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A dynamic logistics coordination model for evacuation and support in disaster response activities

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Abstract

This paper describes an integrated location-distribution model for coordinating logistics support and evacuation operations in disaster response activities. Logistics planning in emergencies involves dispatching commodities (e.g., medical materials and personnel, specialised rescue equipment and rescue teams, food, etc.) to distribution centres in affected areas and evacuation and transfer of wounded people to emergency units. During the initial response time it is also necessary to set up temporary emergency centers and shelters in affected areas to speed up medical care for less heavily wounded survivors. In risk mitigation studies for natural disasters, possible sites where these units can be situated are specified according to risk based urban structural analysis. Logistics coordination in disasters involves the selection of sites that result in maximum coverage of medical need in affected areas. Another important issue that arises in such emergencies is that medical personnel who are on duty in nearby hospitals have to be re-shuffled to serve both temporary and permanent emergency units. Thus, an optimal medical personnel allocation must be determined among these units. The proposed model also considers this issue.

The proposed model is a mixed integer multi-commodity network flow model that treats vehicles as integer commodity flows rather than binary variables. This results in a more compact formulation whose output is processed to extract a detailed vehicle route and load instruction sheet. Post processing is achieved by a simple routing algorithm that is pseudo-polynomial in the number of vehicles utilized, followed by the solution of a linear system of equations defined in a very restricted domain. The behavior and solvability of the model is illustrated on an earthquake scenario based on Istanbul's risk grid as well as larger size hypothetical disaster scenarios.
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Keywords: Logistics in disaster response activities; Integrated location and distribution problem; Integer multi-commodity network flow problem

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1. Introduction: Logistical setting in disaster response activities

Logistics support and evacuation are two major activities in disaster response. Evacuation activities take place during the initial response phase whereas logistics support operations tend to continue for a longer time for sustaining the basic needs of survivors who remain in the affected area. The timely availability of commodities such as food, shelter and medicine and effective transportation of the wounded affect the survival rate in affected areas.

The model proposed here aims to coordinate the transportation of commodities from major supply centers to distribution centers in affected areas and the transport of wounded people from affected areas to temporary and permanent emergency units. The goal is to minimize delay in providing prioritized commodity and health care service. Both wounded people and commodities are categorized into a priority hierarchy, where different types of vehicles are utilized to serve priority transportation needs. The model involves a network flow formulation, where wounded people and vehicles are treated as integer valued commodities. This results in an efficient formulation where vehicles are not tracked individually. Once solved, routes and pick up/delivery instructions of vehicles are constructed from model solution. The model also enables the selection of the best locations of temporary emergency units to be put up without including binary variables for the location problem. Instead, it deals with the location problem implicitly by allocating optimal service rates to medical centers and emergency units according to which patients are discharged from the system. Further, it is assumed that service capacities of temporary emergency centers are supplied by major medical centers in the area. Hence, an optimal equilibrium among re-allocated service levels is identified to minimize transportation delay for patients with different priorities and localities. We can classify the model as an integrated capacitated location-routing model (LRP) with a network flow based routing formulation.

The proposed modeling framework is designed as a flexible dynamic (multi-period) coordination instrument that can adjust to frequent information updates, vehicle re-routing and re-allocation of service capacities. The planning horizon under consideration is short (in days or even hours) due to the fact that information flow is continuous after disasters and initial screening cannot capture the attrition numbers accurately, specially, in earthquakes where many people are under the debris. Continuity of commodity logistics is achieved by incorporating anticipated commodity demand for future periods.

LRP models have been investigated in detail resulting in an abundant literature. These models integrate the discrete facility location (FLP) and vehicle routing problems (VRP). Both FLP and VRP are NP-Hard, however, VRP is usually considered to be more inhibiting for exact methods. The classical FLP selects the best p sites among a range of possible locations with the objective of minimizing total demand-weighted travel distance between demand nodes and facilities (p-median problem). Other objectives considered for FLP are minimizing fixed costs of selected facilities (set covering problem), maximizing the coverage of demand (maximal covering problem), and minimizing maximum distance between demand-facility pairs (p-center problem) (for descriptions of these problems earlier references are, for instance, Hakimi, 1964, 1965; Toregas et al., 1971; Church and ReVelle, 1974, and more recent references are Schilling et al., 1993; Daskin, 1995). In FLP, facilities might have fixed capacities or their sizes might have to be optimized (Mukundan and Daskin, 1991). Extensions to FLP also include the allocation problem where facilities are located simultaneously with the flows between demand nodes and facilities (location–allocation problem). Salhi and Rand (1989) show that when tours are not explicitly considered in this model, distribution costs may increase. Owen and Daskin (1998) provide an extensive survey on FLP and its extensions.

Similar to FLP, VRP has posed a challenge to researchers and practitioners for a long time. In the classical capacitated VRP, a number of customers (each represented as a destination node) are served by m identical capacitated vehicles located at a depot and customers can be visited only once. It is assumed that a vehicle's load capacity exceeds every customer's demand. The aim is to determine vehicle routes resulting in the minimum total travel distance. VRP assumes that sufficient supply is always available at the depot to satisfy all customer demand. Some of these restrictions have been relaxed by researchers to accommodate multiple depots, non-homogeneous fleets, multiple visits to a customer (split delivery, Dror and Trudeau, 1990), and, deliveries/pick-ups on the same tour (mixed delivery approach) where only one order is carried at a time. Among more efficient models proposed for VRP, one can list that of Ribeiro and Soumis (1994) who treat vehicles as commodities. However, in their model delivery cannot be split. Additionally, tours are *predeter-*

mined and the problem is reduced to assigning a vehicle to every tour. Fisher et al. (1995) also adopt the network approach, but the model is simplified by approximating loads to be transported to multiples of truckloads. The model output might also be infeasible and therefore need repair. Surveys on the VRP are found in Desrochers et al. (1988) and Bodin (1990).

The integrated location-routing model (LRP) subsumes both FLP and VRP, and optimizes the locations and capacities of facilities as well as vehicle routes and schedules. A classification of LRP models is given by Min et al. (1998) and it is based on VRP features such as only delivery or mixed delivery, single or multiple vehicles, uncapacitated or capacitated vehicles, and time windows, and FLP features such as uncapacitated or capacitated facilities and facility layers (existence of transshipment nodes-secondary facilities). General features related to demand, such as single- or multi-period (static/dynamic) and deterministic versus stochastic are also factors in model category. Albareda-Sambola et al. (2005) survey LRP models and point out that LRP with primary and secondary facilities and capacitated vehicles is hard to solve. Generally, the VRP assumptions incorporated into LRP describe very simple settings where a facility is served by only one capacitated vehicle that conducts a single tour and a client can only be served by one facility implying a no-split delivery assumption- (Tuzun and Burke, 1999; Albareda-Sambola et al., 2005).

According to the taxonomy provided by Min et al. (1998), our model falls into the following category: mixed split delivery, deterministic dynamic demand (with anticipated multi-period demand and supply), multiple capacitated facilities whose sizes have to be optimized, and heterogeneous capacitated vehicle fleet. This model has some major deviations from standard LRP models with regard to the VRP sub-problem.

(i) Supply availability is limited because the impact of the disaster might be greater than anticipated and there is always transportation delay from major warehouses. It might also be difficult to have high levels of stocks for non-durable commodities at all times before the disaster occurs.
(ii) Vehicle routing and availability conditions are quite different from commercial settings. During the first response and throughout ongoing relief operations, a vehicle is not required to return immediately to a supply node (depot) once its current assignment is completed. It can wait for the next instruction at its last destination or may move towards a depot at the end of the shift if drivers are required to change shifts. Thus, the standard VRP definition of a tour becomes redundant and it is replaced by an itinerary that starts at the beginning of the shift and continues until its completion.

These two features result in a highly flexible model that enables a structure where vehicles are not tracked individually, but are represented as integral commodities crossing arcs in different periods. Hence, the model can deal with large numbers of vehicles deployed in disaster response. This is an advantage over existing LRP models. Furthermore, the service policy does not have restrictions such as the assignment of a single vehicle to each supply node or customer, nor does it restrict the number of times a customer can be visited. Vehicles may execute mixed delivery trips where commodities and injured people are picked up and delivered in an arbitrary sequence.

Since the model output consists of integer arc based flows for vehicles, a pseudo-polynomial algorithm is developed to convert the solution into vehicle itineraries. Load/unload schedules are then obtained by solving a linear set of equations that use vehicle itineraries and the original model output that involves arc based wounded people and commodity flows. This model structure also facilitates the execution of the re-planning procedure that is activated at regular time intervals to incorporate demand, supply and vehicle availability updates.

The FLP sub-problem in the model involves a selection of locations for temporary emergency centers given several site possibilities in affected areas. Such centers are established after the disaster to provide immediate care for injured people whose conditions do not need immediate service from fully equipped hospitals and hence, they reduce congestion at major medical centers. However, especially during the initial response time, these temporary centers utilize the medical resources of hospitals in the area (medical personnel and equipment) and consume some of the total health care capacity in the region. Depending on how fast care can be provided (based on transportation time needed to reach temporary centers and hospitals) and on the number and classification of injured people, the model formulates an equilibrium among all service levels. In our model, binary variables that typically take place in FLP are excluded, since there is no fixed cost incurred when a temporary center is built.

The goal in the proposed model is to minimize the delay in the arrival of commodities at aid centers and in the provision of healthcare for the injured. Heavily and moderately injured people hold the first and second priorities whereas medicine holds the highest priority among all commodities. The remaining commodities are prioritized appropriately. All elements of the objective function including the commodities are represented in an equivalent common unit of number of persons not served. As the latter is the outcome of distance traveled and demand, the objective function is similar to the *p-median* criterion. Other criteria such as number of vehicles utilized in support operations and cost of inventories held can be incorporated in the objective function in the same common unit, however, in the initial response period, these objectives are hardly considered.

2. A two-stage procedure for solving disaster response logistics coordination problem

The complex logistics problem described in the previous section is solved by a two-stage procedure where a compact model that treats vehicles as integer flows is formulated and solved in the first stage. In the second stage, detailed vehicle instructions are obtained using a simple vehicle splitting algorithm that converts integer vehicle flows into binary vehicle itineraries and then by solving a set of linear equations to assign a loading/unloading schedule to each such itinerary.

2.1. Stage I: Formulation of a compact model

The mathematical formulation of the problem and the notation are given below. For the sake of simplicity, we assume a single-transportation mode and a corresponding heterogeneous fleet valid for that mode.

A dummy node, (node θ), is added to the network for dealing with vehicle availability at the time of planning. This node has a link (with a traversal time of zero) to each supply node where vehicles of any type are available at the beginning of the planning horizon or during ongoing operations.

Sets and parameters:

T: length of the planning horizon (e.g., a working shift)
$\mathscr{C}$: set of commodities
$\mathscr{H}$: set of different categories of wounded people (heavy, moderate-light)
$\mathscr{M}$: set of different vehicle types or transportation modes
$\mathscr{A}$: set of arcs
$\mathscr{N}$: set of nodes (Node 'θ' is designated as a dummy node)
$\mathscr{DN}$: set of demand nodes including transshipment nodes, $\mathscr{DN} \subset \mathscr{N}$
$\mathscr{SN}$: set of supply nodes, $\mathscr{SN} \subset \mathscr{N}$
$\mathscr{EN}$: set of potential temporary emergency center sites that can be established in affected area and receive light-moderate category of wounded people, $\mathscr{EN} \subset \mathscr{N}$
$\mathscr{HN}$: set of available hospitals whose emergency units can receive both heavy and light-moderate wounded people, $\mathscr{HN} \subset \mathscr{N}$
t_{op}: time required to traverse arc (o,p) with damage consideration ($t_{\mathrm{op}} = 0$ for $(o,p) \notin \mathscr{A}$)
d_{cot}: amount of demanded or supplied commodity type c at node o at time t, positive for supply and negative for demand
f_c: standard amount of commodity c required to sustain one affected person during a shift
d_{hot}: number of wounded people of category h waiting at node $o \in \mathscr{DN}$ at time t ($d_{hot} \geqslant 0$)
av_{omt}: number of type m vehicles *added* to the fleet at node o at time t
w_c: Unit weight of commodity c
w_h: average weight of a wounded person
cap_m: load capacity of vehicle type m
s^0_{ho}: initial per period service rate for category h wounded people at hospital at node $o \in \mathscr{HN}$
ρ: maximum percentage of total permanent hospital emergency unit capacity that can be shifted to temporary emergency units
ρ', ρ'': allowable expansion/contraction percentage of permanent hospital emergency unit service rate ($\rho' > 1.0$; $\rho'' < 1.0$)

B: a big number
P_c: priority of satisfying demand of commodity type c (set equal to: $P_c = k_c/\left[\sum_t \sum_{o\in\mathscr{DN}} - d_{cot}\right]$, where k_c is a positive subjective parameter)
P_h: priority of serving wounded people of category h (set equal to: $P_h = k_h/\left[\sum_t \sum_{o\in\mathscr{N}\setminus\mathscr{SN}} d_{hot}\right]$, where $k_h \gg k_c$)
A_{opt}: binary parameter matrix: if $t < t_{op}$, then $\mathsf{A}_{opt} = 0$, else $\mathsf{A}_{opt} = 1$

Decision variables:
Q_{copt}: amount of commodity type c traversing arc (o,p) at time t
DEV_{cot}: amount of unsatisfied demand of commodity type c at node o at time t
X_{hopt}: integer number of wounded people of category h traversing arc (o,p) at time t
DEV_{hot}: number of unserved wounded people of category h at node $o \in \mathscr{N}\setminus\mathscr{SN}$ at time t
V_{opmt}: integer number of vehicles of type m traversing the arc (o,p) at time t
SX_{hot}: number of wounded people of category h who are served at node $o \in \mathscr{EN}\cup\mathscr{HN}$ at time t
S_{ho}: service rate of an emergency facility $o \in \mathscr{EN}\cup\mathscr{HN}$

The objective aims at minimizing the weighted sum of unsatisfied demand over all commodities and weighted sum of wounded people waiting at demand nodes and temporary and permanent emergency units. Commodities are represented in their people equivalents. This objective is compatible with the goal of minimizing service delay.

Model P:

$$\text{Minimize} \quad \sum_{c\in\mathscr{C}} \sum_{o\in\mathscr{DN}} \sum_t (P_c DEV_{cot}) \Big/ f_c + \sum_{h\in\mathscr{H}} \sum_{o\in\mathscr{N}\setminus\mathscr{SN}} \sum_t P_h DEV_{hot} \tag{0}$$

Subject to:

Constraint set (1) balances material flow on demand nodes and transhipment nodes and explicitly reports unsatisfied demand, DEV_{cot}, in each time period. Constraint set (2) enforces material flow balance on supply nodes. In both constraints, supply or demand is adjusted according to wounded people transported from or arrived at these nodes. Hence, in the optimal solution, wounded people are not transported to emergency centers and form long queues at a given node if commodities are inadequate. Knowledge on future demand is predicted based on current demand. Confirmed arrivals represent supplies in future periods, thereby enabling continuity of routing plans throughout multiple planning horizons.

$$\sum_{\tau=1}^{t}\left[-\sum_{p\in\mathscr{N}} \mathsf{A}_{po\tau} Q_{cpo\tau} + \sum_{p\in\mathscr{N}} Q_{cop\tau}\right] - DEV_{cot} = \sum_{\tau=1}^{t}\left[d_{co\tau} - \sum_{h\in\mathscr{H}} \sum_{p\in\mathscr{N}} f_c X_{hop\tau}\right] \quad (\forall c\in\mathscr{C}, o\in\mathscr{DN}, t\in T), \tag{1}$$

$$\sum_{\tau=1}^{t}\left[-\sum_{p\in\mathscr{N}} \mathsf{A}_{po\tau} Q_{cpo\tau} + \sum_{p\in\mathscr{N}} Q_{cop\tau}\right] \leqslant \sum_{\tau=1}^{t}\left[d_{co\tau} - \sum_{h\in\mathscr{H}} \sum_{p\in\mathscr{N}} f_c \mathsf{A}_{po\tau} X_{hpo\tau}\right] \quad (\forall c\in\mathscr{C}, o\in\mathscr{SN}\cup\mathscr{EN}, t\in T). \tag{2}$$

Constraints (3) restrict the itinerary of each vehicle type to existing arcs. Constraints (4) restrict transportation quantities by the capacity of vehicles traversing the arc.

$$V_{opmt} \leqslant Bt_{\mathrm{op}} \quad (\forall (o,p)\in\mathscr{A}, m\in\mathscr{M}, t\in T), \tag{3}$$

$$\sum_{m\in\mathscr{M}} V^{*}_{opmt} cap_m \geqslant \sum_{c\in\mathscr{C}} w_c Q_{copt} + \sum_{h\in\mathscr{H}} w_h X_{hopt} \quad (\forall (o,p)\in\mathscr{A},\ t\in T). \tag{4}$$

Constraints (5) ensure that all vehicles leaving supply nodes connected to the dummy node during the planning horizon come back to any of these start nodes at the end of the working shift. The next set of constraints (6) balance the flow of vehicles over each node. Constraints (7) restrict the number of vehicles introduced to the network by their cumulative availability over time. Thus, it is also possible to plan ahead with a dynamic number of available vehicles varying over time.

$$\sum_{\tau=1}^{T}\sum_{p\in\mathscr{N}} V_{p0m\tau} = \sum_{\tau=1}^{T}\sum_{p\in\mathscr{N}} V_{0pm\tau} \quad (\forall m \in \mathscr{M}), \tag{5}$$

$$\sum_{\tau=1}^{t}\sum_{p\in\mathscr{N}} \mathsf{A}_{po\tau} V_{pom\tau} \geqslant \sum_{\tau=1}^{t}\sum_{p\in\mathscr{N}} V_{opmt} \quad (\forall o \in \mathscr{N}\setminus\theta,\ m\in\mathscr{M},\ t\in T), \tag{6}$$

$$\sum_{\tau=1}^{t} V_{\theta om\tau} \leqslant \sum_{\tau=1}^{t} av_{om\tau} \quad (\forall o \in \mathscr{S}\mathscr{N},\ m\in\mathscr{M},\ t\in T). \tag{7}$$

Constraints (8) define wounded people still waiting at demand nodes whereas constraints (9) define those that are waiting in hospital queues. Here, queue size is reduced by those who have already been served and sent out of the emergency system.

$$\sum_{\tau=1}^{t}\left[-\sum_{p\in\mathscr{N}} \mathsf{A}_{po\tau} X_{hpo\tau} + \sum_{p\in\mathscr{N}} X_{hop\tau}\right] + DEV_{hot} = \sum_{\tau=1}^{t} d_{ho\tau} \quad (\forall h\in\mathscr{H},\ o\in\mathscr{D}\mathscr{N},\ t\in T), \tag{8}$$

$$\sum_{\tau=1}^{t}\left[\sum_{p\in\mathscr{N}} \mathsf{A}_{po\tau} X_{hpo\tau} - \sum_{p\in\mathscr{N}} X_{hop\tau}\right] - \sum_{\tau=1}^{t} SX_{ho\tau} = DEV_{hot} \quad (\forall h\in\mathscr{H},\ o\in\mathscr{N}\setminus\mathscr{D}\mathscr{N},\ t\in T). \tag{9}$$

Constraints (10) impose an upper bound on the total service rate of temporary emergency units as a percentile of available permanent hospital service rates. Constraints (10′) restrict the number of wounded served in each period by the allocated service rate of the medical center. Constraints (11) fix total service capacity in the region. Constraints (12) impose upper bounds on service capacity expansion/contraction of permanent hospital service rates. The last set of constraints define variable domains.

$$\sum_{o\in\mathscr{E}\mathscr{N}} S_{ho} \leqslant \rho \sum_{o\in\mathscr{H}\mathscr{N}} s_{ho}^{0} \quad (\forall h\in\mathscr{H}), \tag{10}$$

$$SX_{hot} \leqslant S_{ho} \quad (\forall h\in\mathscr{H}, o\in\mathscr{E}\mathscr{N}\cup\mathscr{H}\mathscr{N}\, t\in T) \tag{10'}$$

$$\sum_{o\in\mathscr{E}\mathscr{N}\cup\mathscr{H}\mathscr{N}} S_{ho} = \sum_{o\in\mathscr{H}\mathscr{N}} s_{ho}^{0} \quad (\forall h\in\mathscr{H}), \tag{11}$$

$$\rho'' s_{ho}^{0} \leqslant S_{ho} \leqslant \rho' s_{ho}^{0} \quad (\forall h\in\mathscr{H}, o\in\mathscr{H}\mathscr{N}), \tag{12}$$

$$SX_{hot}, V_{opmt}, X_{hopt} \geqslant 0 \text{ and integer; } Q_{copt}, DEV_{cot}, S_{ho}, DEV_{hot} \geqslant 0. \tag{13}$$

Similar to sharing a pool of medical service capacities, the model also enables sharing of commodities among all supply nodes. The overall needs of wounded people heading towards supply nodes with less availability can thus be accommodated as well as those of survivors waiting in the affected areas.

2.2. Stage II: An algorithm for generating vehicle routes and load/unload instructions from solution of stage I

In model P, vehicles are treated as commodities, and they are not tracked individually. Details of dispatch orders for vehicles are obtained by executing an algorithm called "*Route*" that reads the optimal solution and generates a pick up/delivery schedule for each vehicle without specifying the load quantities. After *Route* is executed, loaded/unloaded quantities on vehicle routes are calculated by solving a system of linear equations.

Although this system of equations involves integer variables for wounded people, it is simpler to solve as compared to Model P, because it has a special structure and does not involve any routing decisions. Treating vehicle flows as commodity flows increases the efficiency of model P which is important because the number of vehicles utilized in disasters tends to be quite large and the planning horizon rather short.

2.2.1. Algorithm "Route"

According to the multi-depot, split-delivery multiple tour VRP model, a tour is defined as consecutive arcs traversed by the vehicle between two pick-up actions from a supply node. A complete route of a vehicle is composed of all its consecutive tours. However, here this interpretation is modified. A route is assumed to consist of a single tour that starts from any supply node at the beginning of the planning horizon and ends at its completion.

We define V^*_{opmt} as the optimal non-empty set of type m vehicles traversing arc (o,p) at time t. V^*_{opmt} are sorted in ascending order of t. *Route* picks a starting node o with the minimum time index t and identifies the set $\mathscr{V}_{\text{ot}}$, the union of all non-empty subsets of vehicles departing from node o in period $t(\mathscr{V}_{\text{ot}} = \cup_{p,m} V^*_{opmt})$. An arbitrary subset V^*_{opmt} is taken and decomposed into a set of singular unlabelled vehicles λ^i where i is vehicle index. The maximum value that index i can take is equal to the total number of vehicles utilized in the optimal solution, $\kappa(\kappa = \sum_{\tau=1}^{T}\sum_{m\in\mathscr{M}}\sum_{op\in\mathscr{A}}|V^*_{\text{opm}\tau}|)$. However, since each vehicle route consists of more than one consecutive arcs, the number of vehicles to be traced is actually much smaller than κ. Each λ^i is traced till the end of its itinerary (until period T), and all arcs on its itinerary are recorded on its route, r^i. The value of V^*_{jkmt} is decremented whenever a new arc (jk) is identified on the itinerary of λ^i. Once the total route is completed, another vehicle λ^i, that is an element of V^*_{opmt} is selected and its route traced, until all elements of V^*_{opmt} are labeled. This procedure is repeated for all other subsets $V^*_{opmt} \subseteq \mathscr{V}_{\text{ot}}$ until $\mathscr{V}_{\text{ot}}$ is exhausted. Then, another non-empty departing vehicle set, $\mathscr{V}_{q\tau}(q \neq o$ and/or $\tau \neq t)$ is taken from the sorted list. *Route* has a worst case pseudo-polynomial complexity of $O(\kappa|\mathscr{N}^2|T)$. The pseudocode of the algorithm is given below.

Definitions:

$\mathscr{U}\mathscr{N}$: set of nodes that where all departing vehicles have not been traced
V^*_{opmt}: optimal non-empty set of vehicles traversing arc (o,p) at time t
$\mathscr{V}_{\text{ot}}$: set of all vehicles departing from node o at time $t : \mathscr{V}_{\text{ot}} = \cup_{p\in\mathscr{N}\cup m\in\mathscr{M}} V^*_{opmt}$
λ^i: ith vehicle, $i = 1,\ldots,\kappa$, where $\kappa = \sum_{\tau=1}^{T}\sum_{m\in\mathscr{M}}\sum_{op\in\mathscr{A}}|V^*_{\text{opm}\tau}|$
r^i: set of arcs traversed by λ^i

function *Route*:
Read from file: sorted list (in ascending order of t) V^*_{opmt};
Initialize: $\mathscr{U}\mathscr{N}=\mathscr{N}$; $r^i = \phi$ for $i = 1,\ldots,\kappa$; Set $i = 0$;
while($\mathscr{U}\mathscr{N} \neq \phi$){
  Construct $V_{\text{ot}} = \cup_{p\in\mathscr{N},m\in\mathscr{M}} V^*_{opmt}$ for some $o \in \mathscr{U}\mathscr{N}$, with t minimum on list;
  while($\mathscr{V}_{\text{ot}} \neq \phi$){
  Select any non-empty subset $V^*_{opmt} \subseteq V_{\text{ot}}$;
  while($|V^*_{opmt}| \neq 0$){
    $i = i + 1$;
    Select $\lambda^i \in V^*_{opmt}$;
    Update: $r^i = r^i + \{o,p\}$; $|V^*_{opmt}| = |V^*_{opmt}| - 1$;
    Initialize: Tail $= p$; τt;
    do{
      If any non-empty set $V^*_{\text{Tail,km}\tau}$ exists for any arc $\{\text{Tail},k\} \subseteq \mathscr{A}$
        {
          Update: $r^i = r^i + \{\text{Tail},k\}$; $|V^*_{\text{Tail,km}\tau}| = |V^*_{\text{Tail,km}\tau}| - 1$;

```
            Update: τ = τ + t_{Tail,k}; Tail = k;
          }
          else τ = τ + 1;
      }while (τ ⩽ T);
    }/*endwhile(|V*_{opmt}| ≠ 0)*/
    𝒱_ot = 𝒱_ot − {V*_{opmt}};
  }/*endwhile(𝒱_ot ≠ ϕ)*/
    𝒰𝒩 = 𝒰𝒩 − {o};
}/*endwhile(𝒰𝒩 ≠ ϕ)*/
```

2.2.2. Generating vehicle load/unload instructions for each itinerary

This procedure utilizes the routes of every labeled vehicle to determine picked up and delivered quantities on each route. We define a matrix called R whose binary element $\mathsf{R}[i,o,t]$ indicates that a vehicle λ^i is incident to a node $o \in \mathscr{N}$ in time period t. The time period is required because a node can be traversed more than once by vehicle λ^i on its route throughout the planning horizon. This matrix is one of the three sets of parameters transferred from *Route* and from the solution of model P. The other two sets of parameters are the optimal commodity and people flows over each arc in period t, Q^*_{copt} and X^*_{hopt}. Below we provide the system of equations P1 that identify the number of people and quantity of commodities loaded and unloaded by each vehicle at every node on its route.

Parameters:

- κ: total number of vehicle itineraries identified in *Route*
- R: binary matrix of size $[\kappa \times |\mathscr{N}| \times T]$. "$\mathsf{R}[i,o,t] = 1$" indicates that a vehicle λ^i is incident to node o in time period t (identified by *Route*)
- cap_i: capacity of vehicle λ^i
- Q^*_{copt}: optimal amount of commodity type c traversing arc (o,p) at time t identified by Model P
- X^*_{hopt}: optimal number of wounded people of category h traversing arc (o,p) at time t identified by Model P

Decision variables:

- QP^i_{cot}: quantity of commodity c picked up at node o in period t by vehicle λ^i
- QD^i_{cot}: quantity of commodity c delivered at node o in period t by vehicle λ^i
- XP^i_{hot}: integer number of wounded people of category h picked up at node o in period t by vehicle λ^i
- XD^i_{hot}: integer number of wounded people of category h delivered at node o in period t by vehicle λ^i

System of equations and inequalities P1:

$$\sum_{i\in\kappa}\sum_{\tau=1}^{t}[XP^i_{ho\tau} - XD^i_{ho\tau}] = \sum_{\tau=1}^{t}\sum_{p\in\mathscr{N}}[X^*_{hpo\tau} - X^*_{hop\tau}] \quad (\forall h \in \mathscr{H},\ o \in \mathscr{N},\ t \in T), \tag{14}$$

$$\sum_{i\in\kappa}\sum_{\tau=1}^{t}[QP^i_{co\tau} - QD^i_{co\tau}] = \sum_{\tau=1}^{t}\sum_{p\in\mathscr{N}}[Q^*_{cpo\tau} - Q^*_{cop\tau}] \quad (\forall c \in \mathscr{C},\ o \in \mathscr{N},\ t \in T), \tag{15}$$

$$\sum_{o\in\mathscr{N}}\sum_{\tau=1}^{t}[XP^i_{ho\tau} - XD^i_{ho\tau}] \geqslant 0 \quad (\forall i \in \kappa, h \in \mathscr{H},\ t = 1 \ldots T-1), \tag{16}$$

$$\sum_{o\in\mathscr{N}}\sum_{\tau=1}^{t}[QP^i_{co\tau} - QD^i_{co\tau}] \geqslant 0 \quad (\forall i \in \kappa,\ c \in \mathscr{C},\ t = 1 \ldots T-1), \tag{17}$$

$$\sum_{o\in\mathcal{N}}\sum_{\tau=1}^{T}[XP^{i}_{ho\tau}-XD^{i}_{ho\tau}]=0\quad(\forall i\in\kappa,\ h\in\mathcal{H}),\tag{18}$$

$$\sum_{o\in\mathcal{N}}\sum_{\tau=1}^{T}[QP^{i}_{co\tau}-QD^{i}_{co\tau}]=0\quad(\forall i\in\kappa,\ c\in\mathcal{C}),\tag{19}$$

$$\sum_{h\in\mathcal{H}}[XP^{i}_{hot}+XD^{i}_{hot}]+\sum_{c\in\mathcal{C}}[QP^{i}_{cot}+QD^{i}_{cot}]\leqslant \mathsf{R}[i,o,t]^{*}B\quad(\forall i\in\kappa,\ o\in\mathcal{N}\in T),\tag{20}$$

$$\sum_{\tau=1}^{t}\sum_{h\in\mathcal{H}}\sum_{o\in\mathcal{N}}w_h[XP^{i}_{ho\tau}-XD^{i}_{ho\tau}]+\sum_{\tau=1}^{t}\sum_{c\in\mathcal{C}}\sum_{o\in\mathcal{N}}w_c[QP^{i}_{co\tau}-QD^{i}_{co\tau}]\leqslant cap_i\quad(\forall i\in\kappa,\ t\in T)\tag{21}$$

$$XP^{i}_{hot},XD^{i}_{hot}\geqslant 0,\ \text{and integer};\ QP^{i}_{cot},QD^{i}_{cot}\geqslant 0.$$

Constraints (14) state that in each period the net cumulative number of people (category h) picked up from a node o by all vehicles should be equal to the optimal net picked up quantity found in Model P. Constraints (15) impose similar restriction on commodities. Constraints (16) and (17) state that the cumulative net quantity picked up on the route by each labeled vehicle is non-negative for each type of commodity and category of wounded people. Constraints (18) and (19) are ending conditions that ensure that the quantities picked up by each vehicle is equal to the quantities delivered by the end of the planning horizon. Constraints (20) imply that a vehicle can pick up and deliver from/to a node if and only if it is incident to that node in that period. Constraints (21) restrict the net cumulative quantity picked up by a vehicle over all nodes at any time period by its capacity.

The optimal quantities picked up and delivered by each labeled vehicle are identified by Solving P1. P1 is a fairly easy mixed integer system of equations where the feasible space is tightly restricted by the X^{*}_{hpot} and Q^{*}_{cpot} parameters obtained from Model P. Integer variables are compulsory for accommodating wounded people.

2.3. Comparison of the two-stage modeling approach with a VRP based single-stage formulation

The two-stage procedure for finding vehicle routes and load quantities enables us to utilize an efficient formulation (Model P) that does not involve VRP type of routing constraints to keep track of individual vehicles. Considerable computational burden is eliminated by the network flow type routing formulation in Model P. In a typical VRP formulation there would be $[\kappa\Gamma|\mathcal{N}^2|]$ binary variables where $\Gamma=$ maximum number of routes/vehicle. On the other hand, model P has $[|\mathcal{N}^2|\times|\mathcal{M}|\times T]$ integer variables related to vehicles and the latter is easier to manage when the number of vehicles involved is high. In both cases, $|\mathcal{N}^2|$ can be replaced by $|\mathcal{A}|$ to represent unsuppressed integral variables.

In Appendix A, we provide a dynamic VRP based formulation (Model P2) for our problem. Here, binary variables V_{opit} indicate that vehicle i traverses arc (op) in period t. This formulation has the same type of route definition as in Model P and, hence, there is no need to specify the maximum number of routes Γ. Subtour elimination constraints are avoided by including the time index t in V_{opit}, however, the load carried by vehicle i has to be identified with the same index as there is no aggregation of load over vehicles. The number of vehicle related binary variables in P2 is $[\kappa T|\mathcal{N}^2|]$. This model represents a single-stage approach where vehicle itineraries are identified individually with all load/unload instructions eliminating the need for post-processing.

For illustration purposes, constraints related to service rates of medical centers are not included in P2 and evacuation of the wounded is omitted. In model P, this strategy results in Eqs. (0)–(7) with X_{hopt} excluded. Hence, we compare only the logistics part of the problem by omitting location–allocation sub-problem. Truncated model P is compared with model P2 using a problem from Eilon et al. (1971). The distance matrix of the original problem is preserved while demand-supply-vehicle availability-capacity parameters are assigned arbitrarily to generate different instances of various sizes and parameters. Vehicle tightness is indicated by the ratio of min{total supply, total demand} to total available vehicle capacity. All instances are solved by the MIP solver CPLEX 7.5 on a PC with of 3.2 GHz CPU and 512 MB RAM. The results obtained are given in Table 1 and they are self-explanatory both in the number of constraints, the number of binary/integer variables and solution times. CPU times are given separately for both stages of the approach.

Table 1
Comparison of single-stage VRP based model and two-stage procedure

No.	Input size	Tightness = Load–vehicle capacity	Model size (MB)		Number of binary/integer variables		Number of constraints		Computation time (seconds)			Iteration count	
	$\lvert\mathscr{C}\rvert \times \lvert\mathscr{N}\rvert \times \lvert\kappa\rvert \times \lvert\mathscr{M}\rvert \times \lvert T\rvert$		P2	P	P2	P	P2	P	P2	P		P2	P
										First stage	Second stage		
1	(2, 13, 6, 1, 8)	0.867	7	3	8112	1352	12,943	3522	11.0	1.3	0.36	11,524	1194
2	(2, 13, 6, 2, 8)	0.774	7	5	8112	2704	12,943	6435	10.8	4.5	0.34	5579	7893
3	(2, 13, 9, 1, 8)	0.674	9	3	12,168	1352	19,318	3522	47.1	0.23	0.34	16,514	143
4	(2, 13, 9, 3, 8)	0.674	9	7	12,168	4056	19,318	9348	74.6	3.7	0.36	24,189	3418
5	(2, 13, 12, 3, 8)	0.867	12	7	16,224	4056	25,693	9348	125.8	3.8	0.36	39,380	2769
6	(2, 13, 15, 3, 8)	0.758	15	7	20,280	4056	32,068	9348	227.5	2.0	0.38	54,977	1397

3. Illustration of the two-stage approach on an earthquake scenario

We implement the two-stage procedure on a scenario that describes a possible severe Istanbul earthquake that is expected to take place with 65% probability within the next 30 years. Attrition numbers and possible structural damage of Istanbul are provided in the report prepared by the Earthquake Engineering Dept. (Bogazici University, 2002). Structural risk grades are categorized as VII (20% of buildings-moderate damage), VIII (20–60% of buildings-severe damage) and IX (20–60% of buildings-very severe damage). Based on risk grades and population intensity of districts, it is estimated that about 600,000 people will need immediate shelter after the earthquake. Possible attrition numbers are mapped on Istanbul's district partition (Fig. 1). Fig. 2 illustrates districts where aid distribution centers and temporary medical emergency units may be situated. Nodes 1–6 represent districts with sever damage whereas nodes 7–15 represent those that will be less affected. It is assumed that aid in commodities and medical personnel can be supplied from these low risk districts in immediate response duration. Existing hospital emergency units in the latter districts are calculated in aggregate based on information from local municipalities and Turkish Medical Doctors Association's statistics. Nodes 16–17 represent two smaller cities situated across Marmara Sea and accessible from Istanbul by sea as well as by land. They can provide significant amounts of aid to Istanbul. Nodes 18–20 represent possible aggregate sites for temporary emergency units to provide immediate health care for lightly moderately wounded people. The roads drawn in Fig. 2 represent the network of international highway TEM (Trans European Motorway (TEM) and its peripherals that are particularly dense in European side of Istanbul.

Input data used in the scenario are provided in Table 2 where the distribution of commodity demand and wounded people among affected districts are indicated. In Table 3, supply distribution is indicated for less affected districts as well as vehicle transportation capacity and type (helicopters for wounded people and medicine, trucks for food and ambulances for wounded people). We categorize two levels of injury:

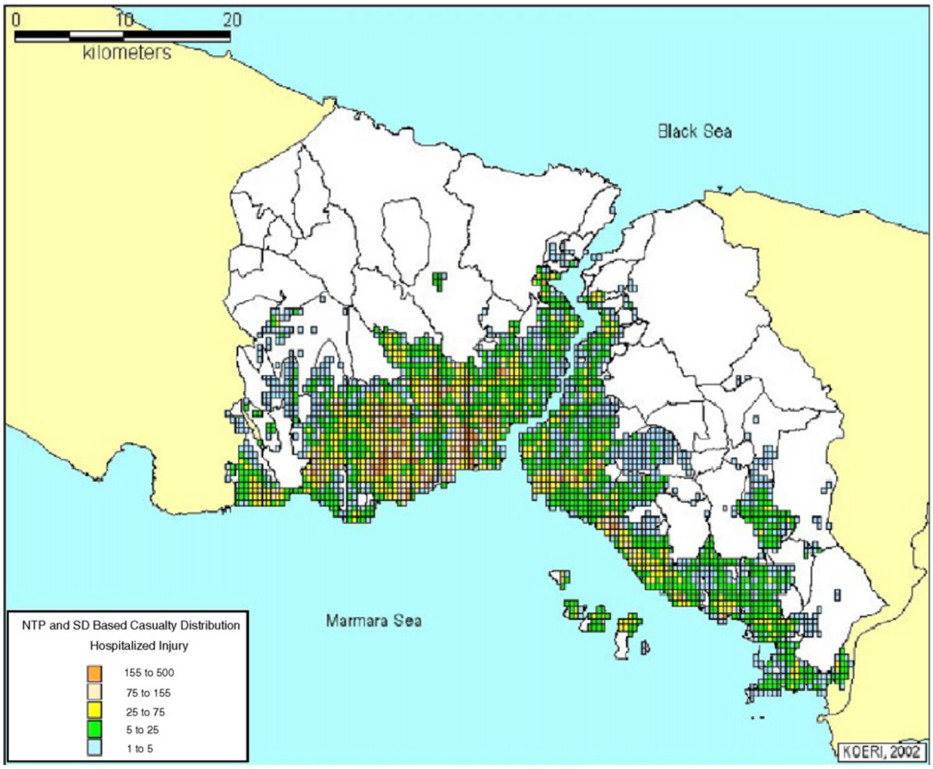

Fig. 1. Expected number of injured people in a possible Istanbul earthquake.

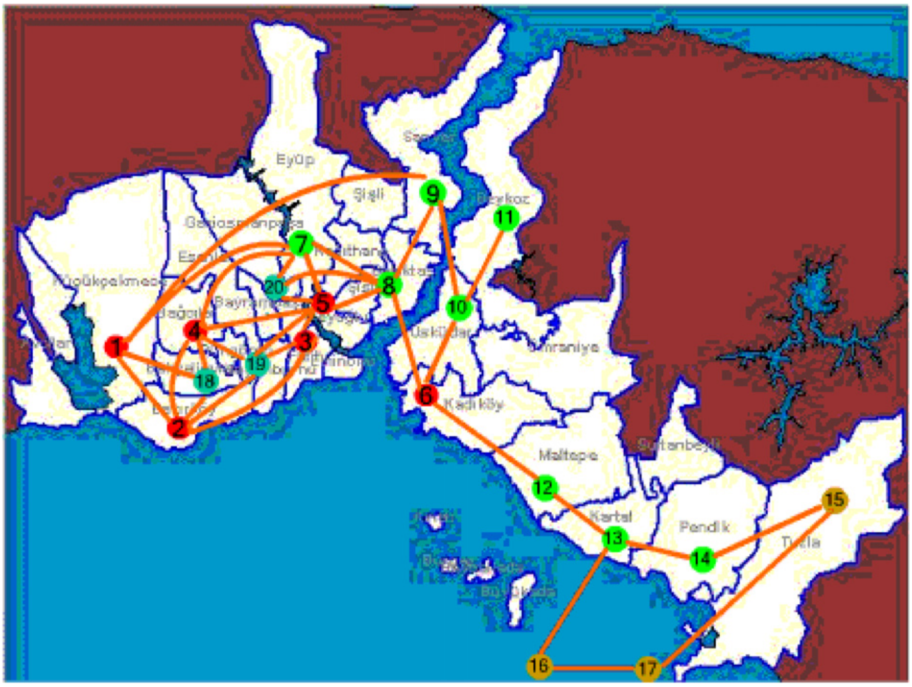

Fig. 2. District based aggregate affected areas (nodes in red), supply distribution centers and hospitals (nodes in green), possible temporary emergency units (nodes in dark green), two aiding cities (nodes in orange). (For interpretation of the references in color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Commodity demand and wounded distribution (people equivalent)

Node	1	2	3	4	5	6	Total demand-commodity
Attrition percentages (period $t = 1$)							(period $t = 1$, people equivalent)
	18	8	26	12	19	17	79,400
Expected attrition percentages (period $t = 5$)							(period $t = 5$, people equivalent)
	15	2	18	12	21	32	18,600 (actual = 20,000)
Wounded people percentages (period $t = 1$)							Total wounded
	8	12	23	25	23	9	4420 (L-M: 2400, H: 2020)
Additional wounded people percentages (period $t = 5$)							
	13	8	20	19	15	23	1290 (L-M: 580, H: 710) (Actual: L-M: 590, H: 680)

Table 3
Supply distribution (people equivalent), fleet composition and total capacity transport

Node	7	8	9	10	11	12	13	14	15	16	17
Average supply percentages in periods t = 1,2,3											
	9.45	2.75	9.2	10.6	13.0	9.63	1.14	9.7	10.43	13.54	10.25
Total supply (People equivalent)				$t = 1$ 35,900	$t = 2$ 27,400	$t = 3$ 21,200					

Vehicle availability % over all nodes		Total transport capacity			
		L-M	H	Food	Medicine
Helicopters	7.82	1500	240		Ample
Trucks	81.43			Ample	
Ambulance	10.75	825	330		

light-to-moderate (L-M) and heavy (H). There are initially a total of 1535 vehicles, the majority of which are trucks. In period 5, an additional quantity of about 20% of the initial fleet is added to the system.

A time bucket of 1 hour is utilized here, and the plan has to be revised every 4 hours (end of $t = 4$ is re-planning time). Hence, during a planning horizon of 8 hours, information is updated once. Two major commodities, medicine and food are considered and their demands and supplies are provided in units of people equivalent for convenience. Supplies keep on arriving by the hour. Service priority weights for heavy, light-moderate wounded, medicine and food are given as 10, 5, 2 and 1, respectively.

The results of the two-stage procedure are summarized in Tables 4–6. In Table 4, initial and finally optimized service rates of medical facilities are indicated. It is observed that one temporary emergency unit (node 20) is established near high attrition demand nodes 3, 4, and 5. Some of existing L-M service capacity is reduced in facilities 11, 13 and 14 that are far away from affected districts and transferred to the temporary emergency unit, 20, and existing hospitals 7, 8, 10. It is noted that node 20 has been selected as the new emergency unit rather than nodes 18 and 19 (despite the fact that they are closer to affected nodes 3–5) because

Table 4
Initial and optimized service rates of medical facilities

Initial service rates of medical facilities	7	8	9	10	11	12	13	14	18	19	20
L-M	25	30	35	30	35	35	30	30	–	–	–
H	15	10	10	15	15	20	20	20	–	–	–
Optimized service rates											
L-M	35	35	35	35	20	35	20	20	–	–	15
H	20	20	20	20	5	20	15	5	–	–	–

Table 5
Queue lengths of wounded people in medical facilities at different time periods and percentage of wounded people discharged out of emergency units

Queue sizes				Percentage of discharged		
Time	Node	H queue length	L-M queue length	Time	% H discharged	% L-M discharged
5	13	5	0	1	0	0
	14	7	0	2	0	0
6	14	2	0	3	5	8
7	20	0	15	4	11	18
	Total	14	15	5	12	23
				6	16	31
				7	21	39
				8	26	48

Table 6
Number of vehicles utilized in each period of planning

Vehicle utilization			
Time	Helicopters	Trucks	Ambulances
1	110	700	115
2	80	900	175
3	115	650	190
4	120	650	175
5	120	600	150
6	115	700	95
7	80	600	55
8	Return	Return	Return

supplying commodities to those nodes for the newly arrived wounded would take more time and delay their treatment further (Constraints (2) couple arrival of commodities and wounded). H service category of facilities 7–9 on the European side have increased while those of 11, 13 and 14 are decreased, because the former are closer to the worst hit zones and temporary units in European side cannot accommodate H patients. Since the number of wounded people is very high as compared to available capacity, all facilities are utilized to the maximum overall service rate. During the first 3 hours medical facilities are mostly not utilized due to transportation delay. The latter takes place because we assume vehicles are in unaffected districts. Hence, we may consider the first three periods spent for stabilizing the system.

We can observe queue lengths in Table 5 where queue sizes that are positive are indicated. The queue sizes are negligible as compared with the total number of people transported because of commodity coupling with the wounded. In Table 5, we also provide the percentages of wounded people who have been served and discharged from the emergency system in each time period. We observe a rise in these percentages after third period, because the system is now stable.

In Table 6, we can observe the number of vehicles utilized throughout the planning horizon. Helicopters and ambulances have high utilization ratios whereas trucks achieve about 75% utilization rate due to supply limitation and the fact that transport capacity of trucks is much larger than transport requirement of available supplies. Utilization rates drop to zero in period 8 when all vehicles return to arbitrary supply nodes for change of shift.

Several comments can be made on the routes generated by the model. In general, wounded people are sent to the nearest hospitals. For example, wounded from node 6 are sent to nodes 10, 12, and some to 13, 14, and, their commodity requirements are satisfied by nodes 10 and 12. Node 12 also receives wounded from other nodes because there is only one nearby affected district. Injured from node 1 are sent to nodes 7, 9, 20 and those from nodes 3, 4 and 5 are sent to nodes 7, 8 and 20. Due to the limitation of overall system service rate, people at node 2 who are located far away from emergency centers (route from node 2 has a high risk grade that causes an accessibility problem) have relatively restricted access to medical care. Similarly, node 1 receives its commodity requirements from node 9 and nodes 4 and 5 receive mostly from nodes 7 and 8. Nodes 15–17 (outside the boundaries of Istanbul) do not receive any wounded but send commodity aid via nodes 13 and 15. Some supply nodes (e.g., node 8) that serve as medical centers and supply storage units start receiving commodities from other supply nodes after the initial hours of the planning horizon because of an increasing influx of wounded people who generate commodity demand.

An overall assessment can be made for the plans generated on the scenario. The model coordinates logistics support and evacuation activities expediting high priority evacuation while maintaining equilibrium among service rates of medical facilities. Scarce resources are exploited to the full extent. However, based on the poor service rate achieved for node 2, we remark that a minimum service rate constraint could be imposed on districts relatively farther away from supply centers at the expense of reduced total service rate.

Scenario plans also show the efficiency of the proposed system in terms of solvability. For this scenario, model P is solved in 15.9 seconds and 11,599 iterations involving 579 nodes. The MIP that has to be solved for route construction and vehicle sheet preparation takes a total of 3.38 seconds in 1622 iterations (10 nodes). The re-planning MIP is solved in 3.11 seconds and in 3732 iterations (375 nodes) and the second one is solved in 2.25 seconds in 429 iterations (6 nodes). These CPU times include the execution of algorithm *Route* that is also coded in GAMS.

4. Additional experiments to show solvability of the two-stage procedure on larger scenarios

The two-stage procedure is applied to 18 randomly generated problems (20–60 nodes) to illustrate model solvability in larger size scenarios. Results are provided in Table 7. Characteristics are represented by the number of nodes, number of possible sites for temporary centers, number of vehicles in the system and the demand structure of demand nodes. The number of arcs in the networks is $2|\mathcal{N}|$ on the average. Problems are categorized into two types, A and B, where demand quantities in type A are generated in the interval $[0.1*cap, 0.3*cap]$, and those in type B are in interval $[0.8*cap, 1.2*cap]$. Here, *cap* is average per vehicle capacity. Supplies are generated randomly in a similar fashion within the same supply intervals matching the demand pattern.

Table 7a
Computational results for Type I scenarios

Problem	Nodes	Possible locations	No. of vehicles	Type	Number of constraints	Number of integer variables	CPU time (seconds)	
							Model P	Second stage
1	20	3	38	I	12,312	2491	16.9	0.8
2	25	3	66	I	18,384	3137	12.9	1.9
3	30	3	85	I	25,608	3729	15.5	3
4	35	5	84	I	34,096	4391	77.9	3.8
5	40	5	115	I	43,752	5019	45.1	31.5
6	45	5	128	I	54,624	5665	102.1	34.6
7	50	8	166	I	66,744	6362	46.2	79.3
8	55	8	167	I	79,952	6936	71.1	80.9
9	60	5	170	I	94,344	7495	65.7	74.6

Table 7b
Computational results for Type II scenarios

Problem	Nodes	Possible locations	No. of vehicles	Type	Number of constraints	Number of integer variables	CPU time (seconds)	
							Model P	Second stage
1	20	3	38	II	12,312	2491	3	0.9
2	25	3	66	II	18,384	3137	5.7	2.1
3	30	3	85	II	25,608	3729	6	3
4	35	5	84	II	34,096	4391	7.7	3.6
5	40	5	115	II	43,752	5019	5.6	15.4
6	45	5	128	II	54,624	5665	32	41.8
7	50	8	166	II	66,744	6362	28.9	86.7
8	55	8	167	II	79,952	6936	37.7	80.5
9	60	5	170	II	94,344	7495	49.4	91

All the problems have a planning horizon of 8 time periods. There are three transportation modes, two types of wounded class and two commodities. Network arcs are created randomly so as to generate connected networks. The network topologies of Types I and II instances are the same for same number of nodes.

In Table 7, we indicate the number of constraints, number of integer variables in model P and CPU times taken for solving model P and the second stage. It is observed that model P is solved more slowly in Type I instances than in the corresponding Type II instances, because of increased complexity of routes involving many nodes. The computational results show that the proposed model can solve relatively larger size problems within an acceptable CPU time.

5. Conclusion

An integrated location-routing model is proposed for coordinating logistics support and evacuation operations in response to emergencies and natural disasters. The aim is maximizing response service level by enabling fast relief access to affected areas and locating temporary emergency units in appropriate sites. The location sub-problem involves sharing of scarce medical resources and achieving service rate equilibrium among different emergency centers. Medical doctors can be shifted from center to center, however, their total number remains fixed for a given period of time. The equilibrium problem and the dynamic setting in which the problem is defined describes a complex but flexible system where queues are minimized through the full exploitation of facility capacities achieved by the interaction of the routing problem with service rate equilibrium and location.

The proposed model is efficient in the sense that the routing sub-problem does not necessitate tracking vehicles individually, but rather, it handles vehicles as general integer flow variables rendering a mixed integer multi-commodity network flow formulation. Model outputs related to the routing problem consist of arc-based integer quantities of vehicles of different types and continuous quantities of aggregate commodity flows carried by vehicles traversing each arc. To convert this output into individual vehicle instruction sheets, an efficient routing algorithm of pseudo-polynomial complexity is proposed. This is coupled by a linear system of equations that are fairly easy to solve due to the structure of the matrix and the tightness of the feasible space. The proposed two-stage approach is generic and can be utilized to solve commercial VRP that suffer from abundant quantities of binary variables. It can handle large numbers of vehicles whose individual routes and pick up/delivery quantities are specified by a subsequent execution of a fast routing algorithm and solution of a LP.

The proposed two-stage procedure is compared with the VRP based single-stage model of the problem and its effectiveness is shown on a number of test instances derived from Eilon et al.'s (1971) test network. Furthermore, model solvability is demonstrated for randomly generated test instances of larger size.

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Appendix A. Single-stage procedure: VRP based formulation (location and evacuation sub-problems omitted)

Additional/adjusted notation:

Sets and parameters:
κ: number of vehicles
av_{oit}: binary number indicating if vehicle i is added to the fleet at node o at time t

Decision variables:
QD_{coit}: amount of commodity type c delivered to node o by vehicle i at time t
QP_{coit}: amount of commodity type c picked up at node o by vehicle i at time t
QL_{cit}: amount of commodity type c carried by vehicle i at time t
V_{opit}: binary variable indicating if vehicle i traverses arc (o,p) at time t

Model P2:

$$\text{Minimize} \quad \sum_{c\in\mathscr{C}} \sum_{o\in\mathscr{DN}} \sum_{t} (P_c DEV_{cot}) \tag{0}$$

$$\text{Subject to:} \quad \sum_{p\in\mathscr{N}} V_{opit} \leqslant 1 \quad (\forall o \in \mathscr{N},\ t \in T,\ i \in \kappa), \tag{1}$$

$$\sum_{p\in\mathscr{N}} V_{poit} \leqslant 1 \quad (\forall o \in \mathscr{N},\ t \in T,\ i \in \kappa), \tag{2}$$

$$\sum_{\tau=1}^{t} \left[av_{oi\tau} + \sum_{p\in\mathscr{N}} \mathsf{A}_{po\tau} V_{poi\tau} \right] - \sum_{\tau=1}^{t-1} \sum_{p\in\mathscr{N}} V_{opi\tau} \geqslant \sum_{p\in\mathscr{N}} V_{opit} \quad (\forall o \in \mathscr{N},\ t \in T,\ i \in \boldsymbol{\kappa}), \tag{3}$$

$$\sum_{\tau=1}^{T} \sum_{p\in\mathscr{N}} [\mathsf{A}_{po\tau} V_{poi\tau} - V_{opi\tau}] = 0 \quad (\forall o \in \mathscr{N},\ i \in \boldsymbol{\kappa}), \tag{4}$$

$$\sum_{c\in\mathscr{C}} QP_{coit} \leqslant \sum_{p\in\mathscr{N}} V^{*}_{opit} cap_i \quad (\forall o \in \mathscr{SN},\ t \in T,\ i \in \boldsymbol{\kappa}) \tag{5}$$

$$QL_{cit} = \sum_{\tau=1}^{t} \sum_{o\in\mathscr{N}} [QP_{coi\tau} - QD_{coi\tau}] \quad (\forall c \in \mathscr{C},\ t \in T,\ i \in \boldsymbol{\kappa}) \tag{6}$$

$$\sum_{c\in\mathscr{C}} QL_{cit} \leqslant cap_i \quad (\forall t \in T,\ i \in \boldsymbol{\kappa}) \tag{7}$$

$$QD_{coit} \leqslant \sum_{p\in\mathscr{N}} \mathsf{A}_{pot} V^*_{poit} B \quad (\forall c \in \mathscr{C},\ t \in T,\ o \in \mathscr{N},\ i \in \boldsymbol{\kappa}) \tag{8}$$

$$DEV_{cot} = \sum_{\tau=t}^{t} \left[-d_{co\tau} + \sum_{i\in\kappa} QD_{coi\tau} \right] \quad (\forall c \in \mathscr{C},\ t \in T,\ o \in \mathscr{DN}) \tag{9}$$

$$\sum_{\tau=1}^{t} d_{co\tau} \geqslant \sum_{\tau=1}^{t} \sum_{i\in\kappa} QP_{coi\tau} \quad (\forall c \in \mathscr{C},\ t \in T,\ o \in \mathscr{SN}) \tag{10}$$

$$V_{opit} \text{ binary; } QD_{coit}, DEV_{cot}, QL_{cit}, QP_{coit} \geqslant 0. \tag{11}$$

Brief explanation of model P2:

Constraints (1) and (2) construct a path for vehicle i, i.e., entry to a node is enabled from a single node and vice versa for departure. Constraints (3) ensure that the cumulative number of entries to a node at any time is greater than or equal to cumulative number of departures by same vehicle. Constraints (4) ensure that the number of departures from a node is equal to entries at the end of the planning horizon. Constraints (5) make sure that commodity can be picked up from a node only if the vehicle departs the node. Constraints (6) define the load carried by the vehicle in each time period. Constraints (7) ensure that load carried is restricted by vehicle capacity. Constraints (8) make sure that commodity can be delivered to a node if the vehicle arrived there in that time period. Constraints (9) define unsatisfied demand while constraints (10) restrict picked up commodity from supply nodes by available quantities.

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