UUQ

M-91

No.

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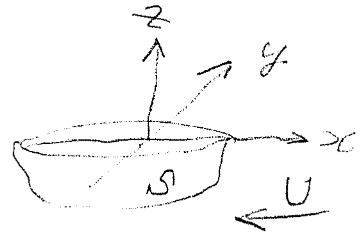
波浪中の増加抵抗公式について(未)

概要

平面波のエネルギー、運動量の式から漸近展開を使って波浪中の増加抵抗を初等的に説明する。

1. 漸近展開

水面条件は



$$\left.\frac{\partial \phi}{\partial z}\right|_{z=0} = -\left(i\omega - U\frac{\partial}{\partial x}\right)\zeta(x,y), \quad (1.1)$$

$$\zeta(x,y) = \frac{1}{g}\left(i\omega - U\frac{\partial}{\partial x}\right)\phi\Big|_{z=0}, \quad (1.2)$$

$$\left(i\omega - U\frac{\partial}{\partial x}\right)^2\phi + g\frac{\partial \phi}{\partial z} = 0,\ z=0, \quad (1.3)$$

核関数を

$$S(x,y,z) = \frac{1}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \frac{g}{4\pi^2}\int_{-\pi}^{\pi}\int_0^\infty \frac{e^{k(z+z') + ik\{(x-x')\cos\theta + (y-y')\sin\theta\}}\,k\,dk\,d\theta}{A(k,\theta)}, \quad (1.4)$$

$$A(k,\theta) = (\omega + kU\cos\theta)^2 - gk - i\mu(kU\cos\theta + \omega), \quad (1.5)$$

とおくと

$$\phi(P) = \iint_{S+F}\left\{\frac{\partial \phi}{\partial n}S - \phi\frac{\partial}{\partial n}S\right\}dS, \quad (1.6)$$

のような表現が出来るから，今

$$H(k,\theta) = \frac{1}{4\pi}\iint_{S+F}\left(\frac{\partial \phi}{\partial n} - \phi\frac{\partial}{\partial n}\right)e^{ik(x\cos\theta + y\sin\theta) + kz}\,dS, \quad (1.7)$$

とおくと

$$\phi(P) = \phi_0(P) + \phi_1(P), \quad (1.8)$$

$$\phi_0(P) = \frac{1}{4\pi}\iint_{S+F}\left(\frac{\partial \phi}{\partial n} - \phi\frac{\partial}{\partial n}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)dS, \quad (1.9)$$

$$\phi_1(P) = -\frac{g}{\pi}\int_{-\pi}^{\pi}\int_0^{\infty} \frac{H(k,\theta)}{A(k,\theta)} e^{kz - ik(x\cos\theta + y\sin\theta)} k\,dk\,d\theta \quad , \qquad (1.10)$$

と書ける。

漸近展開を考える時は ϕ_1 のみとればよい。

またその時 (1.2)により

$$\zeta(x,y) \doteqdot \frac{1}{\pi i}\int_{-\pi}^{\pi}\int_0^{\infty} \frac{H(k,\theta)}{A(k,\theta)} e^{-ik(x\cos\theta + y\sin\theta)} k(\omega + kU\cos\theta)\,dk\,d\theta \quad , \qquad (1.11)$$

(ω_j)

さて $A(k,\theta) = U^2\cos^2\theta\,(k - k_1)(k - k_2)$, (1.12)

$$\zeta(x,y) \doteqdot \frac{2}{U^2}\int_{\varphi-\frac{\pi}{2}}^{\varphi+\frac{\pi}{2}} \frac{H(k_1,\theta)\,e^{-ik_1 r\cos(\theta-\varphi)}\,k_1\omega_1}{(k_1 - k_2)\cos^2\theta}\,d\theta$$

$$+ \frac{2}{U^2}\int \frac{H(k_2,\theta)\,e^{-ik_2 r\cos(\theta-\varphi)}\,k_2\omega_2}{(k_2 - k_1)\cos^2\theta}\,d\theta \quad , \qquad (1.13)$$

$$U^2\cos^2\theta\,(k_2 - k_1) = g\sqrt{1 - \frac{4\omega U}{g}\cos\theta} \quad ,$$

定常点は

$$f(\theta) = k\cos(\theta-\varphi), \quad \cdots \qquad (1.14)$$

とおくと

$$f'(\theta) = \left[k'\cos(\theta-\varphi) - k\sin(\theta-\varphi)\right] = 0, \qquad (1.15)$$

$$\therefore \quad \tan(\theta-\varphi) = \frac{k'(\theta)}{k(\theta)}, \qquad (1.16)$$

次に

$$f''(\theta) = \left[(k''-k)\cos(\theta-\varphi) - 2k'\sin(\theta-\varphi)\right],$$

となるが (1.16)を代入すると

$$f''(\theta) = k\cos(\theta-\varphi)\left[\left(\frac{k'}{k}\right)' - 1 - \frac{k'^2}{k^2}\right], \qquad (1.17)$$

となる。

そこで (1.16)を θで微分すると. (今は φ は θ の関数となる)

$$\left(\frac{k'}{k}\right)' = \sec^2(\theta-\varphi)\left[1 - \frac{d\varphi}{d\theta}\right]$$

$$= 1 + \frac{k'^2}{k^2} - \sec^2(\theta-\varphi)\frac{d\varphi}{d\theta}, \qquad (1.18)$$

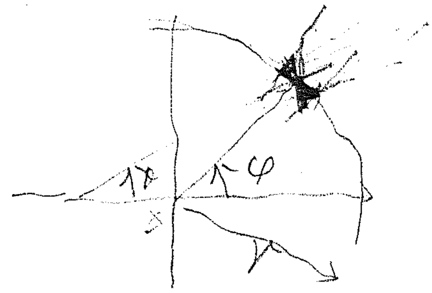
これを (1.17) に代入すると (Newman)

$$f''(\theta) = -\frac{k}{\cos(\theta-\varphi)}\frac{d\varphi}{d\theta}, \qquad (1.19)$$

それ故

$$\zeta = \frac{2}{g}\sqrt{\frac{2\pi}{r}} \sum_{j,n} \pm \frac{H(k_{jn},\theta_n)\, k_{jn}\,\omega_{jn}\, e^{-ik_j r\cos(\theta_n-\varphi) \pm \frac{\pi}{4}i}}{\sqrt{|f''(\theta_n)|}\sqrt{1-\frac{4\omega v}{g}\cos\theta_n}}, \qquad (1.20)$$

無限大の円筒面の面素 $r\,d\varphi$ を単位時間に通じて出てゆく波のエネルギーは



$$dW = \frac{\rho g}{2}\left(\frac{c_j}{2} - U\cos\theta\right)\zeta\bar{\zeta}(\theta-\varphi)\,r\,d\varphi,$$

運動量は

$$dM = \frac{\rho g}{2}\left(\frac{1}{2} - \frac{U}{c_j}\cos\theta\right)\zeta\bar{\zeta}(\theta-\varphi)\,r\,d\varphi$$

$$dM = \frac{\rho g}{2}\left(\frac{1}{2} - \frac{U\cos\theta}{c_j}\right)\frac{8\pi}{g^2}\frac{|H(k_j,\theta)|^2 k_j \omega_j^2}{\left(1-\frac{4\omega U\cos\theta}{g}\right)}d\theta$$

$$= \frac{2\pi\rho\,|H|^2 k^2\,d\theta}{\sqrt{1-\frac{4\omega U\cos\theta}{g}}}$$

$$dW = 2\pi\rho\frac{|H|^2 k_j\omega_j\,d\theta}{\sqrt{1-\frac{4\omega U\cos\theta}{g}}}$$

$$k_j c_j = \omega_j$$

$$\omega_j = \omega + k_j U\cos\theta.$$

x方向の運動量は $dM\cos\theta$.

散逸エネルギー

$$dW = dE + (dR)U$$

抵抗のなす仕事

$$dE = 2\pi\rho\omega\frac{|H|^2 k\,d\theta}{\sqrt{1-\frac{4\omega U\cos\theta}{g}}}$$

附録A

$$A(k,\theta)=(\omega+kU\cos\theta)^2-gk-i\mu(\omega+kU\cos\theta)$$

$$=k^2U^2\cos^2\theta+k(2\omega U\cos\theta-g-i\mu U\cos\theta)+\omega^2-i\mu\omega,\qquad (A.1)$$

$$\therefore\ \left.\begin{matrix}k_1\\k_2\end{matrix}\right\}=\frac{+g-2\omega U\cos\theta+i\mu U\cos\theta\mp g\sqrt{1-\frac{4U}{g}\cos\theta(\omega-\frac{i\mu}{2})}}{2U^2\cos^2\theta},\qquad (A.2)$$

$$k_jU\cos\theta+\omega=\omega_j,\quad j=1,2,\qquad (A.3)$$

$$\left.\begin{matrix}\omega_1\\\omega_2\end{matrix}\right\}=\frac{g+i\mu U\cos\theta\mp g\sqrt{1-\frac{4U\omega}{g}\cos\theta}}{2U\cos\theta},\qquad (A.4)$$

$$\omega_j^2/g=k_j=g/c_j^2,\quad \frac{g}{c_j}=\omega_j,\qquad (A.5)$$

$$\left.\begin{matrix}c_1\\c_2\end{matrix}\right\}=\frac{2U\cos\theta}{1\mp\sqrt{1-\frac{4\omega U\cos\theta}{g}}},\qquad (A.6)$$

$$\left.\begin{matrix}\frac{c_1}{2}>U\cos\theta\\ \frac{c_2}{2}<U\cos\theta\end{matrix}\right\}\begin{matrix}\text{前進}\\\text{不前進}\end{matrix}\quad \text{for } \cos\theta>0.\qquad (A.7)$$

$$\frac{c_1}{2}=\frac{+U|\cos\theta|}{\sqrt{1+\frac{4\omega U}{g}|\cos\theta|}-1}>0,\quad \text{for } \cos\theta<0,$$

$$\frac{c_2}{2}=\frac{-U|\cos\theta|}{\sqrt{1+\frac{4\omega U}{g}|\cos\theta|}+1}<0,\quad \left|\frac{c_2}{2}\right|<U|\cos\theta|,\ \text{不前進}\qquad \text{for } \cos\theta<0$$

$$(k_2-k_1)U^2\cos^2\theta=g\sqrt{1-\frac{4\omega U\cos\theta}{g}},$$

$$=(\omega_2-\omega_1)U\cos\theta,\qquad (A.8)$$

$$(k_1+k_2)U^2\cos^2\theta = g - 2\omega U\cos\theta$$

$$k_1 k_2 U^2\cos^2\theta = \omega^2 \qquad (A\cdot 9)$$

$$(\omega_1+\omega_2)U\cos\theta = g, \quad \omega_1\omega_2 U\cos\theta = g\omega,$$

$$\frac{1}{c_1}+\frac{1}{c_2} = \frac{1}{U\cos\theta},$$

$$\sqrt{1-\frac{4\omega U}{g}\cos\theta} = \frac{2U}{c_2}\cos\theta - 1 = 1 - \frac{2U}{c_1}\cos\theta, \qquad (A\cdot 10)$$

$$\frac{k_j'}{k_j} = 2\frac{\omega_j'}{\omega_j}, \quad \left(\frac{k'}{k}\right)' = 2\left\{\frac{\omega_j''}{\omega_j} - \left(\frac{\omega_j'}{\omega_j}\right)^2\right\}, \qquad (A\cdot 11)$$

$$\frac{\omega_j'}{\omega_j} = \frac{\mp\frac{2\omega U \sin\theta}{g}}{\left(1\mp\sqrt{1-\frac{4\omega U\cos\theta}{g}}\right)\sqrt{1-\frac{4\omega U\cos\theta}{g}}} + \tan\theta = \left(\mp\frac{\frac{\omega}{g}c_j + 1}{\sqrt{1-\frac{4\omega U\cos\theta}{g}}}\right)\tan\theta$$

$$= \left(+1 \mp \frac{\omega}{\omega_j\sqrt{1-\frac{4\omega U\cos\theta}{g}}}\right)\tan\theta$$

$$= \left(+1 \mp \frac{\omega g}{\omega_j(\omega_2-\omega_1)U\cos\theta}\right)\tan\theta$$

$$= \frac{+g\tan\theta}{U\omega_j(\omega_2-\omega_1)}\left\{\omega_j(\omega_2-\omega_1)\frac{U\cos\theta}{g} \mp \omega\right\}$$

$$\therefore \frac{\omega_1'}{\omega_1} = \frac{g\tan\theta}{U\omega_1(\omega_2-\omega_1)}(\omega-\omega_1) = -\frac{\omega_1}{\omega_2-\omega_1}\tan\theta$$

$$\frac{\omega_2'}{\omega_2} = \frac{\omega_2}{\omega_2-\omega_1}\tan\theta \qquad (A\cdot 12)$$

$$\left(\frac{\omega_j'}{\omega_j}\right)' = \frac{\omega_j}{\omega_j-\omega_{j+1}}\sec^2\theta + \tan\theta\left(\frac{\omega_j'}{\omega_j-\omega_{j+1}} - \frac{\omega_j(\omega_j'-\omega_{j+1}')}{(\omega_j-\omega_{j+1})^2}\right)$$

$$= \frac{\omega_j\sec^2\theta}{\omega_j-\omega_{j+1}} + \frac{\tan\theta\,\omega_j\omega_{j+1}}{(\omega_j-\omega_{j+1})^2}\left(\frac{\omega_j\omega_{j+1}'}{\omega_{j+1}} - \frac{\omega_j'\omega}{\omega_j}\right)$$

$$= \frac{\omega_j\sec^2\theta}{\omega_j-\omega_{j+1}} - \frac{\tan^2\theta\,(\omega_j+\omega_{j+1})\omega_j\omega_{j+1}}{(\omega_j-\omega_{j+1})^3} = \frac{\omega_j}{\omega_j-\omega_{j+1}} + \frac{\tan^2\theta\,\omega_j}{(\omega_j-\omega_{j+1})^3}\left[(\omega_j-\omega_{j+1})^2 - \omega_{j+1}(\omega_j+\omega_{j+1})\right]$$

$$= \frac{\omega_j}{\omega_j-\omega_{j+1}} + \frac{\omega_j\tan^2\theta}{(\omega_j-\omega_{j+1})^3}\left(\omega_j^2 - 3\omega_j\omega_{j+1}\right), \quad (A\cdot13)$$

$$\left(\frac{k'}{k}\right)' - \left(1+\frac{k'^2}{k^2}\right) = 2\left(\frac{\omega_j'}{\omega_j}\right)' - \left\{1+4\left(\frac{\omega_j'}{\omega_j}\right)^2\right\}$$

$$= 2\left[\frac{\omega_j}{\omega_j-\omega_{j+1}} + \frac{\omega_j\tan^2\theta}{(\omega_j-\omega_{j+1})^3}\left(\omega_j^2-3\omega_j\omega_{j+1}\right)\right] - 1 - \frac{4\omega_j^2\tan^2\theta}{(\omega_j-\omega_{j+1})^2}$$

$$= \frac{\omega_j+\omega_{j+1}}{\omega_j-\omega_{j+1}} + \frac{2\omega_j^2\tan^2\theta}{(\omega_j-\omega_{j+1})^3}\left(\omega_j - 3\omega_{j+1} - 2(\omega_j-\omega_{j+1})\right)$$

$$= \frac{\omega_j+\omega_{j+1}}{\omega_j-\omega_{j+1}}\left[1 - \frac{2\omega_j^2\tan^2\theta}{(\omega_j-\omega_{j+1})^2}\right], \quad \cdots (A\cdot14)$$

$$f''(\theta)\Big|_{\text{station}} = -k\left[\left(\frac{k'}{k}\right)' - 1 - \frac{k'^2}{k^2}\right]\cos(\theta-\varphi)$$

$$= \pm k\left[\left(\frac{k'}{k}\right)' - 1 - \frac{k'^2}{k^2}\right]\Big/\sqrt{1+\left(\frac{k'}{k}\right)^2}$$

$$= \pm k\left(\frac{\omega_j+\omega_{j+1}}{\omega_j-\omega_{j+1}}\right)\frac{1-\frac{1}{2}\left(\frac{k'}{k}\right)^2}{\sqrt{1+\left(\frac{k'}{k}\right)^2}}, \quad (A\cdot15)$$

2.12 $\dfrac{\omega_1+\omega_2}{\omega_2-\omega_1} = \dfrac{1}{\sqrt{1-\frac{4\omega U}{g}\cos\theta}}$,

$$f'''(\theta) = (k''' - k')\cos(\theta-\varphi) - (k'' - k)\sin(\theta-\varphi) - 2k''\sin(\theta-\varphi)$$
$$- 2k'\cos(\theta-\varphi)$$

$$= (k''' - 3k')\cos(\theta-\varphi) - (3k'' - k)\sin(\theta-\varphi), \quad (A\text{-}16)$$

$$f'''(\theta)\Big|_{f=0} = \left(k''' - 3k' - \frac{(3k''-k)k'}{k}\right)\cos(\theta-\varphi) = \frac{kk''' - 3k'k + kk' - 3k'k''}{k}$$

$$= \left(k''' - 2k' - 3\frac{k'k''}{k}\right)\cos\theta = \left[k\left(\frac{k'}{k}\right)'' - 2k' - \frac{2k'^3}{k^2}\right]\cos(\theta-\varphi)$$

$$= k\left[\left(\frac{k'}{k}\right)'' - 2\frac{k'}{k}\left(1 + \frac{k'^2}{k^2}\right)\right]\cos(\theta-\varphi),$$

$$= k\left[\left\{\left(\frac{k'}{k}\right)' - \left(1 + \frac{k'^2}{k^2}\right)\right\}'\right.$$