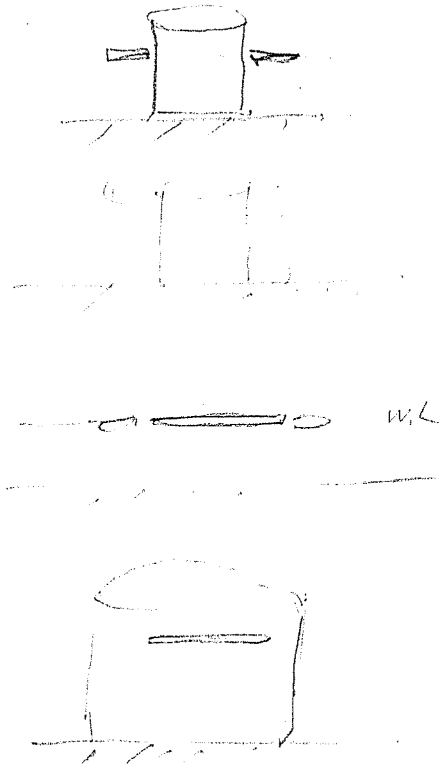
M.A6-7

No.

Date 55.2.17

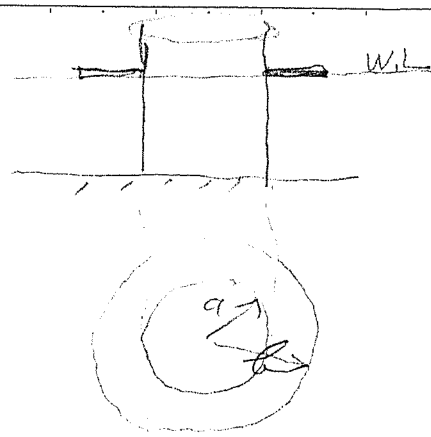
浅水の波

1. 円筒と円輪
2. 円筒と輪筒
3. 円板と円輪
4 円板と輪筒



円筒と円輪

i) 上下動　単位振巾



$$\Delta\phi = \frac{1}{h} \quad \text{for } a<r<b$$

$$h\phi = \frac{r^2}{4} + C\log r + D$$

$$\frac{\partial\phi}{\partial r} = \frac{r}{2} + \frac{C}{r}\Big|_{r=a} = 0$$

$$C = -\frac{a^2}{2}$$

$$h\phi = \frac{r^2}{4} - \frac{a^2}{2}\log r + D, \quad h\frac{\partial\phi}{\partial r} = \frac{r}{2} - \frac{a^2}{2r}$$

$r>b$ では

$$\phi = AH_0^{(2)}(kr), \quad \frac{\partial\phi}{\partial r} = -kAH_1^{(2)}(kr)$$

$$\left.\begin{aligned} hAH_0^{(2)}(kb) &= \frac{b^2}{4} - \frac{a^2}{2}\log b + D \\ -khAH_1^{(2)}(kb) &= \frac{b}{2}\left(1-\frac{a^2}{b^2}\right) \end{aligned}\right\}$$

$$A = -\frac{(b^2-a^2)}{2hkb\,H_1^{(2)}(kb)},$$

$$D = \frac{a^2}{2}\log b - \frac{b^2}{4} - \frac{(b^2-a^2)\,H_0^{(2)}(kb)}{2kb\,H_1^{(2)}(kb)}$$

$$2\pi a\,\phi\Big|_{r=a} = \frac{2\pi a}{h}\left[\frac{a^2}{4} - \frac{a^2}{2}\log a + \frac{a^2}{2}\log b - \frac{b^2}{4} - \frac{(b^2-a^2)H_0^{(2)}}{2kbH_1^{(2)}}\right]$$

$$= \frac{2\pi a}{h}\left[\frac{a^2}{2}\log\frac{b}{a} - \frac{(b^2-a^2)}{2}\left\{\frac{1}{2} + \frac{H_0^{(2)}}{kbH_1^{(2)}}\right\}\right]$$

$$2\pi\int_a^b r\phi\,dr = \frac{2\pi}{h}\int_a^b\left[\frac{r^3}{4} - \frac{a^2}{2}r\log r + Dr\right]dr$$

$$= \frac{2\pi}{h}\left[\frac{b^4-a^4}{16} - \frac{a^2}{4}\left\{b^2\left(\log b - \frac{1}{2}\right) - a^2\left(\log a - \frac{1}{2}\right)\right\} + \frac{D}{2}(b^2-a^2)\right],$$

$$\int r\log r\,dr = \frac{r^2}{2}\log r - \frac{1}{2}\int r\,dr = \frac{r^2}{2}\left(\log r - \frac{1}{2}\right)$$

$r\log r + \frac{r}{2} -$

ii) 縦ゆれ

$$\Delta\phi = \frac{x}{h}$$

$$h\phi = \frac{r^2 x}{8} + \frac{C}{r}\cos\theta + Dx$$

$$h\frac{\partial\phi}{\partial r} = \left(\frac{3}{8}r^2 - \frac{C}{r^2} + D\right)\cos\theta \xrightarrow[r=a]{} 0$$

$$\frac{3}{8}a^2 - \frac{C}{a^2} + D = 0$$

$$h\phi = \left[\frac{r^3}{8} + \left(\frac{3}{8}a^4 + a^2 D\right)\frac{1}{r} + Dr\right]\cos\theta.$$

$$h\frac{\partial\phi}{\partial r} = \left[\frac{3}{8}r^2 - \frac{3a^4}{8r^2} + \left(1 - \frac{a^2}{r^2}\right)D\right]\cos\theta.$$

$$\phi = AH_1^{(2)}(kr)\cos\theta \qquad \text{for } r > b.$$

$$\therefore \left.\begin{aligned} hAH_1^{(2)}(kb) &= \frac{b^3}{8} + \frac{3a^4}{8b} + D\left(b + \frac{a^2}{b}\right), \\ khAH_1^{(2)\prime}(kb) &= \frac{3}{8}\left(b^2 - \frac{a^4}{b^2}\right) + \left(1 - \frac{a^2}{b^2}\right)D, \end{aligned}\right\}$$

ii) 散乱 $\phi_0 = e^{ikx} = \sum_{n=0}^{\infty} i^n \varepsilon_n J_n(kr)\cos n\theta$

$\Delta(\phi_d+\phi_0)=0$

$$\phi_d = -\phi_0 + \sum_{n=0} i^n \varepsilon_n C_n \left(r^n + \frac{a^{2n}}{r^n}\right)\cos n\theta + c_0 \cancel{+D\log r}$$

$$\left.\frac{\partial}{\partial r}(\phi_0+\phi_d)\right|_{r=a}=0$$

$$\phi_d = \sum_{n=0}^{\infty} \varepsilon_n A_n i^n H_n^{(2)}(kr)\cos n\theta \qquad \text{for } r>b$$

$$\frac{\partial}{\partial r}\phi_d = k\sum \varepsilon_n A_n i^n H_n'^{(2)}(kr)\cos n\theta$$

$$\phi_d = -\sum_{n=0} i^n \varepsilon_n J_n(kr)\cos n\theta + \sum_{n=0} i^n \varepsilon_n C_n\left(r^n + \frac{a^{2n}}{r^n}\right)\cos n\theta + c_0$$

$$\cancel{i^n}\varepsilon_n\left[C_n\left(b^n+\frac{a^{2n}}{b^n}\right) - J_n(kb)\right] = \cancel{i^n\varepsilon_n} A_n H_n^{(2)}(kb)$$

$$n C_n\left(b^{n-1} - \frac{a^{2n}}{b^{n+1}}\right) - kJ_n'(kb) = kA_n H_n'^{(2)}(kb) \qquad \text{for } n\geq 1$$

$$-kJ_0'(kb) = kA_0 H_0'^{(2)}(kb) \qquad n=0$$