$$

$$C_m = 2 + \frac{4}{\sqrt{\pi\beta}}\left(1 - \frac{0.895}{\pi^2}K^2\right), \quad K = \pi\xi, \quad (3.10)$$

$$C_D = C_{D1}\left(1 + \frac{.923}{\pi^2}K^2\right), \qquad (3.11)$$

一方, 粘性減衰パワーは（定常流れに等しい）

$$W = \mu\iint \zeta^2\, dx\, dy = \pi\mu\omega^2\xi^4 \int_1^\infty \tilde{\zeta}_2^2(r)\, r\, dr$$

$$= \pi\rho\omega^3\xi^4\left(\frac{\nu}{\omega}\right)\int_1^\infty \tilde{\zeta}_2^2\, r\, dr, \qquad (3.12)$$

$$\int_1^\infty \tilde{\zeta}_2^2\, r\, dr = \int_1^\infty\left(\frac{A^2}{r^4} + \frac{2A\tilde{\zeta}_{2s}}{r^2} + \tilde{\zeta}_{2s}^2\right) r\, dr \doteqdot \int_1^\infty\left(\frac{2A}{r^2}\tilde{\zeta}_{2s} + \tilde{\zeta}_{2s}^2\right) r\, dr,$$

$O(1)$

$$\bar{\zeta}_{2s} = \frac{C_3}{2} e^{-k(r-1)} + \frac{\hat{C}_3}{2} e^{-\hat{k}(r-1)} + D_3$$

$$= \frac{\alpha(1-2i)}{\sqrt{2}} e^{-k(r-1)} + \frac{\alpha(1+2i)}{\sqrt{2}} e^{-\hat{k}(r-1)} - \frac{\alpha}{\sqrt{2}} e^{-(k+\hat{k})(r-1)}$$

$$\frac{2}{\alpha^2}\bar{\zeta}_{2s}^2 = (1-2i)^2 e^{-2k(r-1)} + (1+2i)^2 e^{-2\hat{k}(r-1)} + e^{-2(k+\hat{k})(r-1)}$$

$$+ 2(1-2i)(1+2i) e^{-(k+\hat{k})(r-1)} - 2(1-2i) e^{-(2k+\hat{k})(r-1)} - 2(1+2i) e^{-(k+2\hat{k})(r-1)}$$

$$\frac{1-7i}{2\sqrt{2}\alpha}$$

$$\int_1^\infty \bar{\zeta}_{2s}^2 r\,dr = \frac{\alpha^2}{2}\Big[\frac{-3-4i}{2k} + \frac{-3+4i}{2\hat{k}} + \cancel{\frac{1}{2\sqrt{2}\alpha}} + \frac{10}{\sqrt{2}\alpha}$$

$$+ \frac{1}{2\sqrt{2}\alpha} - \frac{2(1-2i)}{2k+\hat{k}} - \frac{2(1+2i)}{k+2\hat{k}}\Big]$$

$$= \frac{\alpha}{2\sqrt{2}}\Big[1 + 10 + \frac{1}{2} - \frac{12}{5}\Big] = \frac{9.1}{2\sqrt{2}}\alpha$$

$1+3i$

$(3+i)(1+2i)$

$$\frac{2(1+2i)(1-i)}{(1-2i)\sqrt{2}\alpha}$$

$5 \quad 3+9i$

$+3 \quad -3-16i$

$$\frac{2(1+2i)^2(1-i)}{5}$$

$\frac{-13}{12}$

11.5

~~2.4~~

~~7.4~~

9.1

$\frac{6}{5}$

$$\therefore W = \pi\rho\omega^3 \xi^4 \times \frac{9.1}{2\sqrt{2}\alpha} \quad \cdots (3.13)$$

(1.13)と 比べると

$$\frac{W}{W_D} = \frac{9.1}{K}\xi^2 = \frac{9.1}{4\pi^2}\beta^2, \quad (3.14)$$

(3.11)の 第2項と 比べると 係数 .92に対し <u>2.275</u> となり,

より実験値に近づく。

減衰の見積りは こちらの方が精度が高いと考えられる。

この事から附加質量も(41*)を利用する方が精度が高いのでは

4. 振動流れの第2近似解

定常流れを $(4^*)(5^*)$ に代入すると振動流れの第二近似を求める事が出来る。

$$\nabla^4\psi - k^2\nabla^2\psi = -\frac{1}{\nu r}\left[\zeta_r\psi_\theta - \zeta_\theta\psi_r + \zeta_r\bar{\psi}_\theta - \zeta_\theta\bar{\psi}_r\right]$$

$$= -\frac{\omega^2\xi^3}{2\nu r i}\Big[(\sin 3\theta - \sin\theta)(\zeta_{2r}\psi_1 - \zeta_{1r}\bar{\psi}_2) + 2(\sin 3\theta + \sin\theta)(\zeta_{1r}\bar{\psi}_2 - \bar{\zeta}_2\psi_{1r})\Big] \quad (?)$$

$$\frac{1}{\rho}\nabla^2 G = 2\zeta\bar{\zeta} - \left(\bar{\zeta}_r\psi_r + \frac{1}{r^2}\bar{\zeta}_\theta\psi_\theta + \zeta_r\bar{\psi}_r + \frac{1}{r^2}\zeta_\theta\bar{\psi}_\theta\right)$$

$$= i\omega^2\xi^3\Big[2\zeta_1(r)\bar{\zeta}_2\sin\theta\sin 2\theta - \Big(\bar{\zeta}_{2r}\psi_{1r}\sin\theta\sin 2\theta + \frac{2}{r^2}\bar{\zeta}_2\psi_1\cos\theta\cos 2\theta + \zeta_{1r}\bar{\psi}_{2r}\sin\theta\sin 2\theta + \frac{2}{r^2}\zeta_1\bar{\psi}_2\cos\theta\cos 2\theta\Big)\Big]$$

$$= i\omega^2\xi^3\Big[\Big\{\zeta_1\bar{\zeta}_2 - \frac{1}{2}(\bar{\zeta}_{2r}\psi_{1r} + \zeta_{1r}\bar{\psi}_{2r})\Big\}(\cos\theta - \cos 3\theta) - \frac{1}{r^2}(\bar{\zeta}_2\psi_1 + \zeta_1\bar{\psi}_2)(\cos\theta + \cos 3\theta)\Big]$$

なる故，今

$$\left.\begin{aligned}\psi &= \omega^2\xi^3\{\psi_{21}(r)\sin\theta + \psi_{23}(r)\sin 3\theta\},\\ \frac{1}{\rho}G &= \omega^2\xi^3\{g_{21}(r)\cos\theta + g_{23}(r)\cos 3\theta\},\end{aligned}\right\} \quad (4.1)$$

とおくと

$$\left.\begin{aligned}(\nabla_1^4 - k^2\nabla_1^2)\psi_{21}(r) &= \frac{k^2}{2}(\zeta_{2r}\psi_1 - \zeta_{1r}\bar{\psi}_2) - k^2(\zeta_{1r}\bar{\psi}_2 - \bar{\zeta}_2\psi_{1r}),\\ \nabla_1^2 &= \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\end{aligned}\right\} \quad (4.2)$$

$$\left.\begin{aligned}(\nabla_3^4 - k^2\nabla_3^2)\psi_{23}(r) &= -\frac{k^2}{2}\left[(\zeta_{2r}\psi_1 - \zeta_{1r}\bar{\psi}_2) + \zeta_{1r}\bar{\psi}_2 - \bar{\zeta}_2\psi_{1r}\right],\\ \nabla_3^2 &= \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{9}{r^2}\end{aligned}\right\} \quad (4.3)$$

$$\nabla_1^2 g_{21} = i\left[\zeta_1\bar{\zeta}_2 - \frac{1}{2}(\bar{\zeta}_{2r}\psi_{1r} + \zeta_{1r}\bar{\psi}_{2r}) - \frac{1}{r^2}(\underbrace{\bar{\zeta}_2\psi_1 + \zeta_1\bar{\psi}_2}_{\text{neg.}})\right], \quad (4.4)$$

$$\nabla_3^2 g_{23} = -i\left[\psi_1\bar{\zeta}_2 - \frac{1}{2}(\bar{\zeta}_{2r}\psi_{1r} + \zeta_{1r}\bar{\psi}_{2r})\} + \frac{1}{r^2}(\underbrace{\bar{\zeta}_2\psi_1 + \zeta_1\bar{\psi}_2}_{\text{neg.}})\right], \quad (4.5)$$

$$\frac{\pm}{2}(\psi_1\bar{\zeta}_{2r} - \zeta_{1r}\bar{\psi}_2) = (\zeta_{1r}\bar{\psi}_2 - \bar{\zeta}_2\psi_{1r})$$

直接力に関係するのは，$e^{i\theta}$, $k\cos\theta$ の項なので以下それらについてのみ考える事とする。

(4.2), (4.4) をたとえば次のようにおこう。(以下下添字21をとって記す)

$$(\nabla^4 - k^2\nabla^2)\bar{\psi}_2 = \sum_{j=1}^{5} p_j^2 e^{-p_j(r-1)} F_j(r), \quad (4.6)$$

$$\nabla^2 g_2 = \sum_{j=1}^{5} p_j^2 e^{-p_j(r-1)} G_j(r), \quad (4.7)$$

但し $p_1 = k$, $p_2 = \hat{k}$, $p_3 = 2k$, $p_4 = k+\hat{k}$, $p_5 = 2k+\hat{k}$.

ここで $\frac{k^2}{2}($

A. 補助定理 I（重調和関数の特解）

$$\nabla^4 \psi(r) = p^4 F(r) e^{-p(r-1)}, \quad |p| \gg 1, \qquad (A\cdot1)$$

ここに $F(r)$ は エクスポネンシャル項を含まないものとする。

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}, \qquad (A\cdot2)$$

この特解が近似的に

$$\psi(r) = f(r) e^{-p(r-1)}, \qquad (A\cdot3)$$

のように表わされるとすると

$$\zeta(r) = -\nabla^2\psi(r) = -p^2 e^{-p(r-1)}\left\{\overbrace{f(r)}^{(1-\frac{1}{pr})} - \frac{2}{p} f_r(r) + O\left(\frac{1}{p^2}\right)\right\}, \qquad (A\cdot4)$$

$$\nabla^4\psi(r) = p^4 e^{-p(r-1)}\left\{\overbrace{f(r)}^{(1-\frac{2}{pr})x} - \frac{4}{p} f_r(r) + O\left(\frac{1}{p^2}\right)\right\}. \qquad (A\cdot5)$$

となるから，(A·5) を (A·1) と等置して，

$$f(r) \fallingdotseq \underbrace{F(r)}_{(1+\frac{2}{pr})x} + \frac{4}{p} F_r(r) + O\left(\frac{F_r}{p^2}\right), \qquad (A\cdot6)$$

また (A·4) に代入して

$$\zeta(r) = -p^2 e^{-p(r-1)}\left\{\underbrace{F(r)}_{(1+\frac{1}{pr})x} + \frac{2}{p} F_r(r) + O\left(\frac{1}{p^2}\right)\right\}, \qquad (A\cdot7)$$

$$\left(F(1+\frac{1}{pr})F\right)'_r = (1+\frac{1}{pr})F_r - \frac{F}{pr^2}$$

$$\zeta_r(r) = -p e^{-p(r-1)}\left[(1+\frac{2}{pr})F + \frac{4}{p}F_r - \frac{F}{pr^2}\right]$$

$$\left[(1+\frac{2}{pr})F + \frac{3}{p}F_r\right]$$

B. 可逆定理(24*)について

(24*)では流れ関数は攪乱分をとっているので無限遠の検査面の積分は考えていない。

しかし振動流れを含ませて(それを大文字で記し)

$$\Psi_n = \Psi_s = 0 \quad \text{on } C \tag{B.1}$$

としないと積分計算上種々不便である。

そうすると無限遠の積分が表われる。

さて ψ^* としては攪乱分のみとり、

$$\psi^* = \psi_1(r)\sin\theta,\ \psi_1(r) = \frac{1}{r}\left(1+\frac{2}{r^2}\right)\ \cancel{=}\ \frac{\zeta_1(r)}{r^2}, \tag{B.2}$$

とおくと

$$\psi^*_r = ~~\sin\theta~~,\quad \frac{1}{r}\psi^*_\theta = \cos\theta, \tag{B.3}$$

充分遠方では(A.8*)(A.9*)より　sign?

$$\Psi = \varepsilon\omega e^{-i\theta}\Psi(r),\ \Psi(r) = r - \psi(r), \tag{B.4}$$

$$\left.\begin{aligned} \psi(r) &\to \frac{C_X}{2r} \\ C_X &= X/\pi\rho\varepsilon\omega^2 \end{aligned}\right\} \tag{B.5}$$

今は $X=Y$ で、X, Y は振動流れ中の夫々 x, y 方向の力とする。

また $\psi_1(r) \to \dfrac{C_{X1}}{2r}$, $C_{X1} = 2 + \dfrac{4}{k_c}$, (B.6)

$C_{X1} = X_1/\pi\rho a^2$, X_1 は単位振巾の左右振動流れ$\left(\overset{2\pi\omega^2}{-i\omega y}\right)$による左右の力である。

$y \quad x - iy$

$\phi \quad -y - ix = -i(\cos\theta - i\sin\theta)$

No. B.2

Date

一方一般に $G \xrightarrow[r\gg 1]{} -\rho i\omega \bar{\Phi}$, ・・・ (B.7)

であるから (B.4), (B.5) より

$$\left.\begin{aligned} \bar{\Phi} &\doteqdot -i\varepsilon\omega e^{-i\theta}\Psi(r), \\ G &\doteqdot -\rho\omega^2\varepsilon e^{-i\theta}\Psi(r), \end{aligned}\right\} \quad (B.8)$$

ψ^* については

$$\left.\begin{aligned} G^* &\to -\rho i\omega\phi^* = -\rho i\omega\cos\theta\,\psi_1(r), \\ \phi^* &= \psi_1(r)\cos\theta, \end{aligned}\right\} \quad (B.9)$$

さて (24^*) は

$$\int_{C+R}\left[G^*\Psi_s + \mu\zeta^*\Psi_n - (G\psi_s^* + \mu\zeta\psi_n^*)\right]ds$$

$$= \rho\iint_D\left[\zeta(Uv^* - Vu^*) + \zeta(\bar{u}v^* - \bar{v}u^*)\right]dx\,dy, \quad (B.10)$$

であるが 無限大の円 R 上の積分は (B.8) (B.9) により
$\mu\zeta^*$ は充分小さいので

$$\int_R\left[G^*\Psi_s - G\psi_s^*\right]ds = \int_R\left(G^*\bar{\Phi}_n - G\phi_n^*\right)ds$$

$$= \int_0^{2\pi}\left(G\phi_r^* - G^*\bar{\Phi}_r\right)r\,d\theta = -\rho\omega^2\varepsilon\int_0^{2\pi}\left[\Psi(r)\psi_{1r}e^{-i\theta}\cos\theta - \Psi_r\psi_1 e^{-i\theta}\cos\theta\right]r\,d\theta$$

$$= +\pi\rho\varepsilon\omega^2\left[\frac{C_{x1}}{2}(1-\Psi_r) + (r-\Psi(r))\frac{C_{x1}}{2r}\right]$$

$$\doteqdot \pi\rho\omega^2\varepsilon C_{x1} = X_1 \quad , \quad \cdots \quad (B.11)$$

(sign ?)

一方

$$[(B.10)の右辺] = J' = J + K, \qquad (B.12)$$

ここに

$$J = \rho \iint_D \zeta (UV^* - VU^*) + \zeta (\bar{u}V^* - \bar{v}U^*) dxdy, \qquad (B.13)$$

$$K = -\rho \iint_D (\bar{\zeta} V + \bar{v} \zeta) dxdy, \qquad (B.14)$$

ここに

$$\left.\begin{array}{l} \Psi^* = \varepsilon i \theta \, \Psi_1(r), \quad \Psi_1(r) = -r + \psi_1(r), \\ U^* = \Psi^*_y = -1 + u^*, \quad V^* = v^* \\ u^* = \psi^*_y, \; v^* = -\psi^*_x \end{array}\right\} \qquad (B.15)$$

さて

$$K = -\rho \iint_D \left(\Psi'_x \nabla^2 \bar{\psi} + \bar{\psi}_x \nabla^2 \Psi' \right) dx\,dy$$

$$= -\rho \iint_D \left(\bar{\psi}_x \nabla^2 \Psi' - \Psi' \nabla^2 \bar{\psi}_x \right) dxdy,$$

ここで重要な条件により

$$\iint_D \left(\Psi'_x \nabla^2 \bar{\psi} + \Psi' \nabla^2 \bar{\psi}_x \right) dxdy = \int_{C+R} \left(\Psi' \nabla^2 \bar{\psi} \right) dy = 0,$$

よって

$$K = +\rho \int_R \left(\Psi' \bar{\psi}_{xr} - \bar{\psi}_x \Psi'_r \right) r\,d\theta$$

$$= +\rho a^2 \varepsilon^3 A \int \left[r e^{i\theta} \left(-\frac{\cos\theta}{r^2} \right) - \frac{\cos\theta}{r} e^{+i\theta} \right] r\,d\theta = -\pi \rho a^2 \varepsilon^3 A, \qquad (B.16)$$

おいて

$$\bar{\psi}_r \longrightarrow \frac{A}{r}, \quad \left(\frac{e^{-i\theta}}{r} \right)_x = O\left(\frac{1}{r^2} \right)$$

$$\Psi'_{xr} = \left(x - iy - \psi(r) e^{-i\theta} \right)_{xr} = -\left(\psi e^{-i\theta} \right)_{xr} = O\left(\frac{1}{r^3} \right)$$

$$\bar{\psi} = \varepsilon i \varepsilon^2 \bar{\psi}_2$$

従って

$$X = -\int_C (p\cos\theta + \mu\zeta\sin\theta)\,ds$$

$$= X_1 + K + J, \quad \cdots \quad (B.17)$$

$$K = -\pi\rho\omega^2\varepsilon^2 A, \qquad A = 3 + (\ \ )$$

$$X_1 = \pi\rho\omega^2\varepsilon\left(1 + \frac{4}{R}\right),$$

$$J = -\rho(I + II) = -\pi\rho\omega^2\varepsilon^3(I^* + II^*), \quad \cdots \quad (B.18)$$

（以下 小文字の量で 振動流れを含むとする）

$$I = \iint_D \bar{\zeta}(uv^* - vu^*)\,dx\,dy$$

$$= \iint_D \bar{\zeta}(\psi_r^*\psi_\theta - \psi_r\psi_\theta^*)\,dr\,d\theta$$

$$= \pi\varepsilon^3\omega^2 I^*,$$

$$I^* = 2\int_1^\infty \bar{\zeta}_2(r)\psi_{1r}\psi_1\,dr\,d\theta, \qquad (B.19)$$

$$II = \iint_D \zeta(\bar{v}u^* - \bar{u}v^*)\,dx\,dy = \iint_D \zeta(\psi_r^*\bar{\psi}_\theta - \bar{\psi}_r\psi_\theta^*)\,dr\,d\theta$$

$$= \pi\omega^3\varepsilon^3\, II^*,$$

$$II^* = -\int_1^\infty \zeta_1(r)\bar{\psi}_{2r}\psi_1(r)\,dr, \qquad (B.20)$$

(sign ?)

C. 近似積分

$$I = \int_1^\infty e^{-p(r-1)} f(r)\, dr, \quad |p| \gg 1, \qquad (C.1)$$

の近似値は $f(r)$ が slowly varying func. とすれば

$$I \fallingdotseq \frac{f(1)}{p}, \qquad (C.2)$$

それ故 (B.9'), (B.10) の積分において、夫々の関数の exponential term 以外は $r=1$ の値が分か(ってい)れば良い。

附録D 補助定理 II

$$(\nabla^2 - k^2)\zeta(r)e^{-i\theta} = p^2 G(r) e^{-pr-i\theta}, \qquad (D.1)$$

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} - k^2\right)\zeta(r) = p^2 e^{-pr} G(r) \qquad (D.1')$$

この解が
$$\zeta(r) = h(r) e^{-pr}, \qquad (D.2)$$
と書けるならば

$$G(r) = \left(1 - \frac{k^2}{p^2} - \frac{1}{pr}\right)h + \left(-\frac{2}{p} + \frac{1}{p^2 r}\right)h_r + \frac{h_{rr}}{p^2}, \qquad (D.3)$$

$1 - \frac{k^2}{p^2} = \beta \neq 0$ ($p = \hat{k}$ ならば $\beta = 2$, $p = k + \hat{k}$ ならば $\beta = \frac{1}{2}$) ならば

$$G(r) \fallingdotseq \left(\beta - \frac{1}{pr}\right)h - \frac{2}{p}h_r + O\left(\frac{1}{p^2}\right), \qquad (D.4)$$

$$\therefore \quad h(r) \fallingdotseq \frac{1}{\beta}\left[G + \frac{h}{pr} + \frac{2}{p}h_r\right] = \frac{1}{\beta}\left[\left(1 + \frac{1}{\beta pr}\right)G + \frac{2}{\beta p}G_r\right], \qquad (D.5)$$

(この他に $\zeta(r)$には 斉次解が加わらねばならない)

$p \fallingdotseq k$, $\beta = 0$ ならば

$$G(r) = -\frac{h}{kr} - \frac{2}{k}h_r + \frac{h_r}{k^2 r} + \frac{h_{rr}}{k^2}, \qquad (D.6)$$

今,
$$h(r) = h^*(r)/\sqrt{r} \qquad (D.7)$$
とおくと
$$h_r = -\frac{h^*}{2r^{1.5}} + \frac{h_r^*}{\sqrt{r}}, \quad h_{rr} = +\frac{3h^*}{4r^{2.5}} - \frac{h_r^*}{r^{1.5}} + \frac{h_{rr}^*}{\sqrt{r}}$$

$$\therefore kG = -\frac{2}{\sqrt{r}}h_r^* + \frac{1}{kr}\left(-\frac{h^*}{2r^{1.5}} + \frac{h_r^*}{\sqrt{r}}\right) + \frac{1}{k}\left(\frac{3h^*}{4r^{2.5}} - \frac{h_r^*}{r^{1.5}} + \frac{h_{rr}^*}{\sqrt{r}}\right)$$

$$= -\frac{2}{\sqrt{r}}h_r^* + \frac{1}{k\sqrt{r}}\left(\frac{h^*}{4r^2} + h_{rr}^*\right), \qquad (D.6')$$

$$\therefore \quad h_r^* = -\frac{k\sqrt{r}}{2}G(r) + \frac{1}{2k}\left(h_{rr}^* + \frac{h^*}{4r^2}\right)$$

$$\fallingdotseq -\frac{k\sqrt{r}}{2}G - \frac{1}{4}\left([\sqrt{r}G]_r + \frac{1}{4r^2}[\sqrt{r}G]^r\right), \qquad (D.7)$$

上付き $[\ ]^r$ は積分を示す。

一方 (D.1') の斉次解は
$$\frac{e^{-kr}}{\sqrt{r}}\left(1 - \frac{1}{2kr}\right), \qquad (D.8)$$

$$h^*(r) - h^*(1) = -\frac{k}{2}[\sqrt{r}G]^r,$$

これは 斉次解の定数Cを含めれば

$$h(r) \doteqdot \left\{ -\frac{k}{2\sqrt{r}}\left[\sqrt{r}G\right]^r - \frac{1}{4\sqrt{r}}\left(\sqrt{r}G + \frac{1}{4}\left[\frac{[\sqrt{r}G]^r}{r^2}\right]^r\right) + \frac{1}{\sqrt{r}}C\left(1 - \frac{1}{2kr}\right) \right\}, \quad (D.8)$$

次に

$$\nabla^2 \psi(r) e^{-i\theta} = -\zeta e^{-i\theta}, \quad (D.9)$$

であるから

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\right)\psi(r) = -\zeta(r), \quad (D.10)$$

となる。

$\zeta(r)$が(D.2)のように表わされるならば 附録A の計算より

$$\psi(r) = e^{-pr} f(r), \quad (D.11)$$

の形になるとすると

$$f(r) = -\frac{1}{p^2}\left[\left(1 + \frac{1}{pr}\right)h(r) + \frac{2}{p} h_r(r)\right], \quad (D.12)$$

$p \neq k$ として (D.5)を代入すれば

$$f(r) = -\frac{1}{p^2 \beta}\left[\left\{1 + \frac{1}{pr}\left(1 + \frac{1}{\beta}\right)\right\}G + \frac{2}{p}\left(1 + \frac{1}{\beta}\right)G_r\right], \quad (p \neq k) \quad (D.13)$$

$k = p$ では (D.7), (D.8)を代入して

$$f(r) = +\frac{1}{k}\left[+\frac{\left(1 + \frac{1}{kr}\right)}{2\sqrt{r}}\left[\sqrt{r}G\right]^r + \frac{G}{k} + \frac{1}{4k}\left(\sqrt{r}G + \frac{1}{4}\left[\frac{[\sqrt{r}G]^r}{r^2}\right]^r\right)\right] \quad (p = k) \quad (D.14)$$

また (D.8)の斉次解に対しては

$$\psi_s(r) = -\frac{Ce^{-kr}}{k^2}\left[\left(1 + \frac{1}{2kr}\right)\frac{1}{\sqrt{r}} + \frac{2}{k}\left(-\frac{1}{2r^{1.5}}\right)\right] = -\frac{Ce^{-kr}}{k^2\sqrt{r}}\left(1 - \frac{1}{2kr}\right), \quad (D.15)$$

$\psi(r)$には 斉次解として次の周知の関数が加わる。

$$\psi_s(r) \sim \left(r, \frac{1}{r}\right), \quad (D.16)$$

(なお $e^{-i n\theta}$としても解は同じになる)