Games Theoretic Models and Evolutionary Computation in Economics and Finance

Akira NAMATAME
Dept. of Computer Science
National Defense Academy
Yokosuka,239-8686,Japan

E-mail: nama@nda.ac.jp
http//www.nda.ac.jp/~nama/

Outline

A. Introduction to 2x2 Games

B. N-person Games

- Strategic interactions and externalities
- Social Security Games

C. Evolutionary Games

D. Co-evolutionary Learning in Social Games

E. Broad Application Areas

Game Theory

- Game theory is devoted to the logic of rational decision-making in a social context.
- A game is any interdependent situation in which at least two agents interact.
- Game theory is concerned with what happens when multiple agents interact.
- In interdependent situations, the outcomes are described as the payoff functions which depend on all relevant agents. The final outcome is explained by the concept of equilibrium.
- A Nash equilibrium is a combination of strategies that provide the best outcome for each agent, and no agent can obtain a better payoff by unilateral deviating from their strategy.

Basics of Game Theory (1)

- A game is specified by players (agents), actions (strategies), and payoff matrices (functions of joint actions)
- Example: Rock-Paper-Scissors Game

B's action

A's action

$$\begin{array}{cccc} & R & P & S \\ R & 0 & -1 & +1 \\ P & +1 & 0 & -1 \\ S & -1 & +1 & 0 \end{array} \qquad \begin{array}{cccc} & R & P & S \\ R & 0 & +1 & -1 \\ P & -1 & 0 & +1 \\ S & +1 & -1 & 0 \end{array}$$

A's payoff

B's payoff

- If payoff matrices are identical, game is cooperative, else non-cooperative (zero-sum/constant-sum = purely competitive)

Basics of Game Theory (2)

- Games with no states: matrix games
- Games with states: stochastic games, Markov games
  (state transitions are functions of joint actions)

- Games with simultaneous moves: normal form
- Games with alternating turns: extensive form

- Number of rounds = 1: one-shot game
- Number of rounds > 1: repeated game

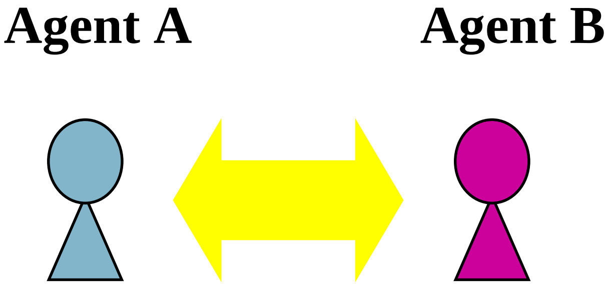
- Deterministic action choice: pure strategy
- Non-deterministic action choice: mixed strategy

Basic Analysis

- A joint strategy **x=** (x_i , x(i)) is Pareto-optimal if no other strategy that improves everybody’s payoffs.
- An agent i’s strategy x_i is a dominant strategy if it’s always best regardless of other agents’ actions.
- A strategy x_i is a best-reponse to others’ strategies x(i) if it maximizes payoff given x(i).
- A joint strategy **x** is a Nash equilibrium if each agent’s strategy is simultaneously a best-response to everyone else’s strategy, i.e. no incentive to deviate.
- A Nash equilibrium always exists.

2x2 Games

- Two agents A and B face a binary decision problem with S_1 or S_2.
- Both agents receive the payoff which depends on the other agent's choice.



The payoff matrix

Agent B Agent A	S_1	S_2
S_1	a_B a_A	c_B b_A
S_2	b_B c_A	d_B d_A

The payoff of A: a_A, b_A, c_A, d_A
The payoff of B: a_B, b_B, c_B, d_B

Classification of 2x2 Games

- Asymmetric games.

- Symmetric games

$a_A=a_B \; b_A= b_B \; c_{A,}= c_{B}, d_A = d_B$

The payoff matrix

Agent B Agent A	S_1	S_2
S_1	a a	c b
S_2	b c	d d

- Doubly symmetric games

$b= c$

Agent B Agent A	S_1	S_2
S_1	a a	b b
S_2	b b	d d

Well-known Games

- Prisoner’s Dilemma Game
- Coordination Game
- Dispersion Game (Chicken Game)
- Hawk-Dove Game
- Vicious-circle Game

Prisoner's Dilemma Game

The other's strategy Own's strategy	S_1 (altruist)	S_2 (egoist)
S_1 (altruist)	R R	T S
S_2 (egoist)	S T	P P

Reward (R)
Sucker's payoff (S)
Temptation (T)
Punishment (P)

The parameters are often set as:
R=3, S=0, T=5, P=1

This game has a dominant strategy (S_2).

- T > R > P > S : S_1 is dominated by S_2.
- 2R > T + S : If both agents commit to a dominated strategy S_1, they are better off.

How do you commit yourself to irrational choice (S_1) *when your partner has incentive to cheat you?*

Coordination Game

Stag-Hunt Game

Agent B **Agent A**	S_1	S_2
S_1	*10* *10*	*x* *0*
S_2	*0* *x*	*x* *x*

$5 < x < 10$

Two agents face the problem of which target to be aimed?

- If they cooperate to hunt “Stag”, they get it.
- They can get “Hare” without cooperation.

- (S_1, S_1): Pareto-dominates (S_2, S_2),
- (S_2, S_1): risk-dominates (S_1, S_1)

In this case, both agents are likely to select (S_2, S_2), which is inferior to (S_1, S_1).

Dispersion Game

Chicken Game

Agent B Agent A	S_1	S_2
S_1	0 0	a 1
S_2	1 a	0 0

$a > 0$

- Both agents get payoffs if they take the distinct action (disperse), otherwise they receive nothing.

Hawk-Dove Game

The other's strategy Own's strategy	S_1 (Hawk)	S_2 (Dove)
S_1 (Hawk)	(V−C)/2 (V−C)/2	0 V
S_2 (Dove)	V 0	V/2 V/2

- Hawk vs. Hawk
  Fighting is occurred.
- Hawk vs. Dove
  Hawk wins and dove flees.
- Dove vs. Dove
  They posture and one of them retreats

- If they fight, the winner gets the prize of the value V, and the loser receives nothing.
- The cost for fighting is C $(0 < V < C)$.

Vicious-circle Game

Seller Buyer	S_1 (honest)	S_2 (cheat)
S_1 (Inspect)	2 3	1 2
S_2 (don't)	3 4	4 1

- If a seller is honest, a buyer does not inspect.
- If a buyer does not inspect, a seller may cheat.
- If a seller cheats, a buyer may inspect.
- If a buyer inspects, a seller should be honest.

A Nash equilibrium of pure strategies does not exist.

An Affine Transformation of a Payoff Matrix

Agent B Agent A	S_1	S_2
S_1	a_B a_A	c_B b_A
S_2	b_B c_A	d_B d_A

→

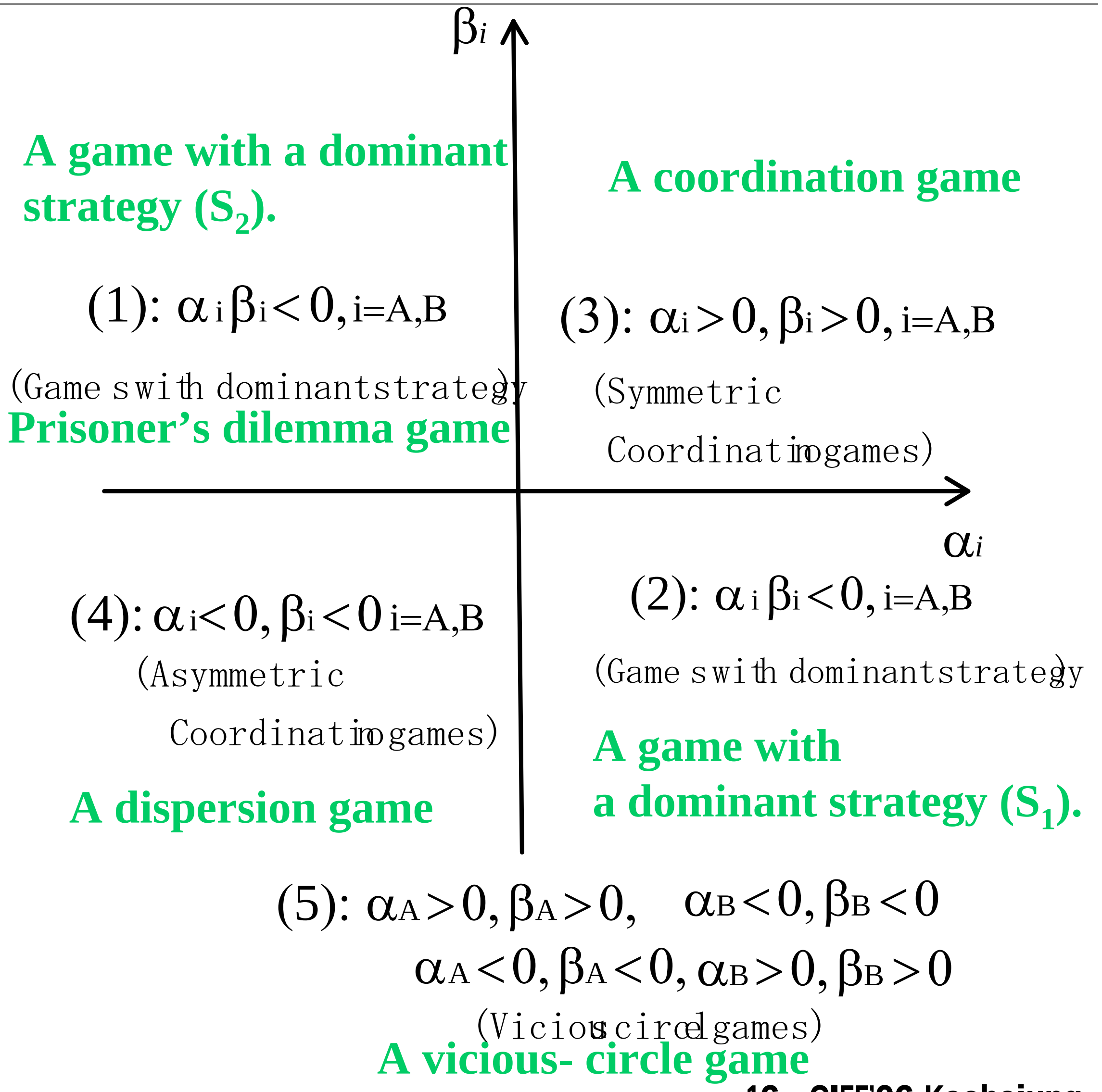
Agent B Agent A	S_1	S_2
S_1	α_B α_A	0 0
S_2	0 0	β_B β_A

A Nash equilibrium remains the same under the following linear transformation.

$$\alpha_i = a_i - c_i, \quad \beta_i = d_i - b_i \quad i = \mathrm{A}, \mathrm{B}$$

Classification of 2x2 Games

Agent B Agent A	S_1	S_2
S_1	α_B α_A	0 0
S_2	0 0	β_B β_A



A game with a dominant strategy (S_2).

(1): $\alpha_i \beta_i < 0$, i=A,B

(Games with dominant strategy)

Prisoner's dilemma game

A coordination game

(3): $\alpha_i > 0, \beta_i > 0$, i=A,B

(Symmetric Coordination games)

(4): $\alpha_i < 0, \beta_i < 0$ i=A,B

(Asymmetric Coordination games)

A dispersion game

(2): $\alpha_i \beta_i < 0$, i=A,B

(Games with dominant strategy)

A game with a dominant strategy (S_1).

(5): $\alpha_A > 0, \beta_A > 0, \quad \alpha_B < 0, \beta_B < 0$

$\alpha_A < 0, \beta_A < 0, \alpha_B > 0, \beta_B > 0$

(Vicious circle games)

A vicious- circle game

Best-Response Functions and Nash Equilibrium

Agent B Agent A	S$_1$ (y)	S$_2$ (1-y)
S$_1$ (x)	α_B α_A	*0* *0*
S$_2$ (1-x)	*0* *0*	β_B β_A

- Mixed strategy: the probability to choose S$_1$ is x(y).

  agent A: x = (x, 1-x),

  agent B: y = (y, 1-y)

- Expected utility with the mixed strategies,

$$U_A(x,y) = (1-y)\beta_A + \{(\alpha_A + \beta_A)y - \beta_A\}x$$

$$U_B(x,y) = (1-x)\beta_B + \{(\alpha_B + \beta_B)x - \beta_B\}y$$

- Best response functions

$$\phi_A(y) = \arg \underset{x \in [0.1]}{Max} U_A(x,y)$$

$$\phi_B(x) = \arg \underset{y \in [0.1]}{Max} U_B(x,y)$$

- The joint optimal（mixed）strategies: Nash equilibrium

$$x^* = (\phi_A(y), 1-\phi_A(y))$$

$$y^* = (\phi_B(x), 1-\phi_B(x))$$

Nash Equilibrium: Coordination Game

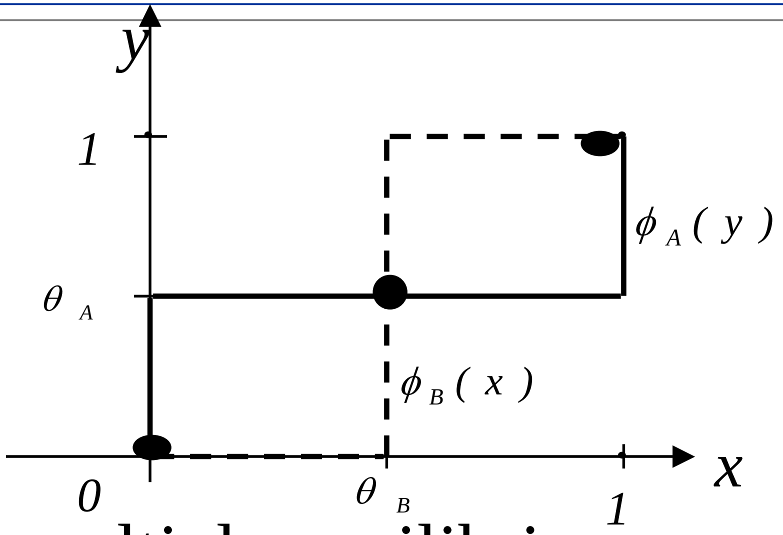
Coordination game:

$$\alpha_A > 0,\ \beta_B > 0\ , \alpha_B > 0,\ \ \beta_B > 0$$

Nash equilibrium

$$x^* = (\phi_A(y), 1 - \phi_A(y))$$

$$y^* = (\phi_B(x), 1 - \phi_B(x))$$



- Coordination games have multiple equilibria.

1. Equilibria of pure strategies: $(S_1, S_1), (S_2, S_2)$
2. Equilibrium of mixed strategies: $x=(\theta_B, 1-\theta_B)$, $y=(\theta_A, 1-\theta_A)$

- Equilibrium selection problem

Which equilibrium can be selected when agents face multiple equilibria?

Nash Equilibrium: Dispersion Game/Hawk-Dove Game

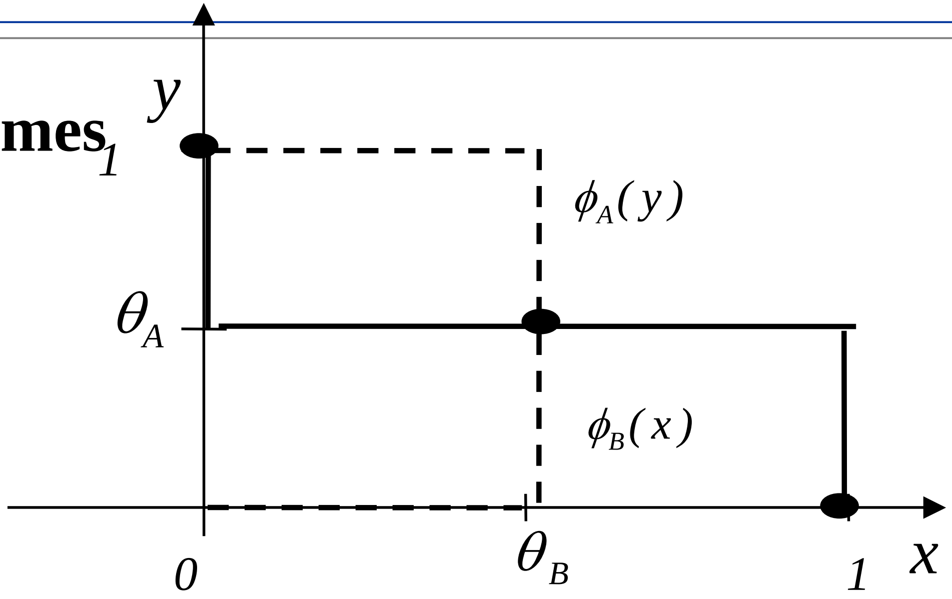
Dispersion and Hawk-Dove games

$\alpha_A < 0,\ \beta_A < 0,\ \alpha_B < 0,\ \beta_B < 0$

Nash equilibrium

$$x^* = (\phi_A(y), 1 - \phi_A(y))$$

$$y^* = (\phi_B(x), 1 - \phi_B(x))$$



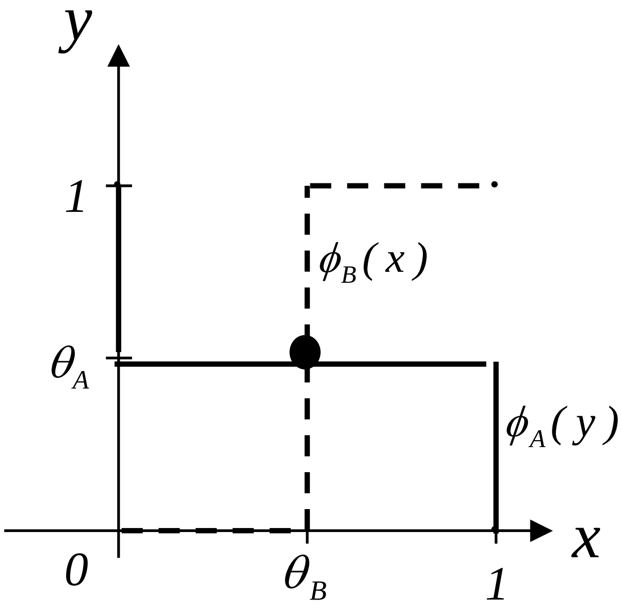
Dispersion and Hawk-Dove games also have multiple equilibria.

- Equilibria of pure strategies: $(S_1, S_2), (S_2, S_1)$
- Equilibrium of mixed strategies: $x=(\theta_B, 1-\theta_B)$, $y=(\theta_A, 1-\theta_A)$

Nash Equilibrium: Vicious Game

Vicious-circle game

$\alpha_A < 0, \beta_B < 0 \quad \alpha_B > 0, \beta_B > 0$



The vicious-circle game has the unique Nash equilibrium of mixed strategies at:

Agent A: $x=(\theta_B,1-\theta_B)$,

Agent B: $y=(\theta_A,1-\theta_A)$

Outline

A. Introduction to Non-cooperative Games

B. N-person Games
  - Implication Externalities
  - Social Security Games

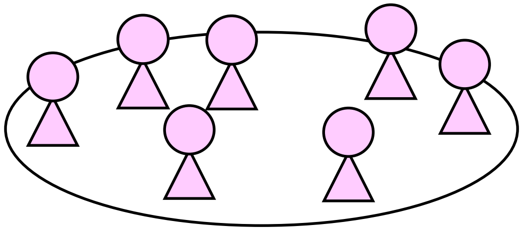
C. Evolutionary Games

D. Co-evolutionary Learning in Social Games

E. Broad Application Areas

Conflict between Individual and Collective Rationality

- Self-interested agents are often faced with the dilemma of acting in their own interest or pursuing a more cooperative course of action.
- Strategic environments are often characterized by a tension between individual and collective rationality.
- A challenging task is to identify conditions under which agents are more cooperative than the Nash equilibrium situation based on the assumption of self-interested agents would predict.



Strategic complementarity vs. Strategic substitutability

- Strategic complementarity:

  The incentive for agents is to move in the same direction.

- Strategic substitutability:

  A change in one agent's choice gives the other agent an incentive to move in the opposite direction.

- Strategic complementarity facilitates coordination compared to strategic substitutability.

N-person Games with Strategic Complementarity

- A binary decision of an agent

  S_1 : volunteer participation on Sunday

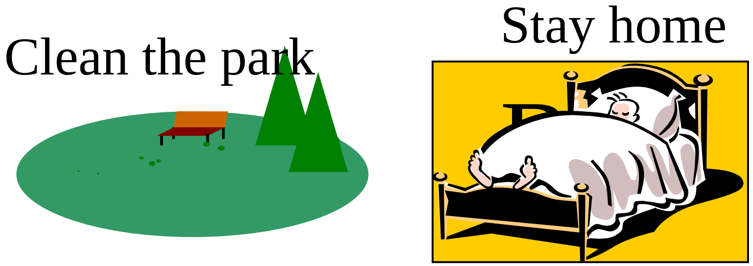
  S_2 : stay at home

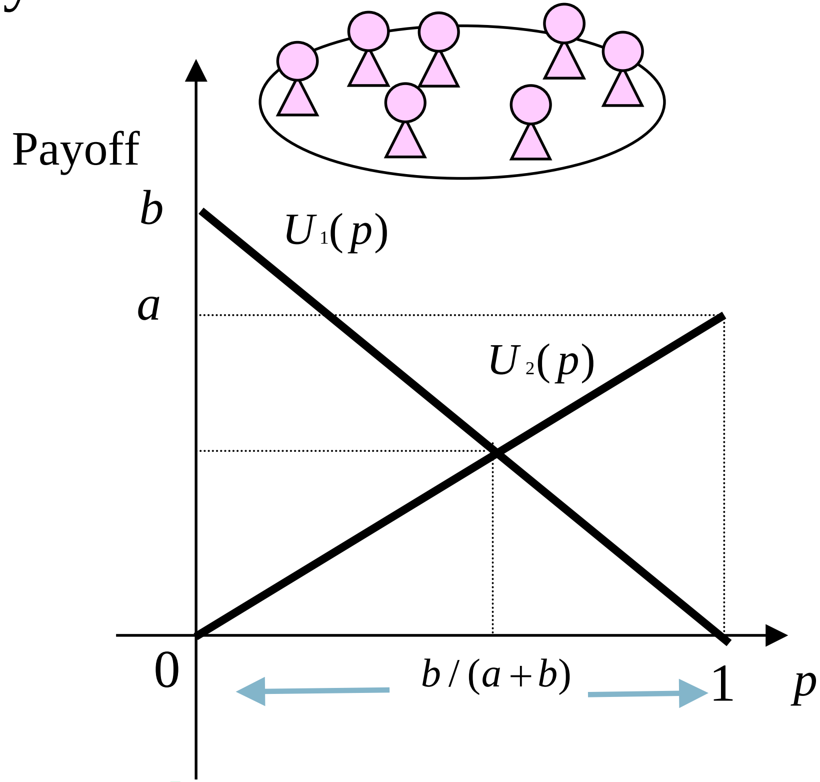
- The utility of each agent

  $S_1 : U_1(p)=ap$

  $S_2 : U_2 (p)=b(1-p)$

p: the proportion of agents choosing S_1

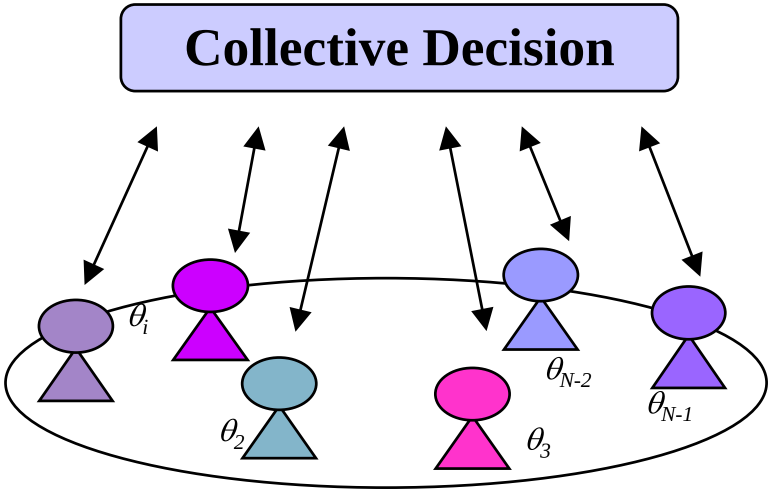


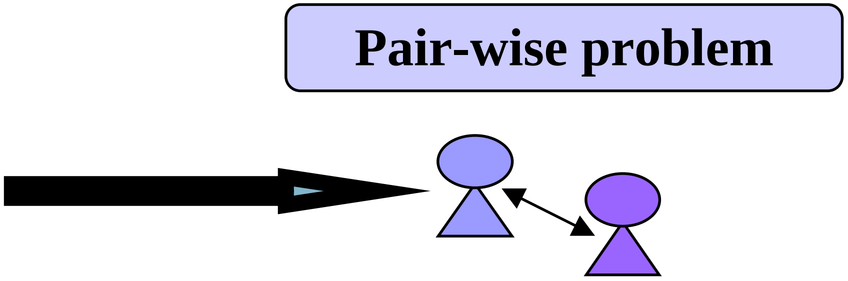


All agents eventually take the same action, S_1 or S_2.

Decomposition into Pair-wise Problems

- The problem of collective decision can be understood as a set of pair-wise problems





The payoff matrix of agent *i*

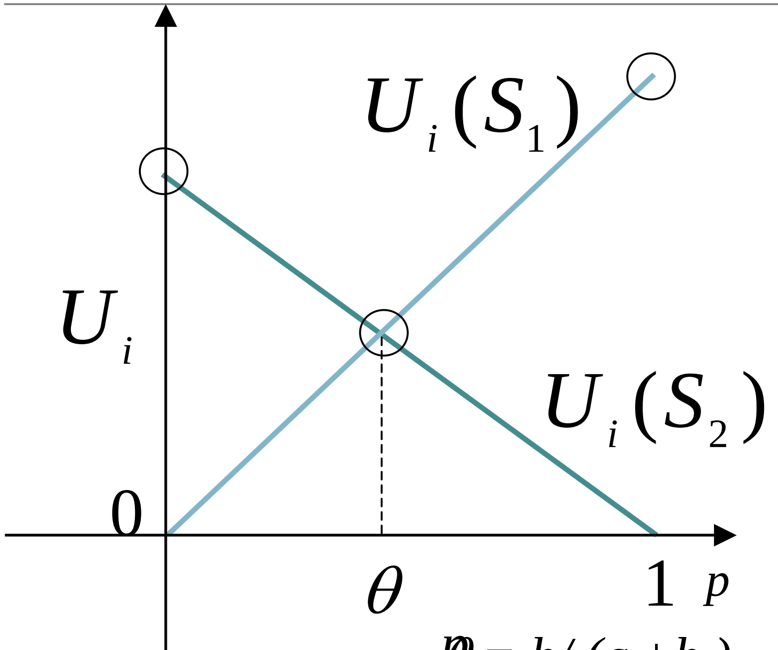
Collectives Agent *i*	S_1 (p)	S_2 $(1-p)$
S_1	a	0
S_2	0	b

$$S_1 : U_1(p)=ap$$

$$S_2 : U_2\,(p)=b(1-p)$$

p: the proportion of agents choosing S_1

Nash Equilibrium and Pareto Optimal Outcome



$\theta = b/(a+b)$

$S_1 : U_1(p)=ap$

$S_2 : U_2(p)=b(1-p)$

The underlying game

Collectives Agent A_i	S_1 (p)	S_2 $(1-p)$
S_1	a	0
S_2	0	b

(1) Nash eqilibria of the underline 2x2 game:

- Pure strategies: $(S_{1,} S_1)$
- Pure strategies: $(S_{2,} S_2)$
- Mixed strategies: $(S_1,S_2)= (\theta,1-\theta)$

(2) Nash eqilibria of N-person game:

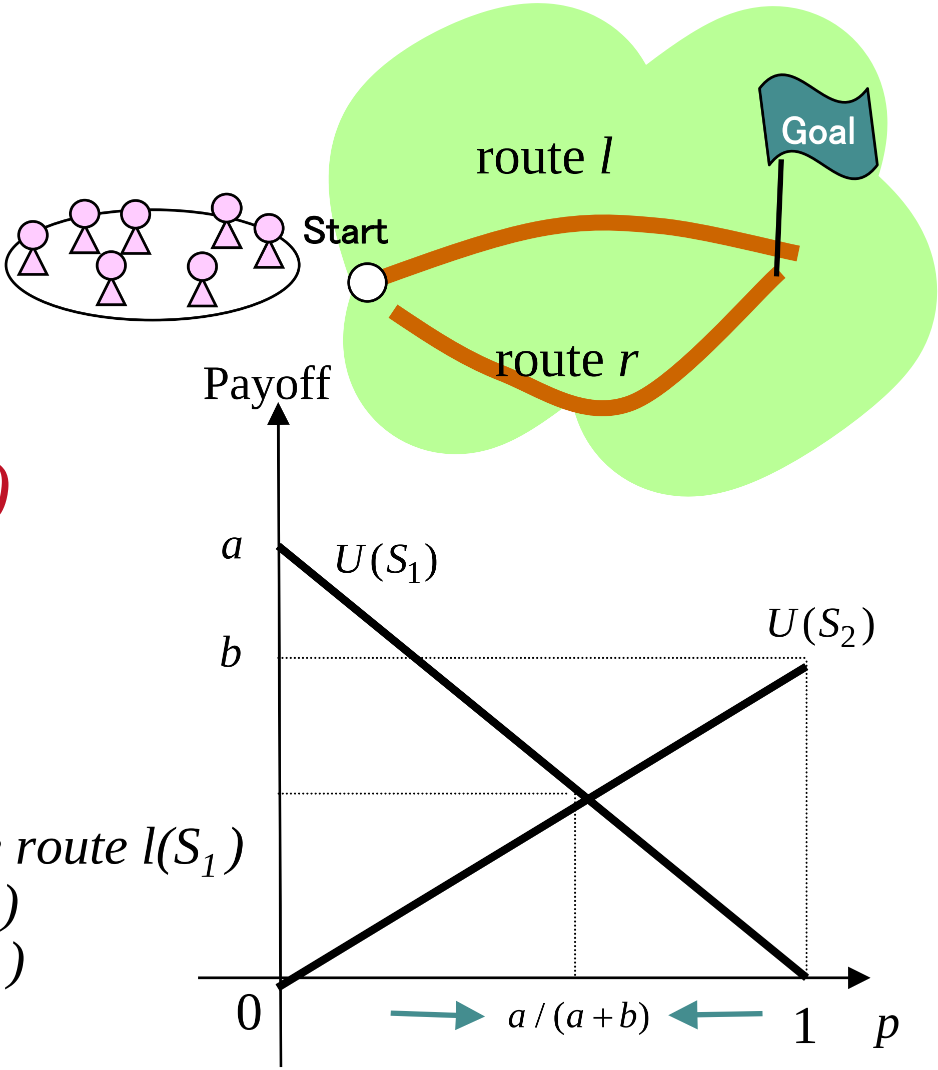
- All choose $S_{1,}$(Pareto-optimal)
- All choose S_2
- The ratio of agents to choose S_1 is θ and that of agents to choose S_2 is $1-\theta$)

N-person Games with Strategic Substitutability(1)

- A binary decision of an agent

  S_1 : Chooses the route l

  S_2 : Chooses the route r



- The utility of each choice

  (utility) = (benefit) – (time)

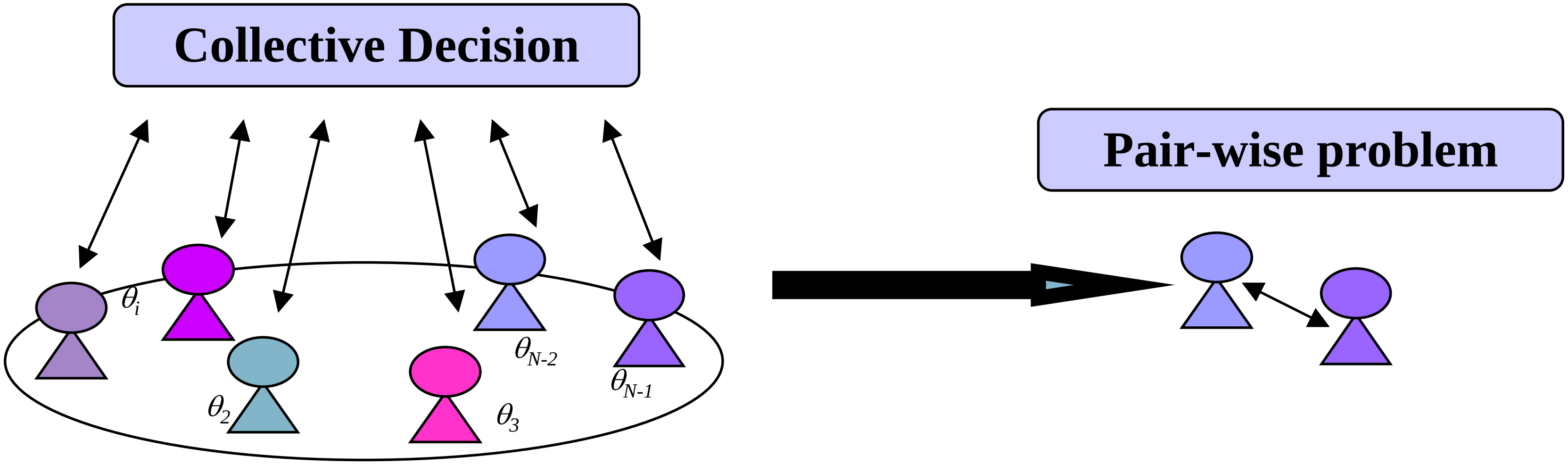
  $S_1 : U_1(p)=a(1-p)$

  $S_2 : U_2(p)=bp)$

p: The proportion of agents choosing the route l(S_1)
a: The benefit of choosing the route l (S_1)
b: The benefit of choosing the route r (S_2)

Decomposition of N-person Dispersion Games

- The problem of collectives can be understood as a set of pair-wise problems


Collective Decision

Pair-wise problem

$$S_1 : U_1(p)=a(1-p)$$

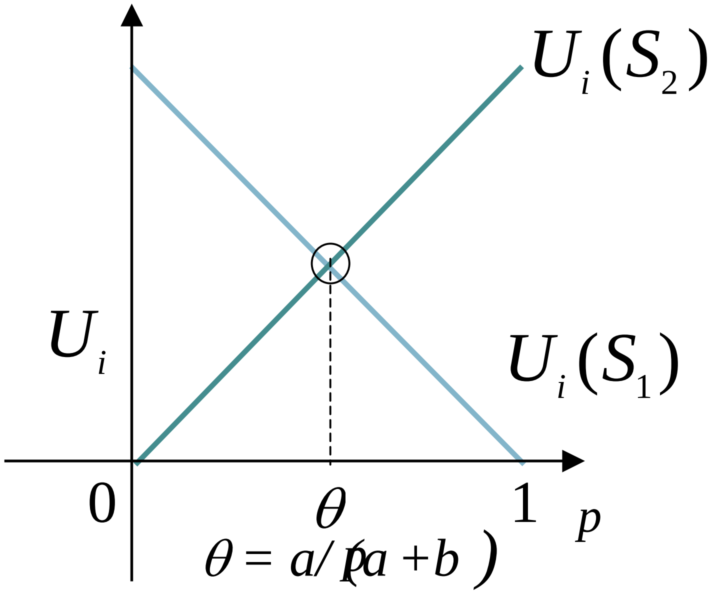
$$S_2 : U_2(p)=bp$$

p: the proportion of agents choosing S_1

The payoff matrix of an agent

Collectives / Agent A_i	S_1 (p)	S_2 $(1-p)$
S_1	0	a
S_2	b	0

Nash Equilibrium



The underlying game

Collectives Agent A_i	S_1 p	S_2 $1-p$
S_1	0	a
S_2	b	0

(1) Nash eqilibria of the underline 2x2 game

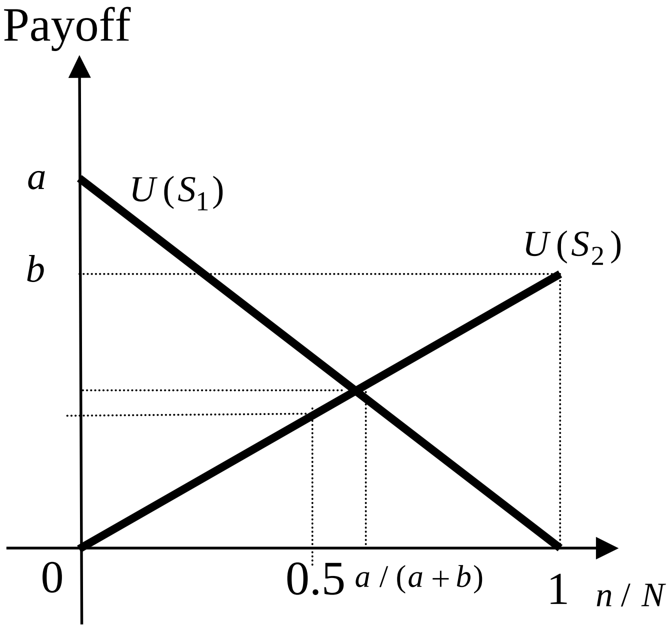
- Pure strategies: (S_1, S_2)
- Pure strategies: (S_2, S_1)
- Mixed strategies: $(S_1,S_2)= (\theta,1-\theta)$

(2) The unique Nash equilibrium of N-person game:

The ratio of agents to choose S_1 is θ and that of agents to choose S_2 is $1-\theta$)

Conflict Between Nash equilibrium and Pareto Optimal Outcome

- Nash equilibrium: $U_1(p)=U_2(p)$
- Pareto optimal outcome: The average utility of the population is maximized



Average utility:

$$E=pU(S_1)+(1-p)U(S_2)$$
$$=(a+b)(p-p^2)$$

Average utility is maximum at p=0.5

- If a=b, Nash equilibrium and Pareto optimal become the same, otherwise they are different.
- At Nash equilibrium all agents receive the same payoff, however, some agents receive higher payoff than the other agents.

Externality

- In economics an ***externality*** is the effect of a transaction between two parties on a third party who is not involved in the carrying out of that transaction.
- An externality occurs when a decision causes costs or benefits to stakeholders other than the person making the decision.
- In other words, the decision-maker does not bear all of the costs or reap all of the gains from his or her action.

Effects of Externality

As a result, in a competitive market too much or too little of the good will be consumed from the point of view of social efficiency.

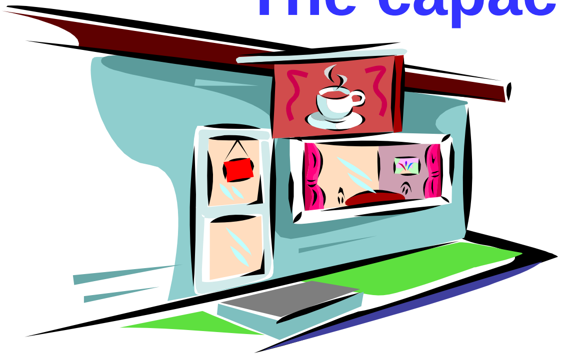
- If the world around the person making the decision benefits more than he does, such as in areas of safety, then the good will be underprovided from society's point of view.

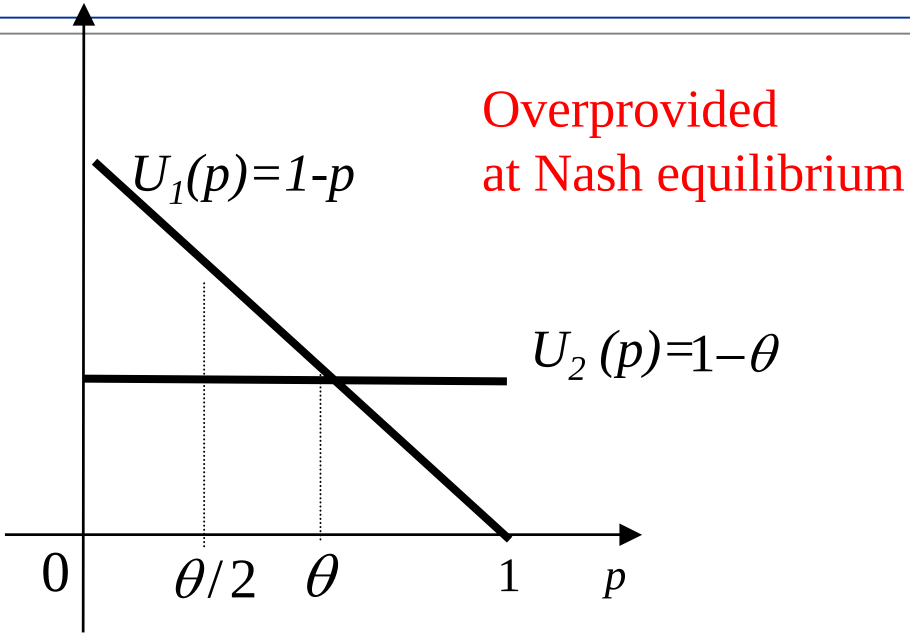
- If the costs to the world exceed the costs to the individual making the choice, such as in areas pollution then the good will be overprovided from society's point of view.

N-person Games with Strategic Substitutability [2]

- **Market entry games**

The capacity is θ





Agents

Nash equilibrium: $S_1{:}\,\theta,\ S_2{:}\ 1-\theta$

The optimal rate of utilization: $S_1{:}\,\theta/2,\ S_2{:}\ 1-\theta/2$

$$\text{Average utility } E=pU_1(p)+(1-p)U_2(p)$$

$$=-p^2+p\theta+1-\theta$$

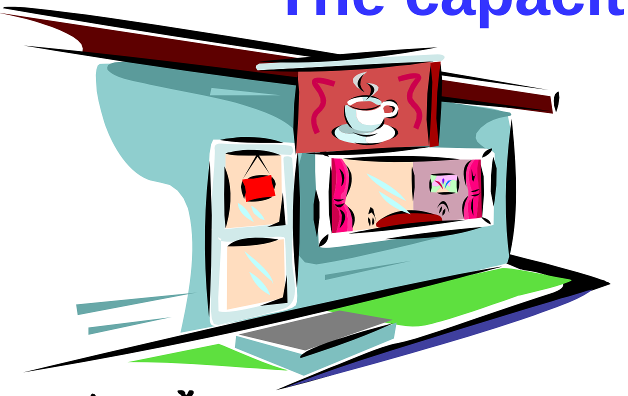
Average utility is maximum at $p=\theta/2$

- How limited resources could be utilized in an efficient and equitable way?

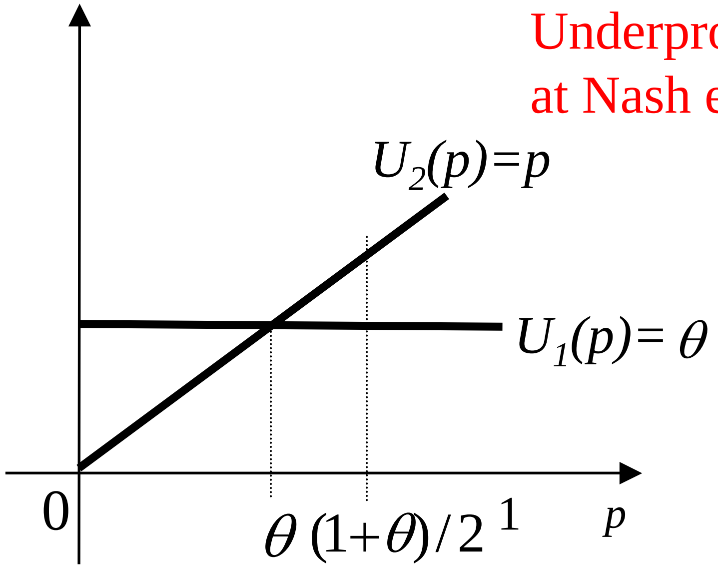
N-person Games with Strategic Substitutability [3]

- **Network congestion games**

The capacity is θ



Agents



Underprovided
at Nash equilibrium

Nash equilibrium: $S_1: \theta, S_2: 1-\theta$

The optimal rate of utilization: $S_1 (1+\theta)/2, S_2: (1-\theta)/2$

$$\text{Average utility } E = pU_1(p)+(1-p)U_2(p)$$
$$= -p^2+p(1+\theta)$$

$$\text{Average utility is maximum at } p=(1+\theta)/2$$

- There are two cases where Nash equilibria are overprovided or underprovided compared with social optimal

Social Security Games

Independent security problems: car insurance, life insurance, home security,..

-In general we can protect against a risk by making a proper investment

: Investment in computer protection against viruses and hackers

: Vaccination can reduce chance of taking flu

Interdependent security problems: computer security, vaccination, airline security,..

-We are also contaminated by others even after investing

: Computer can be attacked by viruses from other computers on the network

: Our risk depends on whether others are vaccinated

Characteristics of the Problems

- Agent can suffer direct and indirect losses

- **Investment no longer buys complete security**

  : Risk faced by one person depends on both its own security investments as well as on the actions of others (externalities).

- Stochastic externalities

  : Indirect losses may be conditioned on the direct loss not occurring (we only die once).

Classes of Social Security Games

Class 1: Partial Protection (airline security)

: Tipping behavior

Class 2: Partial Protection (computer security)

: Investment no longer buys complete security
: Free riding

Class 3: Complete Protection (vaccination)
: Investment buys complete security
: Conflict between stability (equilibrium) and efficiency & fairness

Class 1: Partial Protection (Airline security)

(Kunreuther, H, 2003)

Consider Two Airlines: A_1 and A_2. Y = income of airline A_i before investment on security

- Loss if a bag explodes : L .
- Investment Cost of Baggage Security System: C
- Probability contaminated bag is accepted & explodes : p
- Probability contaminated bag accepted by airline A_i is transferred to the other airline where it explodes : q

Two-Airlines Case

Two Strategies:

(S) : Investing,

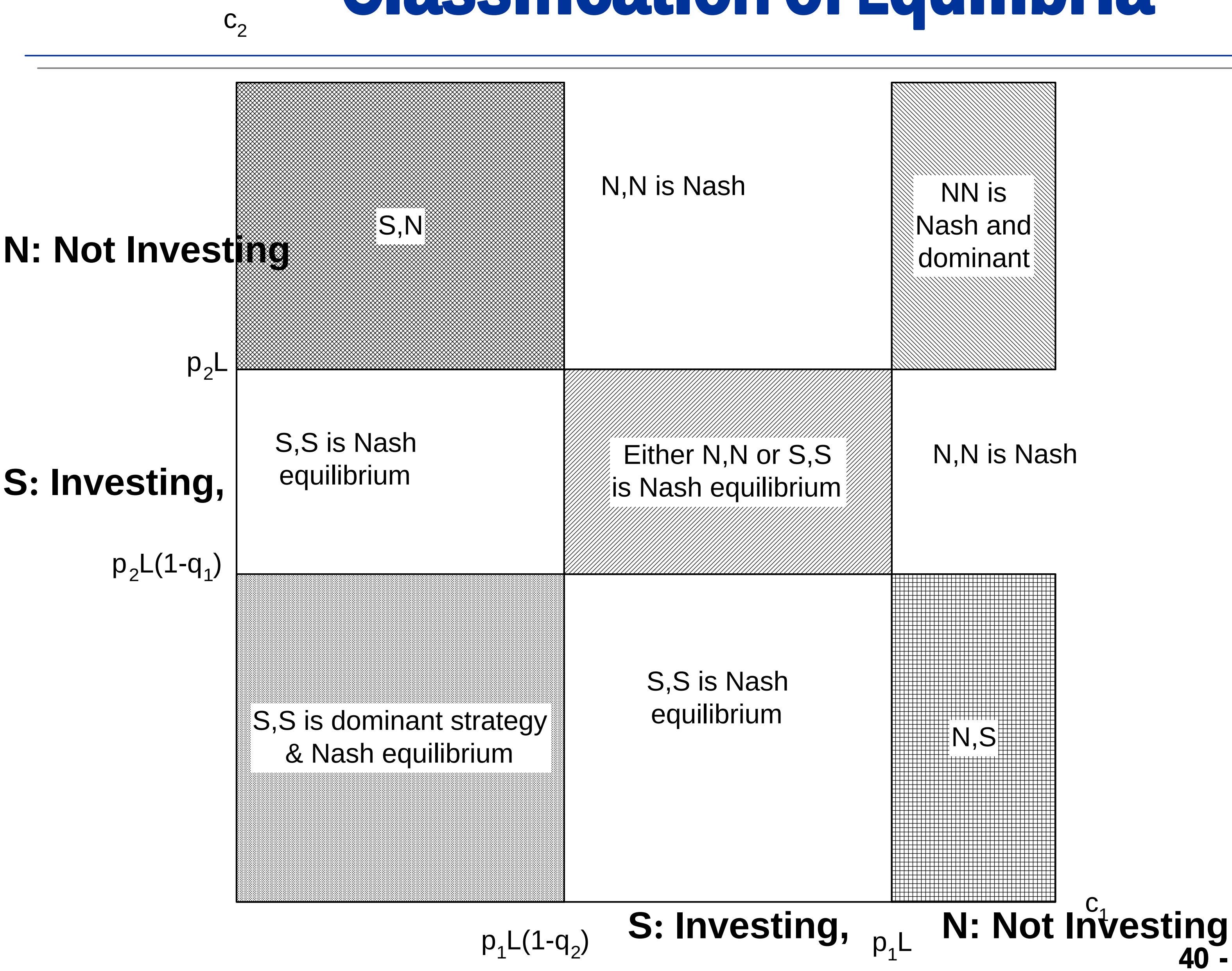
(N): Not Investing, in security systems

		AIRLINE 2	
		S	N
AIRLINE 1	S	Y –C, Y –C_2	Y- C – $qL_{1,}$, Y – pL
	N	Y – pL, Y– C– qL	Y–pL– (1-p)qL, Y–pL– (1-p)qL

Rational decision and outcomes:

- If $C > pL$, no airline will invest: (N,N)
- If $C < pL$, Alone would invest, however, (S, N) may be happen.
- If $C < p_i(1-q)L$, then both airlines will invest: (S,S)

Classification of Equilibria



c_2

N: Not Investing

S,N

N,N is Nash

NN is Nash and dominant

p_2L

S: Investing,

S,S is Nash equilibrium

Either N,N or S,S is Nash equilibrium

N,N is Nash

$p_2L(1-q_1)$

S,S is dominant strategy & Nash equilibrium

S,S is Nash equilibrium

N,S

c_1

$p_1L(1-q_2)$

S: Investing,

p_1L

N: Not Investing

N-Airlines Case

Define $X_i(n,0)$ to be the externalities to Agent i if it invests in security and none of the other agents do (indirect loss) .

(1) What is expected cost to Agent i from **investing** in security if none of the other agents invest in security?

E(Payoff from Investing) = $Y - c – X_i(n,0)$

(2) What is expected cost to Agent i from **not investing** in security if none of the other agents invest in security?

E(Payoff from Not Investing) = $Y- pL - (1-p) X_i(n,0)$

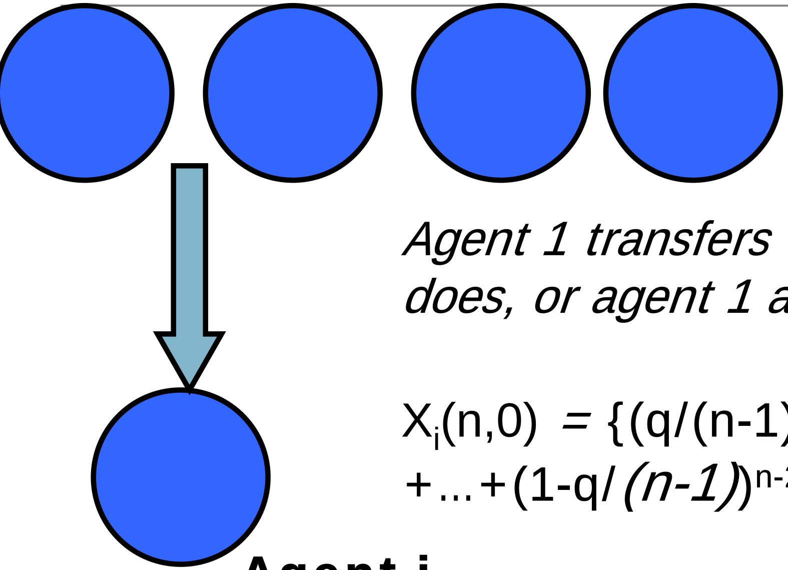
(3) Agent i will only want to invest in security if

$$Y- c – X_i(n,0) > Y- pL- (1-p) X_i(n,0)$$

This implies that $c < p[L- X_i(n,0)]$

The expected direct loss minus indirect loss (externality) should be greater than the cost.

Measurement of Externality (indirect loss)



N-1 agents

Agent i

Agent 1 transfers virus or agent 1 does not transfers but agent 2 does, or agent 1 agent2 do not but agent 3 does, and so on.

$$X_i(n,0) = \{(q/(n-1)+(1-q\,/(n-1))q/\,(n-1)\,)+(1-q\,(n-1))^2 q\,(n-1) +\ldots+(1-q/(n-1))^{n-2}\,q\,(n-1)\,\}L$$

$$= [1-(1-q\,(n-1))^{\,n-1}]\,L$$

Therefore the probability at least one agent is contaminated: $1-(1-q\,/(n-1))^{\,n-1}$

In the limit: $\mathbf{X_i(n,0) = (1 - e^{-q)}\,L}$

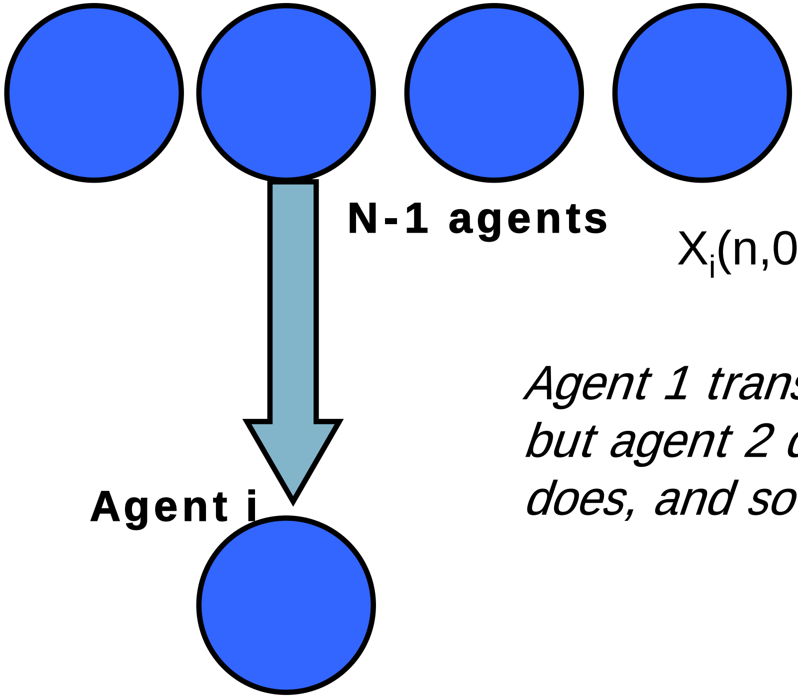
Therefore the externality is measured as the independent value of N, and only depends on q.

q: probability contaminated bag accepted by one agent is transferred to one of the other agents

N-person Security Game: Class 2 (Computer security)

Measurement of externality (indirect loss)

One unprotected computer can infect all the others in the network



$$X_i(n,0) = \{(q+(1-q)q+(1-q)^2q + \ldots + (1-q)^{n-2}\}L$$
$$= [1-(1-q)^{n-1}]L$$

Agent 1 transfers virus or agent 1 does not transfers but agent 2 does, or agent 1 agent2 do not but agent 3 does, and so on.

Therefore the probability at least one agent is contaminated:

$$1-(1-q)^{n-1}$$

N-person Computer Security Games

Expected negative externalities imposed by all other agents on agent i = $X_i(n,0)$)

$$X_i(n,0) = q L \sum_{t=0}^{n-2} [(1-q)^t] = [1-(1-q)^{n-1}] L$$

What is the expected loss *[E(L)]* agent i if it invests in security and none of the others do?

$$E(L) = pL + (1-p) X_i(n,0)$$

In the limit then $X_i(n,0) = L$ then we have $E(L)=L$

Note: $C < p [L - X_i(n,0)]$ for agent i to want to invest in security

In the limit: $C < 0$, so there is no cost incentive to invest in protecting any agent against viruses or hackers if none of the other agents are protected.

N-person Security Games: Tipping Behavior and Free-riding

The same structure as airline security problem and computer security problem with the following key differences:

- airline security---investment by one airline encourages others to also invest and can lead to **tipping behavior**

- computer security---investment by one agent discourages others from following suit and can lead to **free riding**

N-person Security Game: Class 3 [Vaccination Game

- Complete protection:

Individual can protect against a disease by being vaccinated

- Individual concerns some reasons for decision making:

:Reasons for getting vaccine

-Can get disease from an external source

-Can get disease from others who are infected

: Reasons for not getting vaccine

-Cost in money and time

-Negative side effects

Security Game Framework

(Heal,G. 2005)

Y= income of individual before expenditure on vaccine

p= probability of catching disease even if no one else has it

q= probability of catching disease and infecting another person:

L = loss (possibly death) from catching the disease.

c = cost (in time and money) of being vaccinated c

Assume that vaccine provides total protection against the disease.

Two-Agent Case

Two strategies of: being vaccinated (V)
not being vaccinated (NV)

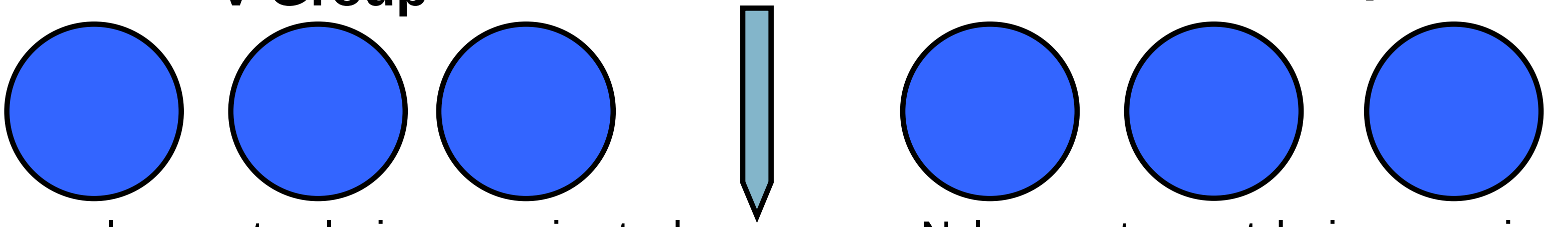
		AGENT 2	
		V	**NV**
AGENT 1	**V**	$Y - C, \quad Y - C$	$Y - C, \quad Y - pL$
	NV	$Y - pL, \quad Y - C$	$Y - pL - (1-p)qL, \quad Y - pL - (1-p)qL$

p: the probability of catching the disease even if no one else has it (environmental risk)

q: the probability of catching the disease from infecting another susceptible agent.

- If $C < pL$, alone would be vaccinated and both will be vaccinated: (V,V)
- If $pL < C < pL + (1-p)qL$, *either* (V, NV) or (NV, V) may be happen.
- If $C > pL + (1-p)qL$, then no one will be vaccinated: (NV,NV)

N-person Vaccination Game



V Group **NV Group**

k agents: being vaccinated N-k agents: not being vaccinated

Q(k): the probability of a non-vaccinated agent catching the disease if the the number of agents being vaccinated is k

		V Group	**NV Group**
agent	**V**	*Y - C*	*Y- C*
	NV	*Y – Q(k-1)L*	Y–*Q(k)L*

- If *C < Q(k-1)L: Agents in V group remain in the same V group.*
- If *C* > Q(k)*L* : *Agents in VP group remain in the same NV group*.

The mixed population with k agents being vaccinated and N-k not being vaccinated if *Q(k)L < C < Q(k-1)L*

Nash equilibrium and Social Optimal(1)

The Payoff Function

$U(S_1) = Y - C$

$U(S_2) = Y - \pi(p)L$

S_1: being vaccinated (V)

S_2: not being vaccinated (NV)

$\pi(p)$: the probability of a non-vaccinated agent catching the disease when the proportion of the agents being vaccinated is p.

(1) Nash equilibrium is achieved at $U(S_1) = U(S_2)$.

Such a mixed situation where the ratio of agents being vaccinated *p is**

$\pi(p^*) = C/L$

(2) Social optimal: The average payoff of the mixed population

$E(p) = pU(S_1) + (1-p)\text{-}U(S_2) = Y - \{pC + (1-p)\,\pi(p)L\}$

How to Estimate the Probability of Catching Disease?

Epidemiology vs. Etiology: Three levels

- ***Microscopic level***

Researchers who try to understand and to kill off new viruses.

- ***Macroscopic level***

Knowing the propagation mode: Statistical analysis in order to understand the propagation of diseases in complex systems (communities, networks, societies)

- ***Mesoscopic level***

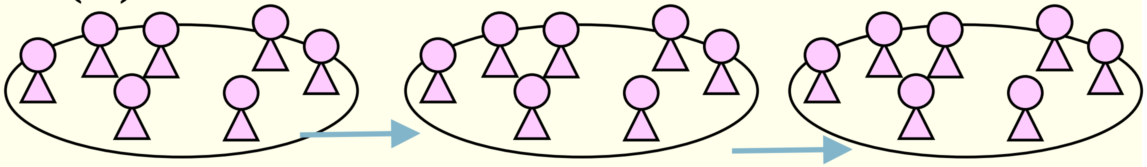
Researchers who try to understand the micro-macro links between individual behavior and the overall viruses spread.

Macroscopic Model of Epidemics: SIR Model

Coarse grained description model.

Individuals exist only in few states such as:

- Healthy or Susceptible (S)
- Infected (I)
- Immune, Dead (R)



$$\frac{dS}{dt} = \mu(1 - p) - \beta SI - \mu S$$

$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I$$

$$\frac{dR}{dt} = \mu p + \gamma I - \mu R$$

μ : the mean birth and death rate

β : the mean transmission rate

p : the vaccine uptake ratio

$1/\gamma$: the mean infection period

The Basic Reproductive Ratio

$S + I + R = 1$ (normalization divided by N)

$$\frac{dS}{d\tau} = f(1-p) - R_0(1+f)SI - fS$$

$$\frac{dI}{d\tau} = R_0(1+f)SI - (1+f)I$$

where

$\tau = \frac{t}{\gamma}$ **: time measured in units of the mean infectious period**

$f = \frac{\mu}{\gamma}$ **:the infectious period as a fraction of mean life time**

$R_0 = \beta/(\gamma + \mu)$ **:the basic reproductive ratio**

The Steady-State Analysis of the SIR Model

(1) If $p \geqq p_{crit}$, the SIR model converges to the disease-free state

$$(S, I, R) \rightarrow (1-p, 0, p)$$

(2) If $p < p_{crit}$, it converges to a stable epidemic state given by

$$\hat{S} = 1 - p_{crit}$$

$$\hat{I} = \frac{f}{1+f}(p_{crit} - p)$$

$$\hat{R} = p$$

where

$$P_{crit} = \begin{cases} 0 & if \quad R_0 \le 1 \\ 1 - 1/R_0 & if \quad R_0 > 1 \end{cases}$$

Estimation of Probability: π(p)

π(p): the probability of a non-vaccinated agent catching the disease when the proportion of the agents being vaccinated is p.

$$\pi(p) = \frac{\hat{I}}{\hat{S}} = \frac{R_0(1+f)\hat{S}\,\hat{I}}{R_0(1+f)\hat{S}\,\hat{I} + f\,\hat{S}} = 1 - \frac{1}{R_0(1-p)}$$

- Healthy or Susceptible (S)
- Infected (I)
- Immune, Dead (R)

$R_0 = \beta/(\gamma+\mu)$ **:the basic reproductive ratio**

Nash equilibrium and Social Optimal (2)

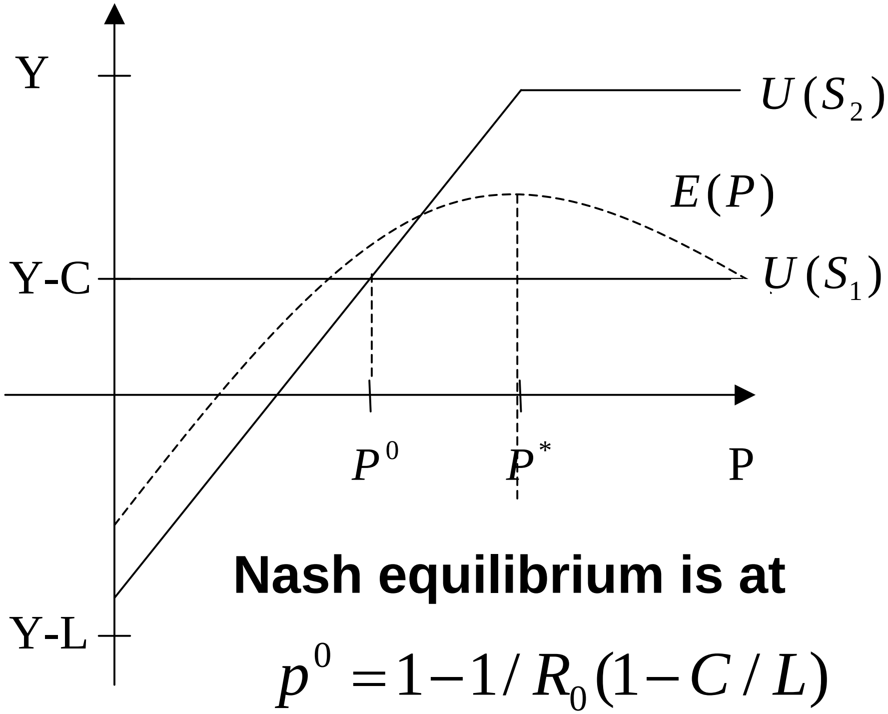
S_1: being vaccinated (V)

S_2: not being vaccinated (NV)

The payoff function:

$$U(S_1) = Y - C$$

$$U(S_2) = Y - \pi(p)L$$



Nash equilibrium is at

$$p^0 = 1 - 1/R_0(1 - C/L)$$

Social optimal is at

$$p^* = 1 - 1/R_0$$

The price of anarchy:

$$\frac{NashEquilibrium}{Social\ optim\hat{a}l} = \frac{Y - C}{Y} = 1 - C/Y$$

Characteristics at Social Optimal

- Underprovided:
  Socially optimal vaccination level will be greater that to the Nash equilibrium level

- *Unfair outcome at social optimal:*
  Not being vaccinated group receives the gain from a more secure situation

At the socially optimal vaccination level, an agent of not being vaccinated receives a greater gain to that of an agent of being vaccinated.

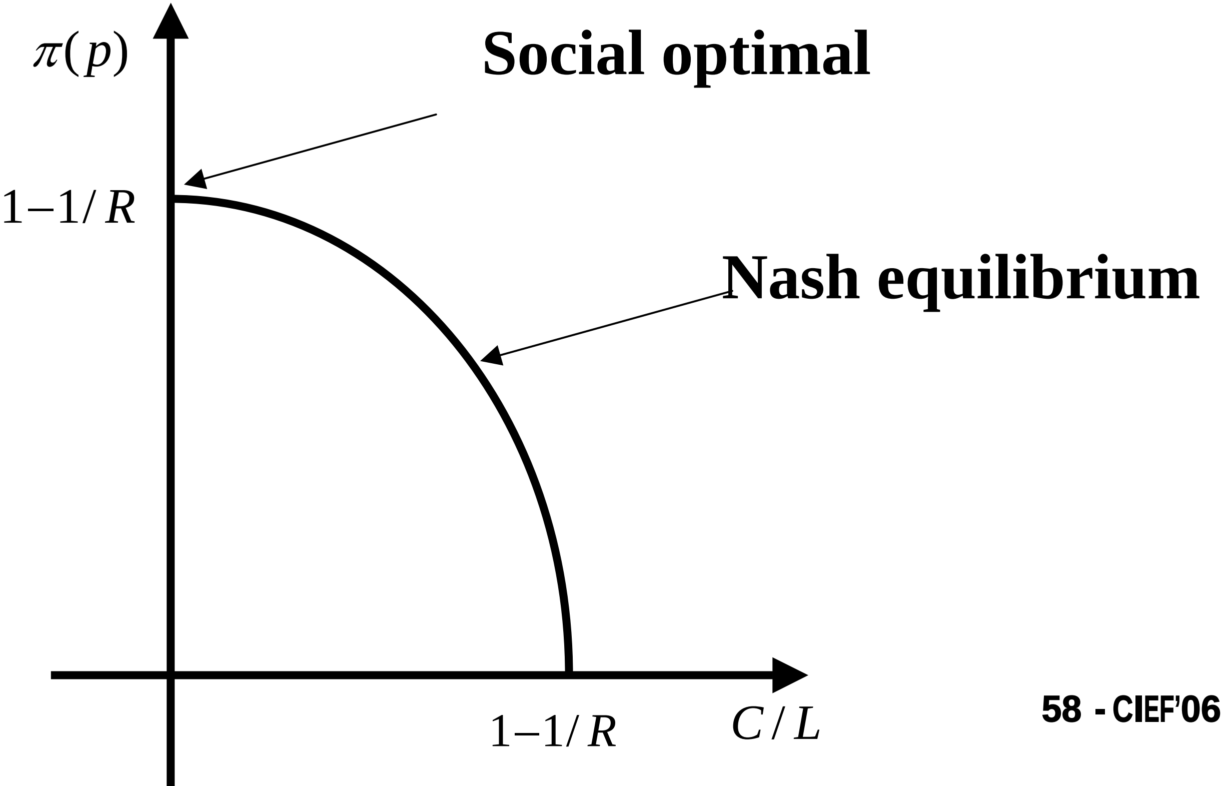
How to Minimize the Price of Anarchy

The Payoff Function

$U(S_1)=Y - C,\ U(S_2)=Y- \pi(p)L$

Nash equilibrium is achieved at $U(S_1)=U(S_2)$.

Nash equilibrium: a mixed population where the ratio of agents being vaccinated *p** **is** at $\pi(p^*)=C/L$



Outline

A. Introduction to Non-cooperative Games

B. N-person Games

- Implication Externalities
- Social Security Games

C. Evolutionary Games

D. Co-evolutionary Learning in Social Games

E. Broad Application Areas

Introduction

•Game theory relies on
: the rational-choice model,
: the best-response based on individual rationality,
: the myopic adjustment.

•A more powerful analysis can be developed by including,
: learning at individual levels,
: selection mechanism: a more fitter survives.

What is Evolutionary Game Theory?

- Origins: Genetics and Biology
- Explain strategic aspects in evolution of species due to the possibility that individual fitness may depend on population frequency
- Evolutionary Model as a dynamic model whose law of motion reflects three basic forces:

1. **Selection**: Objects with higher fitness tend to spread
2. **Mutation**: Introduces new objects (variety) in the system
3. **Inheritance**: Transfers (successful) behavior across time

Why are evolutionary Games?

- Equilibrium and efficiency are possible outcomes of an evolutional process.

- Evolutionary game theory provides general formal models of evolutionary processes.

- Strategic interaction over time such that
  - **monotone:** higher payoff strategies displace lower
  - **inertial:** population takes time to adjust
  - **price-takers:** agents don’t try to influence others’ future choices

Framework of Evolutionary Game

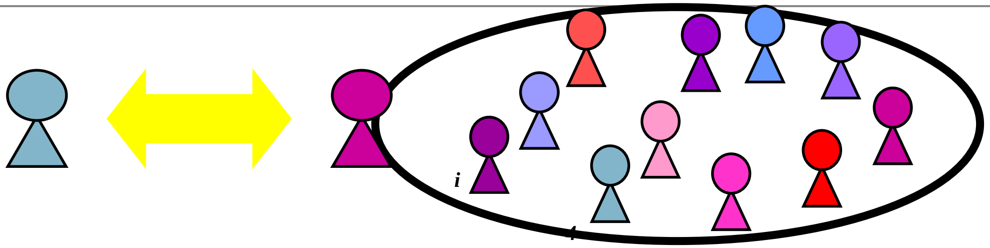
- Represents various behavioral/cognitive properties of individuals as strategies in a game.

- Examines how each strategy performs in the game against other strategies in terms of net profit.

- More fit strategies proliferate in the population gradually (via social/cultural learning).

- Different from game theory, and it does not assume agents with intelligent information processing ability.

Formulation of Evolutionary Game



- A pair of randomly chosen agents choose one of the two possible strategies.
- The population state represents the population share of individuals who adapt each possible strategy.

Underlying game: doubly symmetric payoff matrix.

Agent B Agent A	S_1	S_2
S_1	*a* *a*	*c* *b*
S_2	*b* *c*	*d* *d*

Evolutionarily Stable Strategy (ESS)

- Current **state:** p = strategy distribution in population(s).
- Agent chosing (mixed) strategy *x* has fitness $f(x,p)$.
- Perturbed state (small **invasion** of *x*-mutants) is:

$$s' = \varepsilon x + (1-\varepsilon) s$$

- State *s* is an **ESS** if all small invasions fail, i.e.,

  if $f(s,s') > f(x,s')$ for all $\varepsilon > 0$, and $x \neq s$.

Sources of Dynamics

- **Learning:**

  Agents change action as experience accumulates

- **Endogenous market share changes**

- **Entry and exit.**

Replicator Dynamics: One Population Case

- The fundamental equation of Replicator Dynamics(RD)

$$\dot{x}(t) = \{U(e_1, x) - U(x, x)\}x(t)$$

x(t): The proportion of $S_{1,}$ at t

- Pure strategy choice
- **S_1: e_1 = (1, 0), S_2: e_2 = (0, 1)**

- Expected payoff

$$U(e_1, x) = ax + b(1 - x), \quad U(e_2, x) = cx + d(1 - x)$$

$$U(e_1, x) - U(x, x) = (1 - x)\{U(e_1, x) - U(e_2, x)\}$$

$$= (1 - x)\{(a + d - b - c)x + d - b\}$$

- The derived RD

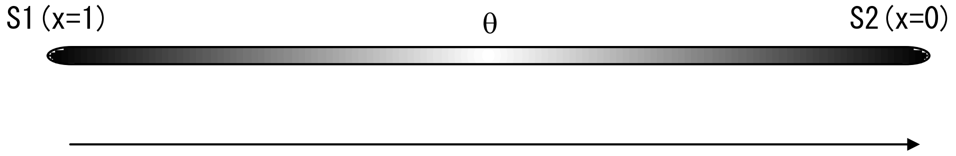
$$\dot{x}(t) = (\alpha + \beta)\{x(t) - \theta\}x(t)\{1 - x(t)\} \quad (\theta \equiv \beta/(\alpha + \beta))$$

$$(\alpha \equiv a - c, \quad \beta \equiv d - b)$$

Stability Analyses of RD

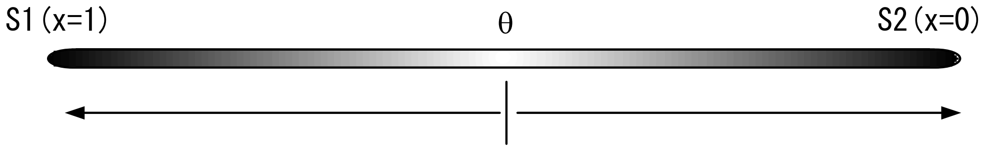
x(t): The proportion of strategy S_1 at t

(1) Prisoner's dilemma game



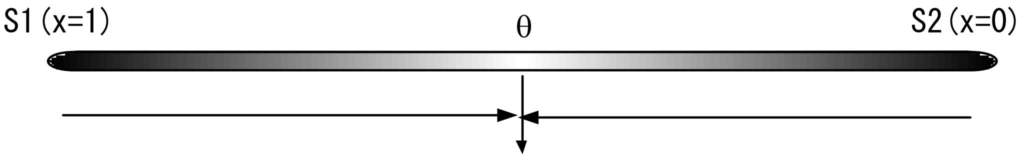
The dominant strategy S_2 is selected. All agents choose S2

(2) Coordination game



All agents choose either the pareto-optimal strategy S_1 or the risk-dominant strategy S_2 .

(3) Anti-coordination (dispersion) game & vicious circle game



The mixed situation, some agents choose S_1 and the others choose S_2 , is selected.

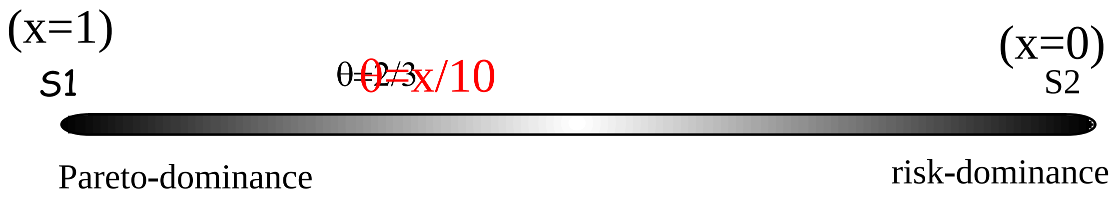
Equilibrium Selection in Coordination Games

Stag-hunt Games

B A	S_1 (Stag)	S_2 (Hare)
S_1 (Stag)	*10* *10*	*x* *0*
S_2 (Hare)	*0* *x*	*x* *x*

(Stag: big anmal, Hare: small animal) (5 < x < 10)

- Risk-dominant equilibrium has the larger basin for convergence



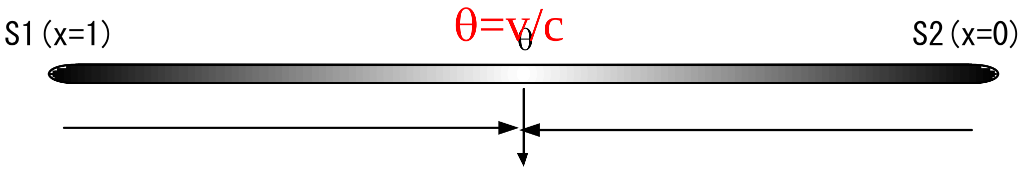
All agents are forced to select the risk-dominant strategy S_2 even when the other strategy S_1 is Pareto-preferred.

Equilibrium Selection in Hawk-Dove Games

Hawk-dove games

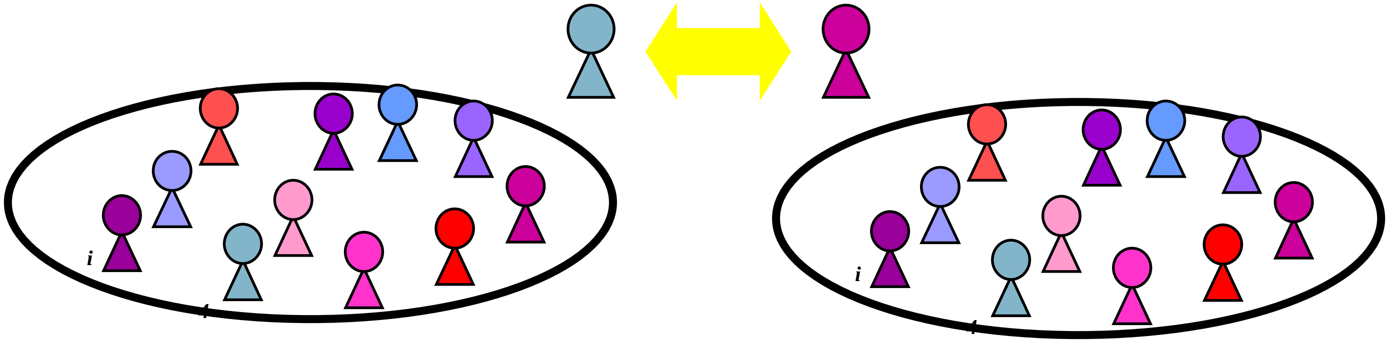
The other's strategy Own's strategy	S_1 (Hawk)	S_2 (Dove)
S_1 (Hawk)	(V−C)/2 (V−C)/2	0 V
S_2 (Dove)	V 0	V/2 V/2

$v < c$



- The equilibrium with the pure strategies (S_1,S_2) or (S_2,S_1) cannot be selected.
- Natural selection selects the mixed situation, in which the ratio of the hawks is *v/c*, and that of the doves is *1- (v/c)*.

Evolutionary Games: Two-Population Case



Underlying game: asymmetric game.

Agent B Agent A	S_1	S_2
S_1	a_B a_A	c_B b_A
S_2	b_B c_A	d_B d_A

Agent B Agent A	S_1	S_2
S_1	α_B α_A	0 0
S_2	0 0	β_B β_A

Replicator Dynamics: Two-Population Case

- Expected payoff

(1) An agent in the population A:

$$U_A(e_1,y) = \alpha_A y(t), \qquad U_A(e_2,y) = \beta_A(1-y(t))$$

$$U_A(x,y) = xU_A(e_1,y) + (1-x)U_A(e_2,y) = \alpha_A xy + \beta_A(1-x)((1-y)$$

(2) An agent in the population B:

$$U_B(e_1,x) = \alpha_B x \quad U_B(e_2,x) = \beta_B(1-x)$$

$$U_B(x,y) = yU_B(x,e_1) + (1-y)U_B(x,e_2) = \alpha_B xy + \beta_B(1-x)((1-y)$$

- RD of two populations

$$\dot{x}(t) = (\alpha_A + \beta_A)\{y(t) - \theta_A\}x(t)\{1-x(t)\}$$

$$\dot{y}(t) = (\alpha_B + \beta_B)\{x(t) - \theta_B\}y(t)\{1-y(t)\}$$

Application: Role of Emotion in Dilemma Games

The other's strategy Own's strategy	S_1 (altruist)	S_2 (egoist)
S_1 (altruist)	R R	T S
S_2 (egoist)	S T	P P

The other's strategy Own's strategy	S_1 (altruist)	S_2 (egoist)
S_1 (altruist)	$3+a$ $3+a$	$5-c$ b
S_2 (egoist)	b $5-c$	1 1

- $c > 0$: The guilty of an egoist.
- $b > 0$: Praise of an altruist
- $b < 0$: Loss of an altruist when the other behaves as an egoist.

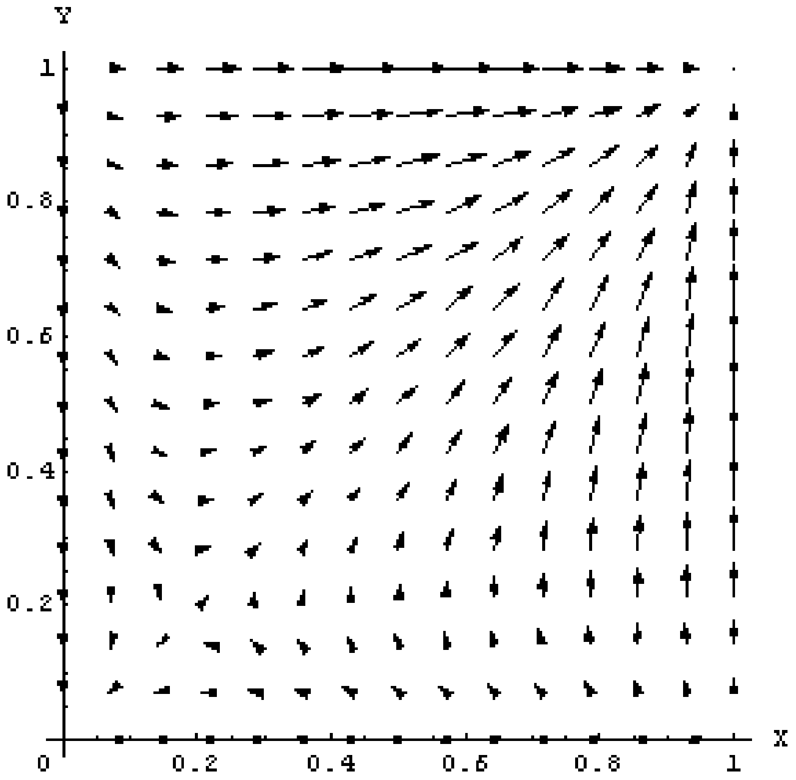
We set : $R=3, S=0, T=5, P=1$

Stability Analysis: Two-Population Case [1]

<Dilemma Games>

The other's strategy Own's strategy	S_1 (altruist)	S_2 (egoist)
S_1 (altruist)	3+a 3+a	5–c b
S_2 (egoist)	b 5–c	1 1

We set: *a=0, b=1.5, c=2.5*



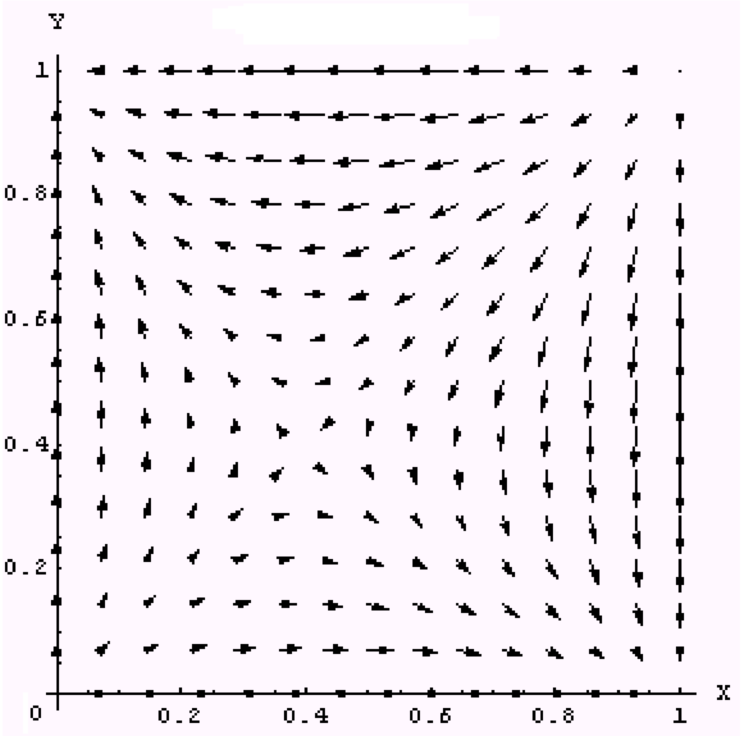
The basin of converging (x,y)=(1,1) becomes large.

Both populations become to behave as altruists.

Stability Analysis:Two-Population Case [2]

<Hawk-Dove Games)

The other's strategy Own's strategy	S_1 (Hawk)	S_2 (Dove)
S_1 (Hawk)	(V−C)/2 (V−C)/2	0 V
S_2 (Dove)	V 0	V/2 V/2



- In <u>one population:</u> the proportion (V/C) of agents play Hawks and the rest (1-V/C) play Doves.(Mixed strategy of 2x2 games)

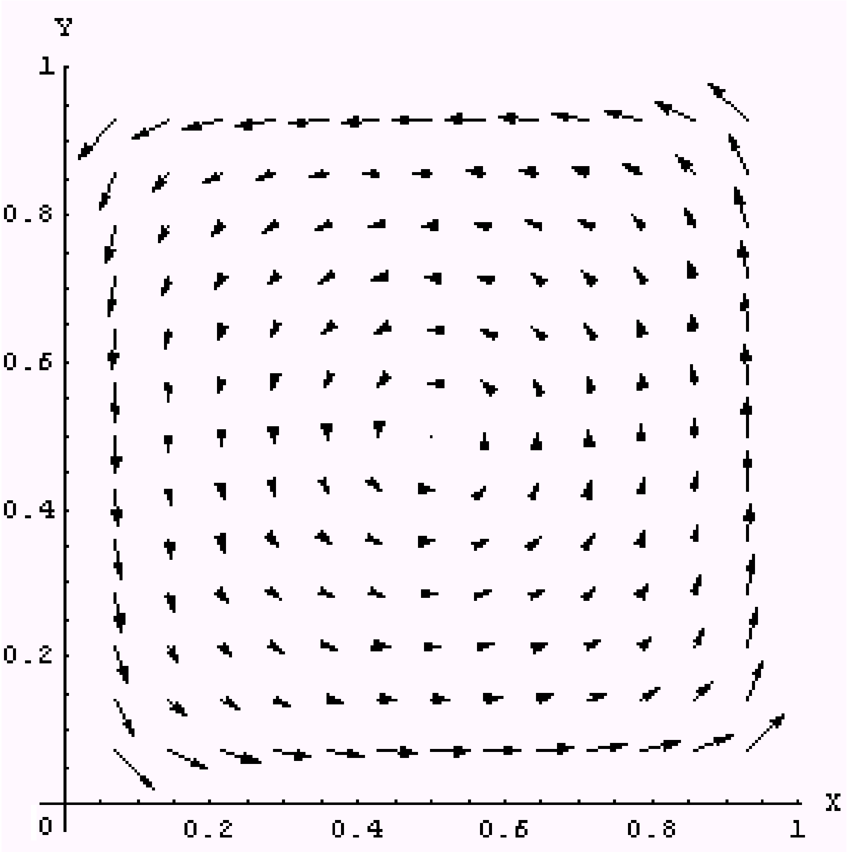
- Two populations: converges to either, (x,y)=(1,0) or (0,1).

: All agents of one population behave as Hawks, and the other population behaves as Doves.

Stability Analysis of RD:Two Populations(3)

<Vicious-circle game>

Seller Buyer	S_1 (honest)	S_2 (cheat)
S_1 (Inspect)	2 3	1 2
S_2 (don't)	3 4	4 1



The RD does not converge, and has the limiting circle.

Outline

A. Introduction to 2x2 Games

B. N-person Games

- Strategic interactions and externalities
- Social Security Games

C. Evolutionary Games

D. Co-evolutionary Learning in Social Games

E. Broad Application Areas

Concept of Social Games

✓ A locally networked agents

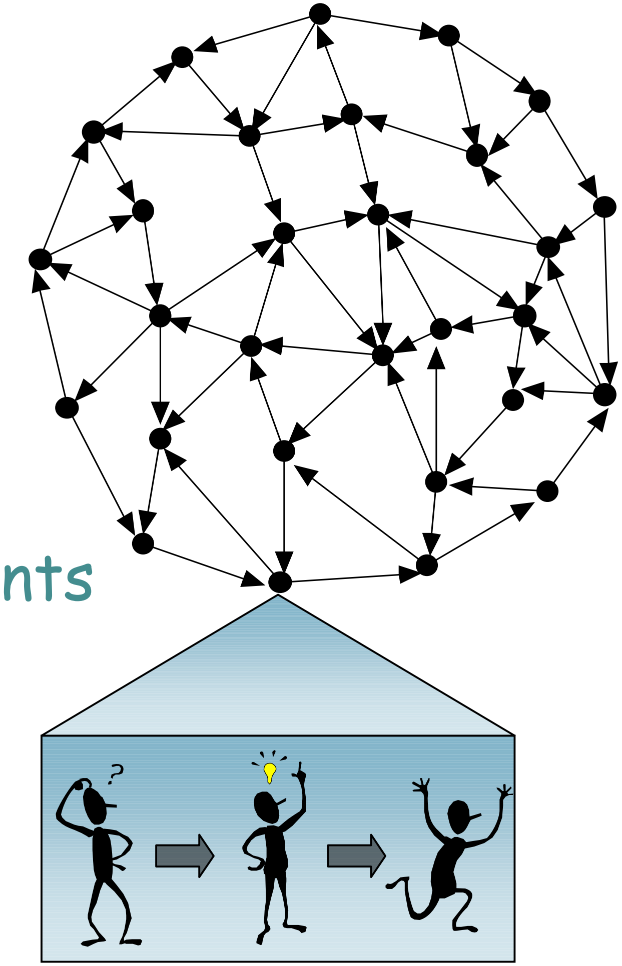
✓ Types of pair-wise interaction

☐ Dilemma game

☐ Coordination game

☐ Dispersion game

☐ Hawk-dove game



✓ Agents evolve their strategies

☐ Whether a society of interacting agents can lead to desirable situations?

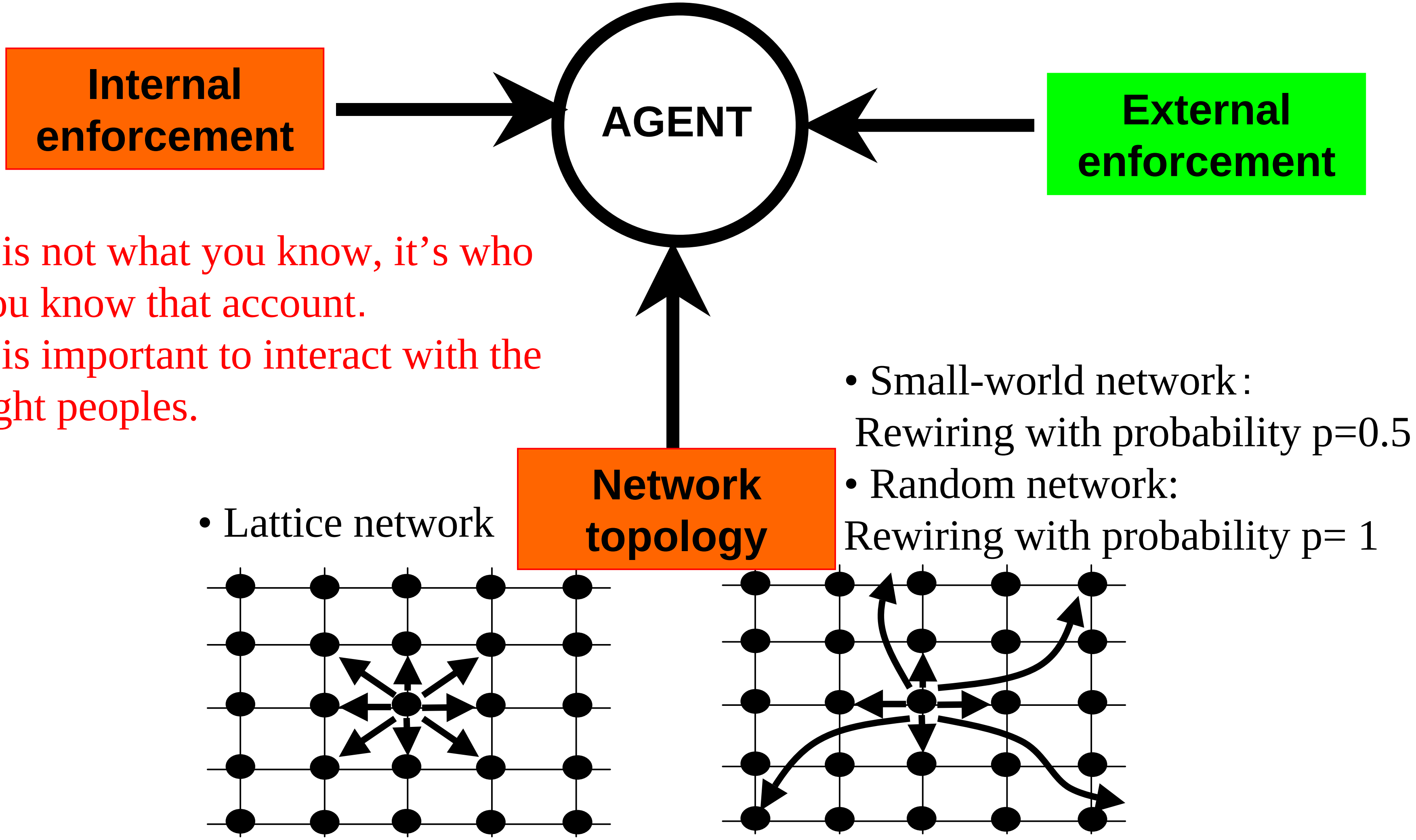
☐ We seek a proper learning model that lead to efficient social outcomes.

Collective Learning/Evolution

- Nash equilibrium is the phrase of the day, but is it a good solution?
- Can we do better than Nash Equilibrium?
- Perhaps we want to just learn some good policy in a decentralized manner. Then what?

Collective learning/evolution)

Matching Models



It is not what you know, it's who you know that account.
It is important to interact with the right peoples.

- Lattice network

- Small-world network: Rewiring with probability p=0.5
- Random network: Rewiring with probability p= 1

Interaction Rules and Memory Length

- Reactionary action: Memory → A_t (action at time t)
- Reactionary action with memory one: $(h_{t\text{-}1}) \rightarrow A_t$

  <Tit-for-Tat Policy>:

  (. , *Defect*) → *Defect*

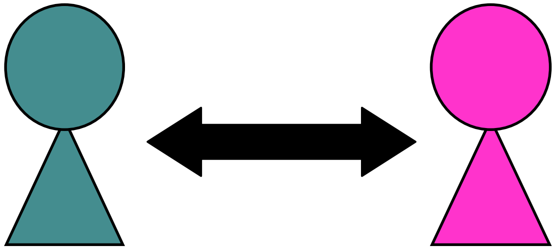
  (. , *Cooperate*) → *Cooperate*
- Finite Memory: $\{ (h_{t\text{-}n} , \ldots , h_{t\text{-}2} , h_{t\text{-}1}) \} \rightarrow A_t$

Is a short memory enough or a longer memory may be necessary in more complex games?

Definition of Interaction Rules

- **Coupled learning:**

: Strategy choice is driven by joint actions



	past strategy		strategy at t
	Own	Opp	
	0	0	#
	0	1	#
	1	0	#
	1	1	#

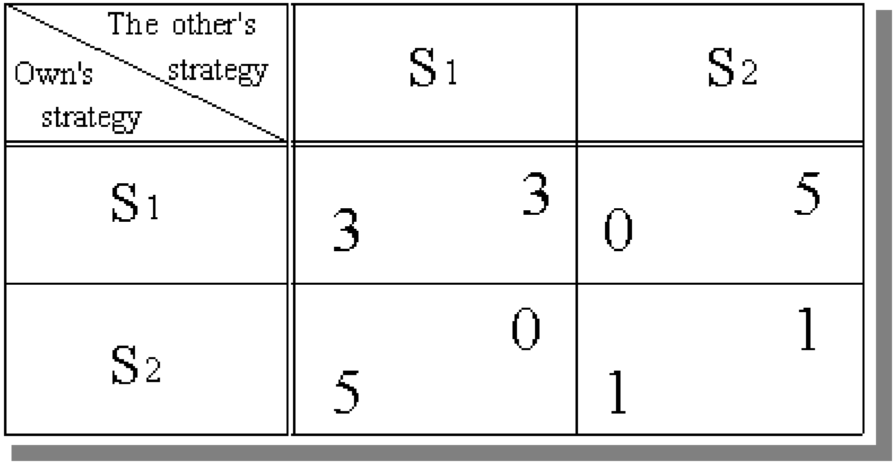
Own: own strategy

Opp: opponent's strategy

•Agents decide the next strategy based on the interaction rule which is the combination of the previous strategies of own and opponent .

•Agents learns strategies marked by # in the interaction rule.

Well-known Rules in Dilemma Game



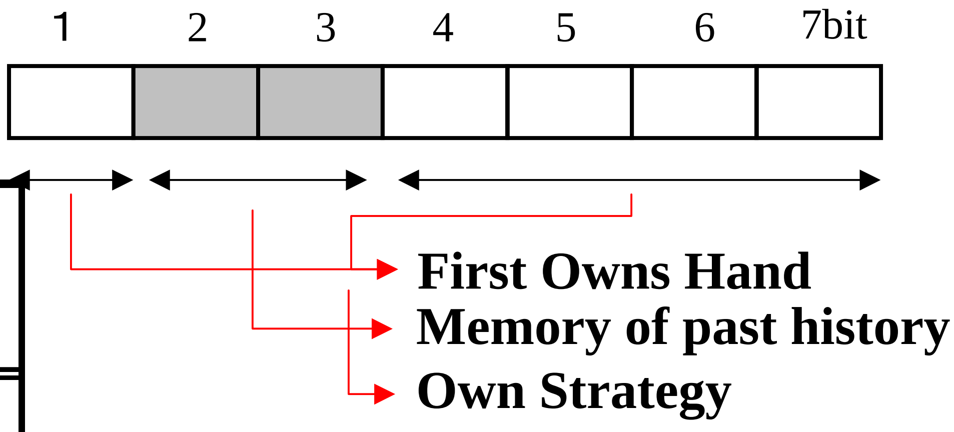
The other's strategy Own's strategy	S_1	S_2
S_1	3 3	5 0
S_2	0 5	1 1

S_1: 0
S_2: 1

Past strategy		TFT	PAVLOV	ALL C(D)
Own	Opp			
0	0	0	0	0(1)
0	1	1	1	0(1)
1	0	0	1	0(1)
1	1	1	0	0(1)

Learnable Rule with Memory of 2

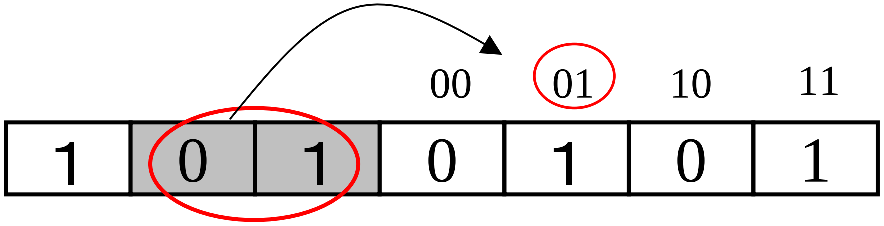
S_1: 0
S_2: 1



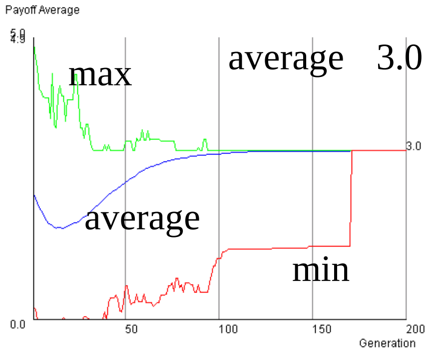
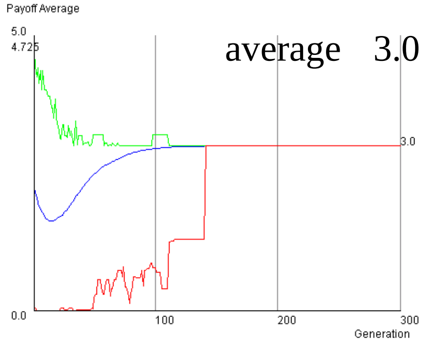
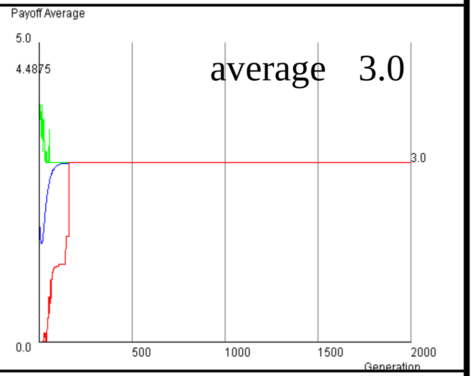
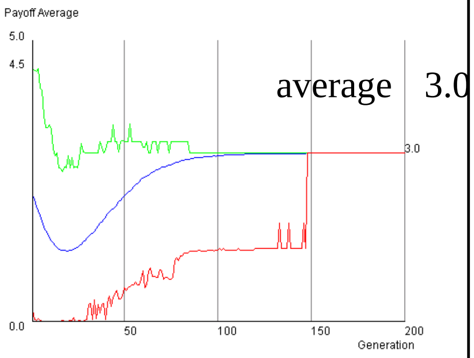
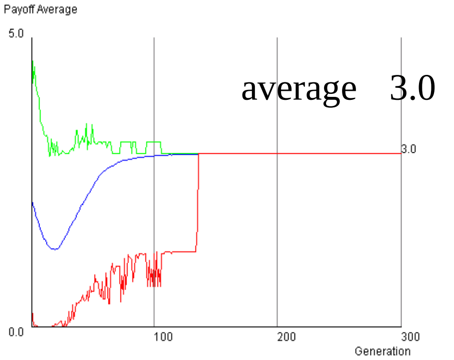
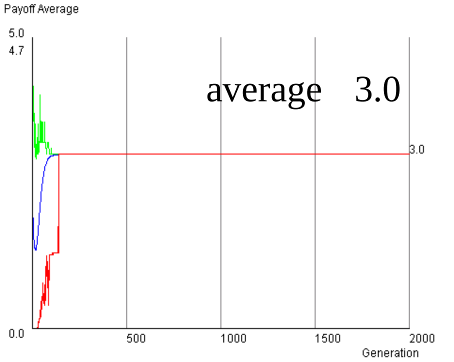
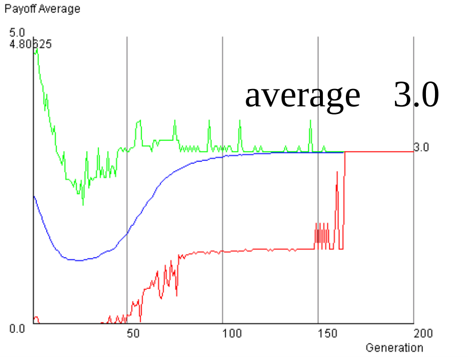
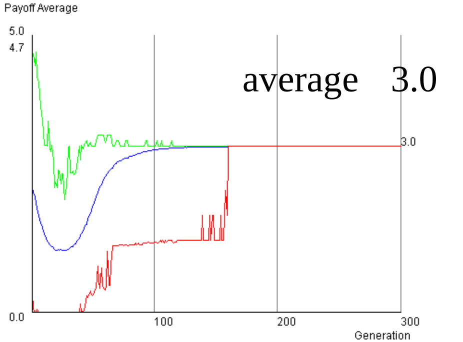
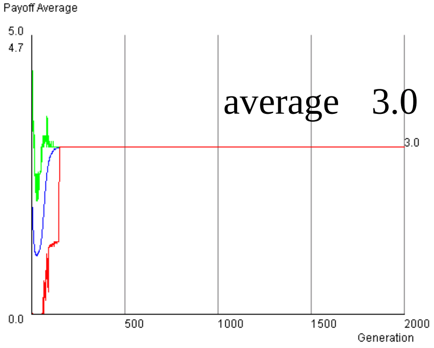
bit	past strategy		strategy at t
	Own	Opp	
4	0	0	#
5	0	1	#
6	1	0	#
7	1	1	#

#: 0 or 1

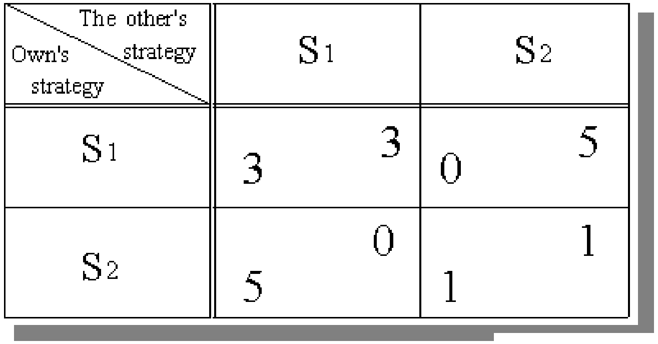
Example:



Simulation Results: Dilemma Game

	generation 200	generation 300	generation 2000
Local model			
Small-world model			
Random model			

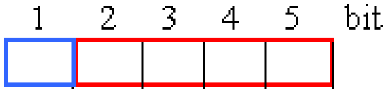
What did Agents Learn in Dilemma Game?



The other's strategy / Own's strategy	S1	S2
S1	3 3	5 0
S2	0 5	1 1

S_1: 0
S_2: 1

Array location
1: Initial strategy
2: Choice when outcome is (0, 0)
3: Choice when outcome is (0, 1)
4: Choice when outcome is (1, 0)
5: Choice when outcome is (1, 1)



1 initial strategy
2~5 rule type

- The rules of 2,500 agents are aggregated into a few rules.

local model

0	0	1	1	1
0	0	1	0	1
0	0	1	1	0
0	0	0	1	1
0	0	0	0	1
0	0	1	0	0
0	0	0	1	0

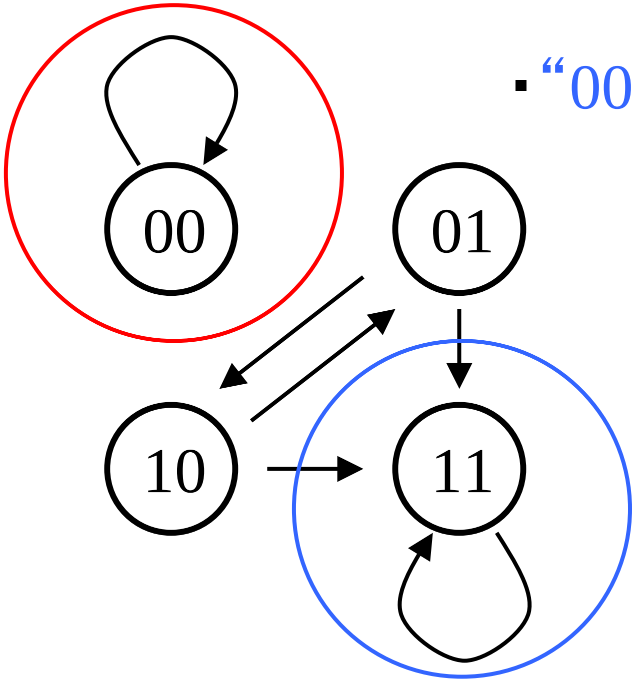
small-world model

0	0	1	1	1
0	0	1	0	1
0	0	0	1	1
0	0	1	1	0
0	0	0	1	1

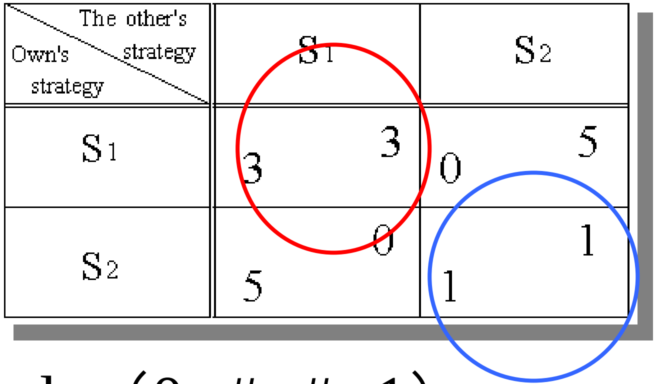
random model

0	0	1	1	1
0	0	0	1	1
0	0	1	0	1
0	0	0	0	1

Interpretation of Co-evolved Rules



- “00”, “11”are the two absorbing stattes.

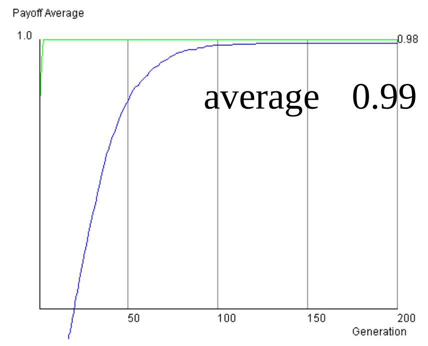
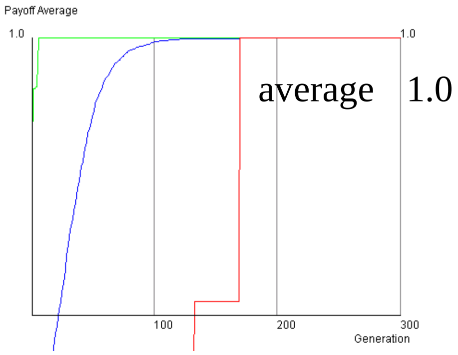
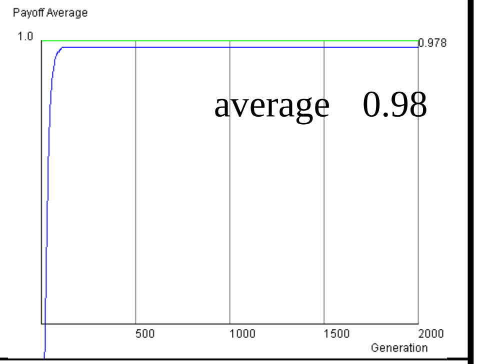
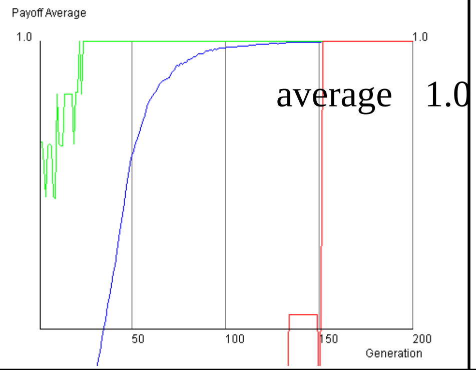
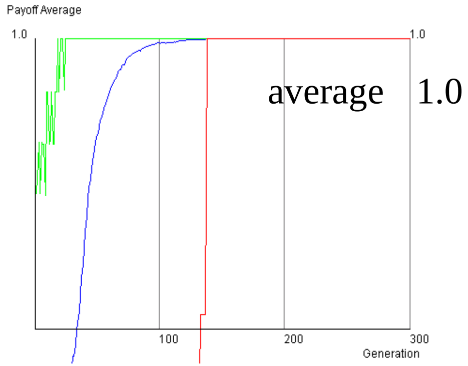
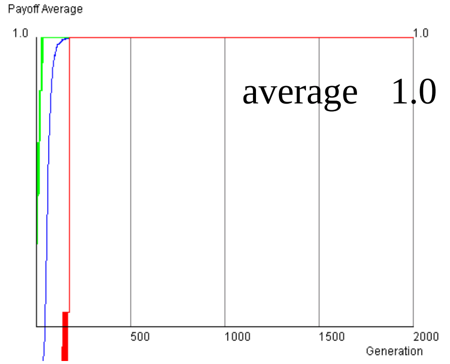
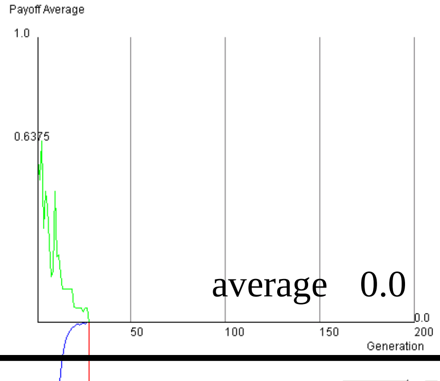
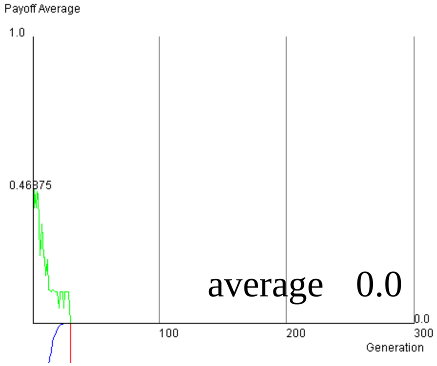
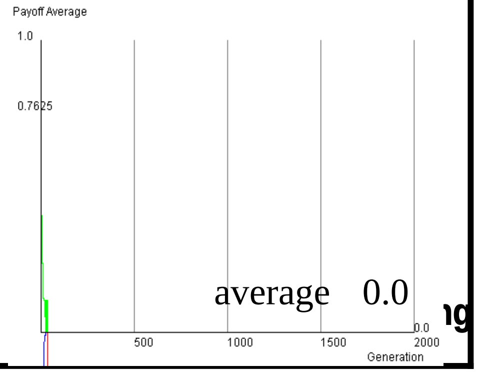


The other's strategy Own's strategy	S1	S2
S1	3 3	5 0
S2	0 5	1 1

Learned rule: (0 # # 1)

Past strategy		TFT	PAVLOV	Learned rule
Own	Opp			
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	1	0	1

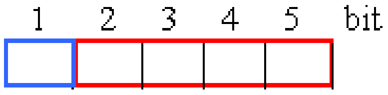
Simulation Results: Coordination Game

	generation 200	generation 300	generation 2000
Lattice network	 average 0.99	 average 1.0	 average 0.98
Small-world network	 average 1.0	 average 1.0	 average 1.0
Random network	 average 0.0	 average 0.0	 average 0.0

What did Agents Learn in Coordination Game?

The other's strategy Own's strategy	S1	S2
S1	1 1	0 -9
S2	-9 0	0 0

S_1: 0
S_2: 1



1 initial strategy
2~5 rule type

Array location
1: Initial strategy
2: Choice when outcome is (0, 0)
3: Choice when outcome is (0, 1)
4: Choice when outcome is (1, 0)
5: Choice when outcome is (1, 1)

- The rules of 2,500 agents are aggregated into a few rules.

lattice network

0	0	1	1	1
0	0	1	0	1
0	0	0	1	1
0	0	1	1	0
0	0	0	0	1
0	0	0	0	0
0	0	1	0	0

small-world network

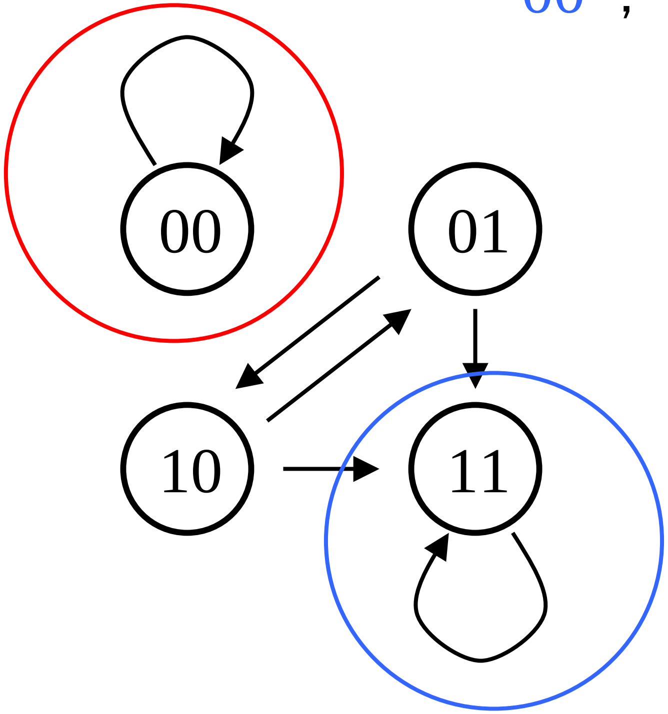
0	0	1	1	1
0	0	0	1	1
0	0	1	0	1
0	0	0	0	1
0	0	1	1	0

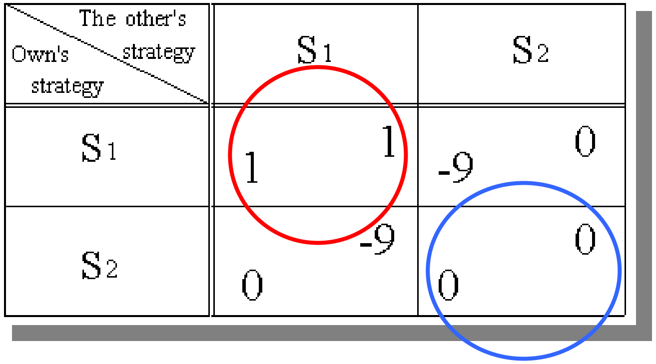
random network

1	0	1	1	1
1	0	0	1	1
1	1	1	1	1
1	1	0	1	1
1	1	1	0	1
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1

Interpretation of Learned Rules

- “00”, “11”are the two absorbing stattes.

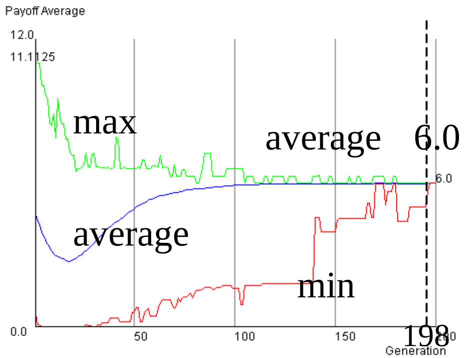
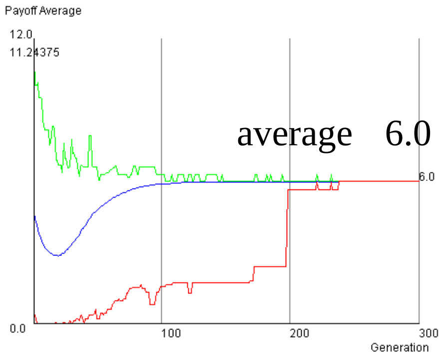
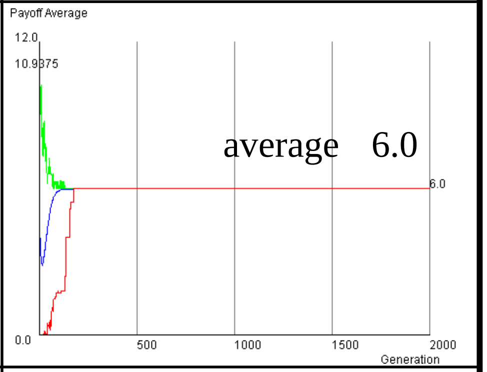
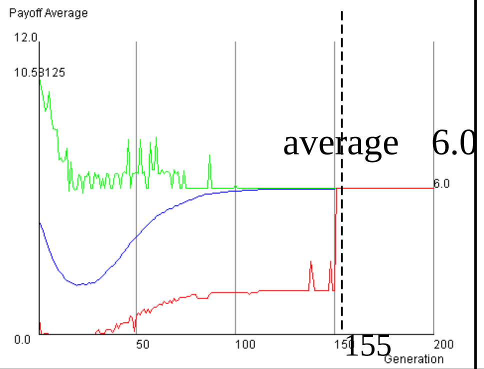
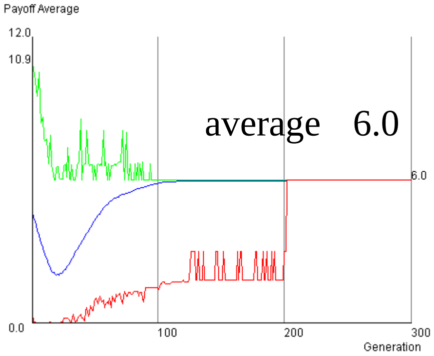
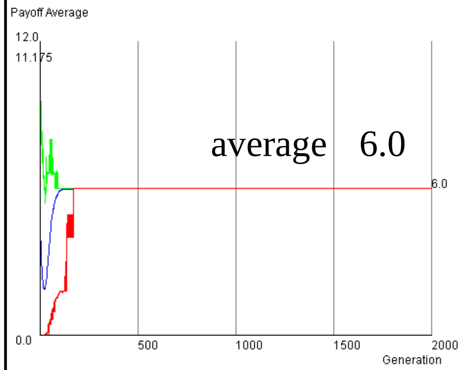
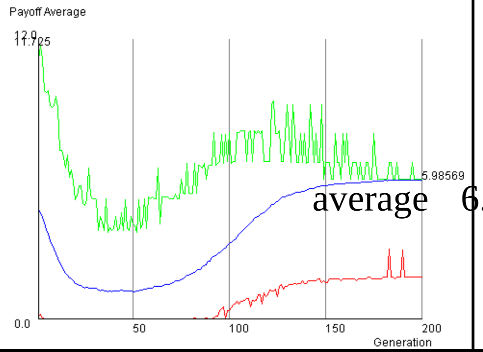
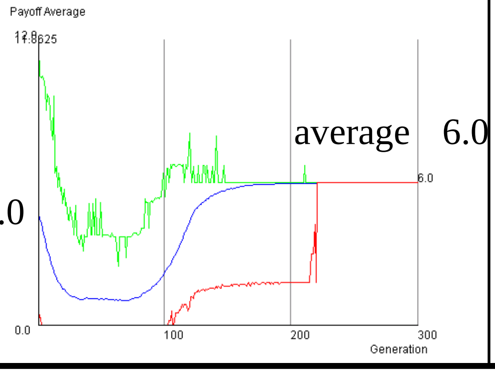
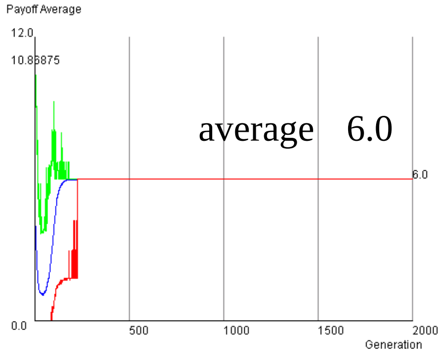




The other's strategy / Own's strategy	S_1	S_2
S_1	1 1	0 -9
S_2	-9 0	0 0

Learned rule: (0 # # 1)

Simulation Results: Hawk-Dove Game

	generation 200	generation 300	generation 2000
Lattice network			
Small-world network			
Random network			

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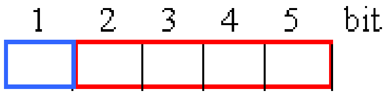
What did Agents Learn in Hawk-Dove Game?

（V,C＝10,12）

The other's strategy Own's strategy	S_1 (Hawk)	S_2 (Dove)
S_1 (Hawk)	-1 -1	0 2
S_2 (Dove)	2 0	1 1

S_1: 0
S_2: 1

Array location
1: Initial strategy
2: Choice when outcome is (0, 0)
3: Choice when outcome is (0, 1)
4: Choice when outcome is (1, 0)
5: Choice when outcome is (1, 1)



1 initial strategy
2~5 rule type

- The rules of 2,500 agents are aggregated into a few rules.

Lattice network

0	0	1	1	1
0	0	1	0	1
0	0	0	1	1
0	0	1	1	0
0	0	1	0	0
0	0	0	0	1
0	0	0	1	0

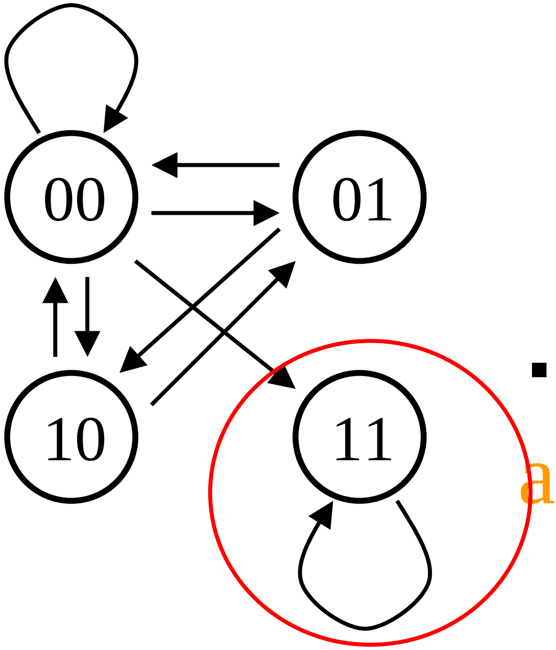
small-world network

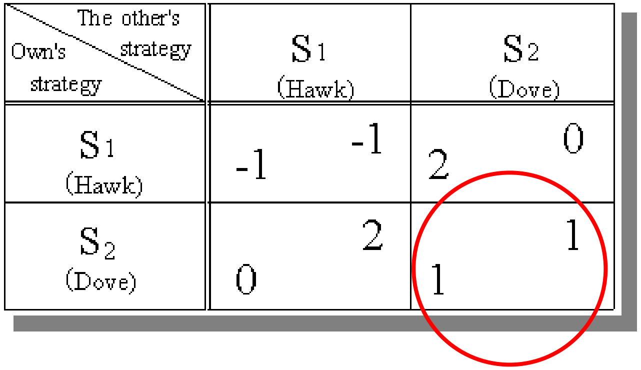
0	0	1	1	1
0	0	1	0	1
0	0	0	1	1
0	0	1	1	0
0	0	0	0	1
0	0	0	1	0

random network

0	0	1	1	1
0	0	1	1	0

Interpretation of Learned Rules



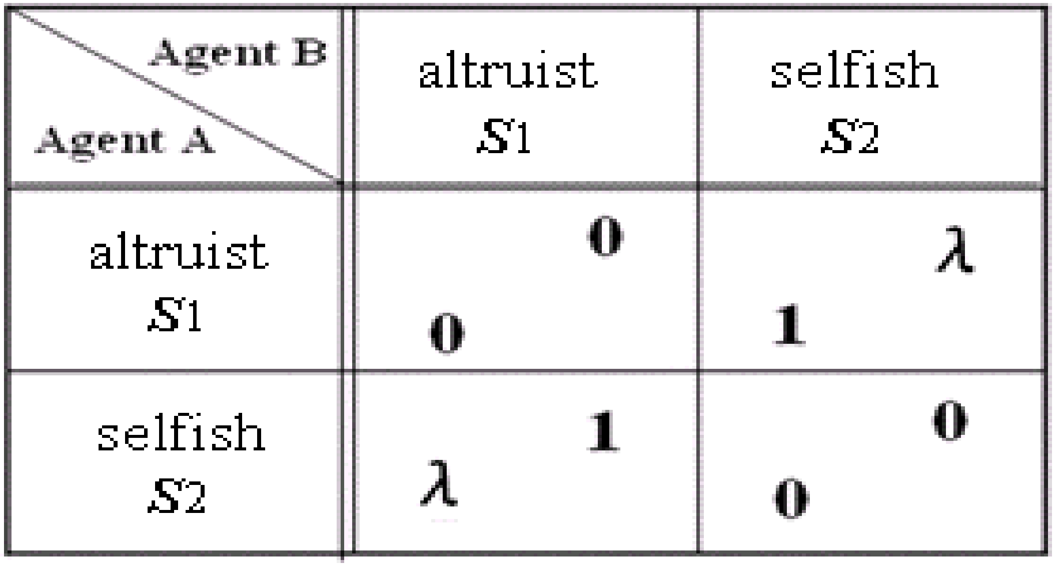


The other's strategy / Own's strategy	S$_1$ (Hawk)	S$_2$ (Dove)
S$_1$ (Hawk)	-1 -1	0 2
S$_2$ (Dove)	2 0	1 1

- (Dove, Dove) is the unique absorbing state.

Learned rule: （# # 0 1）

Dispersion Games



Agent B Agent A	altruist S_1	selfish S_2
altruist S_1	0 0	λ 1
selfish S_2	1 λ	0 0

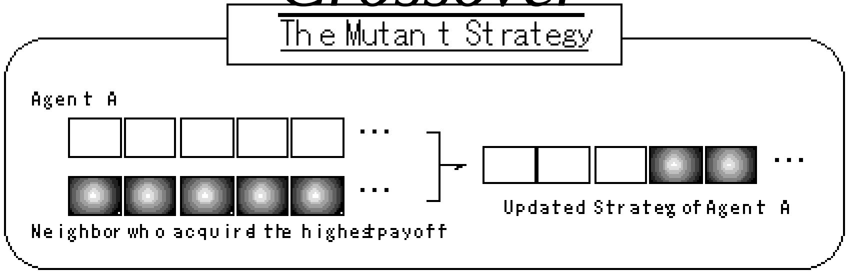
(1) $\lambda = 1$: Symmetric game

(2) $\lambda = 3$: Asymmetric game

Requirements for Agents

- Agents need to “create efficient behavioral rules”.
- Agents crossover with their behavioral rule with that of the most successful neighbor.

- *Crossover*



- Agents need to “spread out efficient interaction rules”.

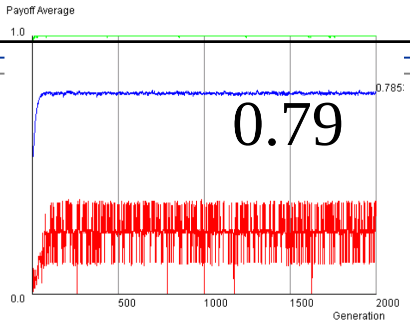
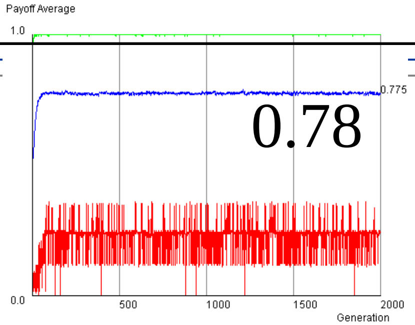
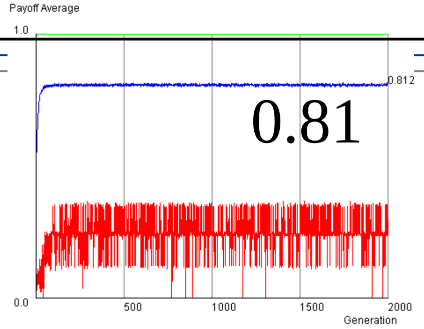
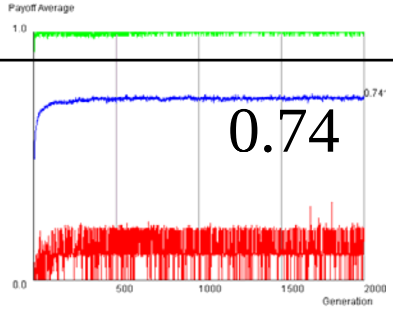
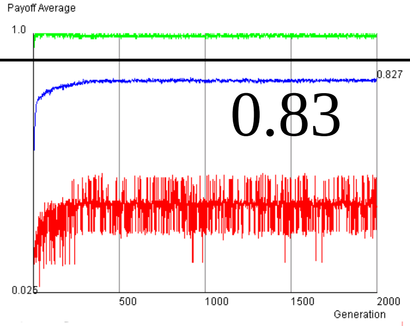
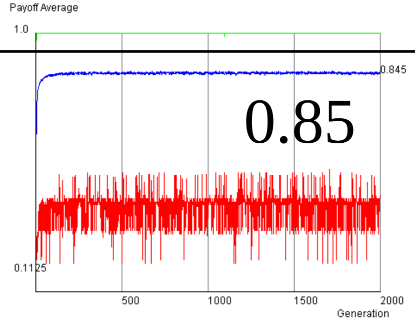
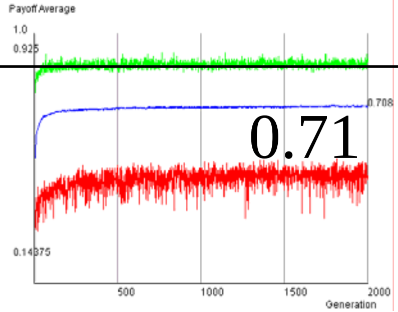
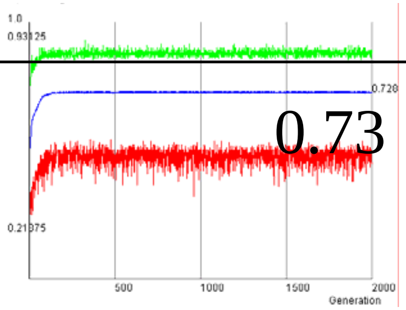
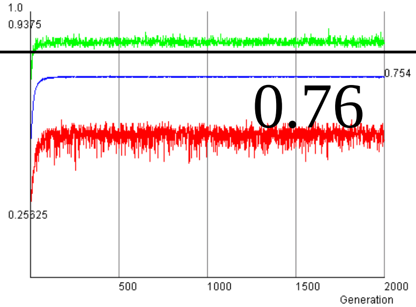
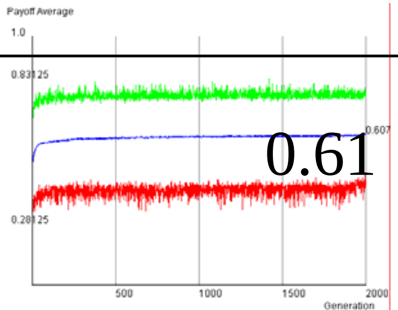
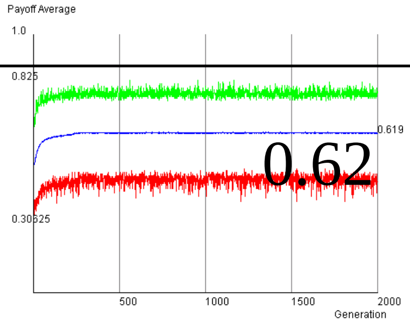
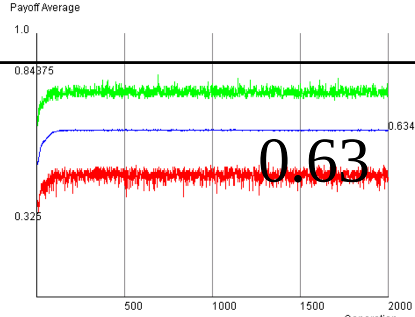
- Implementation Error: Agents occasionally make mistakes when they implement strategy specified by the rule.

Learnable Rules with Memory of 4

At t-2		At t-1		Strategy at t
own	opponent	own	opponent	
0	0	0	0	#
0	0	0	1	#
0	0	1	0	#
0	0	1	1	#
0	1	0	0	#
0	1	0	1	#
0	1	1	0	#
0	1	1	1	#
1	0	0	0	#
1	0	0	1	#
1	0	1	0	#
1	0	1	1	#
1	1	0	0	#
1	1	0	1	#
1	1	1	0	#
1	1	1	1	#

Possible rules :

$$2^{16} = 65,536$$

λ =1: Symmetric game	Lattice networks	Small-world networks	Random networks
m=2 noise: 0%	 0.79	 0.78	 0.81
m=4 noise: 0%	 0.74	 0.83	 0.85
m=4 noise 10%	 0.71	 0.73	 0.76
m=4 noise 20%	 0.61	 0.62	 0.63

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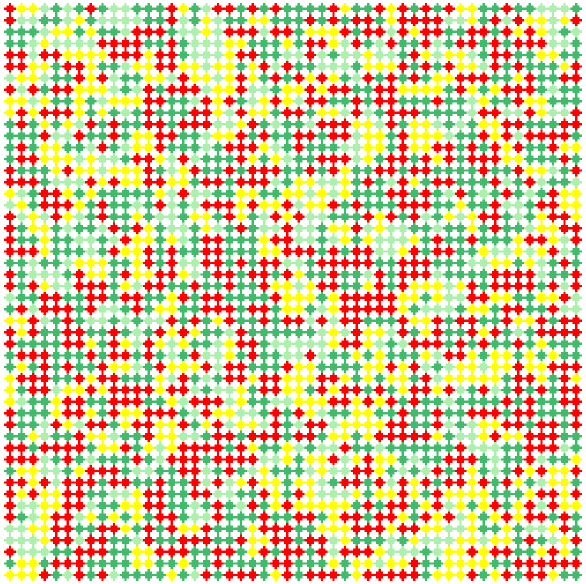
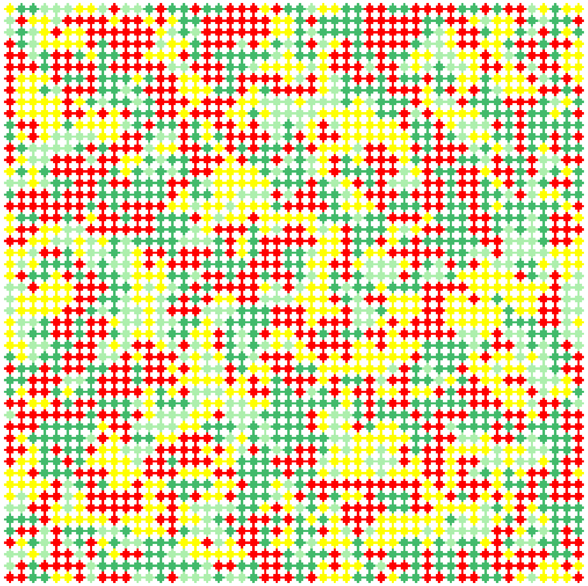
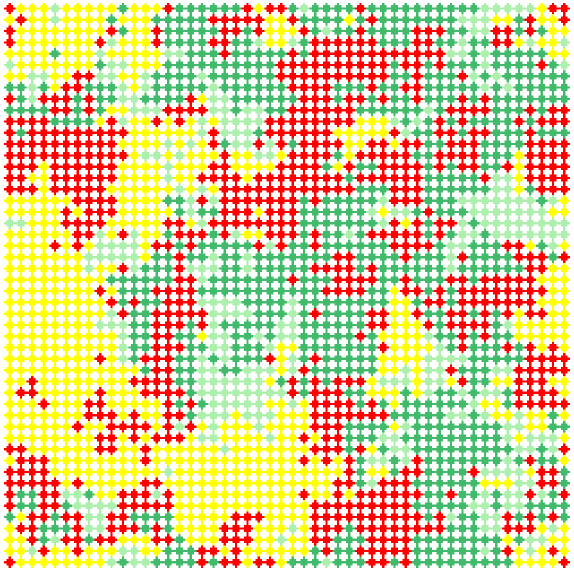
Locations of Agents with Different Rules


Lattice Networks

type1 = 1,0,1,1,0 (533)
type2 = 1,0,1,0,0 (817)
type3 = 0,0,1,1,1 (379)
type4 = 0,0,1,0,0 (771)


Small-world Networks

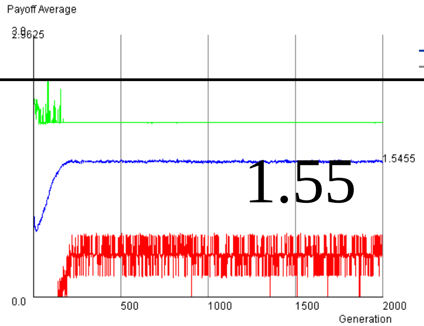
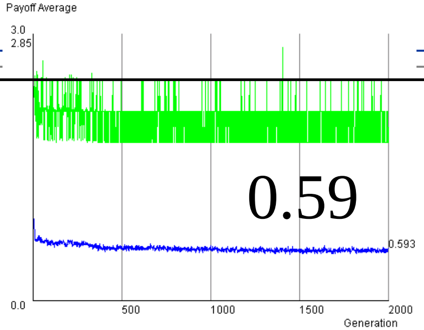
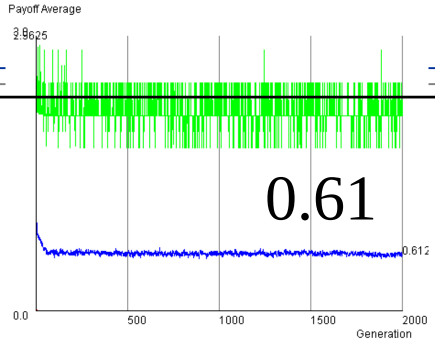
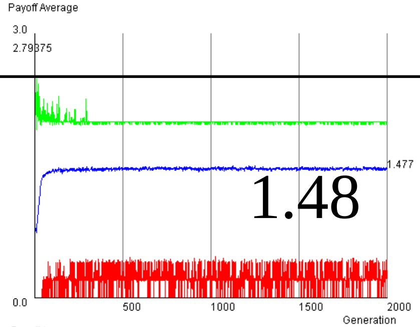
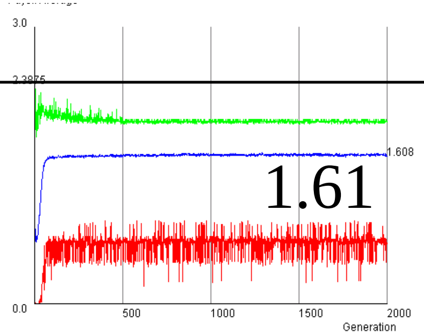
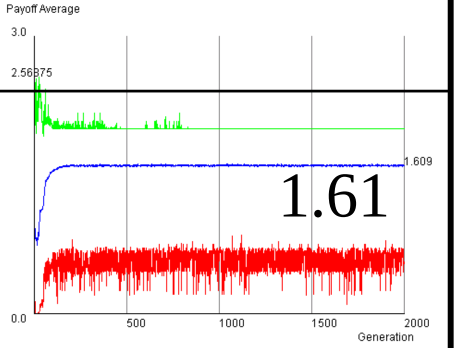
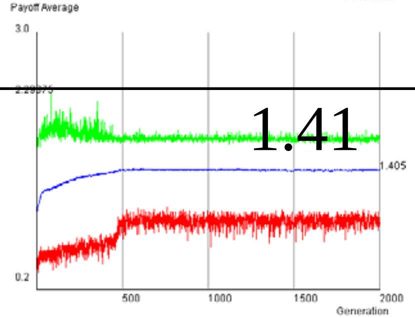
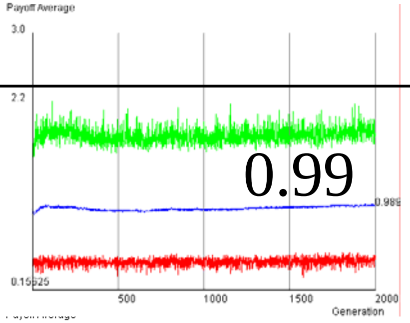
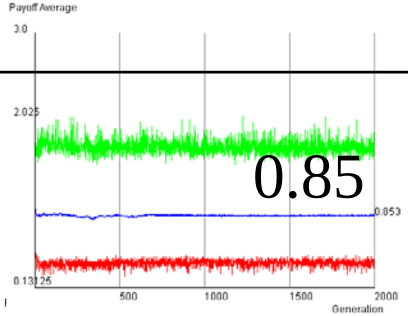
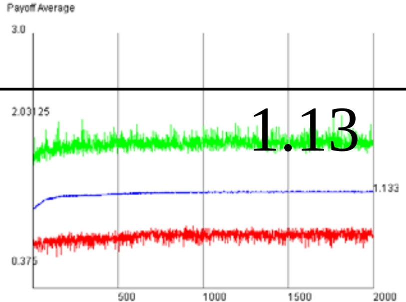
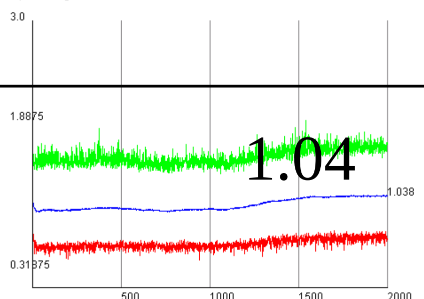
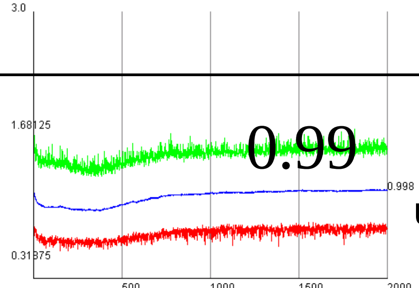
type1 = 1,0,1,1,1 (611)
type2 = 0,0,1,1,0 (381)
type3 = 1,0,1,0,1 (841)
type4 = 0,0,1,0,0 (667)


Random Networks

type1 = 1,0,1,1,1 (683)
type2 = 0,0,1,1,0 (322)
type3 = 1,0,1,0,1 (762)
type4 = 0,0,1,0,0 (733)

(the number of agents who share the same rule)

Co-evolved rule: Win-stay, lose-shift

λ =3: Asymmetric game	Lattice networks	Small-world networks	Random networks
m=2 noise: 0%	 1.55	 0.59	 0.61
m=4 noise: 0%	 1.48	 1.61	 1.61
m=4 noise: 10%	 1.41	 0.99	 0.85
m=4 noise: 20%	 1.13	 1.04	 0.99

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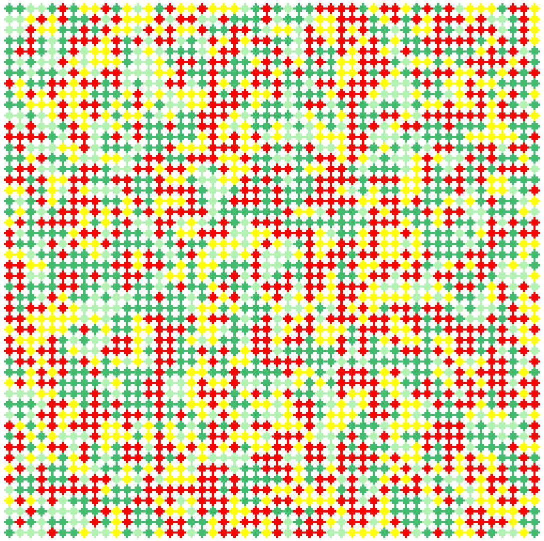
Location of Agents

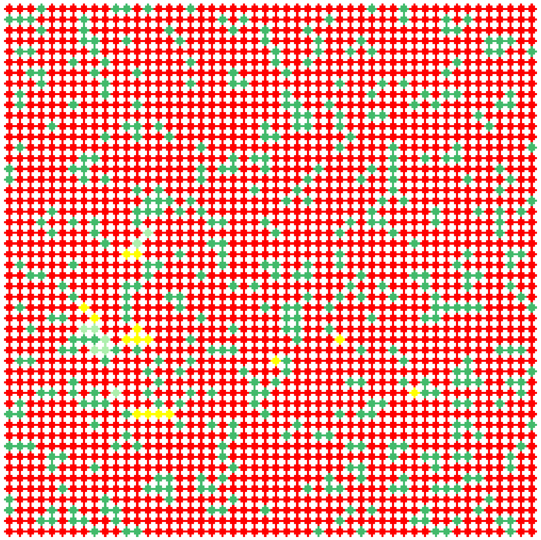
Lattice Networks



type1 = 1,1,0,1,0 (719)
type2 = 0,1,0,1,1 (380)
type3 = 1,1,0,0,1 (842)
type4 = 0,1,0,0,1 (559)

Learned rules:
Give-and-take
Turn-taking

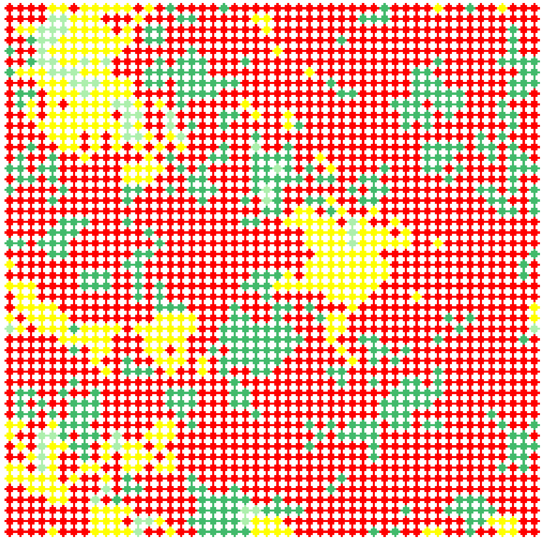
Small-world Networks



type1 = 1,0,1,1,1 (2036)
type2 = 1,0,1,0,1 (441)
type3 = 0,0,1,1,1 (15)
type4 = 0,0,1,0,1 (8)

Learned rules:
Win-stay, lose-shift

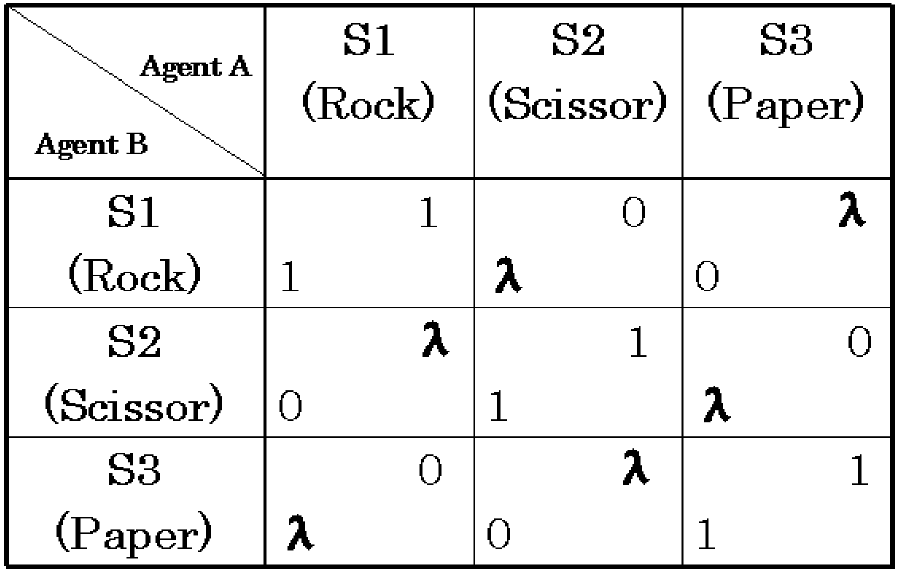
Random Networks



type1 = 1,0,1,1,1 (1695)
type2 = 1,0,1,0,1 (431)
type3 = 0,0,1,1,1 (321)
type4 = 0,0,1,0,1 (53)

Learned rules:
Win-stay, lose-shift

Generalized Rock-Scissor-Paper Games



Agent A Agent B	S1 (Rock)	S2 (Scissor)	S3 (Paper)
S1 (Rock)	1 1	0 λ	λ 0
S2 (Scissor)	λ 0	1 1	0 λ
S3 (Paper)	0 λ	λ 0	1 1

(1) λ =2: The conventional R-S-P game

(2) λ >2: Anti-coordination games

An agent gains the payoff if she takes distinct strategy of her opponent.

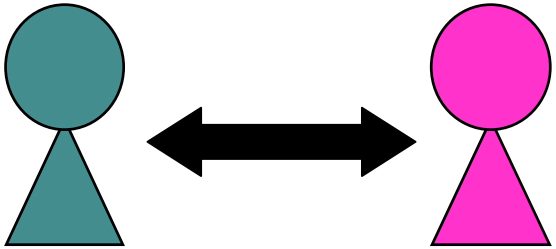
- The unique Nash equilibrium is the mixed strategy with:
  (S_1: 1/3, S_2: 1/3, S_3: 1/3)

- The expected payoff at a mixed Nash equilibrium : (λ +1)/3

Coupling Rules: Three-Strategy Case

Interaction rule

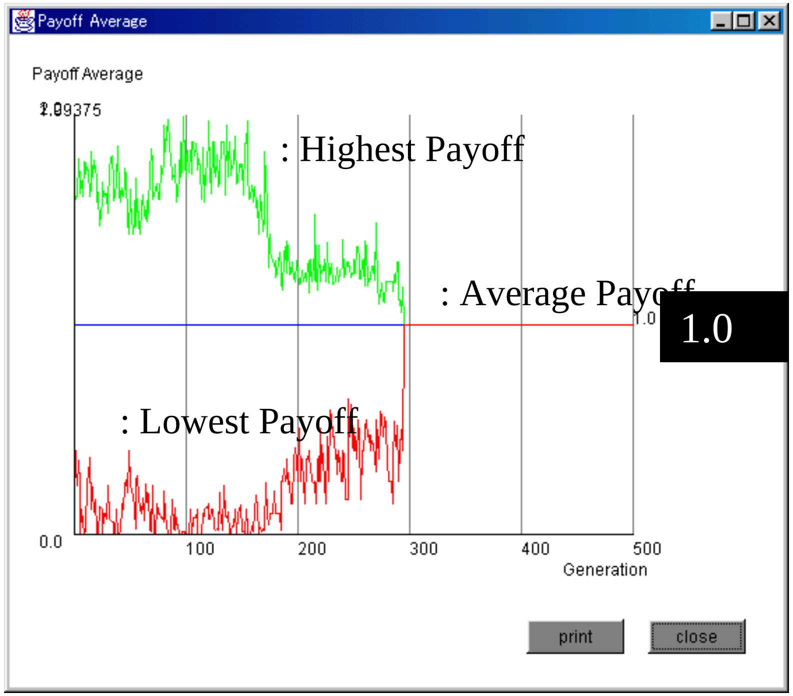
•Previous Strategy		•Next Strategy
•Own	•Opponent	
•0	•0	•#
•0	•1	•#
•0	•2	•#
•1	•0	•#
•1	•1	•#
•1	•2	•#
•2	•0	•#
•2	•1	•#
•2	•2	•#



0 : Rock,
1: Scissor,
2: Paper
#: 0, 1 or 2

Simulation Result (1): λ =2

No implementation error



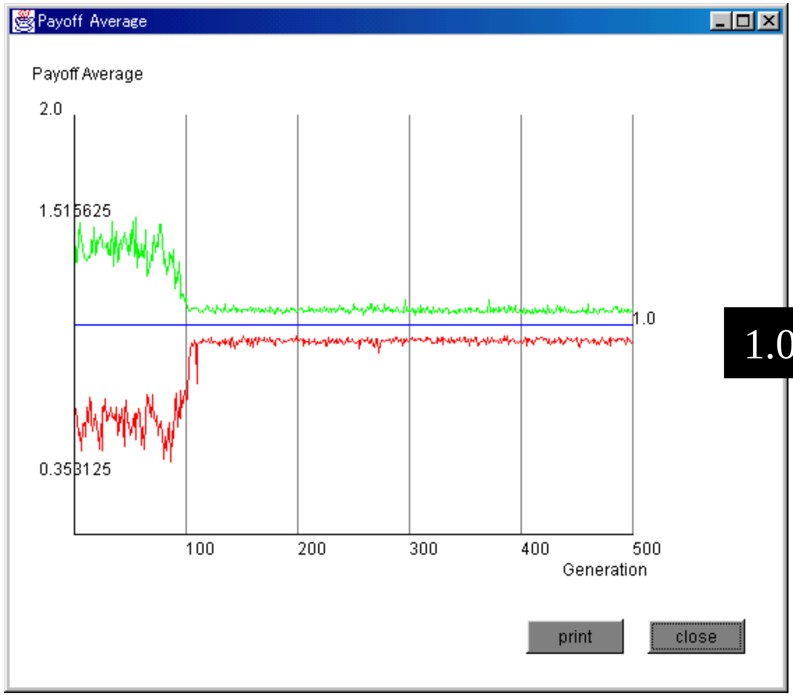
: Highest Payoff

: Average Payoff

: Lowest Payoff

1.0

Error rate: 10%



1.0

Nash equilibrium: 1

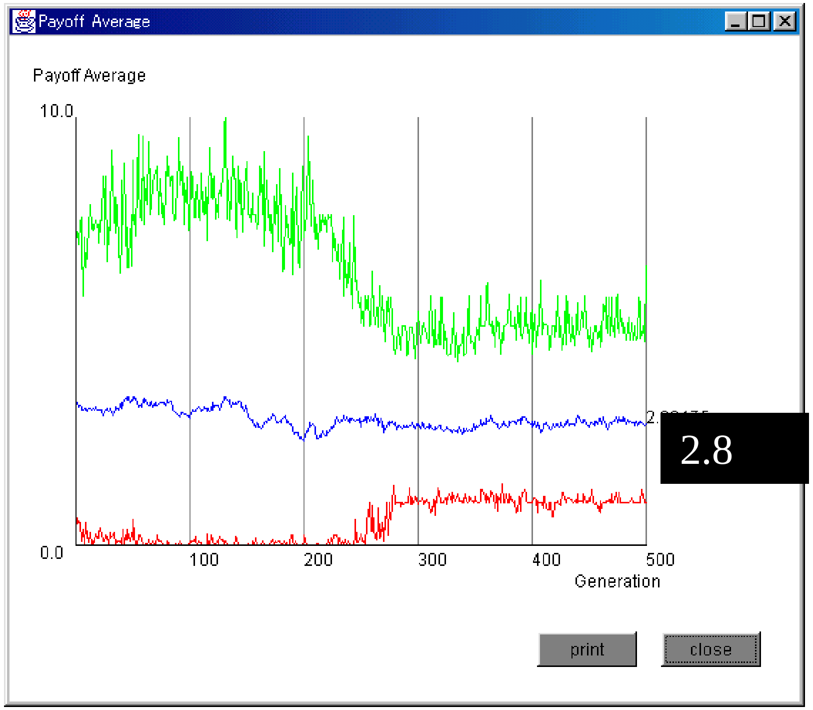
Pareto optimal : 1

X-axis: Generation

Y-axis: Payoff Average

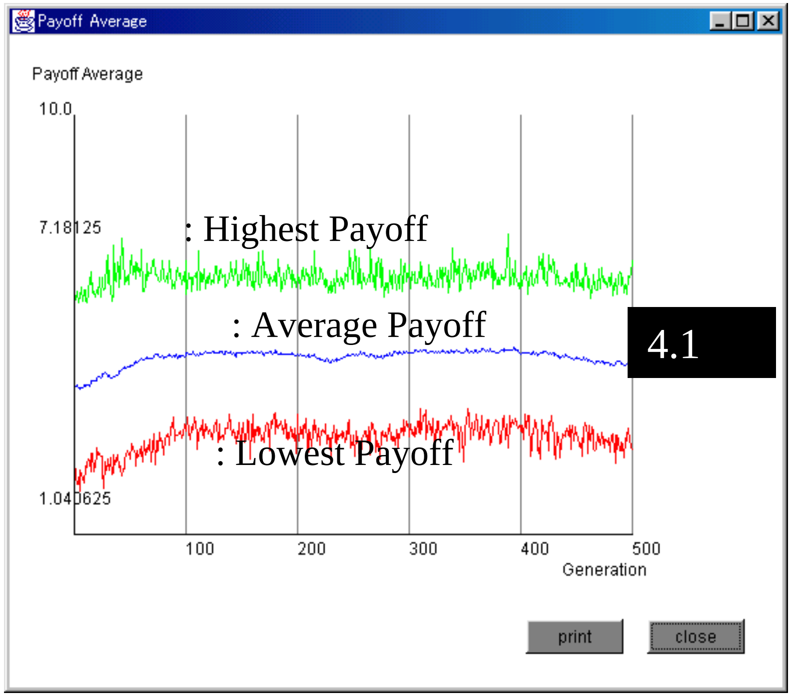
Simulation Result (3) : λ =10

No implementation error



2.8

Error rate: 10%



: Highest Payoff

: Average Payoff

: Lowest Payoff

4.1

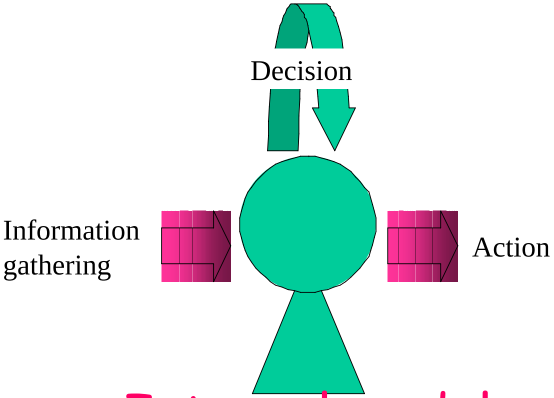
Nash equilibrium: 3.7

Pareto optimal : 5

Types of Learned Rules

	Error Rate: 10%	Error Rate: 0%
Generation	Number of Agents with the Same Rule	Number of Agents with the Same Rule
500th	400	400
1000th	250	400
1500th	30	368
2000th	8	238

Agents learned to share the same rules if they occasionally make mistakes.



Internal model

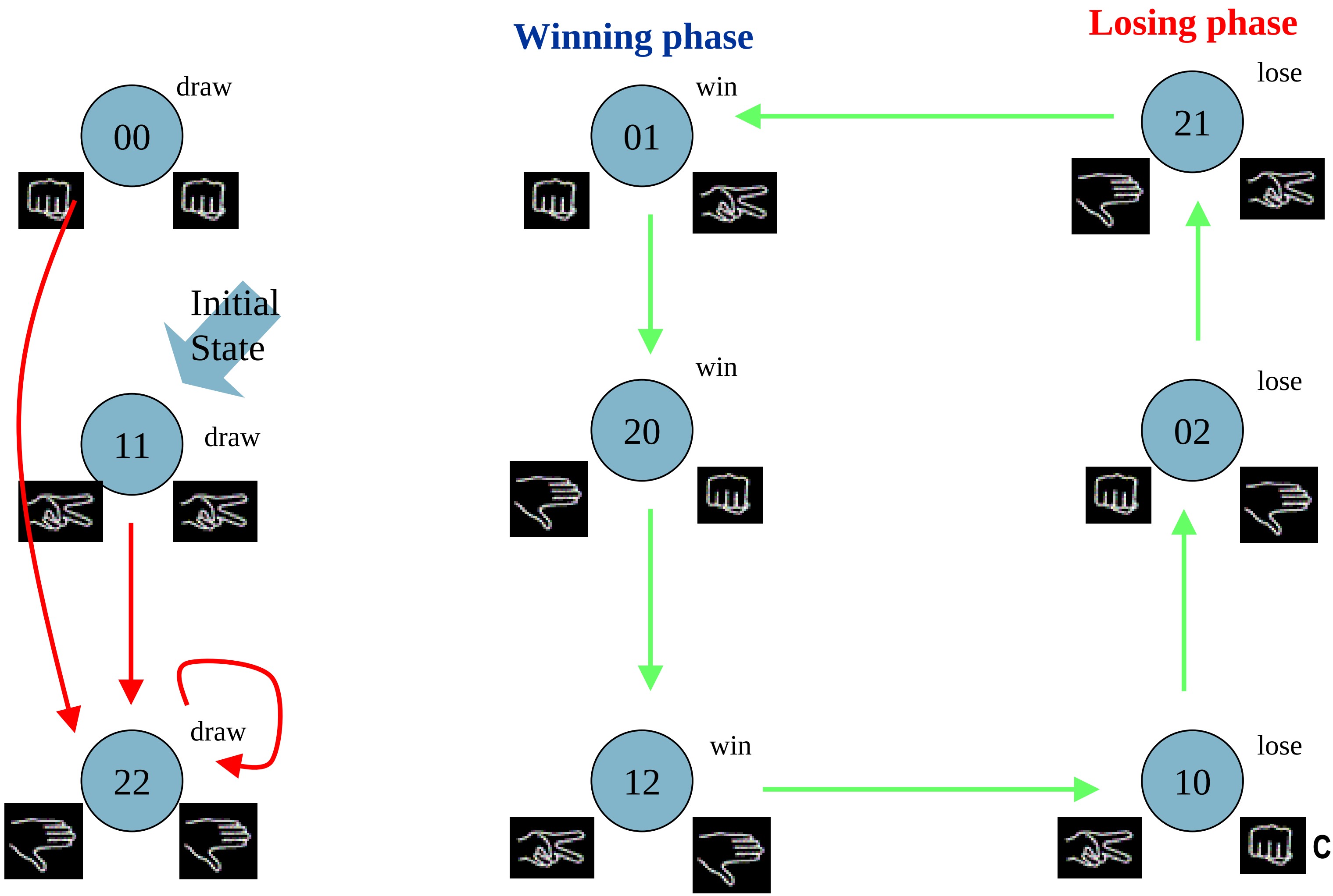
Commonality of Learned Rules

	00	01	02	10	11	12	20	21	22	Number of Agents with the Same Rule
Rule Type 1	2	2	2	0	2	1	1	0	2	126
Rule Type 2	2	2	2	0	2	1	1	0	0	76
Rule Type 3	1	2	2	0	2	1	1	0	2	58
Rule Type 4	1	2	2	0	2	1	1	0	0	54
Rule Type 5	2	2	2	0	0	1	1	0	2	40
Rule Type 6	1	2	2	0	0	1	1	0	2	19
Rule Type 7	2	2	2	0	0	1	1	0	0	17
Rule Type 8	1	2	2	0	0	1	1	0	0	10

400 agents learned to share 8 rules which have some common values.

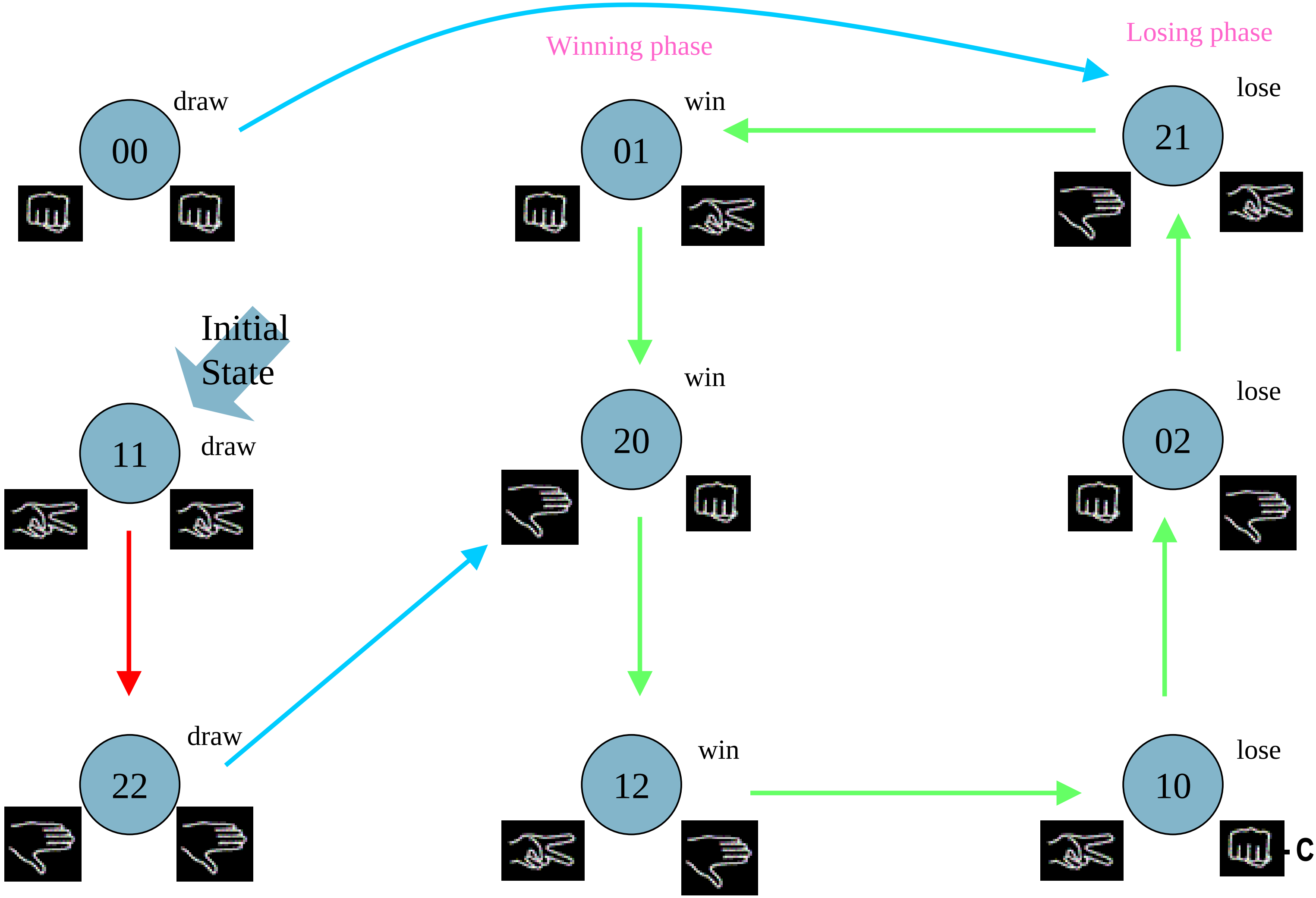
State Transitions of Agents with the Same Rule

If agents with the same rule play the game, they absorb in the drawing cycle. Their payoffs are lower than the Pareto-optimal solution.



State Transitions of Agents with Different Rules

Agents win three times by adapting each strategy, then they lose three times.



How Did Agents Play with the Repeated R-S-P Games?

Agent A Agent B	S1 (Rock)	S2 (Scissor)	S3 (Paper)
S1 (Rock)	1 1	0 λ	λ 0
S2 (Scissor)	λ 0	1 1	0 λ
S3 (Paper)	0 λ	λ 0	1 1

○ **Unfair Pareto-optimal outcome**

- With the mixed Nash equilibrium strategy, (S_1: 1/3, S_2: 1/3, S_3: 1/3), the expected payoff is 1, which is also Pareto-efficient.
- However,there exist lucky agents with the payoff 1, and unlucky agents with the payoff 0.

❑ Efficiency and equity (fairness) are achieved with turn-taking

Outline

A. Introduction to 2x2 Games

B. N-person Games

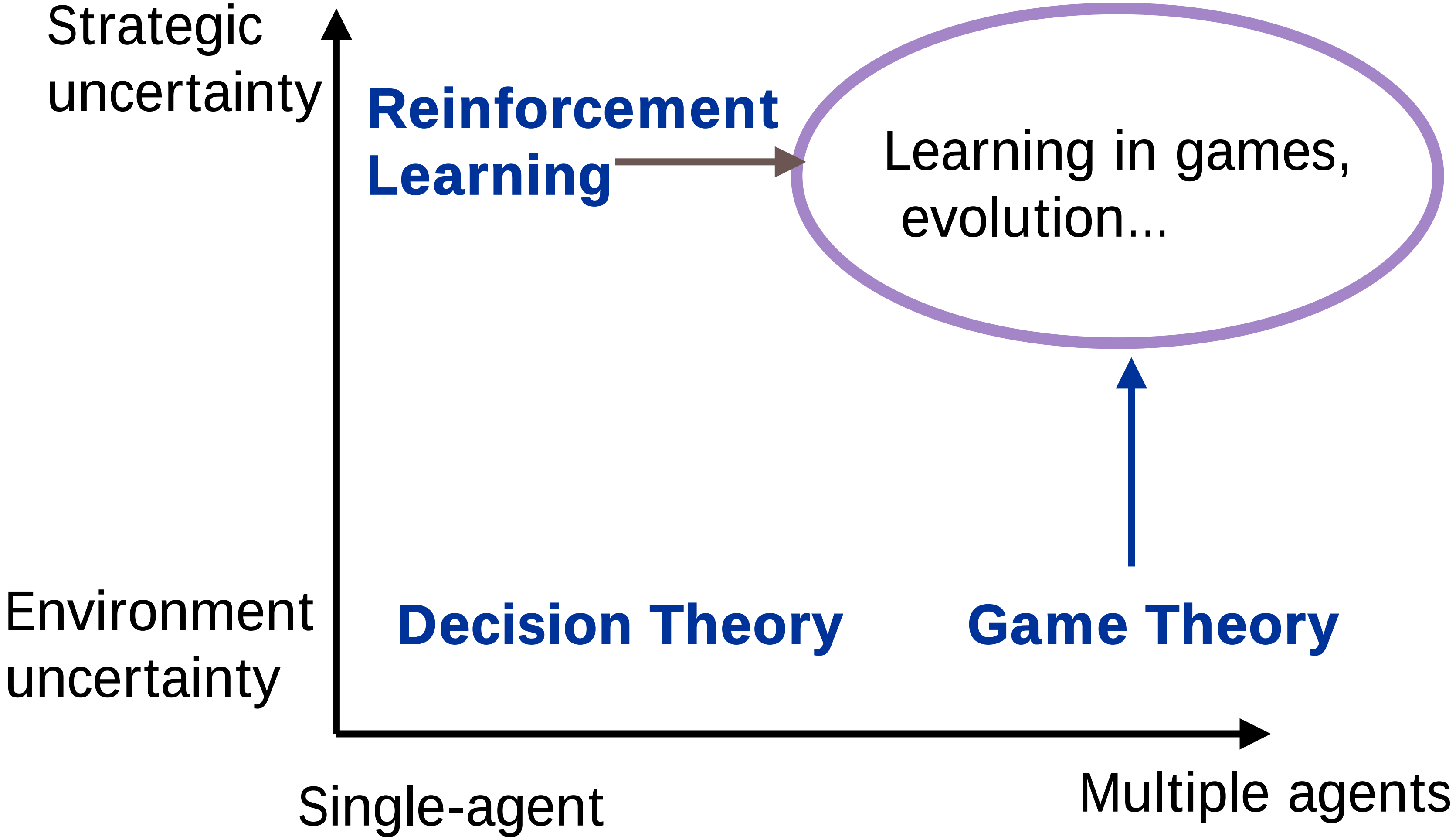
- Strategic interactions and externalities
- Social Security Games

C. Evolutionary Games

D. Co-evolutionary Learning in Social Games

E. Broad Application Areas

Learning in Other Learners



Strategic uncertainty

Reinforcement Learning

Learning in games, evolution...

Environment uncertainty

Decision Theory

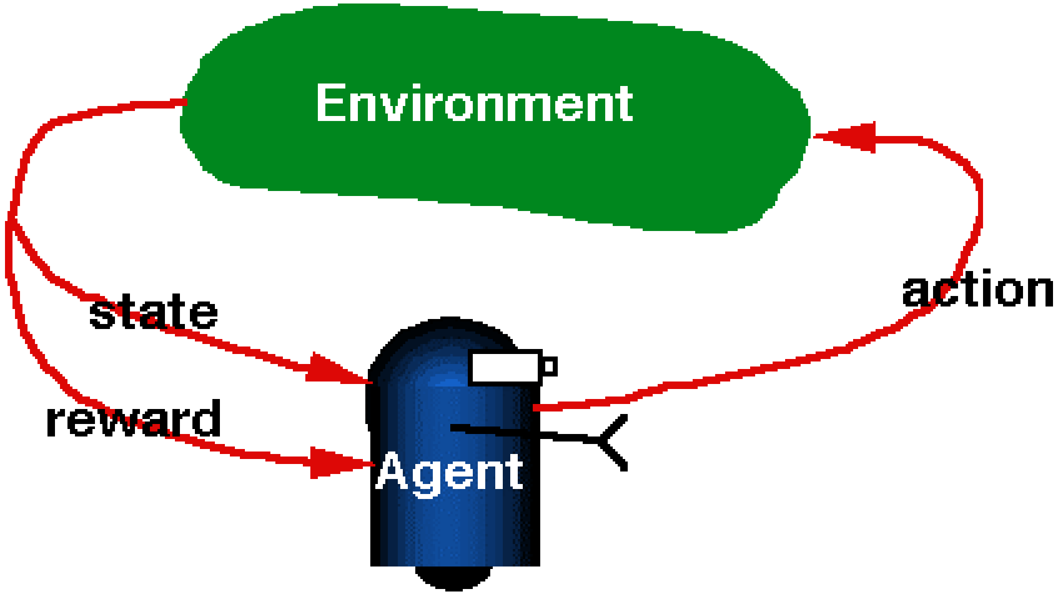
Game Theory

Single-agent

Multiple agents

Single-agent Learning

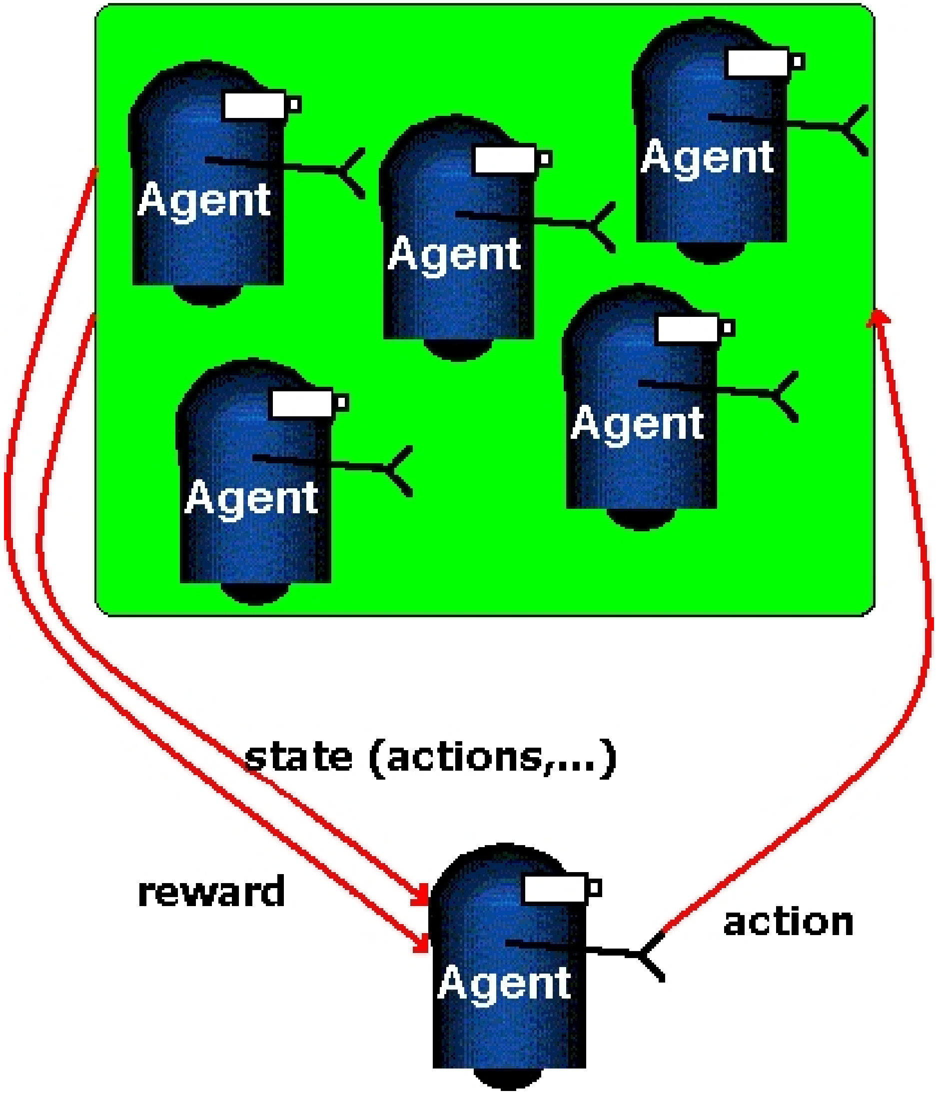
- Assume that environment has observable states, expected rewards and state transitions, and all of the above is stationary
- Learning: solve by trial and error without a full specification.



Multi-agent Learning

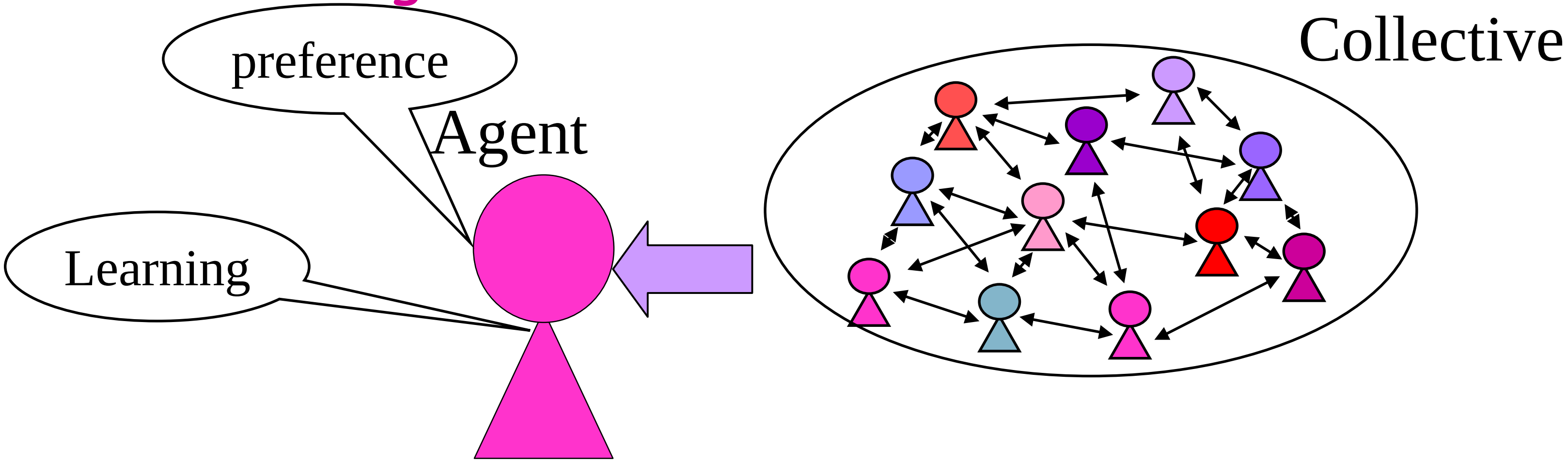
- Non-learning, theoretical solution to fully specified problem.

- Agent tries to solve its learning problem, while other agents in the environment also are trying to solve their own learning problems.



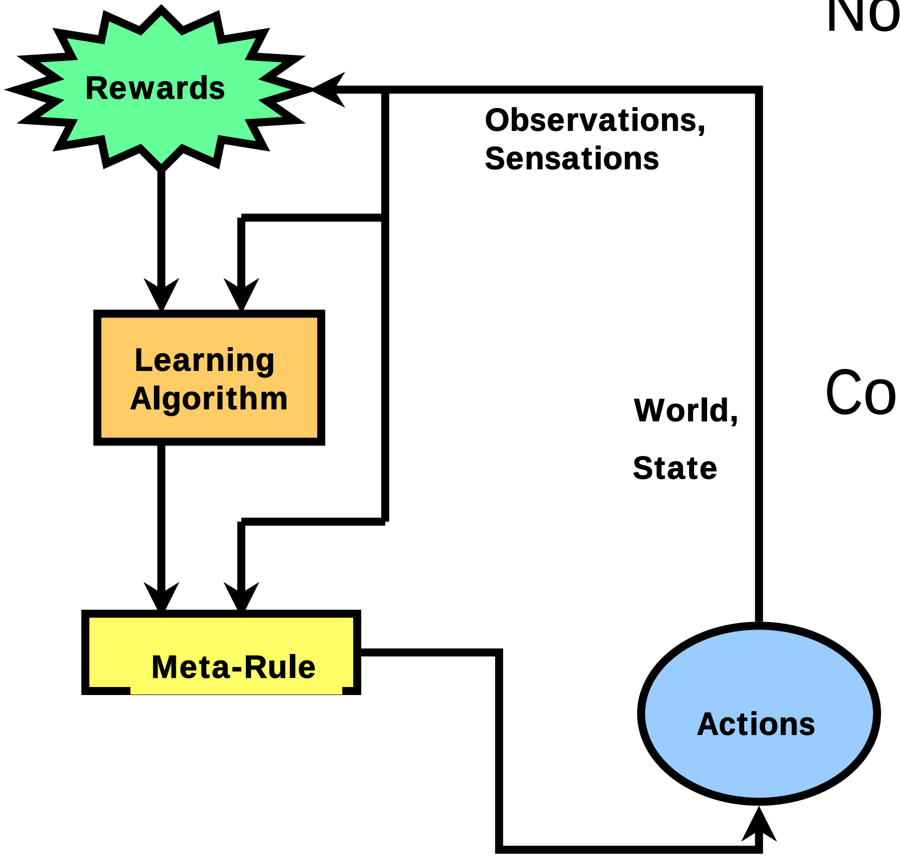
Learning Issues in Multi-agent Environments

✓ How should agents learn in the context of other learners?



- ☐ Equilibrium agenda (Game theory):
  How simple adaptive rules lead the agents to an equilibrium?
  (It is not required any optimal requirement).
- ☐ AI agenda (Multi-agents learning):
  What is the best learning algorithm?
- ☐ Collective learning agenda:
  How should agents learn to realize a "desired collective"?

Action Choice Based on Meta-Rule



Non-cooperative fixed rule:

We want to play our best response to the observed play of the world

Cooperative learnable rule:

Approximate a global optimal using only local information or less computation

Two Broad Application Areas

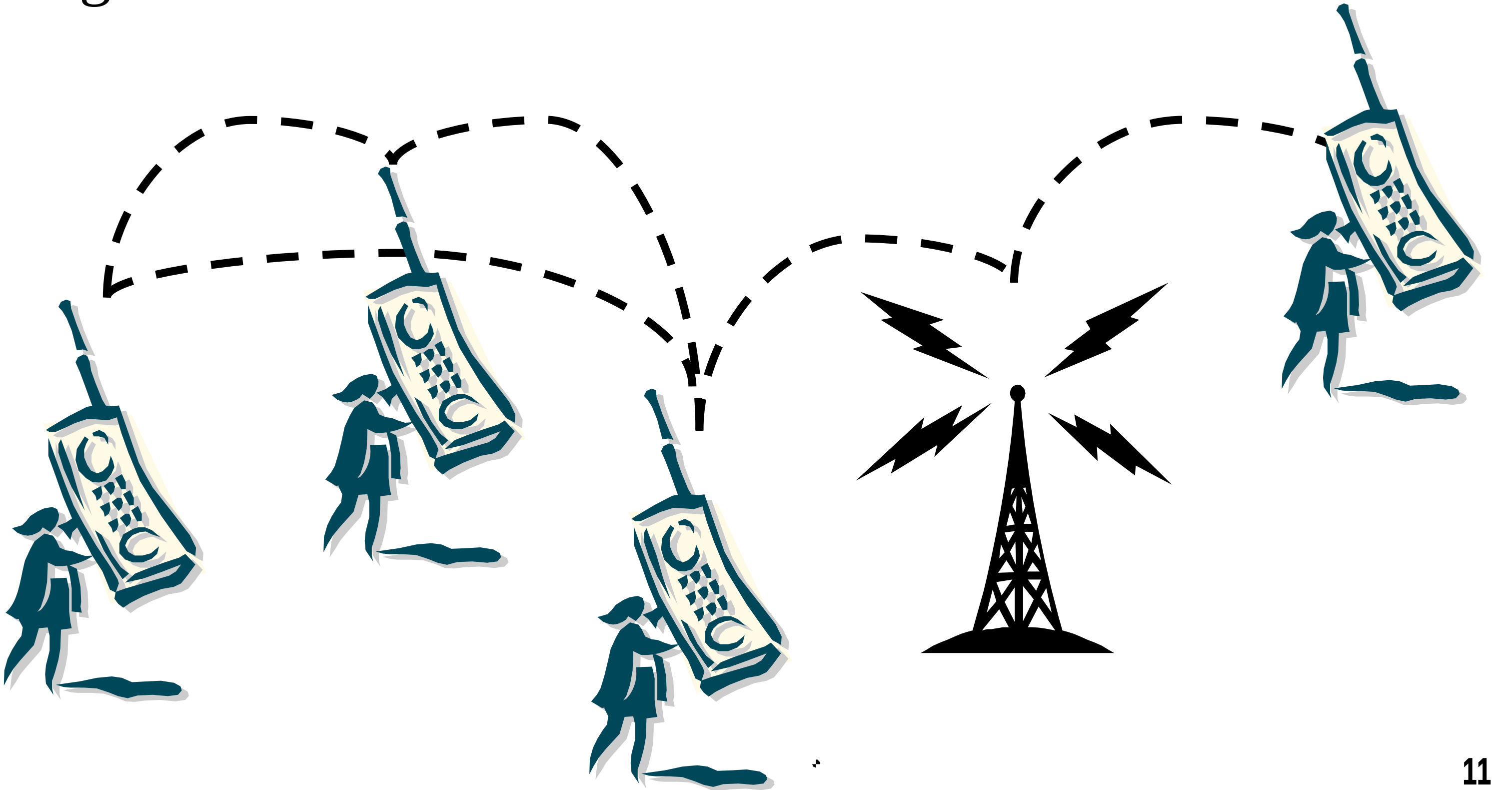
- E-commerce
- Mobile networks
  - Large-scale competitive Multiple agents
  - Billions of economically motivated agents
    - Buying and selling information goods and services
    - Adaptive, and coupled directly and indirectly

- Today: Lots of good practical techniques for single agent to learn about a static agent or environment, with solid theory to back it up.

- Challenge: Establish theoretical foundation for understanding and performing learning and optimization in multi-agent systems.

Multi-agent Learning in Network Environments

- Mobile ad-hoc networks
- Mobile sensors, tracking agents, …
- Generally a distributed system that wants to optimize some global reward function



Scientific Challenges

: Routing

- Dynamic environment: neighbor nodes moving in and out of range, source and receivers may also be moving
- Limited bandwidth: channel allocation, limited buffer sizes

: Moving

- What is the globally optimal configuration?
- What is the globally optimal trajectory of configurations?
- Can we learn a good policy (protocol) using only local knowledge?

Today's Internet

: There are indications that the amount of non-congestion-reactive traffic is on the rise.

- Most of this misbehaving traffic does not use TCP (**T**ransport **C**ontrol **P**rotocol).
- e.g. Real media, network games, other real time multimedia applications.

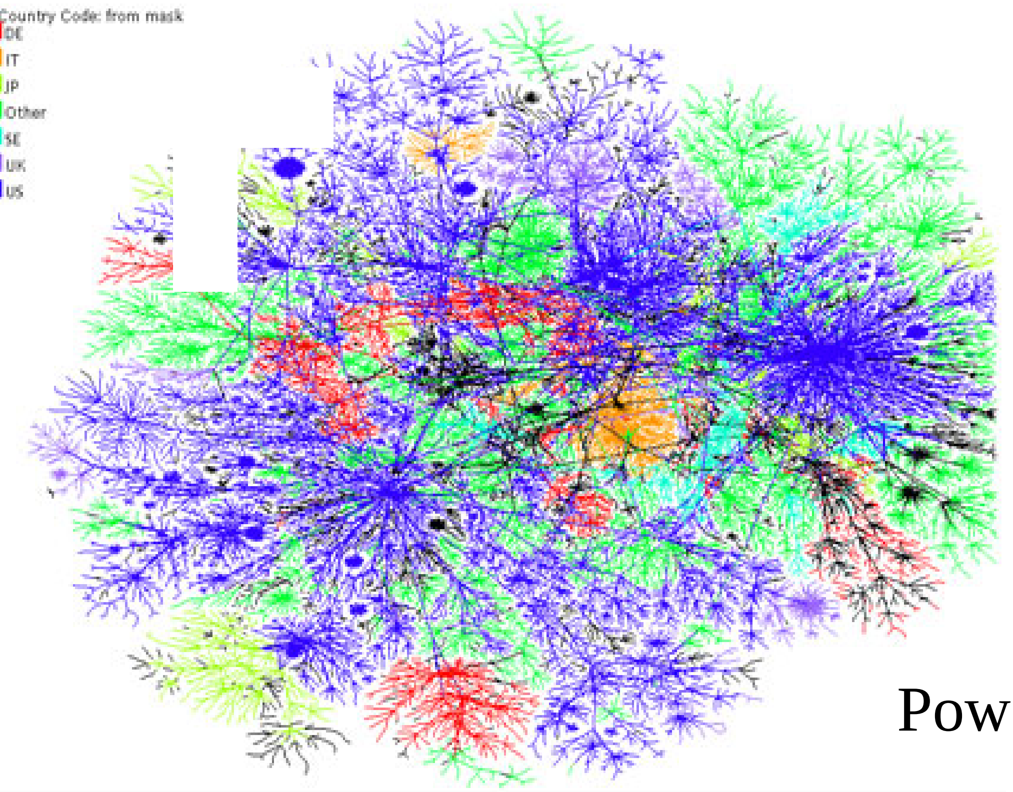
: The unresponsive behavior can result in both unfairness and congestion collapse for the Internet.

: The network itself must now participate in controlling its own resource utilization.

Protocol Equilibrium

- TCP does not guarantee good performance in the face of aggressive, greedy users (who are willing to violate the protocol to obtain better performance).
- ***Protocol Equilibrium*** – A protocol which leads to an efficient utilization and a somewhat fair distribution of network resources (like TCP does), and also ensure that no user can obtain better performance by deviating from the protocol.
- If protocol equilibrium is achievable, then it would be a useful tool in designing robust networks.

Emerging Science: Complex Network Science



- Power Law Random Graphs

  Bollobas 80's, Molloy&Reed 90's, Chung 00's.

- Preferential Attachment

  Simon, 1955, Barabasi-Albert,1999, .

Power Law: $\Pr[d=i] \sim i^{-\alpha}$

- What is the impact of the interaction structure on the performance of the multi-agent system?

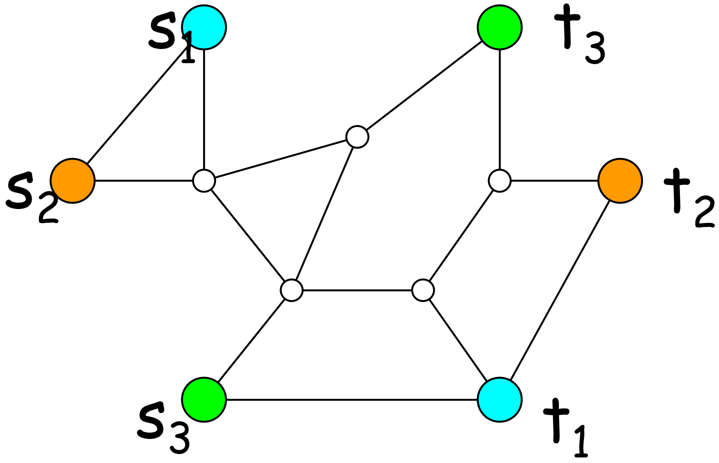
Network Design Game

: Problem

- Selfish agents share network building *cost* to make their sets of *terminals* connected

: Focus

- Behavior of selfish agents
- Structure of the network generated by selfish agents



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