Dynamic Analysis of Knowledge Sharing of Agents with Heterogeneous Knowledge

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Abstract
In this paper, we consider knowledge sharing as basic constitutes of social interaction. The problem of knowledge sharing of self-interested agents with heterogeneous knowledge is formulated as knowledge trading games. Agents consider to trade a set of heterogeneous knowledge on which they have different value judgments. The knowledge transactions are formulated as symmetric coordination games. We can aggregate their idiosyncratic value judgments with threshold. We investigate how knowledge types held by agents may influence knowledge sharing.

1. Introduction
Many social interactions can be treated as transaction [1] [7]. We focus on knowledge sharing as the constitute part of social interactions and formulate knowledge trading game as a basic methodology. For instance, let consider a group of agents $G = \{A_i : 1 \leq i \leq N\}$ with a set of heterogeneous knowledge to be transacted. Agents desire to exchange their knowledge on which they may have different value judgments. They exchange their private knowledge with other agents of interests, and they benefit by exchanging their knowledge if their utility will be increased. They do transaction on the basis of their own utility through acquiring the new knowledge [3].

In Fig 1, we show a conceptual framework for the creation of new knowledge. If they succeed to share knowledge at the high level, it creates the positive feedback to the creation of new knowledge at the individual level. Such agents are rational in the sense that they only do what they want to do and what they think is in their own best interests. With the knowledge transaction among self-interested agents, they mutually exchange their private knowledge such a way that their utilities can be improved. Each agent needs to reason about the value of knowledge held by the other agent before the transaction. The factors such as the value (worth) of the knowledge possessed by each agent, the utility through acquiring the item, and the transaction cost also provides effect on the mutual an agreement for knowledge transaction.

The goal of our research is to formalize an economic model of knowledge creation by focusing the quantitative aspects of the value of knowledge. And the central issue in this paper is the relationship between the value of knowledge which agents have and the properties of knowledge trading. As agents trade and receive knowledge, they are able to integrate it with their existing stock, and create new knowledge. But this view can be beneficial to those agents who are at least partly capable of understanding and integrating it [5]. We investigate the knowledge trading between individuals and how the various types of knowledge influences knowledge trading.
2. Knowledge Sharing through Knowledge Trading

In this section, we formulate knowledge transaction as knowledge trading games. Agent \( A \) and agent \( B \) have the following two strategies:

- \( S_1 \) : Trade a piece of knowledge
- \( S_2 \) : Does not trade

We assume that the utility function for agent \( i \), \( i = A, B \), with the private knowledge \( \Omega_i \) and the common knowledge \( K \) in Fig 2 is given as the semi-linear function as follows.

\[
U_i(\Omega_i, K) = \Omega_i + v_i(K), \quad i = A, B, \tag{2.2}
\]

We consider a trading situation in which agent \( A \) trades with his knowledge \( X \) and agent \( B \) trades with his knowledge \( Y \). The utility function defined over the common knowledge \( v_{i}(K) \) can be classified as the following three types:

**Definition:** For a pair of knowledge \( X \) and \( Y \), \( (X \neq Y) \)

- (1) If \( v_i(X \lor Y) = v_i(X) + v_i(Y) \), then the value function \( v_i(X) \) is linear.
- (2) If \( v_i(X \lor Y) \geq v_i(X) + v_i(Y) \), then the value function \( v_i(X) \) is convex.
- (3) If \( v_i(X \lor Y) \leq v_i(X) + v_i(Y) \), then the value function \( v_i(X) \) is concave.

Factors such as the value (worth) of knowledge possessed by each agent, the loss for disclosing the knowledge to the other should be considered. Each agent has the different value judgment. The associated payoffs of both agents when they choose the strategy \( S_1 \) or \( S_2 \) are given as shown in Table 1.
Table 1 The payoff matrix

<table>
<thead>
<tr>
<th>Agent A</th>
<th>S₁</th>
<th>S₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>$U_A^1$</td>
<td>$U_B^2$</td>
</tr>
<tr>
<td>S₂</td>
<td>$U_A^3$</td>
<td>$U_B^4$</td>
</tr>
</tbody>
</table>

Each payoff in Table 1 is given as follows:

$$U_A(S_1, S_1) = \Omega_A - X + v_A(X \lor Y) \equiv U_A^1,$$
$$U_A(S_2, S_1) = \Omega_A + v_A(Y) \equiv U_A^3,$$
$$U_B(S_1, S_1) = \Omega_B - Y + v_B(X \lor Y) \equiv U_B^1,$$
$$U_B(S_2, S_1) = \Omega_B + v_B(X) \equiv U_B^3,$$

(2.3)

$$U_B(S_1, S_2) = \Omega_B - Y + v_B(X \lor Y) \equiv U_B^1,$$
$$U_B(S_2, S_2) = \Omega_B + v_B(Y) \equiv U_B^3,$$

(2.4)

The above associated payoffs are interpreted as follows: Once they decide to transact their private knowledge, it is disclosed to the other agent, and it becomes common knowledge. When both agents decide to trade their private knowledge, the payoffs of both agents are defined as their values of common knowledge minus their values of private knowledge. If agent $A$ trades his private knowledge $X$ and agent $B$ does not trade, his private knowledge $X$ becomes common knowledge, and he may lose some value from this change. On the other hand, if agent $A$ does not trade and agent $B$ trades his private knowledge $Y$, he receive some payoff since the private knowledge $Y$ becomes common knowledge. This is distinguished difference of knowledge trading from physical commodity. With the trade of knowledge, agents do not lose the value of his traded item. Furthermore they may receive some value even if they do not trade and their partner trades.

Subtracting $U_A^3$ from $U_A^1$, and $U_B^3$ from $U_B^1$ in the payoff matrix of Table 1, we define the following payoff parameters:

$$\alpha_A \equiv U_A^1 - U_A^3 = -X + v_A(X \lor Y) - v_A(Y)$$
$$\beta_A \equiv U_A^3 - U_A^1 = X - v_A(X)$$

$$\alpha_B \equiv U_B^1 - U_B^3 = -Y + v_B(X \lor Y) - v_B(X)$$
$$\beta_B \equiv U_B^3 - U_B^1 = Y - v_B(Y)$$

(2.5)

If agent $A$ does not trade and agent $B$ trades, he receives the positive payoff by acquiring new knowledge $Y$. If both agents do not trade, they receive nothing. The parameter $\alpha_i$ represents the merit of trading. On the other hand, the parameter $\beta_i$ represents the risk of trading. By aggregating those payoffs, we define the following parameters, which represent the values of integrating two independent knowledge $X$ and $Y$.

$$\alpha_i + \beta_i = v_i(X \lor Y) - v_i(X) - v_i(Y) \quad i = A, B,$$

(2.6)

If the value function $v_i(K), i = A, B$, defined over their trading knowledge $X$ and $Y$ are convex, then we have $\alpha_i + \beta_i > 0, i = A, B$. And we assume the parameter $\beta_i, i = A, B$, is not negative.

We introduces the following parameters defined as thresholds.
\[
\theta_A = \frac{\beta_A}{(\alpha_A + \beta_A)} = \frac{\{X - v_A(X)\}}{\{v_A(X \lor Y) - v_A(Y)\}} \quad \theta_B = \frac{\beta_B}{(\alpha_B + \beta_B)} = \frac{\{Y - v_B(Y)\}}{\{v_B(X \lor Y) - v_B(X)\}} 
\] (2.7)

By using these parameters, the payoff matrix in Table 1 can be transformed the payoff matrix in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Agent B</th>
<th>S₁</th>
<th>S₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agent A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₁ (Trade)</td>
<td>1−θ₁</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1−θ₄</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₂ (Not to trade)</td>
<td>0</td>
<td>0</td>
<td>θ₂</td>
</tr>
</tbody>
</table>

Table 2 The Payoff Matrix of Agent

If the probability of the other agent to trade is given by \( p \), the expected utility of agent \( i \) when he chooses the strategy \( S₁ \) or \( S₂ \) is given as follows:

\[
\bar{U}_i(S₁) = p(1 - \theta_i), \quad \bar{U}_i(S₂) = (1 - p)\theta_i. \quad (2.8)
\]

The optimal trading rule is obtained as the functions of their threshold \( \theta_i \), \( i = A, B \), as follows:

(i) If \( p \geq \theta_i \), then trade \( S₁ \).
(ii) If \( p < \theta_i \), then do not trade \( S₂ \). \quad (2.9)

Since each agent has different threshold \( \theta_i \) reflecting his idiosyncratic value judgment over heterogeneous knowledge, the optimal transaction rule of each agent in (2.9) differs in general.

3. Characterizing Heterogeneous Knowledge by Threshold

In this section, we characterize knowledge by threshold. The threshold defined in (2.7) is associated with these pieces of knowledge, which reflects their value judgment at knowledge trading. Here we assume that each agent reasons the value of other agents in term of that of his own knowledge, and the value of knowledge of his partner can be approximated in terms of the value of his own knowledge. For instance, an agent with knowledge of the value \( X \) reasons the value of the other agent of knowledge \( Y \) as \( Y = \alpha X (\alpha > 0) \). If \( 0 < \alpha < 1 \), that agent transacts with the agent of having low value knowledge, and if \( \alpha > 1 \), that agent transacts with the agent of having high value knowledge.

As example, we specify the value function of an agent as follows:

Convex function : \( v_i(X) = kX \ln(X) \) \quad (3.1)

If the value function of agent \( A_j \) is convex as given in (3.1), we can approximate it as follows:

\[
v(X \lor \alpha X) - v(X) - v(\alpha X) = kX(\ln(1 + \alpha) + \alpha \ln(1 + 1/\alpha)) \quad (3.2)
\]

\[
X - v(X) = X(1 - k \ln X) \quad (3.3)
\]

Therefore, if value function of agent \( A_i \) is convex, his threshold in (2.7) can be approximated by

\[
\theta(X, \alpha) \equiv \frac{1/k - \ln X}{\ln(1 + \alpha) + \alpha \ln(1 + 1/\alpha)} \quad (3.4)
\]

The threshold of agent \( A_i \) is the function of both the value of his own knowledge \( X \) and the relative value of his trading partner given \( \alpha \). Fig 3 shows the relation between the value of knowledge and threshold. In this figure, we find that if the value of his own knowledge \( X \) increases, then her threshold decreases. This implies that the agent of convex value function is willing to trade his knowledge if he has knowledge of high quality.
Furthermore, in Fig 3 if agent estimates that trading partner have more valuable knowledge, threshold decreases, and agent willing to trade his knowledge. The agent of convex value function decides whether he trades or not from the value of own knowledge and trading partner's.

We consider the knowledge transaction between agent $A$ with the set of knowledge $\Omega_A = \{X_i: 1 \leq i \leq N\}$ and agent $B$ with the set of knowledge $\Omega_B = \{Y_i: 1 \leq i \leq N\}$. Each agent makes his decision on each piece of his knowledge, whether he trades it or does not trade as the order listed in the sets $K_A$ and $K_B$. As examples, we illustrate several threshold distributions defined over the sets of heterogeneous knowledge, which are approximated as the continuous functions.

Type 1 is the agent with high value of knowledge. This agent has a lot of knowledge of low threshold and he or she willing to disclose his knowledge. Type 2 is the agent with knowledge of intermediate value. He or she has only knowledge of mean value. Type 3 is the agent with knowledge of low value. This agent has a lot of knowledge of high threshold and he or she does not trade.

**Fig 3** The relation between the value of knowledge and threshold

We approximate the discrete functions $n_i(\theta)/N$, $i = A, B$ by the continuous function $f_i(\theta)$, $i = A, B$, which are defined as the density function of threshold. Then the proportions of knowledge which threshold is less than $\theta$ are given by

**Fig 4 Knowledge distribution in thresholds**

4. **The Dynamic Process of Knowledge Sharing of Agents with Heterogeneous Knowledge**

In this section, we characterize repeated knowledge transaction between heterogeneous agents. Because each agent has various types and level of knowledge. That means both agents have approximately different value of knowledge. We show the property of knowledge transaction changes by the trading partner and own knowledge value. We denote the proportion of knowledge which has the same threshold $\theta$ by $n_i(\theta)/N$, $i = A, B$. We approximate the discrete functions $n_i(\theta)/N$, $i = A, B$ by the continuous function $f_i(\theta)$, $i = A, B$, which are defined as the density function of threshold. Then the proportions of knowledge which threshold is less than $\theta$ are given by
\[ F_i(\theta) = \int_{x_i \leq \theta} f(\lambda) d\lambda, \quad i = A, B, \] (4.1)

which are defined as the accumulative distributions of threshold of agent \( \theta_i, \ i = A, B \).

We denote the proportion of the successful trading by the \( t \)-th transaction by \( x(t) \) for agent \( A \), by \( y(t) \) for agent \( B \). Since the optimal transaction rule was given in (2.9), agent \( A \) will transact his knowledge which threshold satisfies \( y(t) \geq \theta_A \). Similarly, agent \( B \) will transact his knowledge with the threshold satisfying \( x(t) \geq \theta_B \). The proportion of knowledge by the next time period \( t+1 \) are given by \( F_A(y(t)) \) for agent \( A \) and by \( F_B(x(t)) \) for agent \( B \). Then the proportions of knowledge to be traded are described by the following dynamics:

\[
\begin{align*}
x(t+1) &= F_A(y(t)) \\
y(t+1) &= F_B(x(t))
\end{align*}
\] (4.2)

The dynamics are at the fixed point

\[
\begin{align*}
x^* &= F_A(y^*) \\
y^* &= F_B(x^*)
\end{align*}
\] (4.3)

We consider the knowledge transaction of heterogeneous agents who are characterized threshold distribution in terms of value of knowledge.

**Case 1** Agent with knowledge of intermediate value and agent with knowledge of high value

First, we consider the knowledge transaction between agent with knowledge of intermediate value and agent with knowledge of high value. Fig 5(c) denotes the portrait of the dynamic process of knowledge transactions. The x-axis represents the proportion of trading for agent \( A \) \( (x(t)) \), and y-axis represents the proportion of trading for agent \( B \) \( (y(t)) \). The dynamics have two stable equilibriums \( E_0 \) and \( E_3 \). At the lowest equilibrium \( E_0 \) where \( (x, y) = (0,0) \), both agents do not trade any knowledge. On the other hand, at the highest equilibrium \( E_3 \) where \( (x, y) = (1,1) \), both agents trade all their knowledge. If the initial estimation \( (x(0), y(0)) \) is in the area of the region (I), the dynamics converge to \( E_0 \). On the other hand, if it is in the region (IV), the dynamics converge to \( E_3 \). In this case, the proportion of converging to \( E_3 \) is so high, because agent trade with the partner who has high valuable knowledge. The partner actively trades his knowledge, agent also willing to trade.

![Diagram](image)

(c) The portrait of the dynamic process

**Fig 5 The portrait of the dynamic knowledge transaction process**
(case 2) Agent with knowledge of intermediate value and agent with knowledge of low value

Next, we consider the knowledge transaction between agent with intermediate value of knowledge and agent with low value of knowledge. In comparing a previous example, we find that the region (I) is larger, and the region (IV) is so smaller. Because agent B with knowledge of low value does not trade positively. If agent A trade so larger proportion of his knowledge, that means only if agent A also trades knowledge of low value, agent B have same level of knowledge and he also trade his knowledge. In this case, we find that it is difficult to trade with each other.

(a) agent A (Knowledge of intermediate value)

(b) agent B (Knowledge of low value)

(c) The portrait of the dynamic process

Fig 6 The portrait of the dynamic knowledge transaction process

(case 3) Agent with knowledge of high value and agent with knowledge of low value

Finally, we consider the knowledge transaction between the agent with knowledge of high value and the agent with knowledge of low value. In this case, these agents have knowledge of completely different level. We find that it is difficult for them to integrate their knowledge. They repeat the miss-coordination because the existence of small asymmetry in the initial stage.

(a) agent A (Knowledge of high value)

(b) agent B (Knowledge of low value)

(c) The portrait of the dynamic process

Fig 7 The portrait of the dynamic knowledge transaction process
5. Conclusion

We investigate knowledge transaction between heterogeneous agents and characterize the knowledge transaction in terms of the value of knowledge. Agents are heterogeneous in many ways. We characterize them by threshold distribution in terms of value of knowledge which agents have. We usually change the attitude by the partner who they interact. We find the same attitude through knowledge transaction. Agents can get new knowledge through knowledge transaction, and they can integrate it with own existing knowledge and create new knowledge. But we find that knowledge level plays a significant role in sharing and creating knowledge.

Reference