相互作用系：
順問題と逆問題

生天目　章
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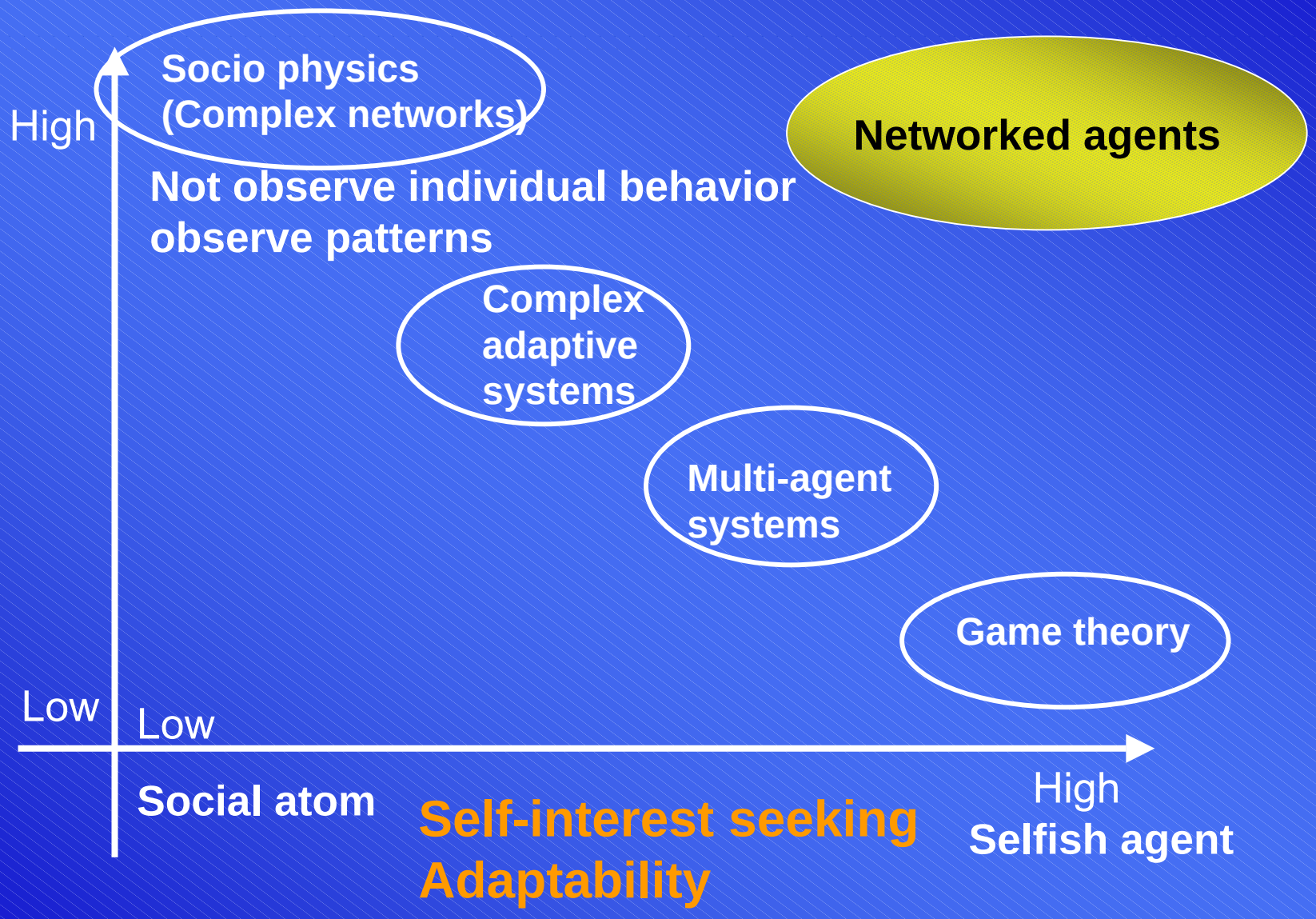
さまざまな研究分野


Scale
High
Low
Socio physics
(Complex networks)
Not observe individual behavior
observe patterns
Networked agents
Complex
adaptive
systems
Multi-agent
systems
Game theory
Low
Social atom
Self-interest seeking
Adaptability
High
Selfish agent

概要

(1) 社会物理学

:Ball, P “The Physical Modeling of Human Social Systems” Complexus", vol.1,(2003)

:Ball, P. Critical Mass, Farrar, Straus and Giroux (2004)

:Buchanam, M The Social Atoms, Reed Elsevier Inc. (2007)

:その他

(2)相互作用系における創発現象

(3) 相互作用系における順問題と逆問題

:求心力（集中）が働く系

:排斥力 (分散）が働く系

:求心力と排斥力が共に働く系

(4)今後の課題など：相互作用と複雑ネットワーク（輪講のテーマ）

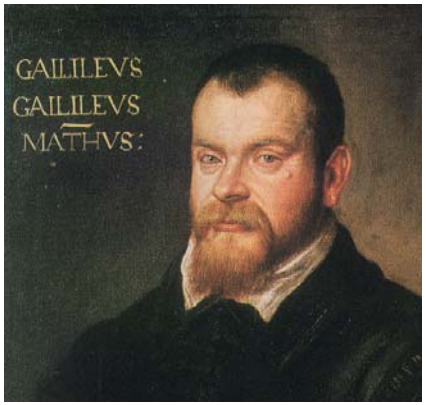
物理学による社会問題への接近法（17世紀頃）

- Sabine
  - 秩序や慣習の問題：システム的アプローチの展開
- Hobbes
  - 社会の働きを，時計のような機械的な部品の働きによって説明
- Descartes
  - 全体を部分に分け，部分の機能を理解し，それらの集合体として社会現象が生成される（デカルト法）
- Galileo
  - 社会を原子(人間)の集合体とし，全体に現れる法則性に着目（ガリレオによる物理的な世界観）
- Petty
  - 統計物理的な手法を用いた大規模な社会問題の分析（人口や経済取引）

その他の数学者と物理学者：Laplace, Poisson, Maxwell，Boltzmann

哲学者：Kant, Comte, Mill，Marx

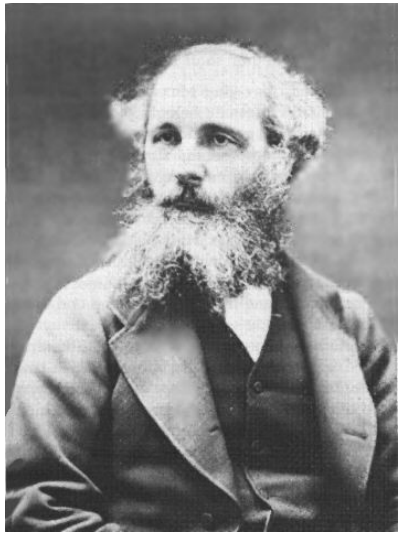
多大な貢献をした物理学者，数学者



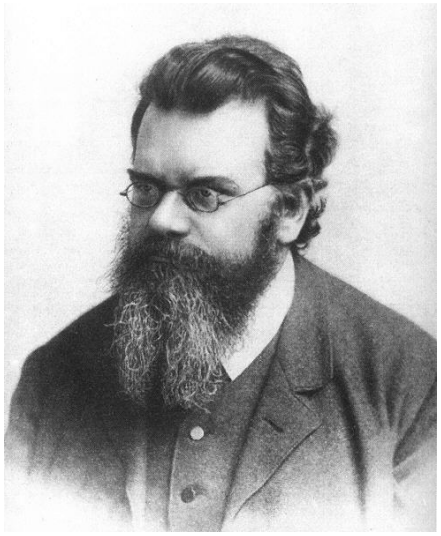
Galileo Galilei

Siméon Denis Poisson

Pierre-Simon Laplace



James Clerk Maxwell



Ludwig Boltzmann

物理学による社会問題への接近法（近年～現在）

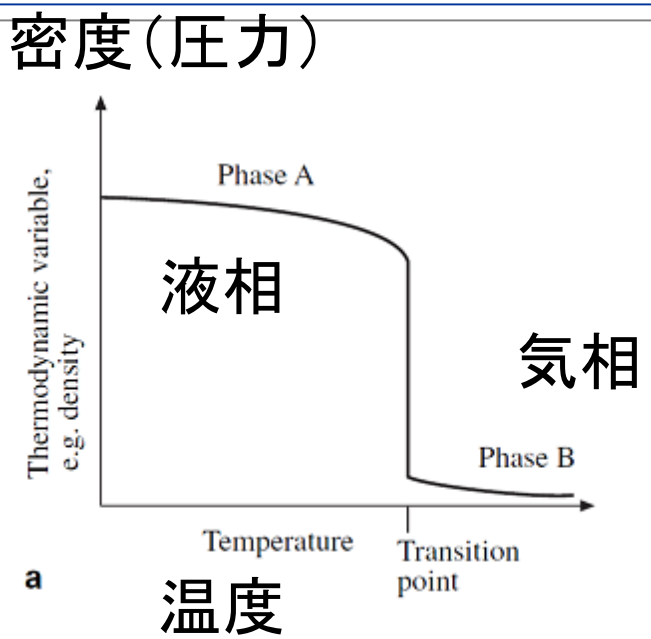
- ２０世紀初期
  - 軟弱な社会問題へ物理学の手法を応用することは，あまりふさわしくないという認識が広まる
  - 個人の心理や脳の働きなどを扱うアプローチの展開
  - 社会科学と経済学は，科学的な扱いが最も困難で複雑である（H. Simon）
- 現在
  - 複雑適応系：個人は適応し，時間の推移とともに，その行動ルールも変化
  - 複雑系と社会物理学
    - 交通流：L.William(1955)
    - 格子モデル：T. Schelling　(1978)
    - 物理経済学
    - ネットワーク科学

複雑系のキーワード

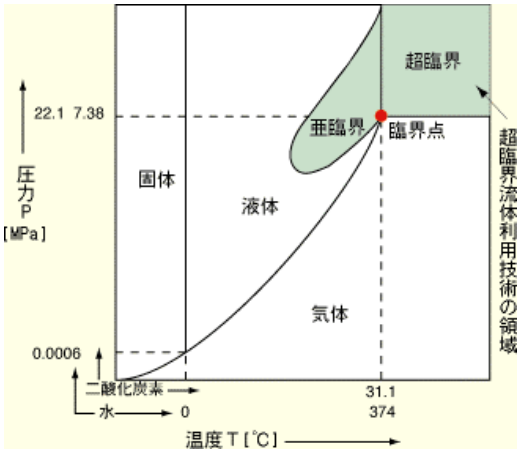
- 多数の要素の集団的な振舞いの統計的性質を明らかにすること

  - 相転移
  - 準平衡
  - ヒステリシス
  - 臨界点
  - ゆらぎ
  - べき乗則

相転移



Johannes Diderik van der Waal



超臨界相では気相と液相が共存

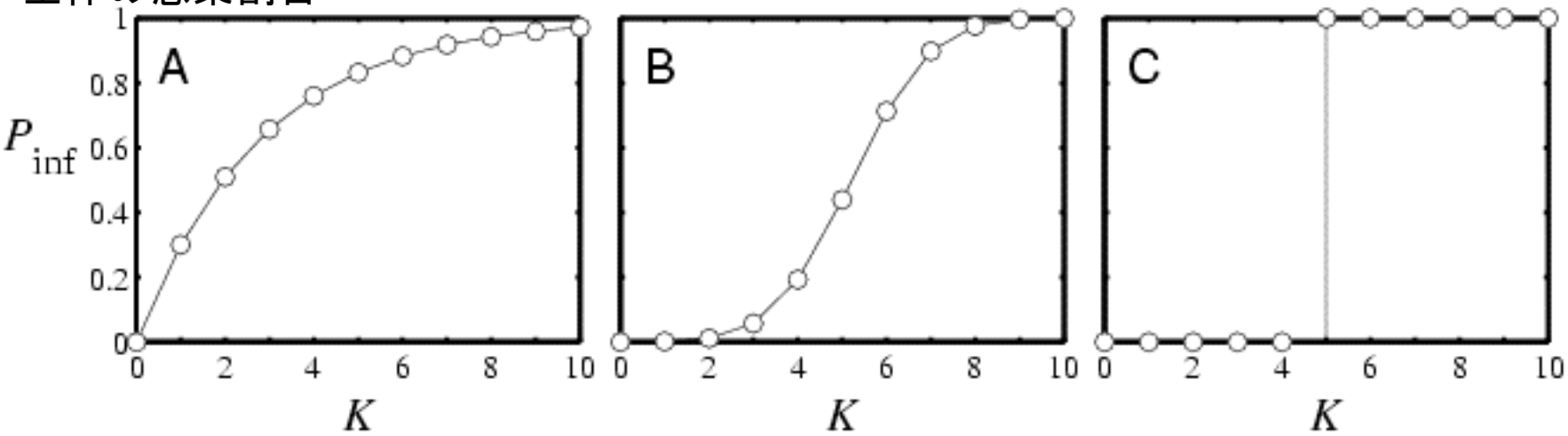
応用:感染症や流行の伝播

Poisson分布

Stochastic Threshold

Deterministic Threshold(閾値)

全体の感染割合

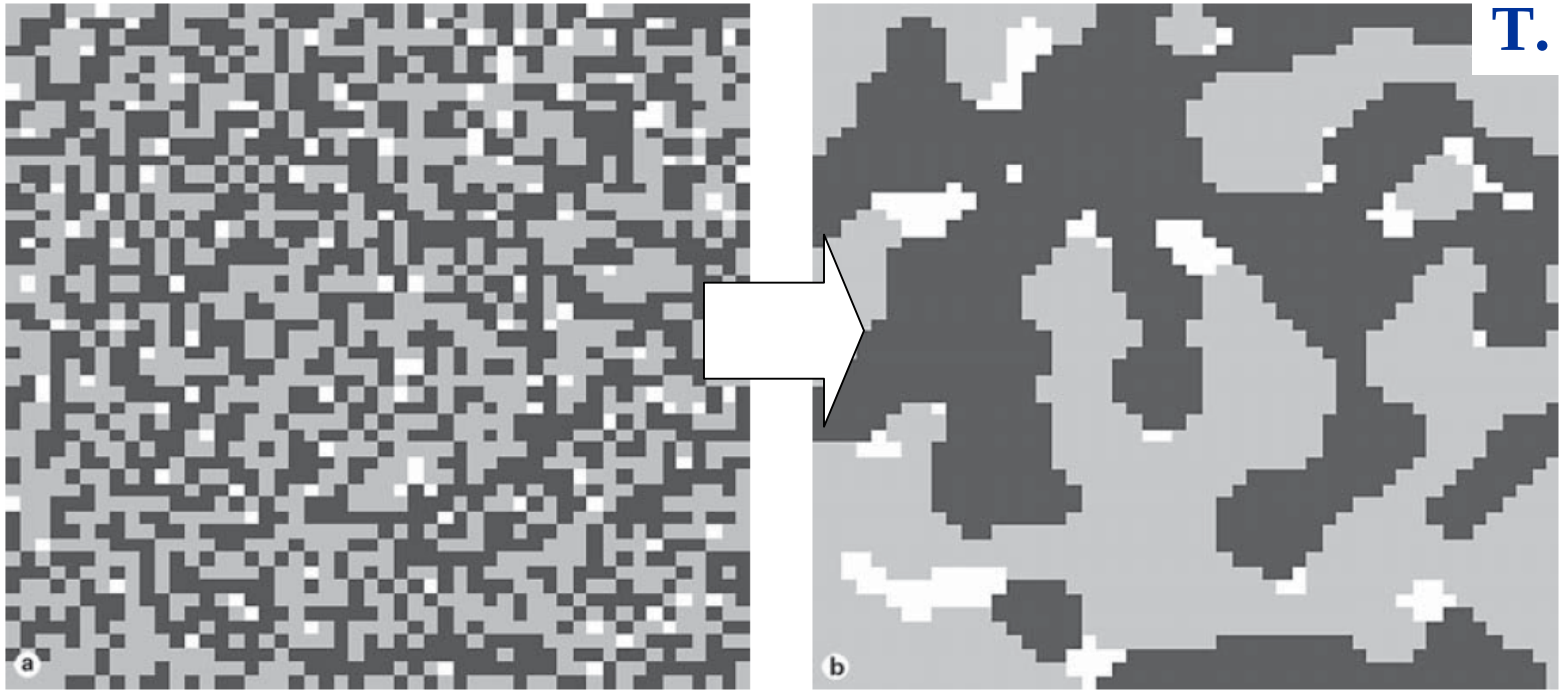


K: 初期の感染割合

Schelling の格子モデル

T. Schelling

- ２種のエージェント（黒，灰）と空き地（白）
- 行動ルール：隣接する異種エージェントの割合が一定の値（閾値）を超えると，空いている場所に移動



初期状態：ランダムに分布　　同じタイプのエージェントのニッチ形成

T.C. Schelling:Micromotives and Macrobehavior, Norton and Company, 1978

文化の流布モデル

Axelrod R: The dissemination of culture. J Confl ict Resol 1997; 41: 203–226.

Robert Axelrod

- 格子モデル，4近傍と作用，非周期的境界条件
- 5つの特徴(feature)（5桁の数値）

8 2 3 3 0 ← 各traitの性質（0～9段階）

- ルール
  - ランダムに格子(区域)とそれに隣接する区域を1つ選択
  - 隣接区域と特徴の一致率に応じて，異なる特徴を一致させる

N
E

74741	87254	82330
01948	09234	67730
49447	46012	42628

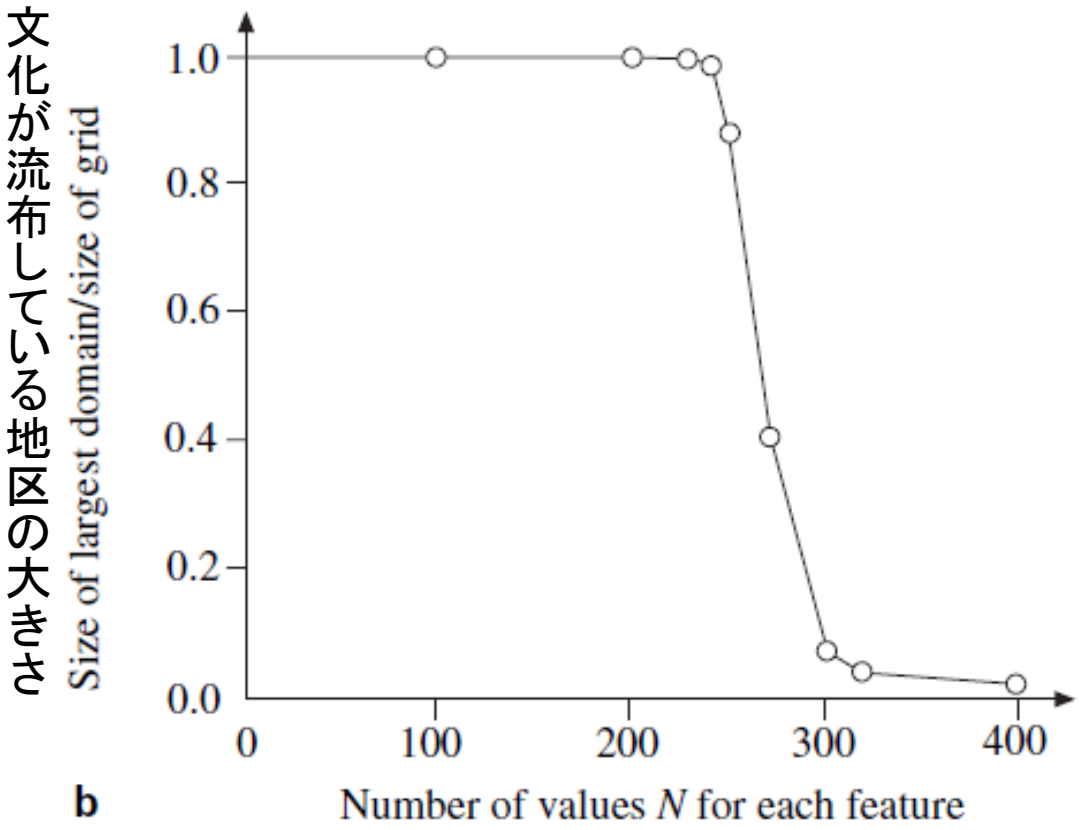
⇒

74741	87254	62330
01948	09234	67730
49447	46012	42628

- 82330の区域に注目すると，その南側の区域と2つの特徴が一致
- 残りの3つの特徴を2/5の確率で南側の区域と合わせる.

⇒この操作により，8が6になるとすると，両区域の類似性は3/5に向上

文化の流布における相転移現象

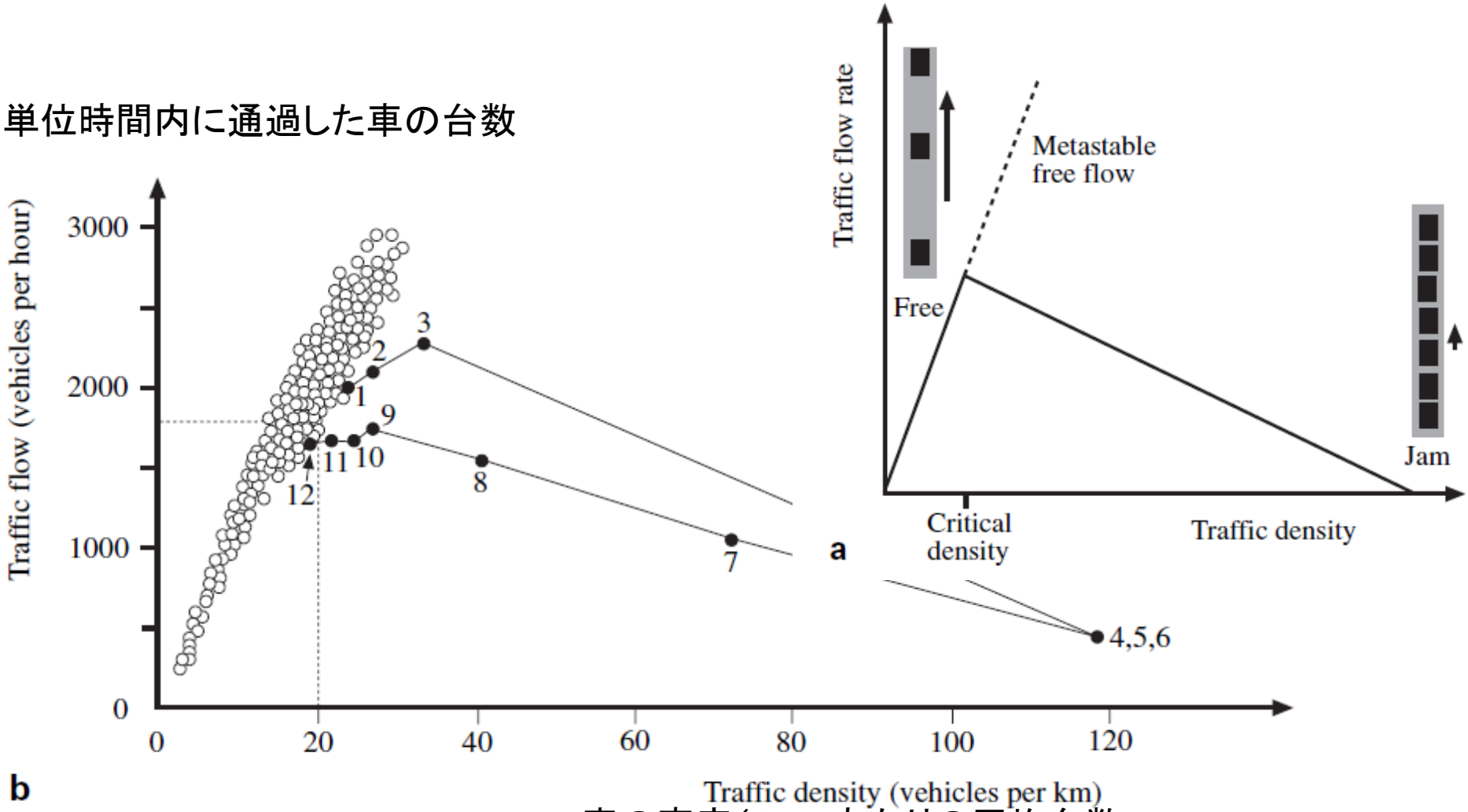


文化的特徴の多さ (複雑度)

Castellano C, Marsili M, Vespignani A: Nonequilibrium phase transition in a model for social influence. Phys Rev Lett 2000; 85: 3536–3539.

交通流における相転移現象

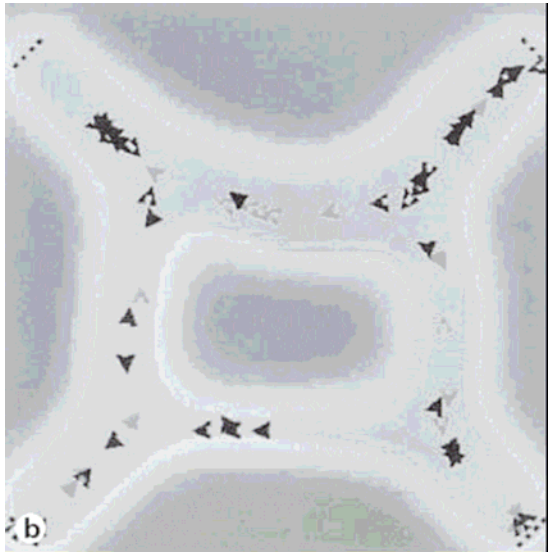
単位時間内に通過した車の台数



車の密度（1kmあたりの平均台数

人の通り易いところに，道はできる

Helbing D, Keltsch J, Molnar P: Modeling the evolution of human trail systems. Nature 1997; 388: 4

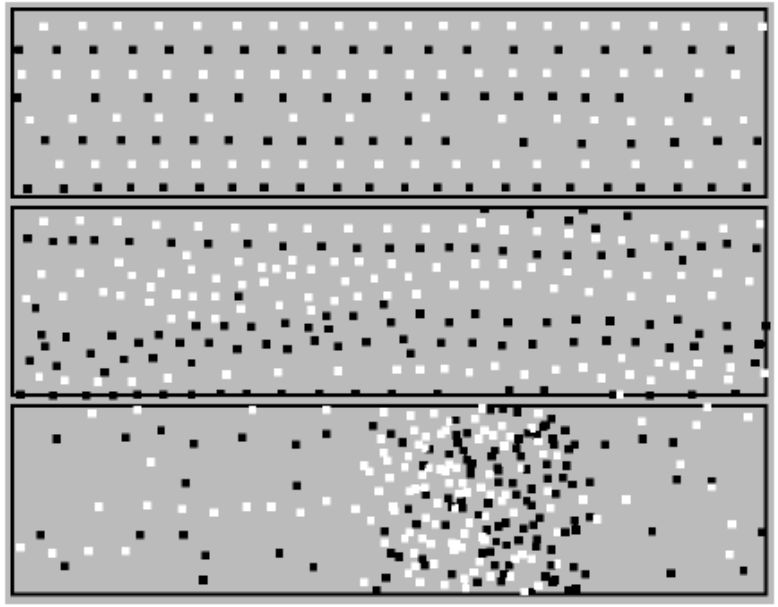


シミュレーション

行動ルール

- 目的地に向かう
- 歩く速さを優先
- もし，意図的に減速したり，障害物（他の歩行者）を回避したりしなければ，方向と速さを維持

歩行者シミュレーション



Helbing D, Keltsch J, Molnar P:

Modelling the evolution of human trail systems.

Nature, 1997; 388: 47–49.

•行動ルール

–目的地に向かう

–歩く速さを優先

–もし，意図的に減速したり，障害物（他の歩行者）を回避したりしなければ，方向と速さを維持

- 黒点が左へ動く歩行者，白点が右へ動く歩行者.
- 上からN = 150, 200, 250.

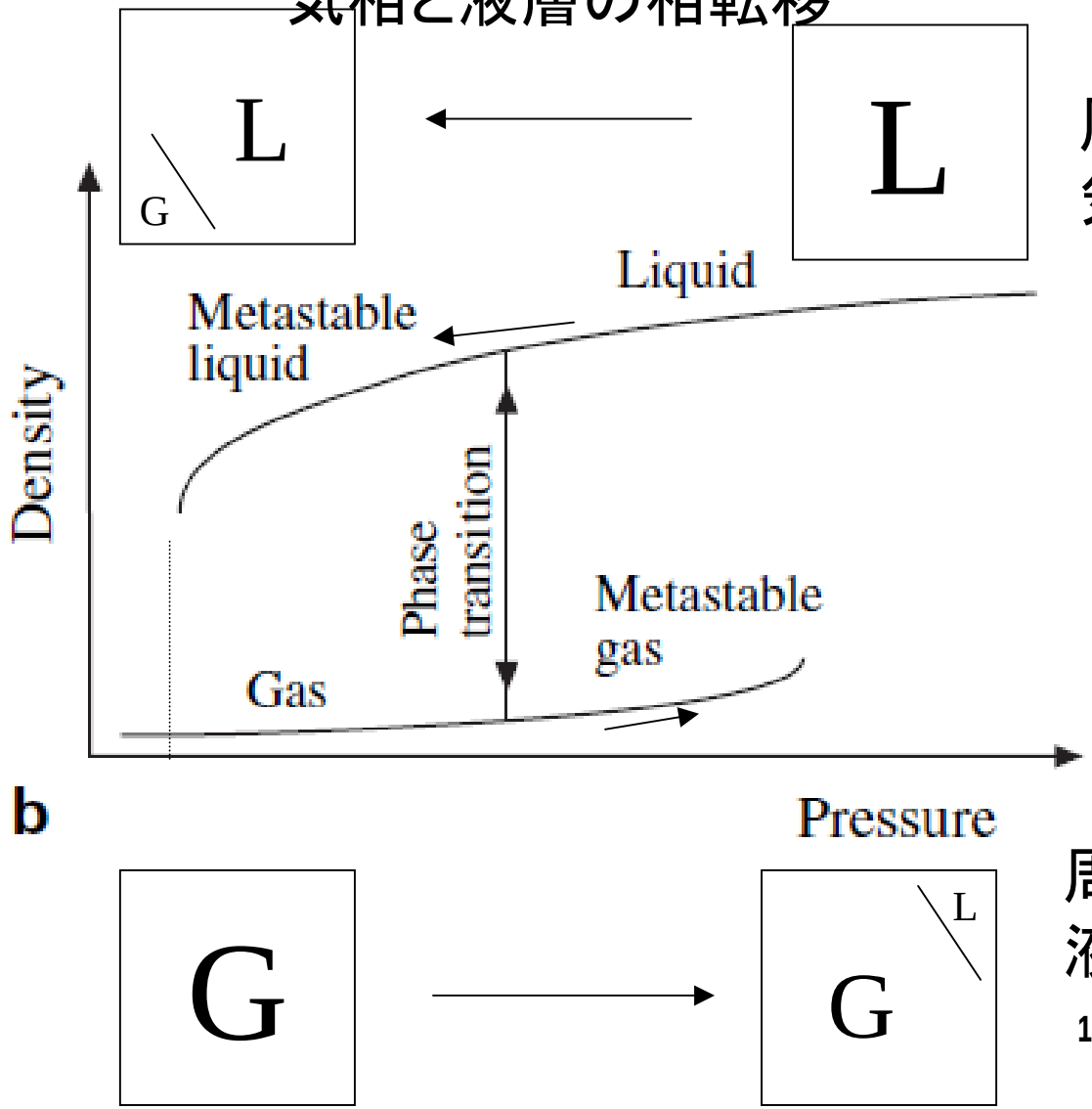
Y. Sugiyama, A. Nakayama: Modeling, Simulation and Observations for Freeway Traffic and Pedestrian, to appear in *the Proc. of Computational physics of transport and interface dynamics* (2003).

歩行者と熱力学的特性との関係

動きが活発	<=>	温度上昇
歩行者増加	<=>	気圧（密度）上昇
渋滞	<=>	活発

ヒステリシス (hysteresis)現象

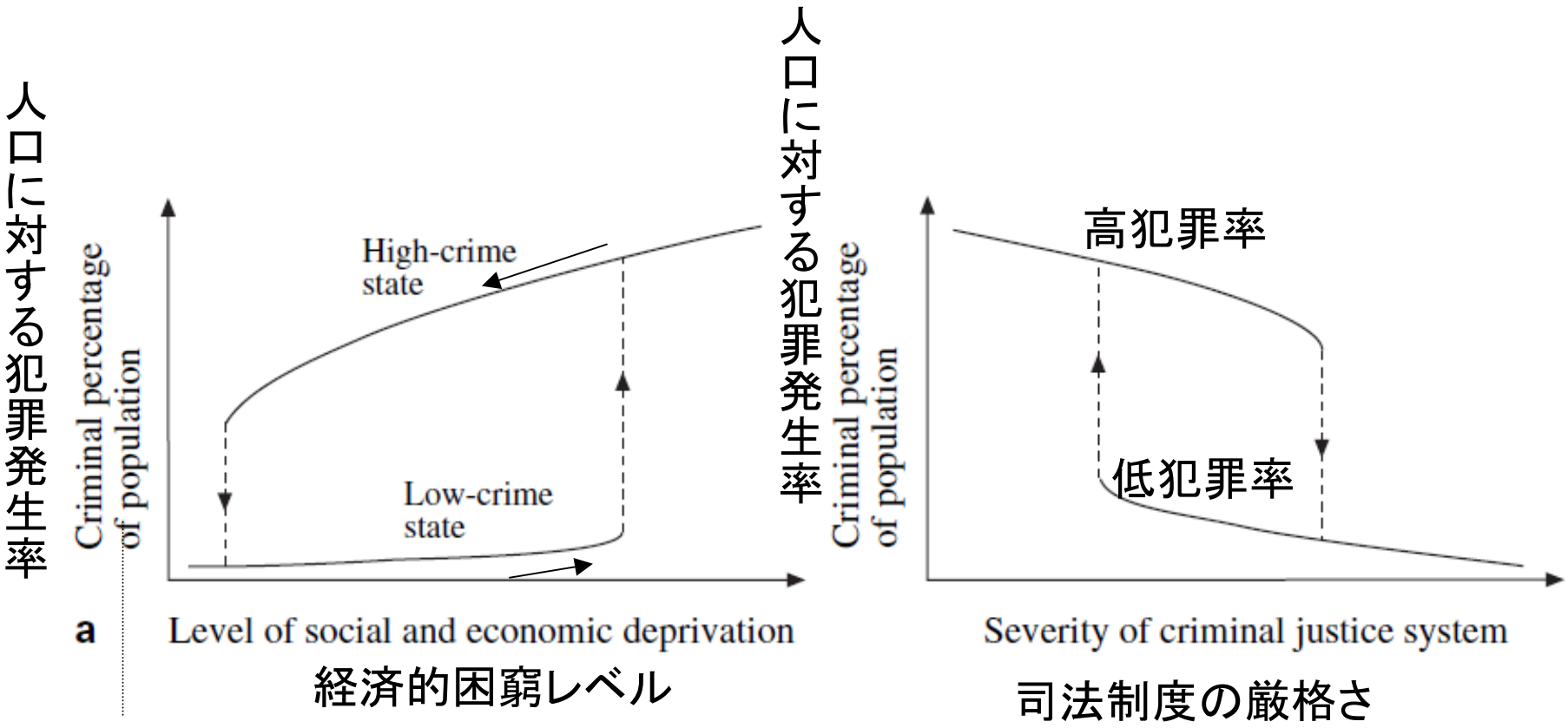
気相と液層の相転移



周りが液体のとき，
気体にはなりにくい

周りが気体のときは
液体になりにくい

相転移のヒステリシスの例：
社会の豊かさ（困窮度）と犯罪発生率の関係



ガウス分布とべき分布

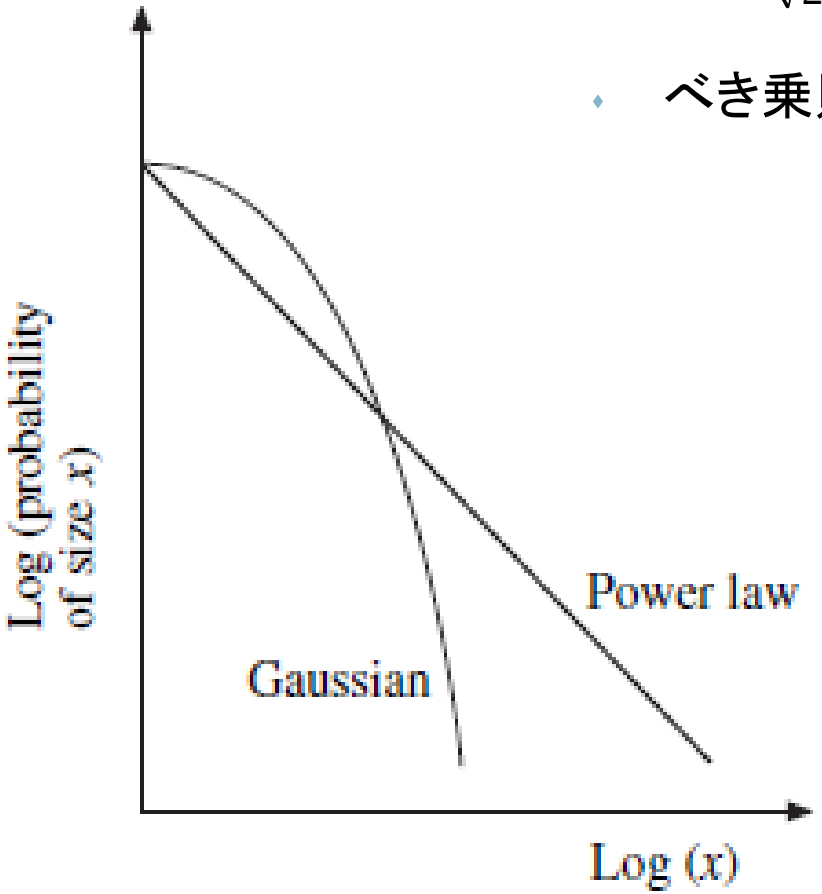
- ガウス分布

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{(x-\mu)^2}{2\sigma^2} \right\}$$

- べき乗則

$$f(x) = Cx^{-\lambda}$$

Carl Friedrich Gauss



λの値

Zipfの法則:1（英単語、都市の大きさ）

タンパク質ドメイン:1 (van Noort et al.)

タンパク質相互作用:2.1-2.5 (Yook et al.)

遺伝子発現量:2 (Ueda et al.)

代謝経路:2.2 (Jeong et al.)

wwwのリンク:2.1 (Barabasi et al.)

競演映画俳優:2.3 (Barabasi et al.)

送電線:4 (Barabasi et al.)

論文の引用:3 (Redner et al.)

べき分布．パレート分布，対数正規分布

- べき分布 $\Pr[X \geq x] \sim cx^{-\alpha}$
- パレート分布 $\Pr[X \geq x] = \left(x / k \right)^{-\alpha}$
  - Log-complementary cumulative distribution function

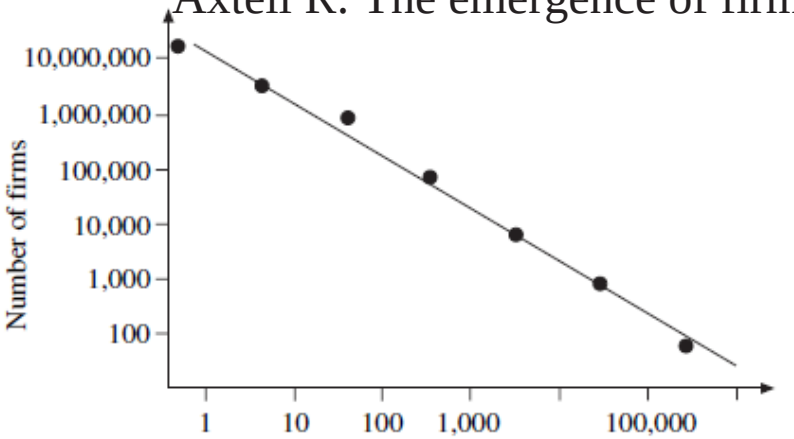
$$\ln \Pr[X \geq x] = -\alpha \ln x + \alpha \ln k$$

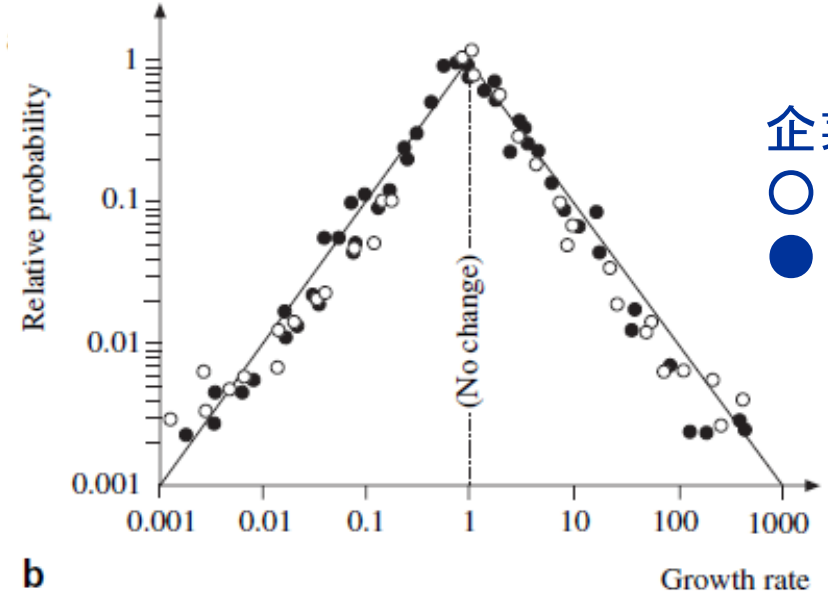
- 対数正規分布 $f(x) = \dfrac{1}{\sqrt{2\pi}\sigma x} e^{-(\ln x - \mu)^2 / 2\sigma^2}$

企業規模と成長率との関係

会社の規模(従業員数)の分布：べき分布

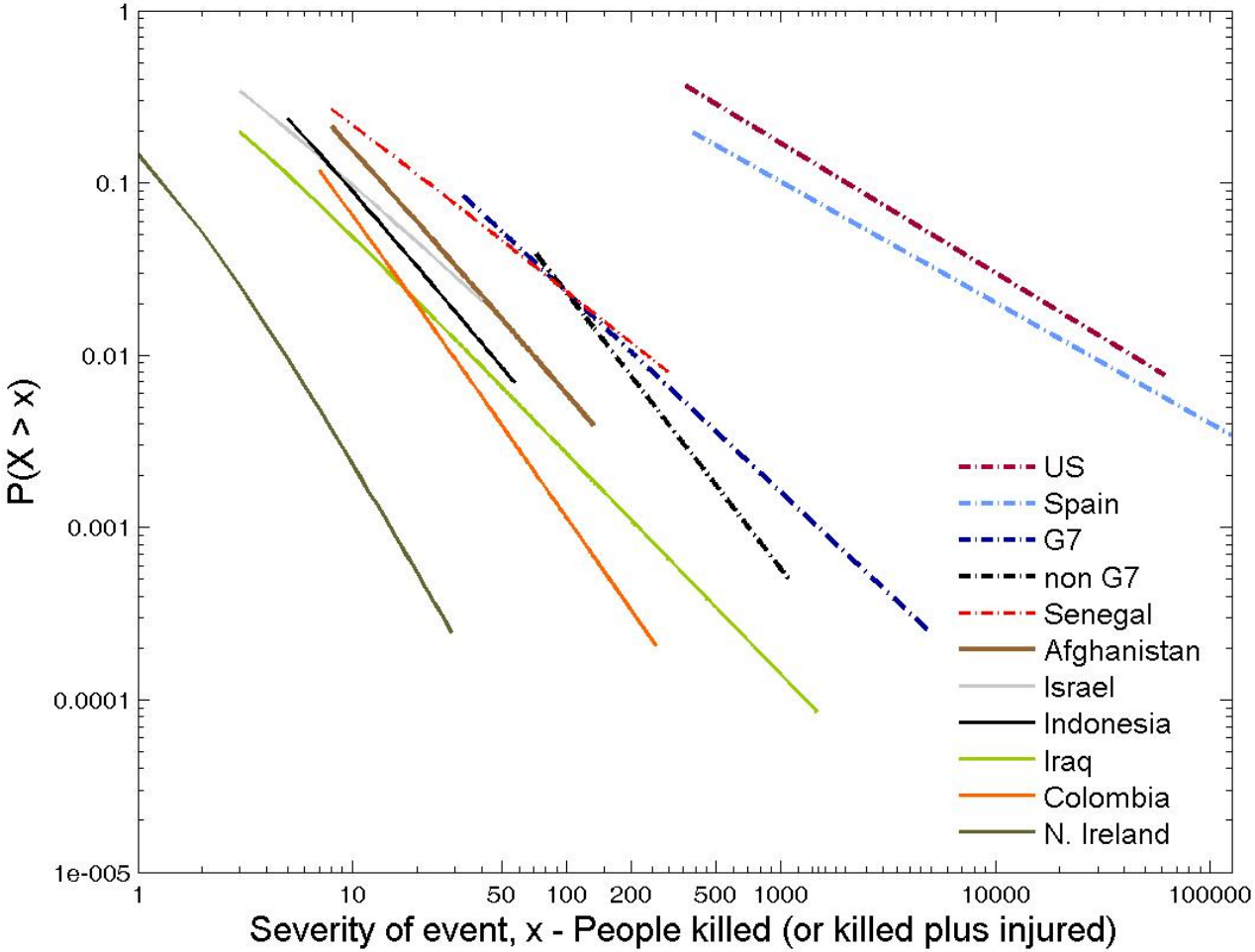
Axtell R: The emergence of firms in a population of agents（1999）.





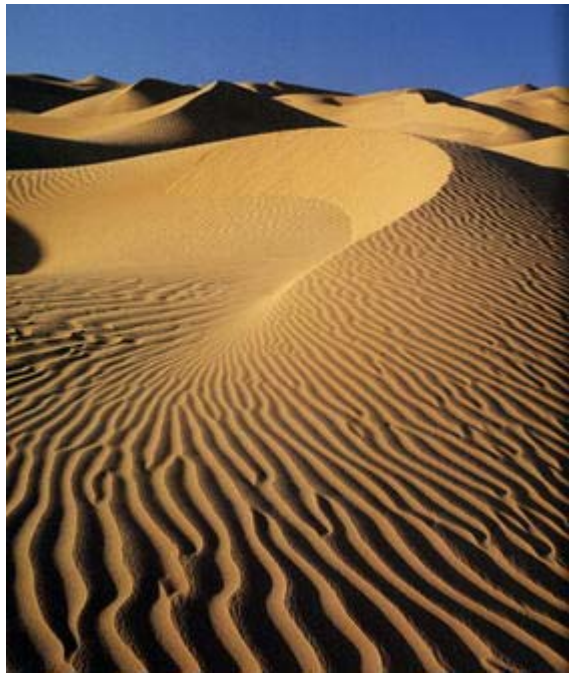
企業成長率の分布
○：従業員数
●：売り上げ

戦争犠牲者（死者と負傷者：べき分布



U. Javeriana, Colombia

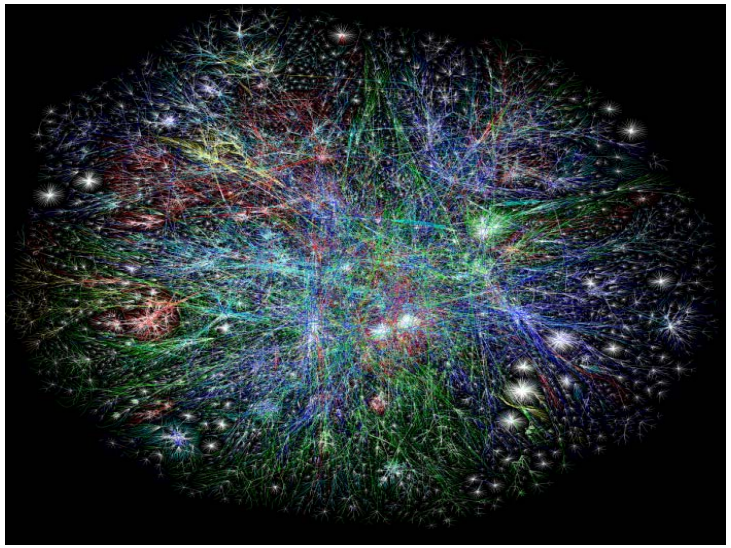
自然界における創発現象



- 西部大砂丘の風紋 「砂漠の世界」より
  - 砂丘の表面には「風紋」（ふうもん）といって、風によって作られた規則的な模様がついており、太陽の位置によって美しい陰影（いんえい）が変化してゆきます。そうしている間にも、沙漠を渡る強 めの風は、砂を運んでその斜面を駆（か）け上がり、稜線（りょうせん）の部分から下へ落として行きます。これが休みなくおこなわれる結果、砂山自体が移動するようになるのです。この砂山も、一年後にふたたび訪れてみると、10kmも風下方向に移動していました。

社会に見られる創発現象

Internet



創発:二つの見方

- Emergence by nature (empirical view)

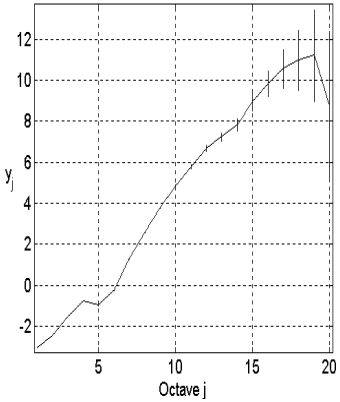
  : Some researchers view emergence as an “innate” property of natural systems and inspires research to discover and explain emergent behaviors

- Emergence by design (operational view)

  : Some researchers view emergence as a property that is “designed” into systems and inspire research into techniques to generate desired emergent behaviors

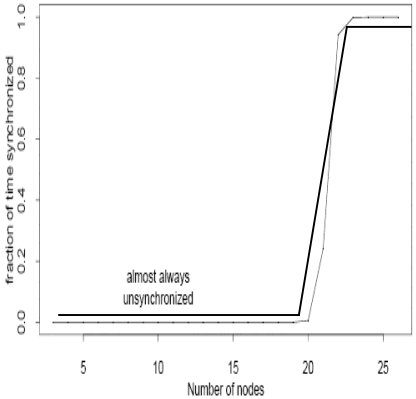
情報ネットワークに見られる創発現象

自己相似性



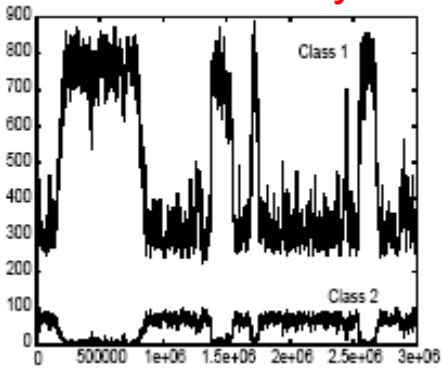
Internet throughput

相転移



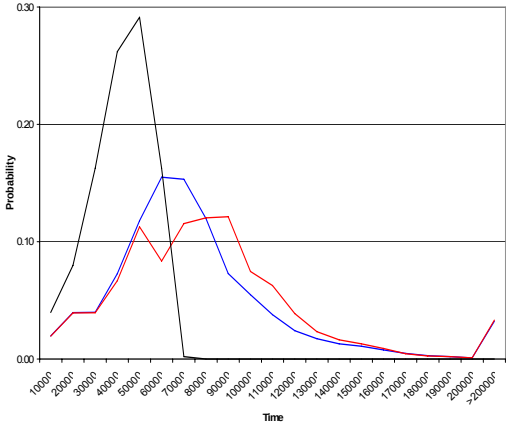
synchronization among Internet routers

Meta-stability



distribution of call types in wireless cells

予期しない動作

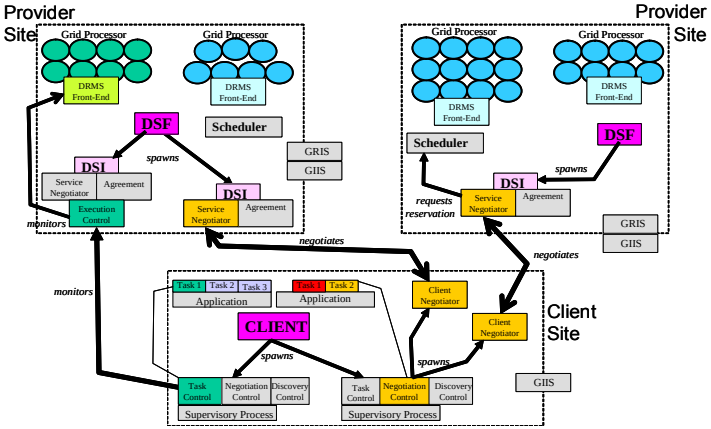


grid job completions

Meta-stability is the ability of a non-equilibrium state to persist for some period of time.

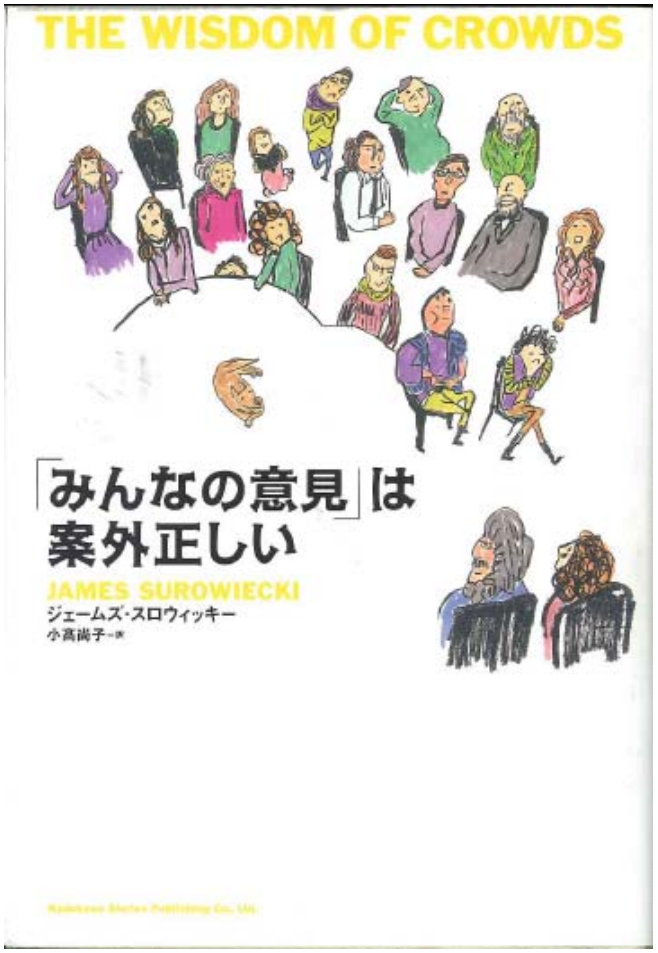
Emergence by Design

Distributed Systems



相互作用系：順問題と逆問題

Madness of crowds



集団の無知 vs 集合知の創発

どのような条件の下，
どのようなメカニズムの下，
どのような英知が生まれるとき，
大衆は，一部のエリート（専門家）の
よりも賢い存在となるか？

社会的相互作用:タイプ1

Type 1: Coordination problems

Agents are better off if they take the same action.

: Cascade / Herding (economics)

: Gossip algorithm (computer science)

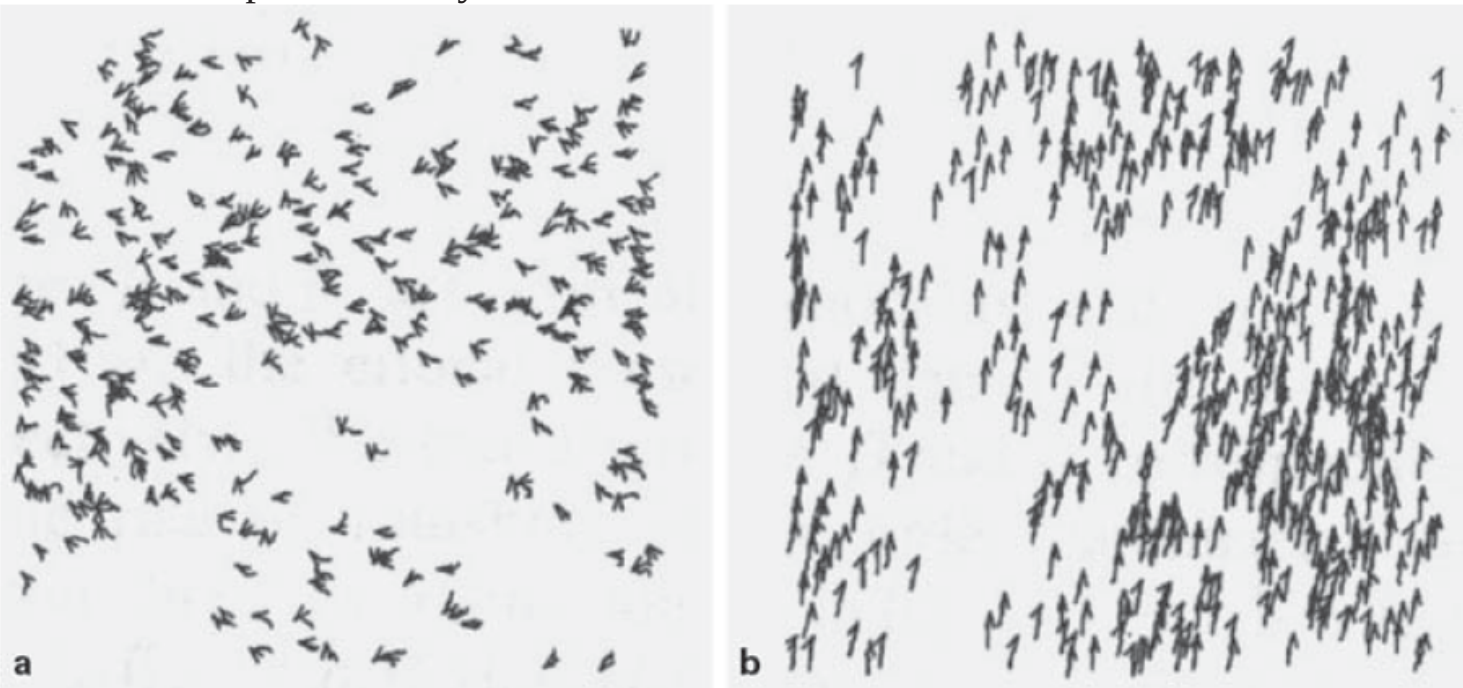
: Consensus problem (control theory)

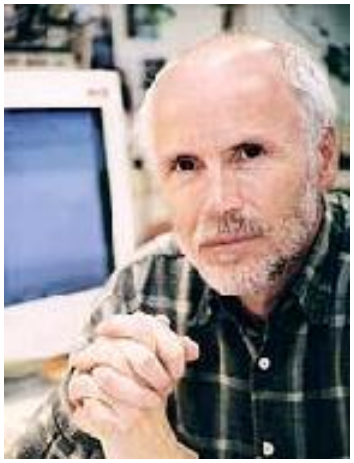
: Synchronization (physics/complex networks)

: Coordination game (game theory)

群れ行動と相転移

Vicsek T, Czirók A, Ben-Jacob E, Cohen I, Shochet O: Novel type of phase transition in a system of self-driven particles. Phys Rev Lett 1995; 75: 1226–1229.





T. Vicsek

システム内のノイズ量(方向のランダム性)が減ると自己駆動粒子(self-propelled particle)は，ランダムな状態から凝集状態へ相転移

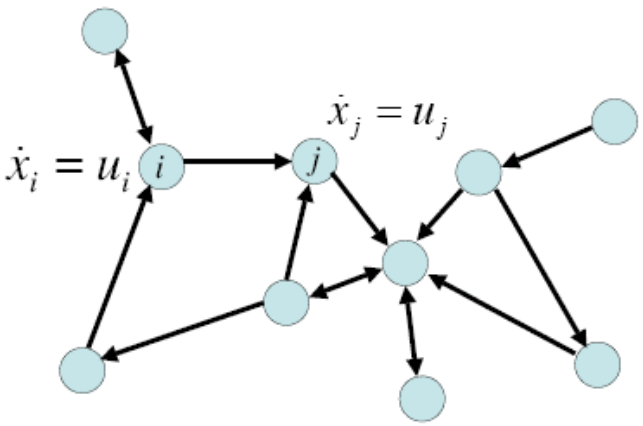
群れ行動		統計力学
ノイズ	<=>	熱運動

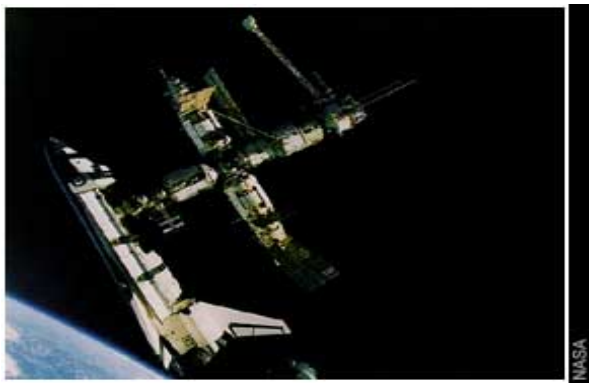
Consensus Problems

- "*Consensus means to reach an agreement regarding a certain quantity of interest that depends on the state of all agents.*
- More specific, a consensus algorithm is a decision rule that results in the convergence of the states of all network nodes to a common value.

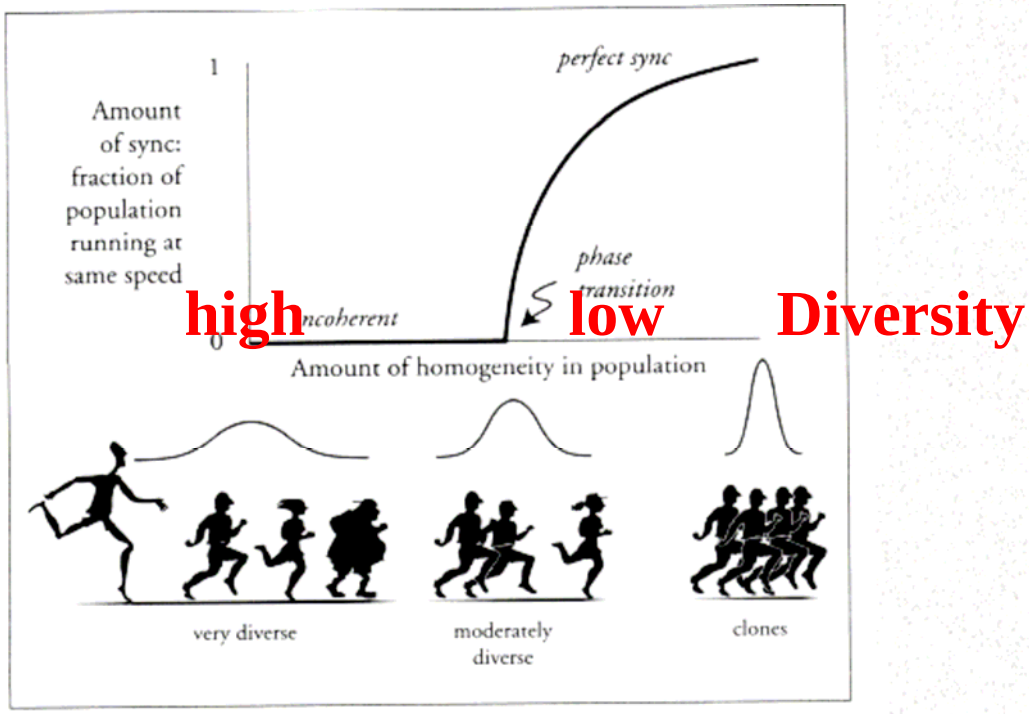
Consensus: Convergence of the states of all agents to a common value

$$x_i = x_j = \ldots = x_{consensus}$$





Synchronization in Complex Networks



Synchronization: Prevalent appearance in physics and biology

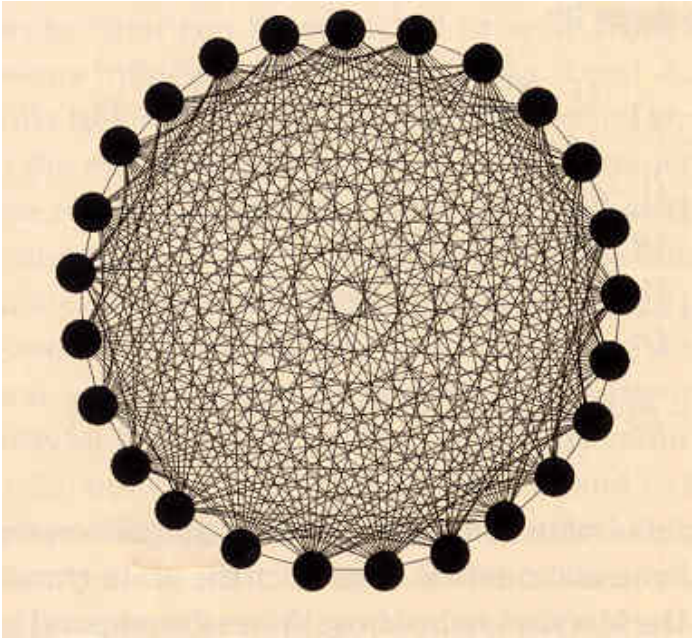
Homogeneity is important for better synchronization

Synchronization in Globally Connected Networks

Observation:

No matter how large the network is, a globally coupled network will synchronize if its coupling strength is sufficiently strong

Good – if synchronization is useful



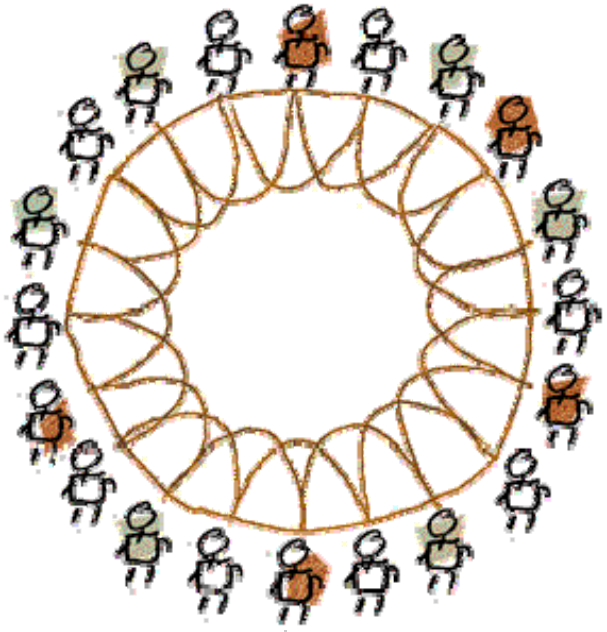
G. Ron Chen (2006)

Synchronization in Locally Connected Networks

Observation:

No matter how strong the coupling strength is,
a locally coupled network will not synchronize if its size is sufficiently large

Good - if synchronization is harmful



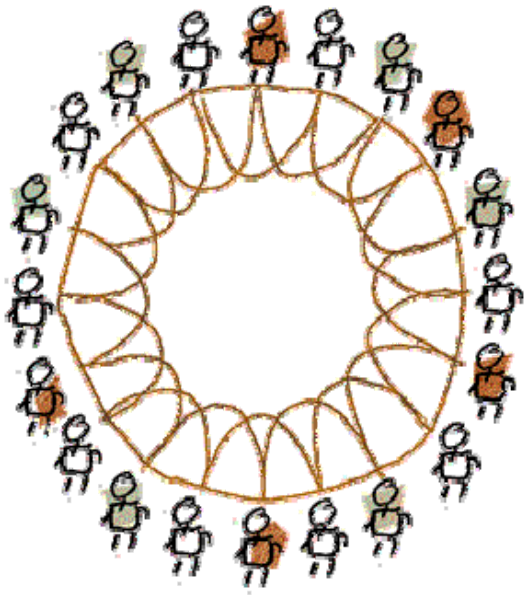
G. Ron Chen (2006)

Synchronization in Small-World Networks

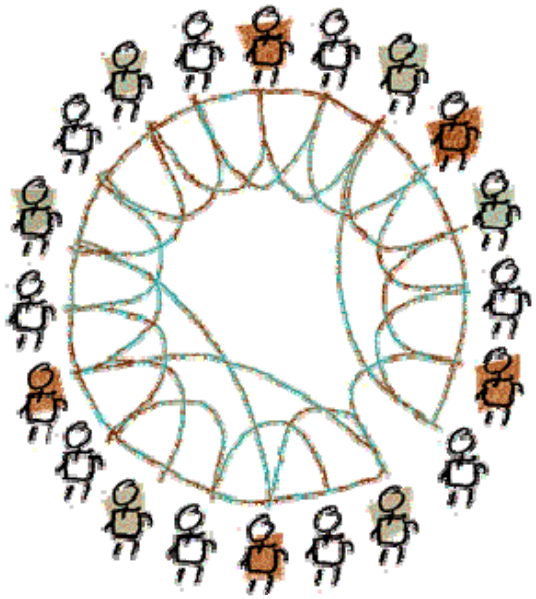
Start from a nearest neighbor coupled network

G. Ron Chen (2006)

small-world network



Add a link, with probability p, between a pair of nodes



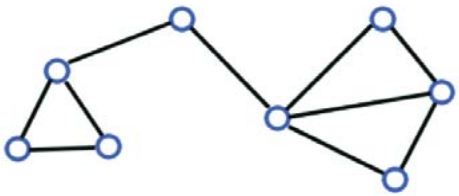
Good news: A small-world network is easy to synchronize!

X.F.Wang and G.R.Chen: Int. J. Bifurcation & Chaos (2001)

Consensus & Synchronization: Network Topology

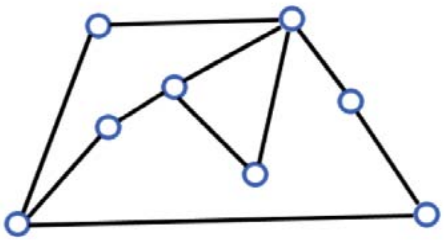
Connectivity of networks does matter for synchronization

Network A



$\lambda_2 = 0.238$

Network B



$\lambda_2 = 0.925$

Laplacian matrix

$$\begin{bmatrix} k_1 & & & \{0,-1\} \\ & k_2 & & \\ & & \ddots & \\ \{0,-1\} & & & k_n \end{bmatrix}$$

- **Laplacian matrix = Degree – Adjacency matrix**
- **$\lambda_1 = 0$ is always an eigenvalue of a Laplacian matrix**
- **λ_N / λ_2 : algebraic connectivity is a good measure of synchronization.**

Fiedler, "Algebraic connectivity of graphs,"

Czechoslovak Mathematical Journal, 1973, 23: 298-305

社会的相互作用:タイプ2

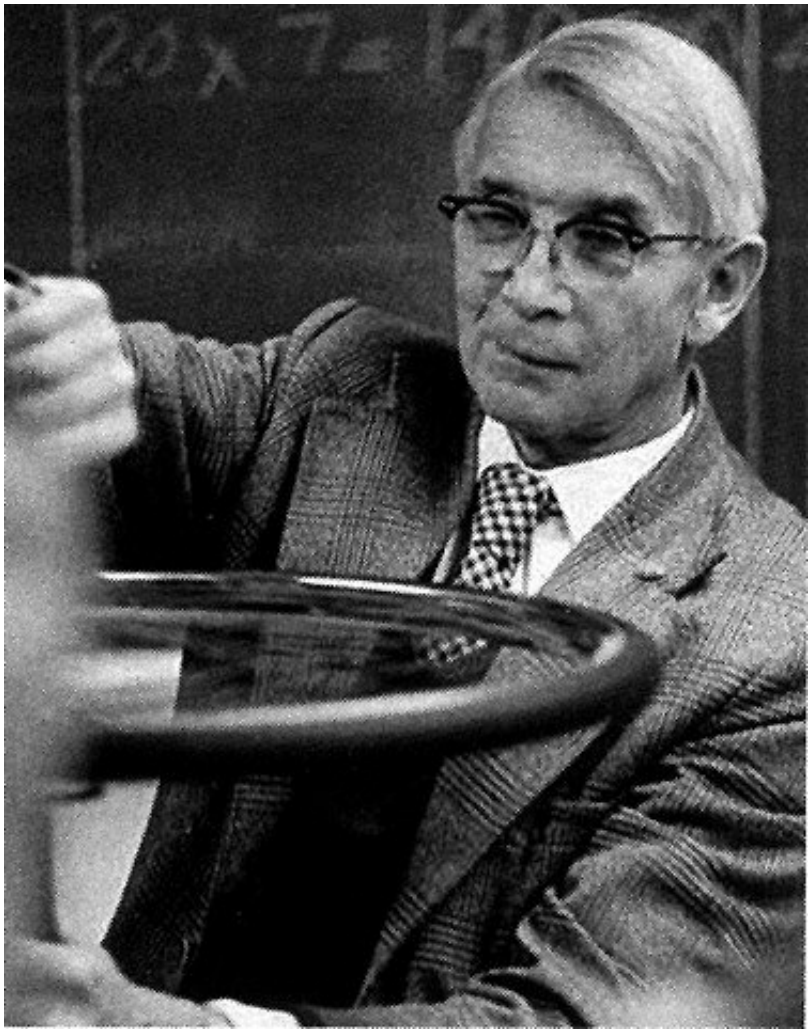
- **Type 2: Dispersion problems:**

Agents are better off if they take the distinct actions.

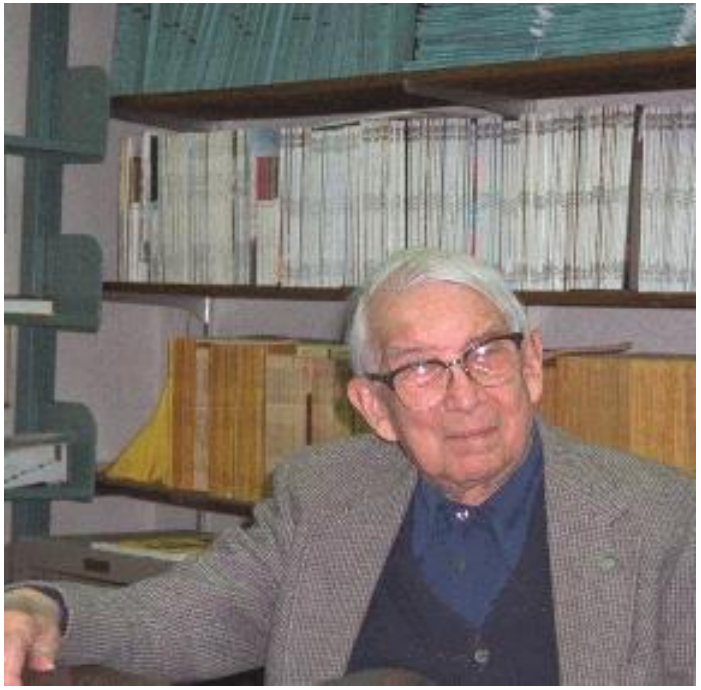
: Stock markets

: Minority games

Ising Model

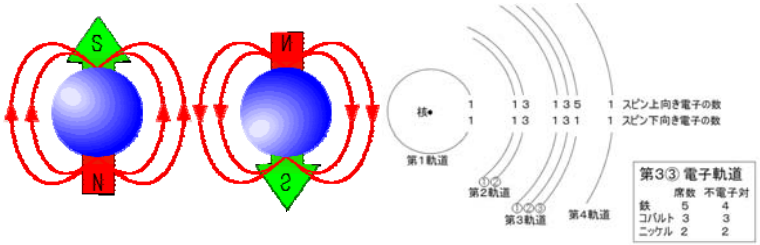


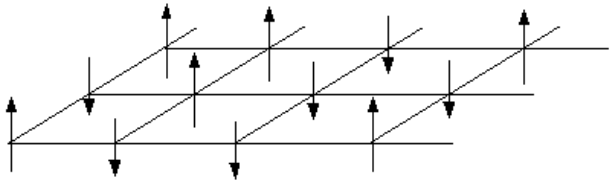
Ernst Ising



Ising Model

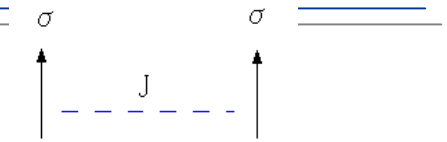
電子のスピン(磁性の向き)





外部磁場との相互作用項

近傍との相互作用項

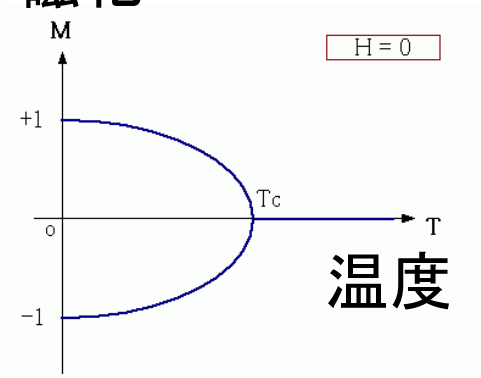


$$E = -H\sum_i \sigma_i - \frac{J}{2}\sum_{(i,j)} \sigma_i\sigma_j$$

$$\sigma_i = \begin{cases} 1 & \text{(up)} \\ -1 & \text{(down)} \end{cases}$$

外部磁化H=0のとき,

磁化



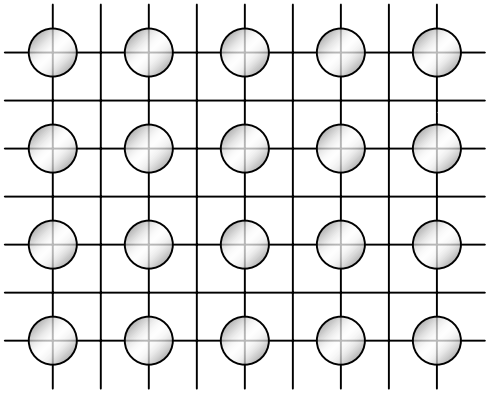
- *キュリー点* T_Cで相転移
  - 自発磁化
- 温度$T > T_C$では，エントロピー(熱振動)が優勢で 磁化$M = 0$
- 温度$T < T_C$では、磁気エネルギーが優勢で $M = \pm 1$

参考: http://www.geocities.jp/hiroyuki0620785/k2jiki/magspin.htm
http://jc.maxwell.jp/statisticalmechanics/ising2D/index.html

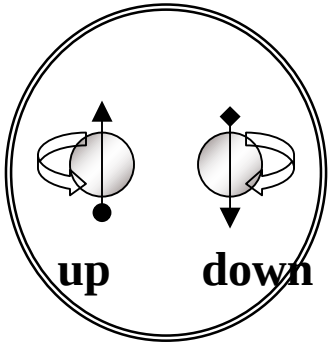
Ising model の応用（1）

投資家の行動と原子の状態を同じと考える！

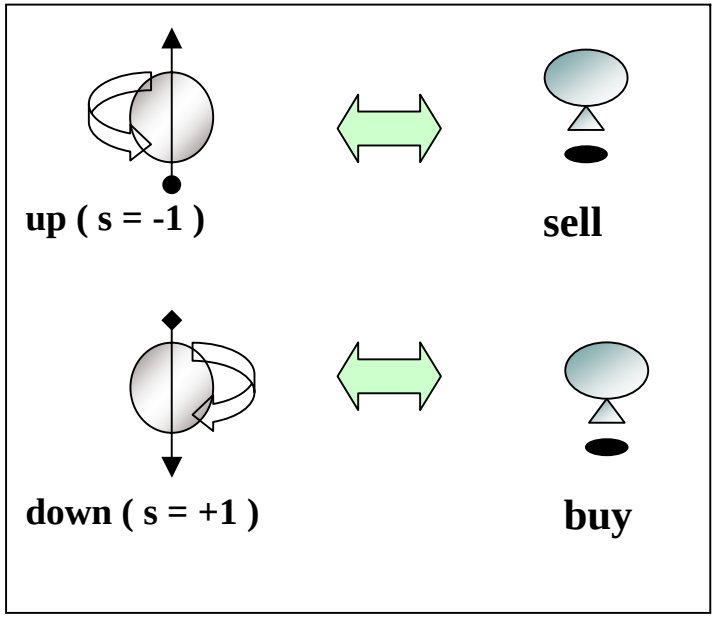
原子配列



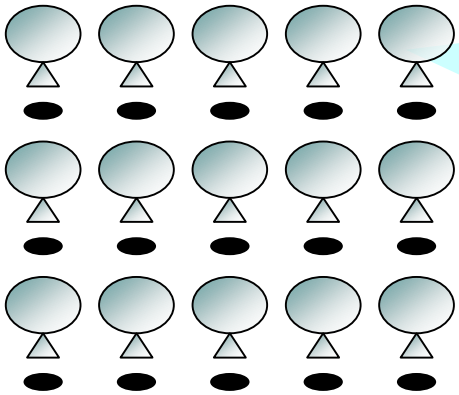
スピンの取り得る2個の状態



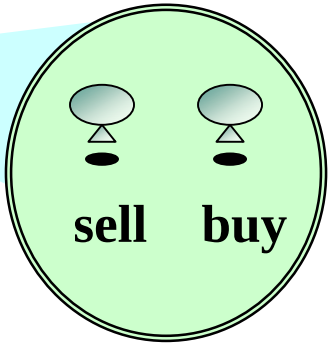
Ising model



経済市場

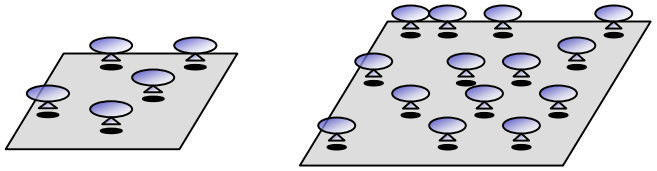


投資家の取り得る行動

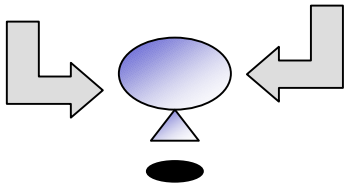


Ising model の応用（2）

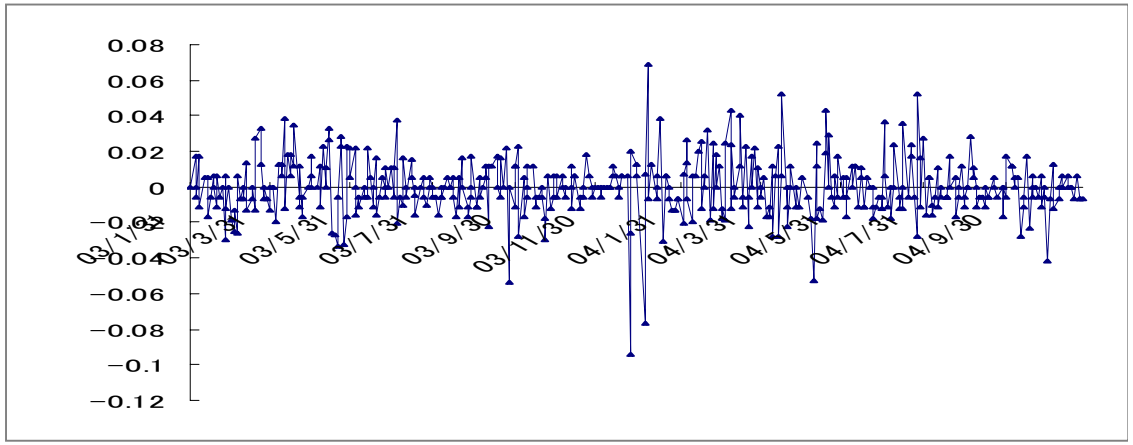
投資行動ルール



$\Sigma J_{ij}S_j(t)$:近傍からの影響　M(t):場の作用



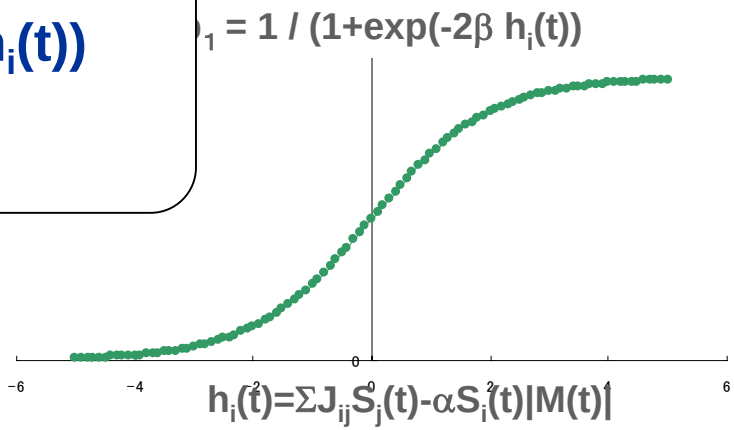
Emergent market behavior



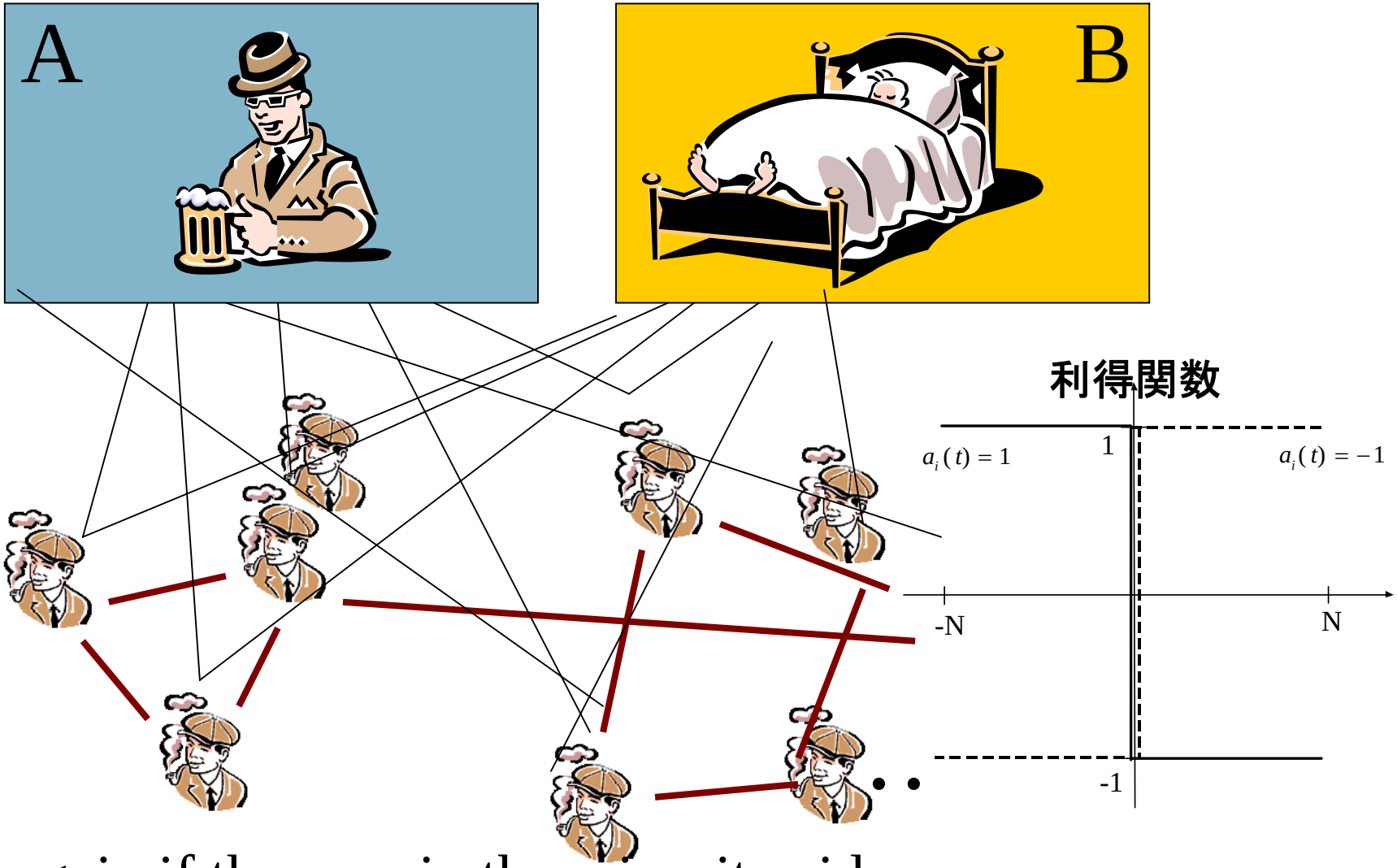
・買いの確率

$p_1 = 1 / (1+\exp(-2\beta\ h_i(t))$

$h_i(t)=\Sigma J_{ij}S_j(t)-\alpha S_i(t)|M(t)|$



The El Farol bar problem



Agents gain if they are in the minority side.

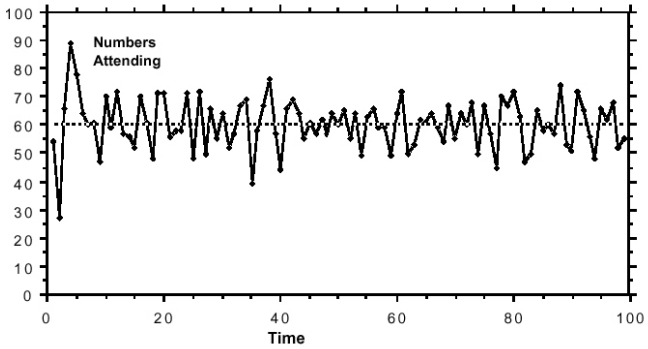
Minority Game

El Farol bar at Santa Fe

D. Challet and Y.-C. Zhang,
Physica A **246**, 407 (1997)

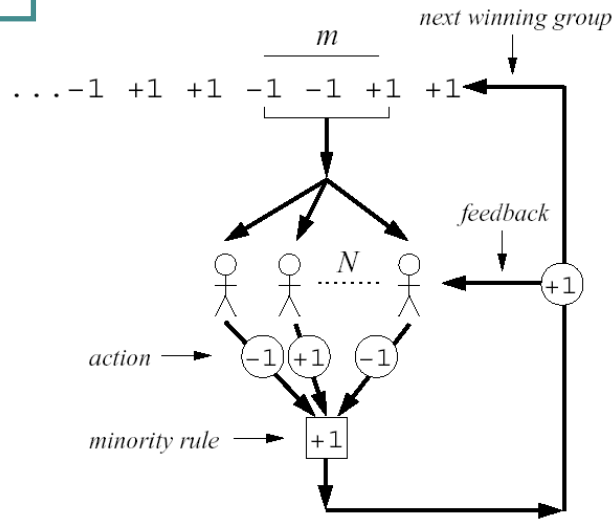
Behavioral **Rules** in Minority Game

input	output
-1 -1 -1	-1
-1 -1 +1	-1
-1 +1 -1	+1
-1 +1 +1	-1
+1 -1 -1	-1
+1 -1 +1	+1
+1 +1 -1	-1
+1 +1 +1	+1

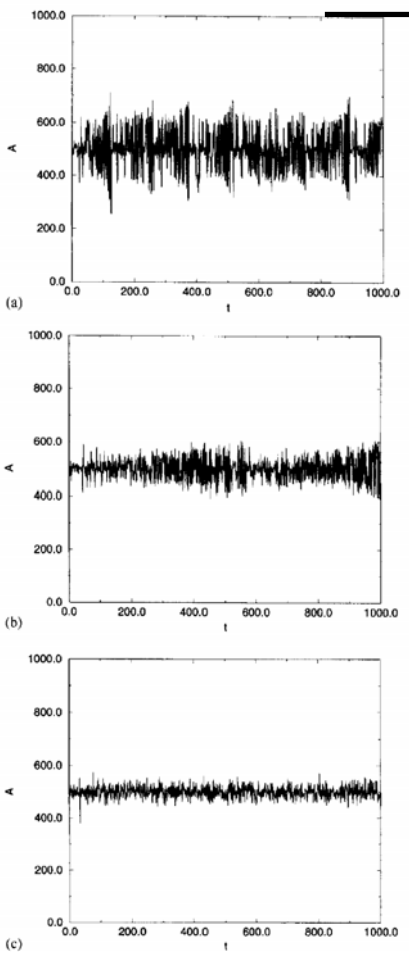


The number of people in the bar

W. B. Arthur,
Amer. Econ. Review **84**, 406 (1994).

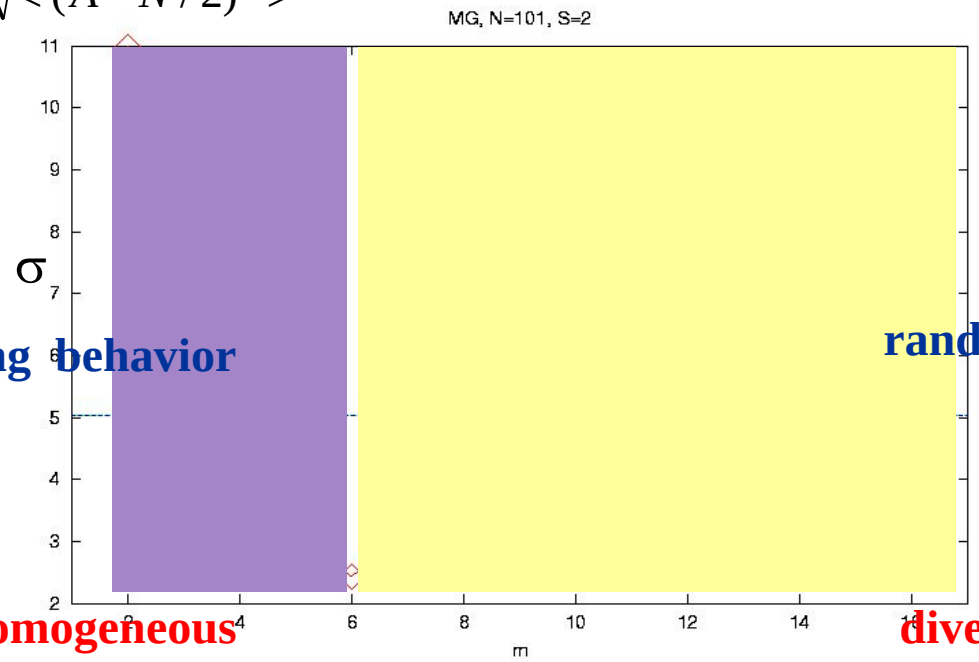


均質性と多様性



Volatility

$$\sigma = \sqrt{<(A - N/2)^2>}$$



herding behavior

random behavior

Nash equilibrium

(S1: 0.5, S2: 0.5)

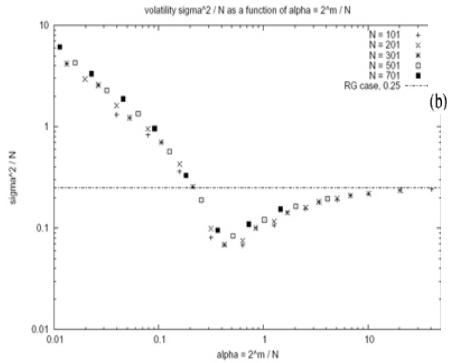
homogeneous

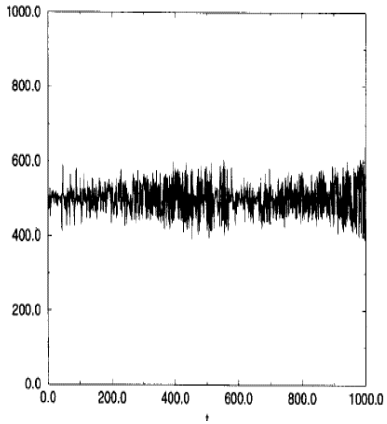
diversity

各エージェントがもつルールの数

暗黙の協調の創発

The networked Minority Game





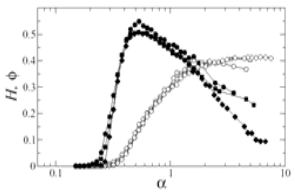
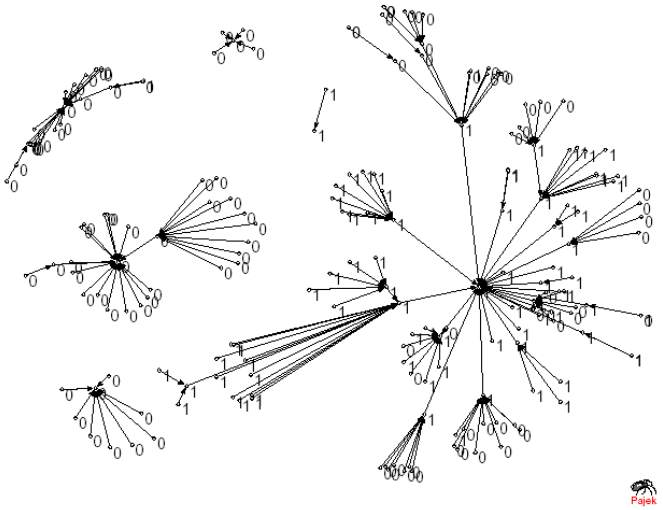


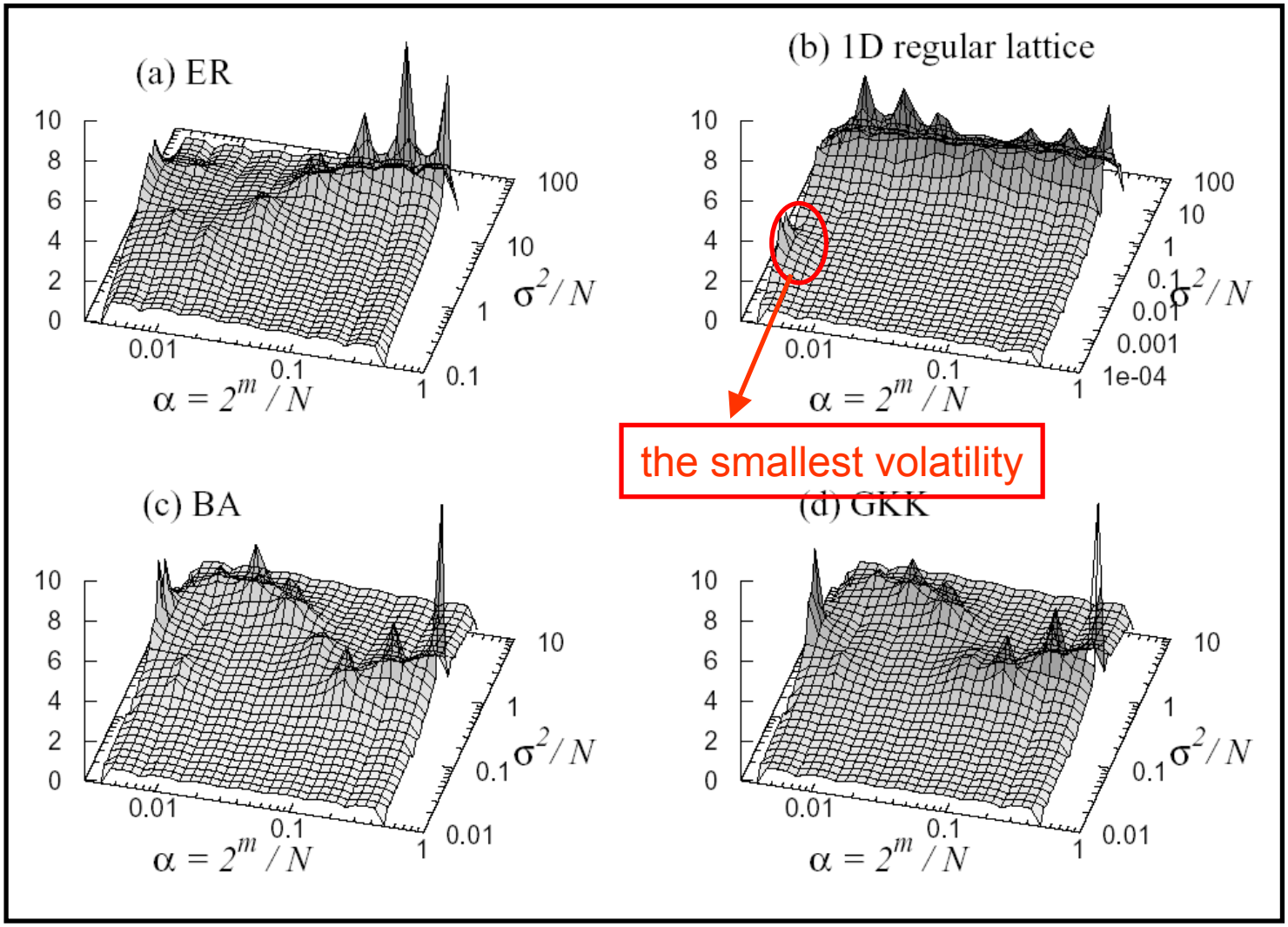
Figure 4: Information H (open symbols) and fraction of frozen agents ϕ (full symbols) as a function of the control parameter $\alpha = 2^m/N$ for $s = 2$ and $m = 5, 6, 7$ (circles, squares and diamonds respectively).

The Minority Game



What if we connect the players by **networks**?

Substrate networks determines volatility



S. Lee and H.Jeong (KAIST) 2005

社会的相互作用と規範の創発（1）

(1) 学習（強化学習）や進化(自然淘汰）

うまくいった方法は，再度繰り返す
（多くの利得をもたらした戦略を次回も高い確率で選択）

(2) 社会的規範：譲り合い，（情けは人のためにあらず）

行動の選択は，獲得した自分の利得でなく，他人を含む全体の結果に依存して決定
（利得を獲得できたときは，次回は，他の人に譲ることを念頭に，戦略を変える）

Give-and-Take（譲り合い）

（1）利得を獲得できたときは（相手に譲る），次回は戦略を変更

（2）利得を獲得できなかったときは，次回はランダムに選択

$$(\omega(t)=0)\bigcap(a_i(t)=1)\Rightarrow a_i(t+1)=0 \qquad \text{Gain}$$

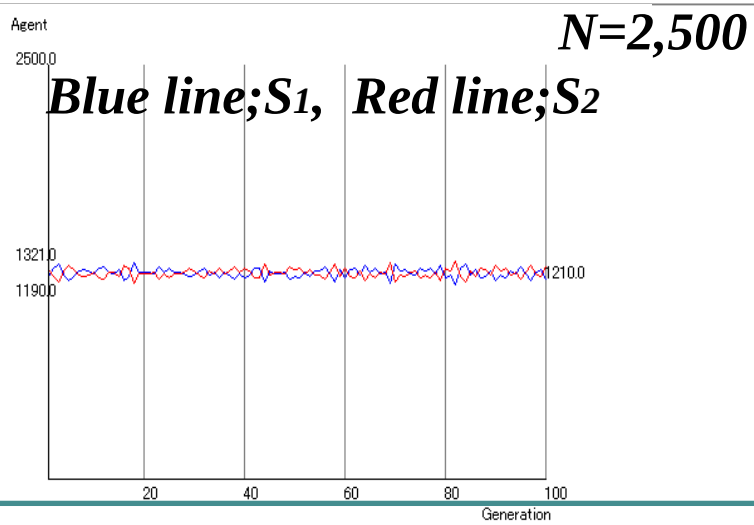
$$(\omega(t)=1)\bigcap(a_i(t)=0)\Rightarrow a_i(t+1)=1 \qquad \text{Gain}$$

$$(\omega(t)=1)\bigcap(a_i(t)=1)\Rightarrow a_i(t+1)=random \qquad \text{No gain}$$

$$(\omega(t)=0)\bigcap(a_i(t)=0)\Rightarrow a_i(t+1)=random \qquad \text{No gain}$$

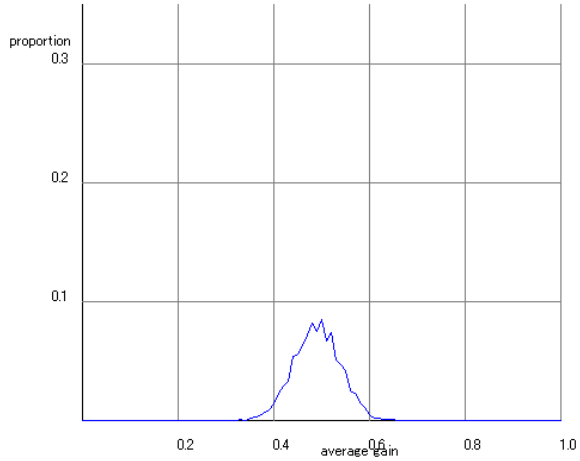
ナッシュ均衡（ランダム）戦略との比較

ナッシュ均衡戦略

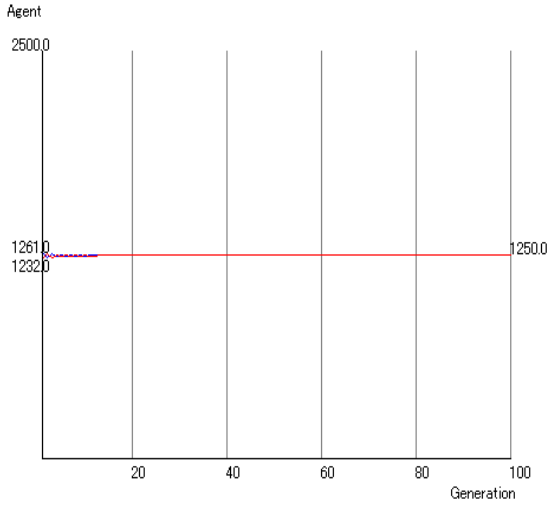


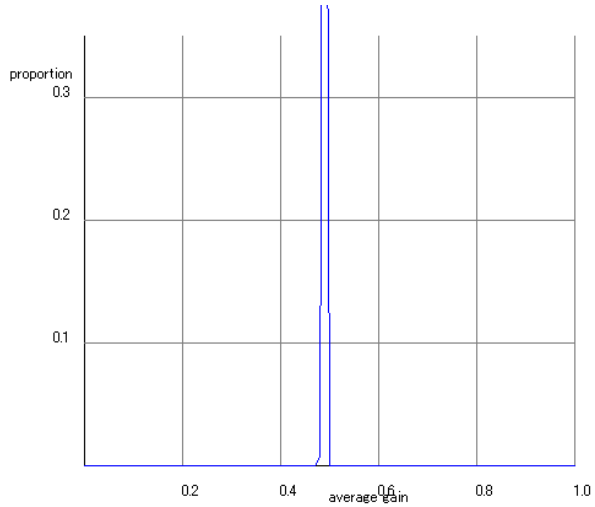
N=2,500

Blue line;S1, Red line;S2



譲り合い戦略

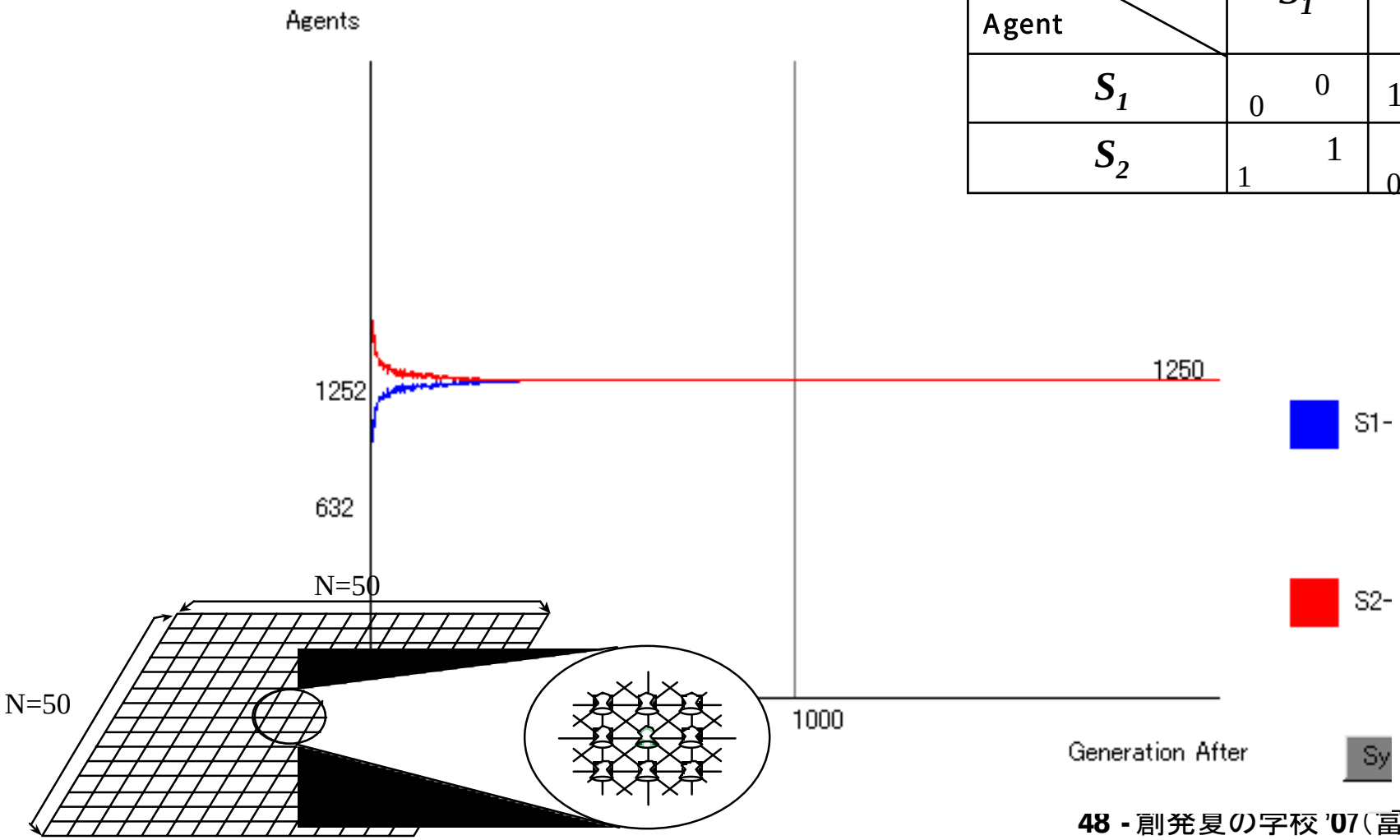




4近傍との少数派ゲーム(1)

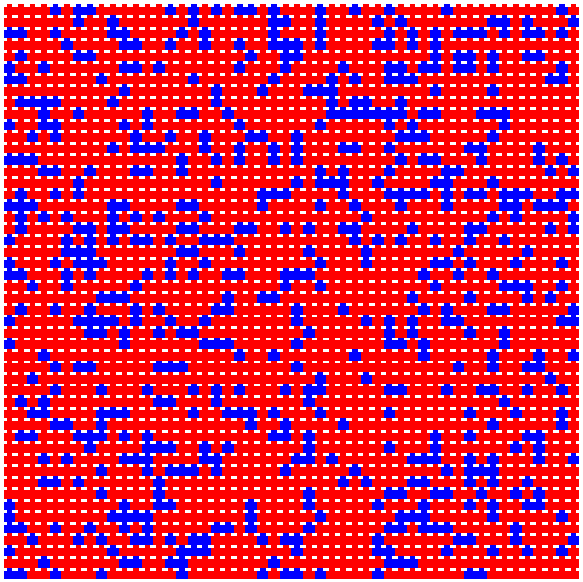
S1,S2を選択するエージェント数

Other agent / Agent	S_1	S_2
S_1	0 0	1 1
S_2	1 1	0 0

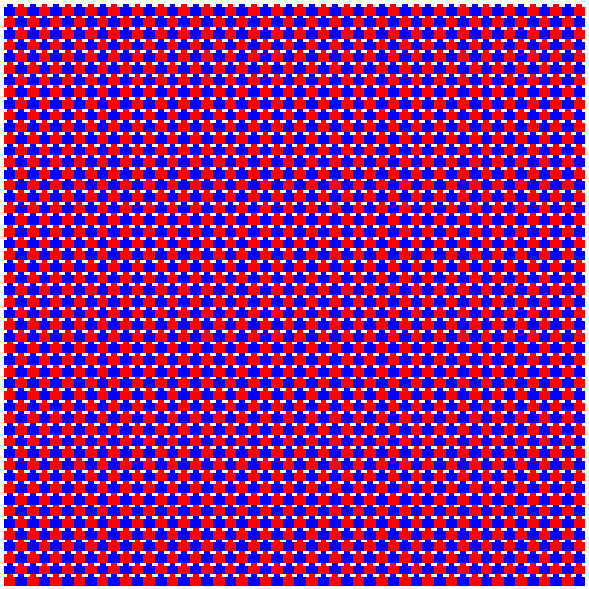


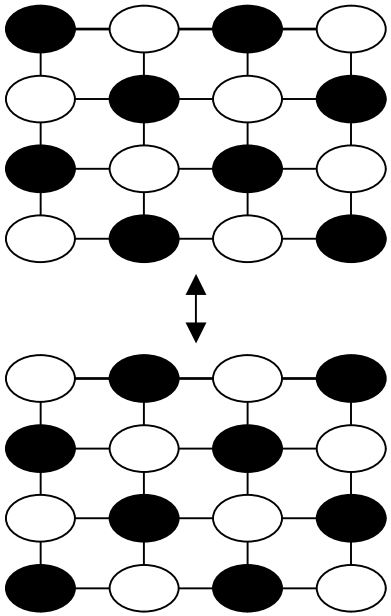
4近傍との少数派ゲーム(2)

初期戦略の配置



動的パターンの創発



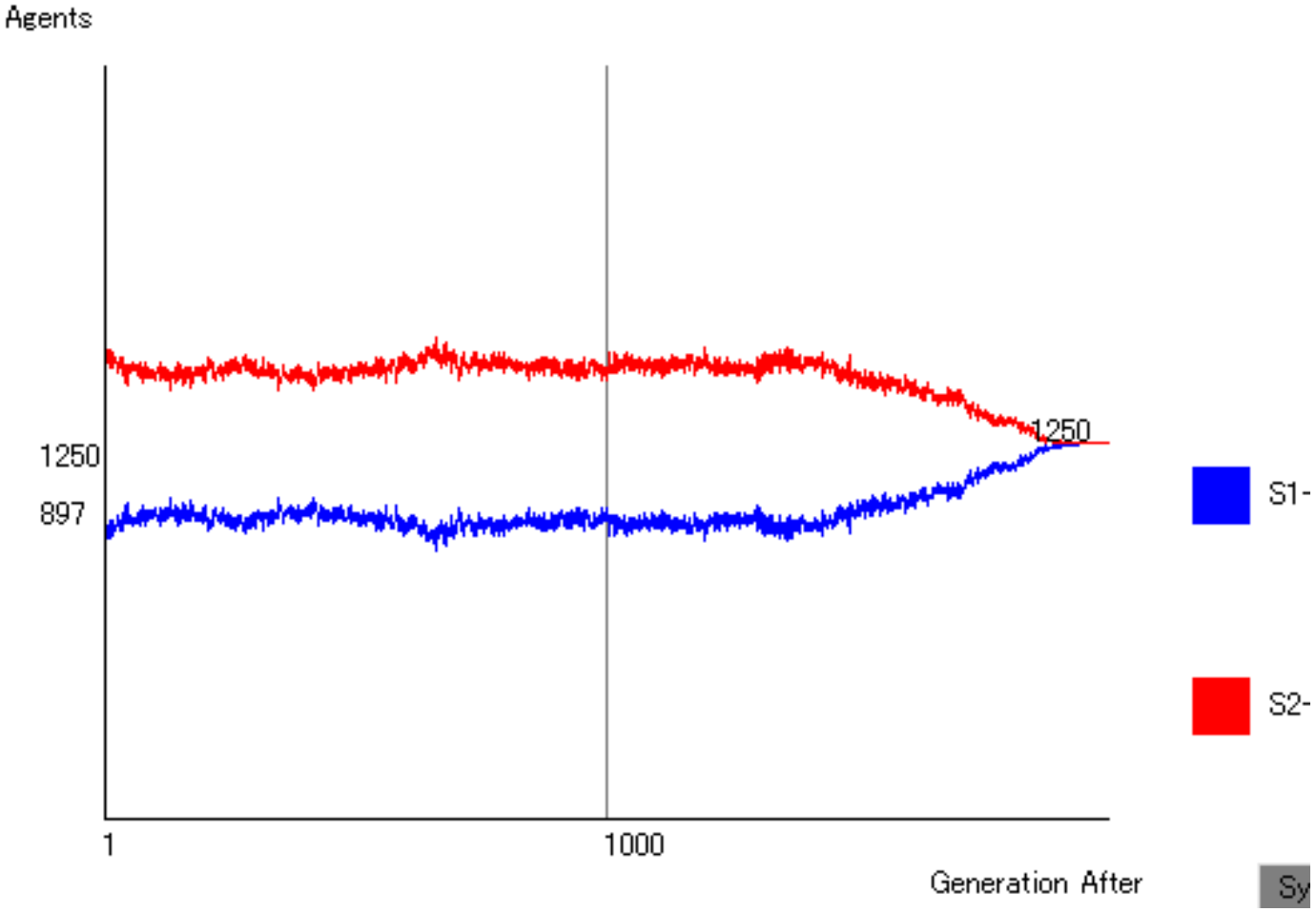


(Final pattern)

(Pattern B)

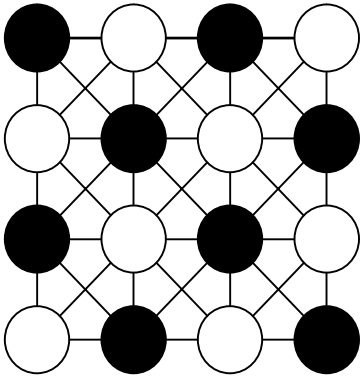
8近傍との少数派ゲーム

S1,S2を選択するエージェント数

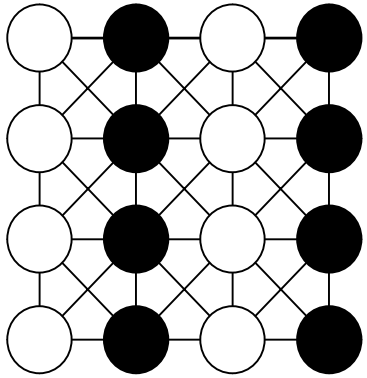


８近傍との少数派ゲーム(2)

最適でないパターン



最適なパターン

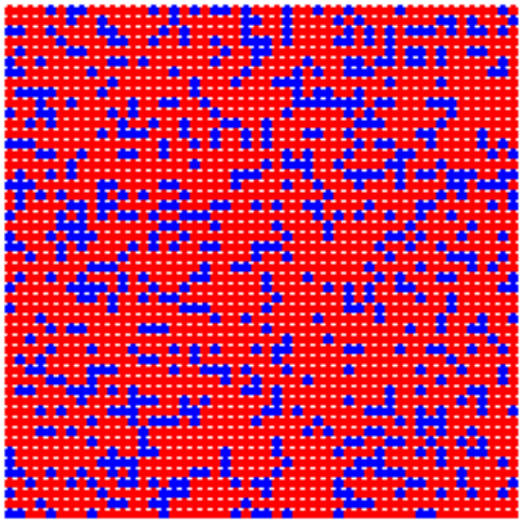


Other agent / Agent	S_1	S_2
S_1	0 0	1 1
S_2	1 1	0 0

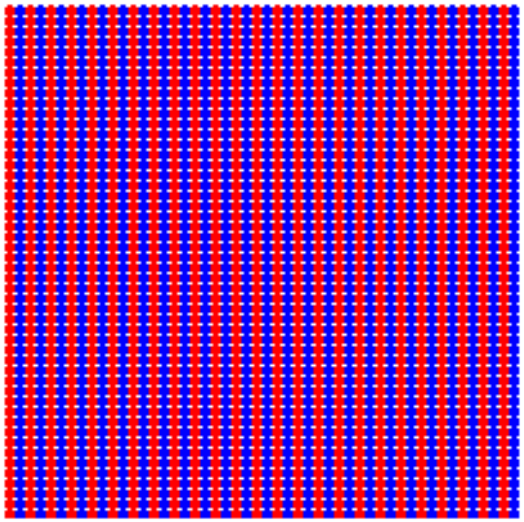
●

○

初期戦略の配置

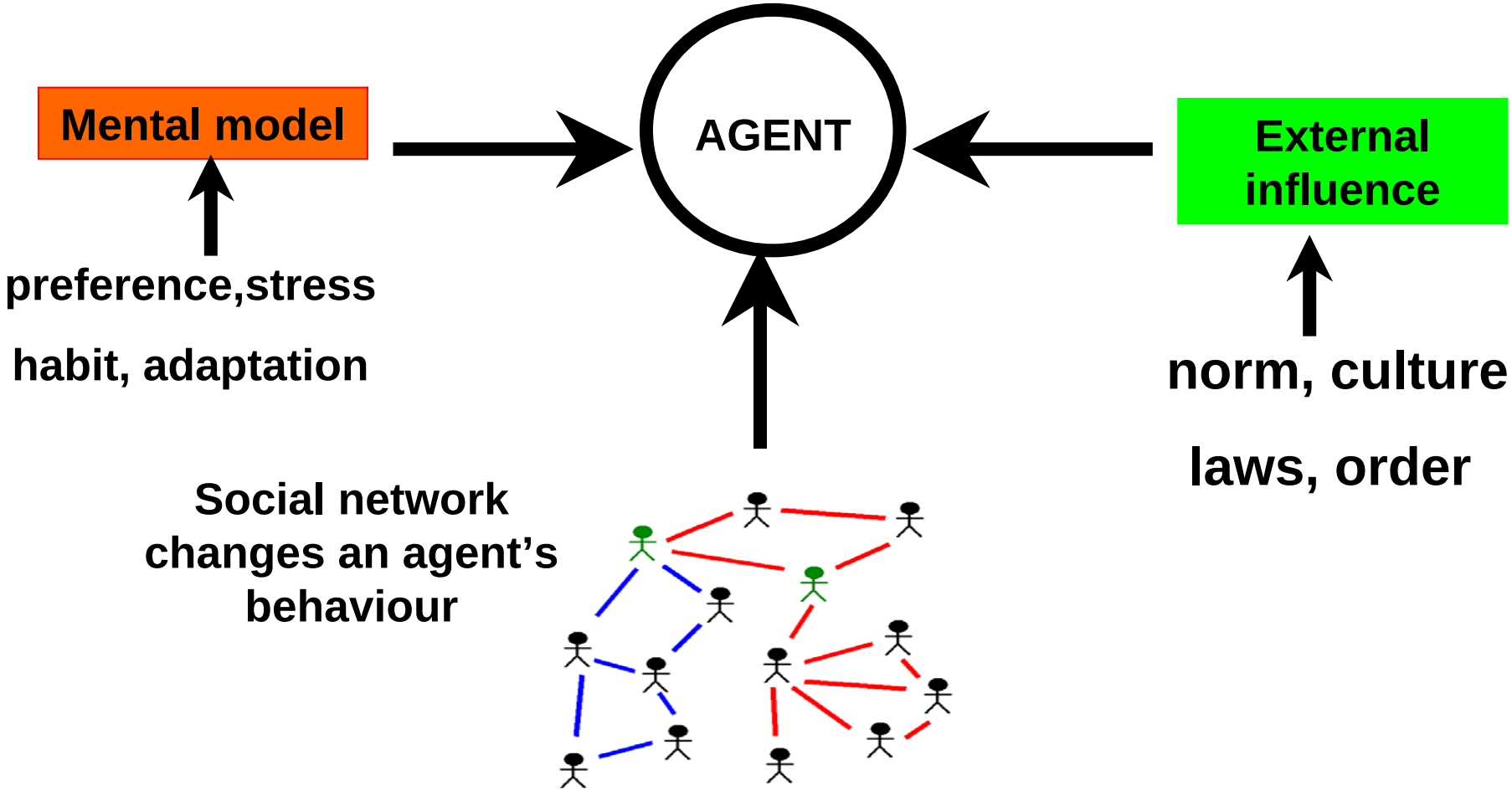


創発した動的パターン



Agent vs. Social Atom

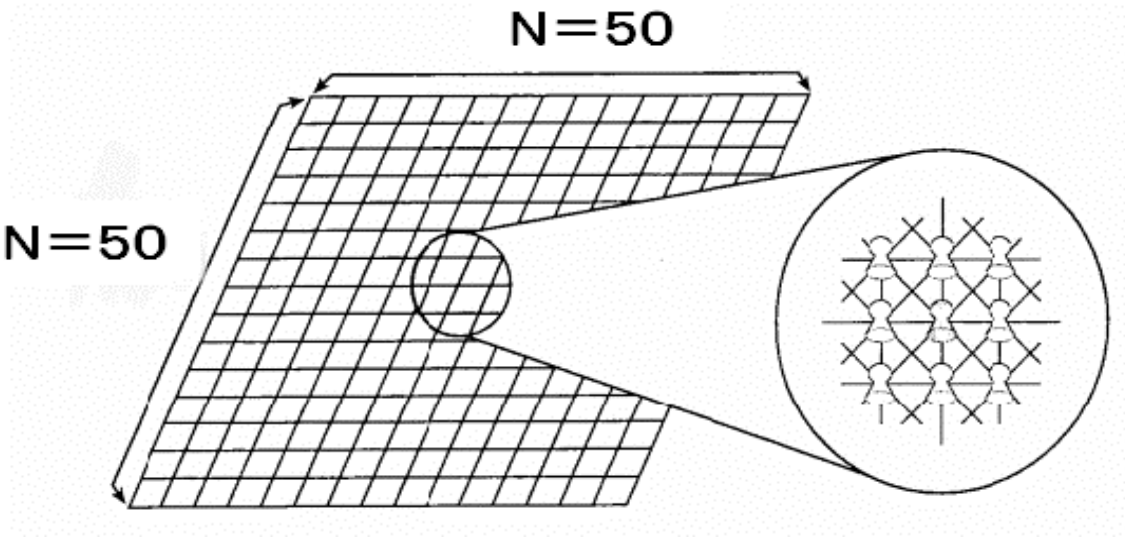
D. Green (2006)

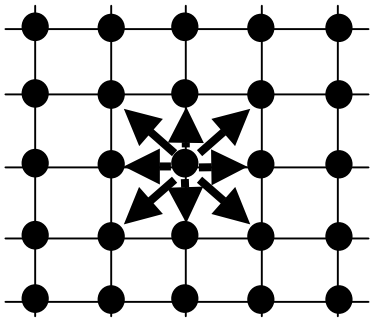


Repeated Games on Networked Agents

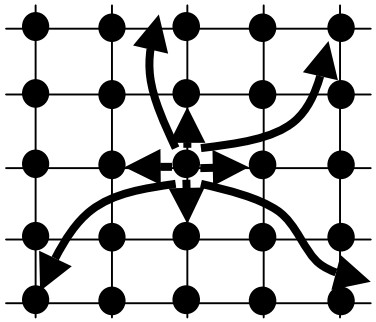
✓Types of pair-wise interactions

- ❑Prisoner's dilemma game
- ❑Coordination game
- ❑Hawk-dove game
- ❑Chicken game
  (battle-of-sexes game)

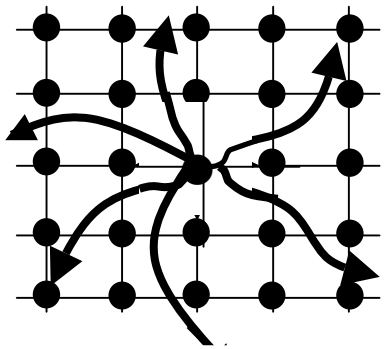




Local model



Small-world model



Random model

Simulation Results

(1) Dilemma game

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

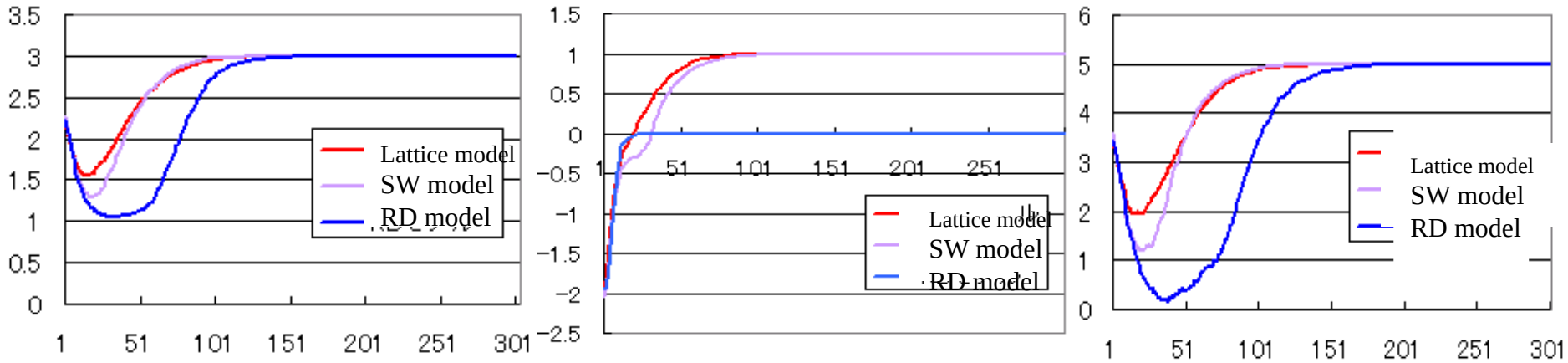
(2) Coordination game

	C	D
C	1, 1	-9, 0
D	0, -9	0, 0

(3) Hawk-Dove game

	C	D
C	5, 5	0, 10
D	10, 0	-1, -1

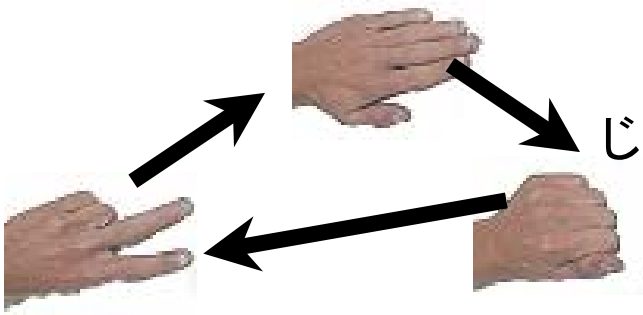
The average payoff per generation



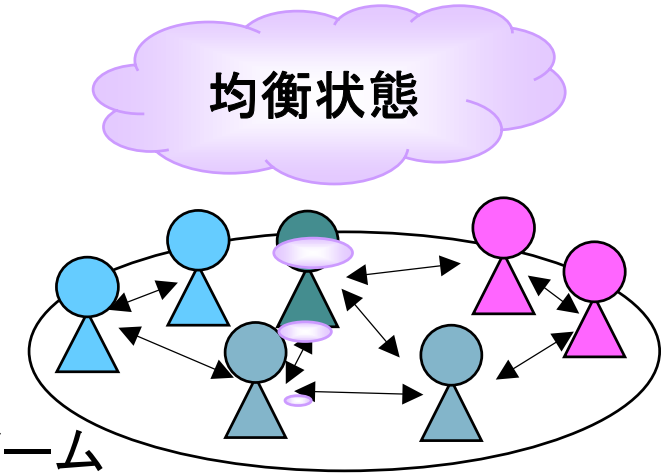
社会的相互作用と規範の創発（2）

無秩序な相互作用（競争状態の均衡解）
：グー、チョキ、パーをの割合はほぼ同じ

（1）それは、果たして望ましい状態か？
（2）そうでない場合、望ましい解から
どの程度乖離しているか？



じゃんけんゲーム



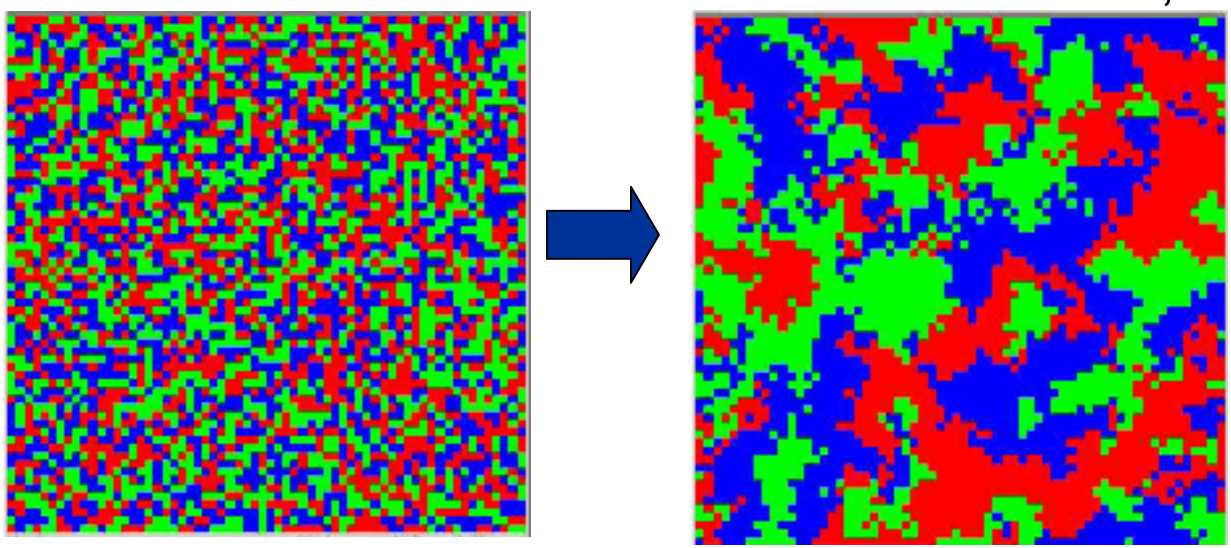
RSP Games：生命体の多様性，共生関係

Kerr *et al.* Nature (2002)
(1) a pair plays Rock-Paper-Scissors,
(2) winner replaces loser

- **R + P→2P (Paper wins)**
- **P + S→2S (Scissors win)**
- **S + R→2R (Rock wins)**

Agent *B* / Agent *A*	S_1 Rock	S_2 Scissor	S_3 Paper
S_1 Rock)	0 0	−1 1	1 −1
S_2 Scissor)	1 −1	0 0	−1 1
S_3 Paper	−1 1	1 −1	0 0

coexistence of 3 trains, R,S,P



: Rock

: Scissor

: Paper

非ゼロ和ゲームとしてのジャンケンゲーム

Agent *B* / Agent *A*	S_1 (Rock)	S_2 (Scissor)	S_3 (Paper)
S_1 (Rock)	1, 1	0, λ	λ, 0
S_2 (Scissor)	λ, 0	1, 1	0, λ
S_3 (Paper)	0, λ	λ, 0	1, 1

1. ナッシュ均衡戦略：各戦略を等確率で選択

$(S_1, S_2, S_3) = (1/3、1/3、1/3)$

2. ナッシュ均衡戦略の下での期待利得

$(\lambda + 1)/3$

3. パレート最適解：
両者の利得の和を最大にする戦略の組

両者の利得の和：λ

(S_1,S_2), (S_1,S_3), (S_2,S_1),
(S_2,S_3), (S_3,S_1), (S_3,S_2)

相互作用ルールの定義

前回の戦略		次回にとる戦略
自分	相手	
0	0	#
0	1	#
0	2	#
1	0	#
1	1	#
1	2	#
2	0	#
2	1	#
2	2	#

0：Rock, 1: Scissor,2: Paper、#: 0, 1 or 2

カップリングルールの種類： $3^9 = 19{,}683$

エージェントは、より良いカップリングルールを求め学習をする

カップリングルール：

自分と相手が前回選択した戦略の組み合わせから，次回の戦略を決定

Agent *B* / Agent *A*	S_1 Rock	S_2 Scissor	S_3 Paper
S_1 Rock	1 / 1	0 / 2	2 / 0
S_2 Scissor	2 / 0	1 / 1	0 / 2
S_3 Paper	0 / 2	2 / 0	1 / 1

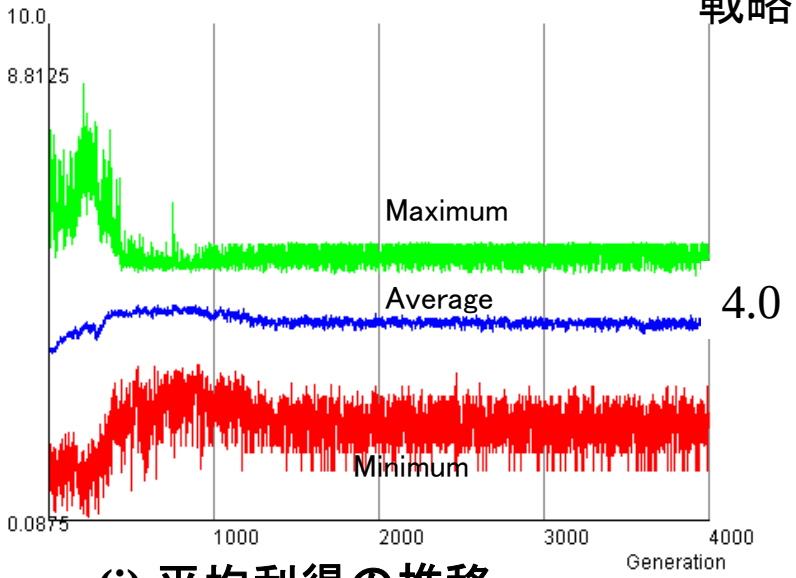
Outcome(自分と相手の戦略)---戦略決定

合理的なルール(最適反応)：

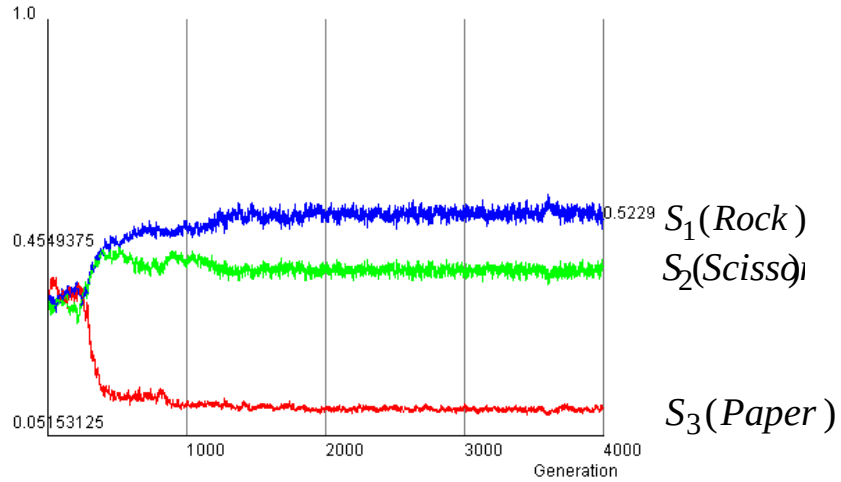
相手の戦略---戦略決定

シミュレーション結果（λ=10）

戦略の誤り率：0%

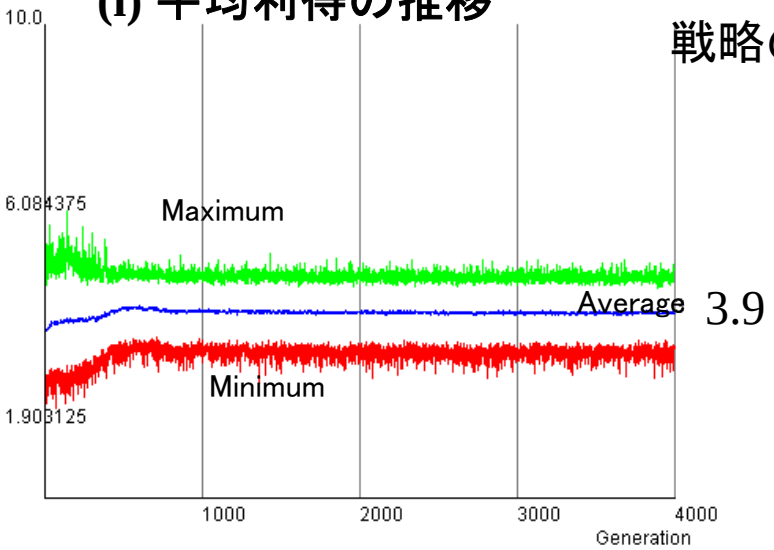


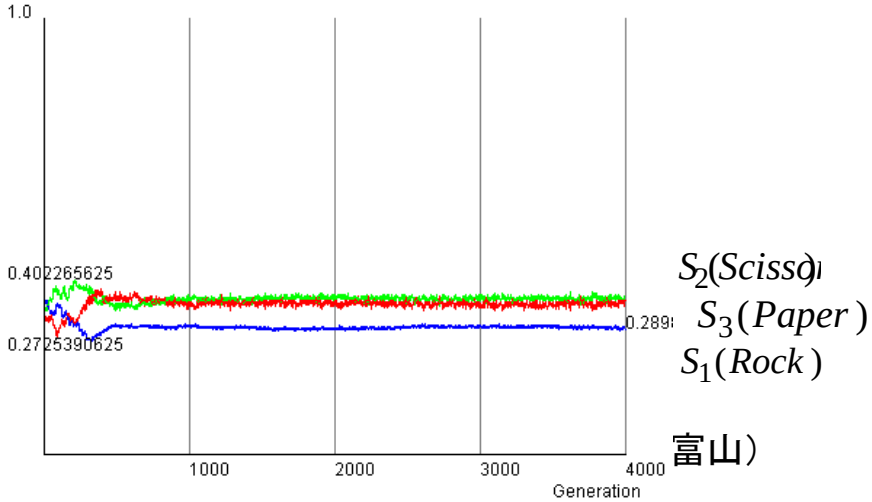
(i) 平均利得の推移



(ii) 戦略の分布

戦略の誤り率：10%





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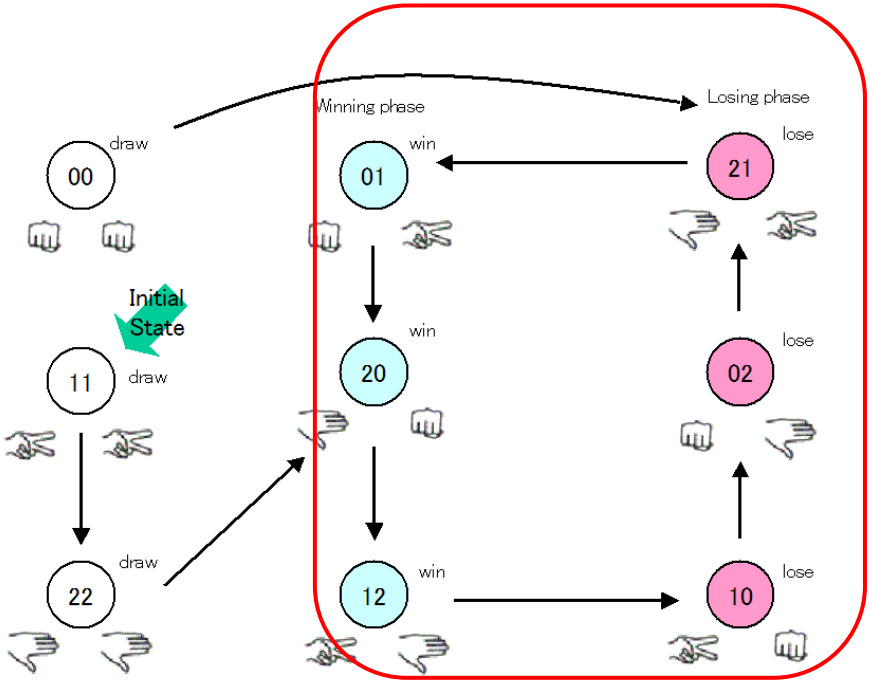
エージェントは何を学習したのか？

λ ＝10、誤り率:10%のとき

	00	01	02	10	11	12	20	21	22	同じルールをもつエージェントの数
タイプ1	2	2	2	0	2	1	1	0	2	126
タイプ2	2	2	2	0	2	1	1	0	0	76
タイプ3	1	2	2	0	2	1	1	0	2	58
タイプ4	1	2	2	0	2	1	1	0	0	54
タイプ5	2	2	2	0	0	1	1	0	2	40
タイプ6	1	2	2	0	0	1	1	0	2	19
タイプ7	2	2	2	0	0	1	1	0	0	17
タイプ8	1	2	2	0	0	1	1	0	0	10

・ 400人のエージェントのカップリングルールは、8タイプに集約され、かつ8つルールの間には共通性がうまれた.

学習後，どのようにプレーするか？



Agent *B* / Agent *A*	S_1 (Rock)	S_2 (Scissor)	S_3 (Paper)
S_1 (Rock)	1 / 1	0 / λ	λ / 0
S_2 (Scissor)	λ / 0	1 / 1	0 / λ
S_3 (Paper)	0 / λ	λ / 0	1 / 1

社会的相互作用:タイプ3

混在する相互作用系:

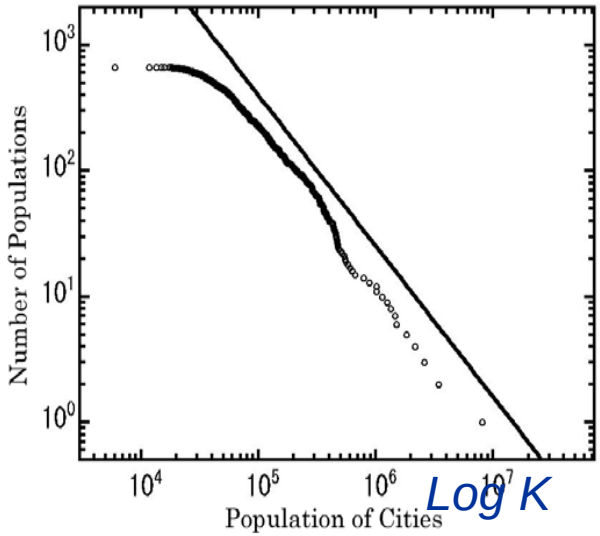
- 集中と分散
- 上昇と下降
- 繁栄と衰退

例1:群れ行動

例2:人口移動や都市の形成

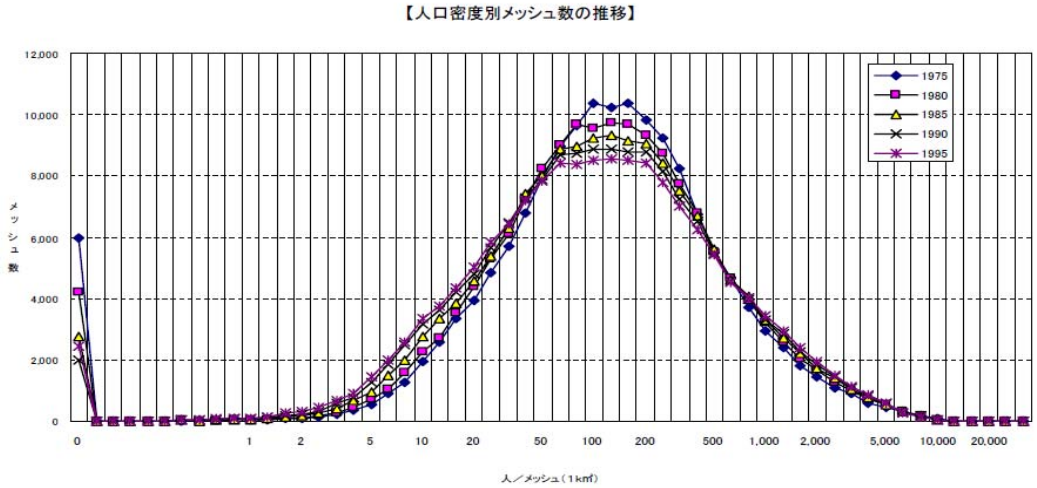
人口分布：べき分布 vs 対数正規分布

都市の人口分布：べき分布



$N_T=672$
$a=4*10^8$
$b=1.2$

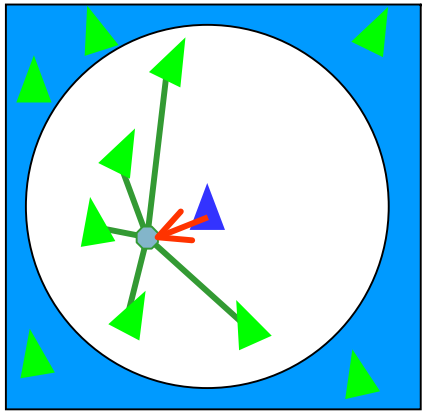
1kmあたりの人口分布：対数正規分布：日本（2000）



BOIDモデル: Flocking Formation

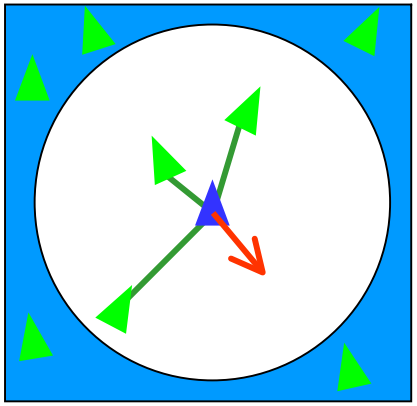
Craig Reynolds, "*Steering Behaviors For Autonomous Characters*", Game Developers Conference, 1999

個々のエージェントの**3つの行動ルール**から群れ行動が創発



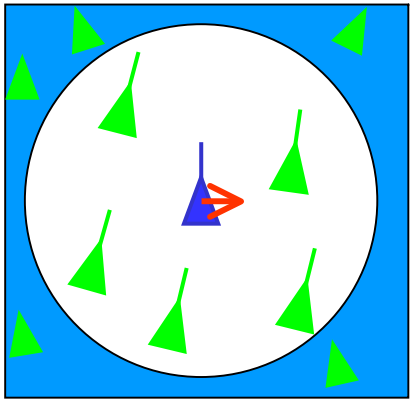
Cohesion

仲間の
近くにいたい
引力 $\vec{F}_c$



Separation

あまり
近づきたくない
斥力 $\vec{F}_s$



Alignment

仲間に
あわせたい
速度調整 $\vec{F}_a$

群れ行動に関する力 $\vec{F}_f = \vec{F}_c + \vec{F}_s + \vec{F}_a$

なぜ群れは創発されるか？

Agent に働く力

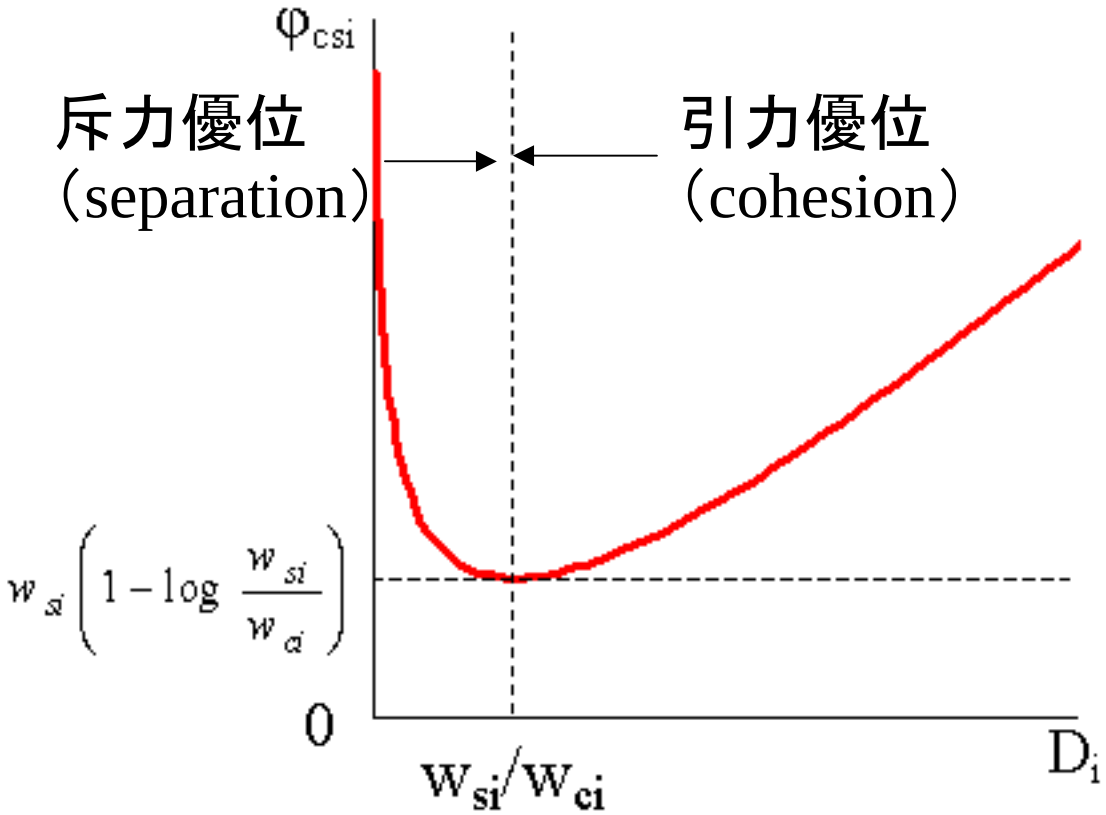
$$\vec{F}_{fi} = \vec{F}_{ci} + \vec{F}_{si} + \vec{F}_{ai} = \left(w_c - \frac{w_s}{D} \right) \vec{e}_D + w_a \vec{e}_V$$

凹関数型ポテンシャル

$$\phi_{if} = \int \vec{F}_{ics} dD_i$$

$$= w_{ic} D_i - w_{is} \log D_i$$

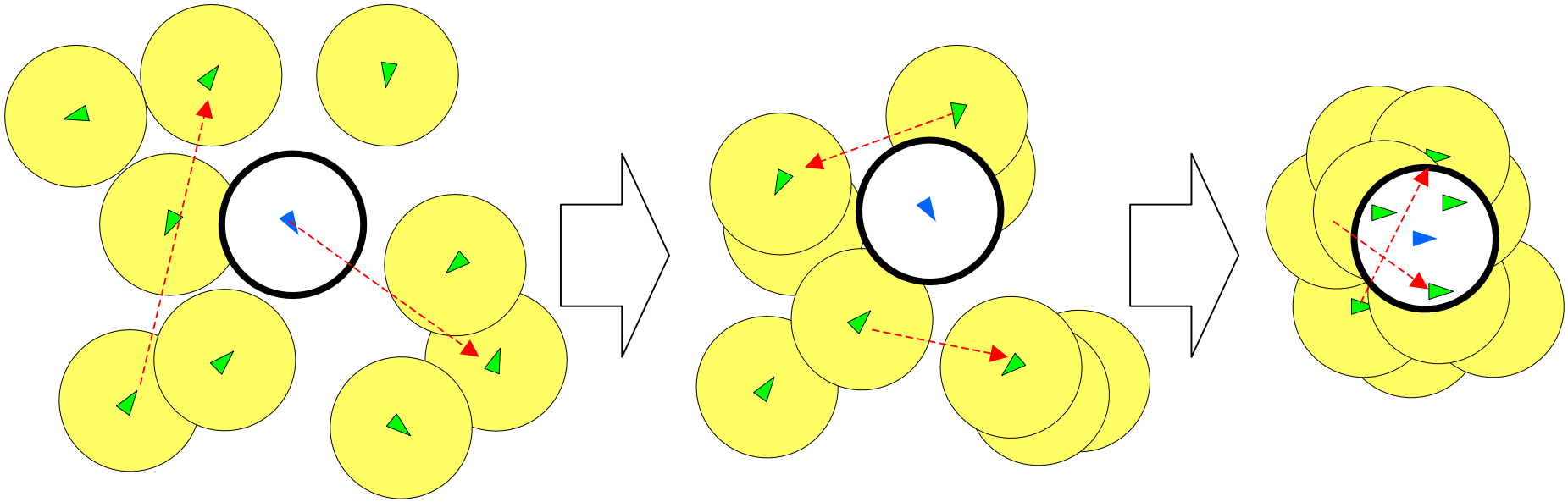
cohesion項　separation項



エージェントと他のエージェントとの中心間距離

群れ行動：根底にあるネットワーク構造の推移

初期状態：
相互に遠隔(視界外)，位置と速度：ランダム



疎なrandom graph

ランダムな遠隔リンク

small-world

疎な局所（近傍）リンク
＋ランダムな遠隔リンク

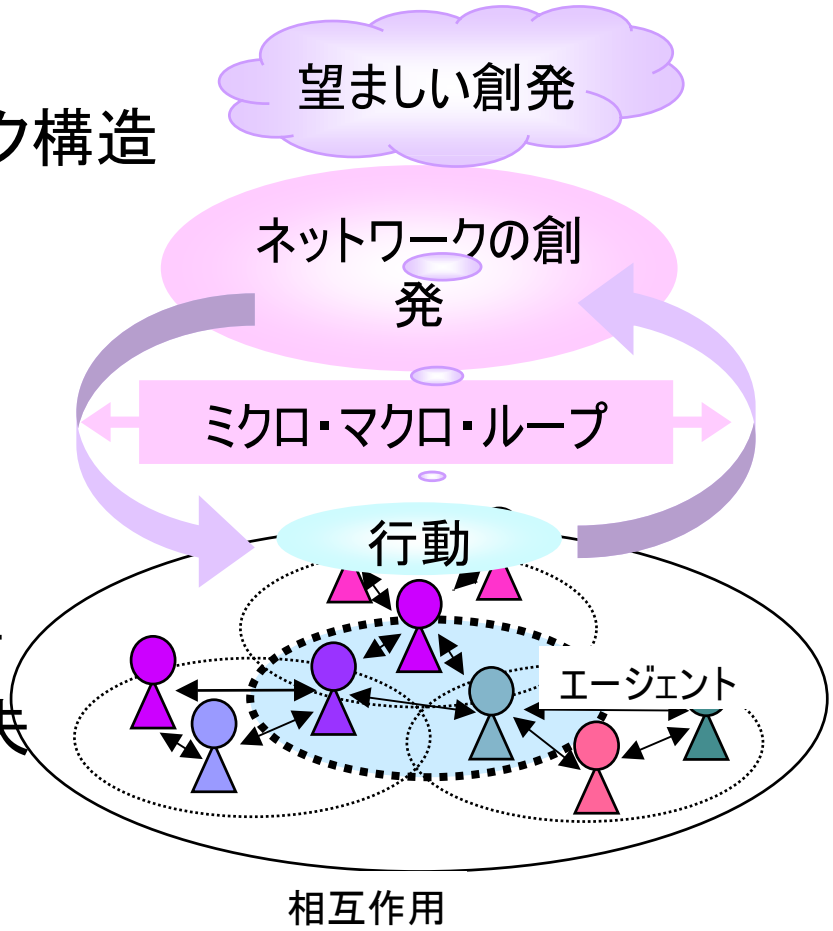
連結グラフ

密な近傍リンク

相互作用系：順問題と逆問題

＜順問題＞
　どのような性質をもったネットワーク構造が創発されるのか？

＜逆問題＞
望ましい性質をもつネットワーク構造を創発するには？
(1)最適なネットワークの進化的設計
(2) 相互作用のミクロレベルでの工夫



Optimization in Complex Networks

- The fitness function
  - The minimization of E(λ)

$$E(\lambda) = \lambda d + (1-\lambda)\rho \quad 0 \le \lambda, d, \rho \le 1$$

  - It involves the simultaneous minimization of distance and number of links.
  - λ : weight of two objects d and ρ

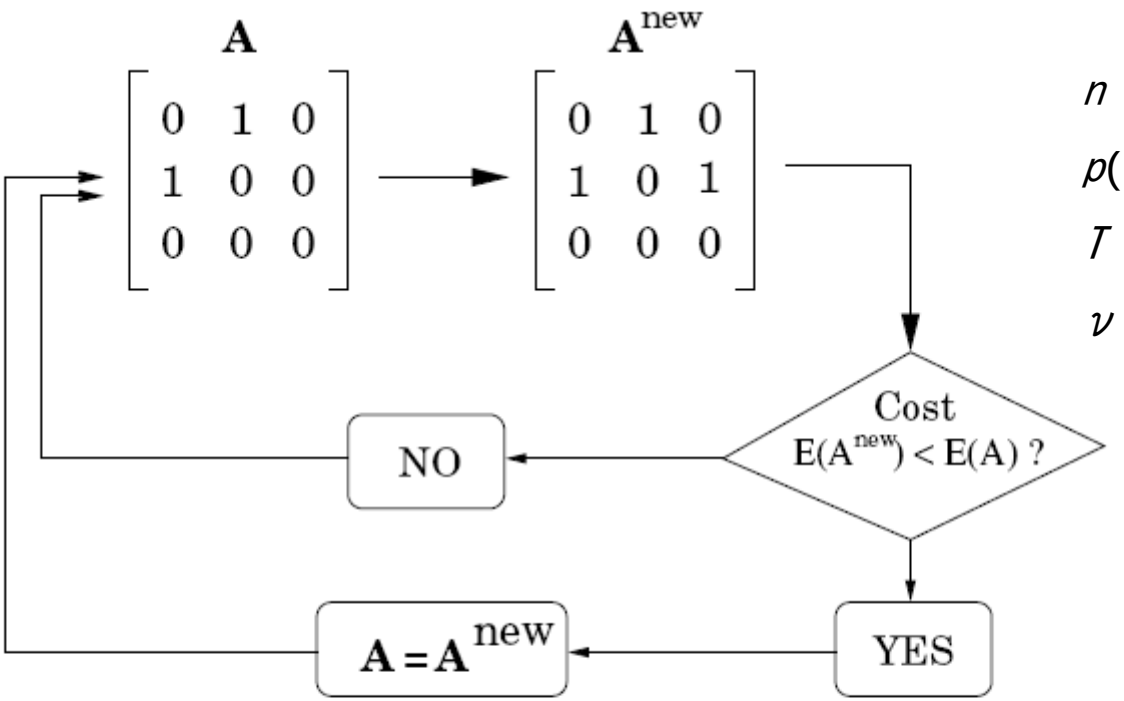
$$\rho = \frac{1}{\binom{n}{2}} \sum_{i<j} a_{ij} = \frac{\langle k \rangle}{n-1}$$

  - ρ : network density
  - d : The normalized vertex-vertex distance

$$d = D / D^{linear} \qquad D = \frac{1}{\binom{n}{2}} \sum_{i<j} d_{ij} \qquad D^{linear} = (n+1)/3$$

  - d_{ij}: The minimum distance between vertices

The optimization algorithm



$n = 100$

$p(0) = 0.2$

$T = \binom{n}{2}$

$\nu = 2 / \binom{n}{2}$

Fig. 1. Basic scheme of the minimization algorithm. Starting from a given adjacency matrix **A** the algorithm performs a change in a small number of bits (specifically, with probability ν, each a_{ij} can flip). The energy function e is then evaluated and the new matrix is accepted provided that a lower cost is achieved. Otherwise, we start again with the original matrix. At the beginning, **A** is set up with a fixed density $\rho(0)$ of ones.

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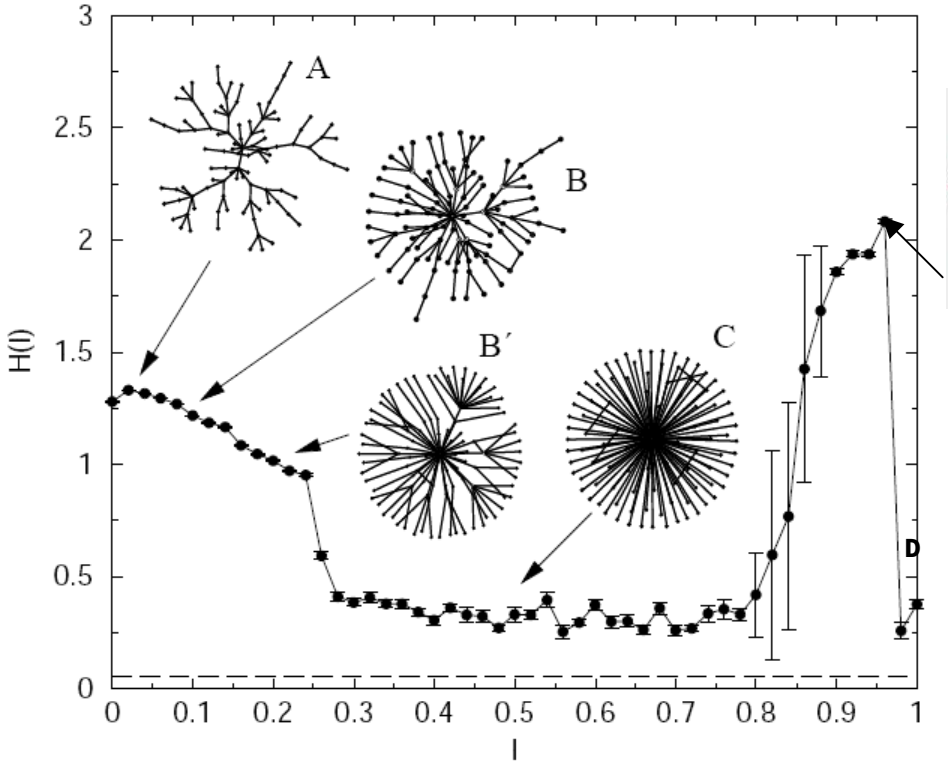
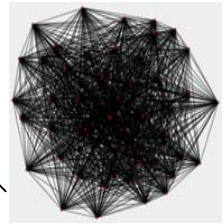
Optimized Networks

- The degree entropy $$H(\{p_k\}) = -\sum_{k=1}^{n-1} p_k \log p_k$$
  - p_k : the frequency of vertices having degree k





The topology ranking

C: Star networks

B: Scale-free networks

A: Exponential networks

D: Dense networks

Network Creation Game

[Fabrikant, etc, 2005]

- A simple model for formation of networks
  - n players which correspond to vertices in a graph
  - Each player can decide to add edges to the graph.
  - The cost of function for each player is:

$$c_i = \alpha \cdot |s_i| + \sum_j d_G(i, j)$$

Weight

Number of edges built by i

Distances of i from the rest of the graph

Social Cost

- Social cost when the strategies s_i are mutually disjoint:

$$C(G) = \sum_i c_i = \sum_i (\alpha \cdot |s_i| + \sum_{j=1}^{n} d_G(i,j))$$
$$= \alpha\,|E| + \sum_{i,j} d_G(i,j),$$

satisfied by all Nash equilibria and social optima.

- A trivial lower bound:

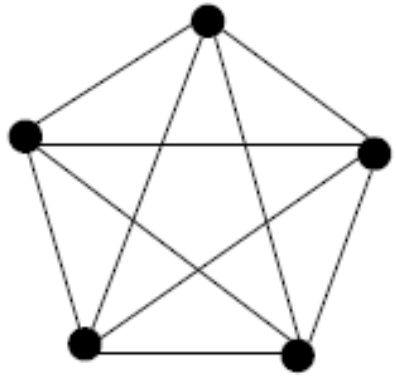
$$C(G) \geq \alpha\,|E| + 2\,|E| + 2(n(n-1) - 2\,|E|)$$
$$= 2n(n-1) + (\alpha - 2)\,|E|\,,$$

the equality is achieved by any graph of diameter at most 2.

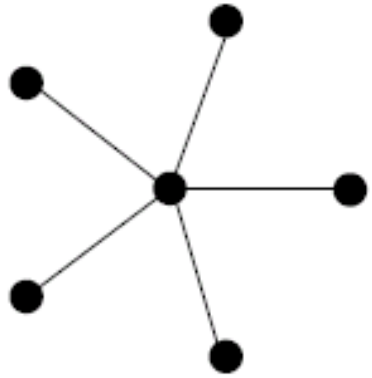
Basic Results: Social Optimum

$$C(G) \geq 2n(n-1) + (\alpha - 2)\,|E|$$

- $\alpha < 2$
  - The social optimum is the complete graph — maximize $|E|$



- $\alpha \geq 2$
  - The social optimum is the star — minimize $|E|$



Basic Results: Nash Equilibrium

$$c_i(s) = \alpha \cdot |s_i| + \sum_{j=1}^{n} d_{G[s]}(i,j)$$

Any Nash equilibrium cannot miss any edge whose addition would reduce the second term by more than α

- $\alpha < 1$
  - The complete graph is the only Nash equilibrium — of diameter 1
- $1 \leq \alpha < 2$
  - The worst Nash equilibrium is the star — of diameter at most 2, $|E|$ minimized
  - The price of anarchy is

$$\rho(\text{star}) = \frac{C(\text{star})}{C(K_n)} = \frac{(n-1)(\alpha - 2 + 2n)}{n(n-1)(\frac{\alpha-2}{2} + 2)}$$
$$= \frac{4}{2+\alpha} - \frac{4-2\alpha}{n(2+\alpha)} < \frac{4}{2+\alpha} \leq \frac{4}{3}$$

- $\alpha \geq 2$
  - The star is a Nash equilibrium, but worse one may exist

おわりに: Five Stages of Research

1) Observe: Gather data to demonstrate power law behavior in a system.
2) Interpret: Explain the import of this observation in the system context.
3) Model: Propose an underlying model for the observed behavior of the system.
4) Validate: Find data to validate (and if necessary specialize or modify) the model.
5) Control: Design ways to control and modify the underlying behavior of the system based on the model.

Focus on control issues (guidance) : Lots of open research problems in the study of emergence (complex systems)

今後の課題

How to control smart agents with ICT

(ICT:Information/Communication Technology)

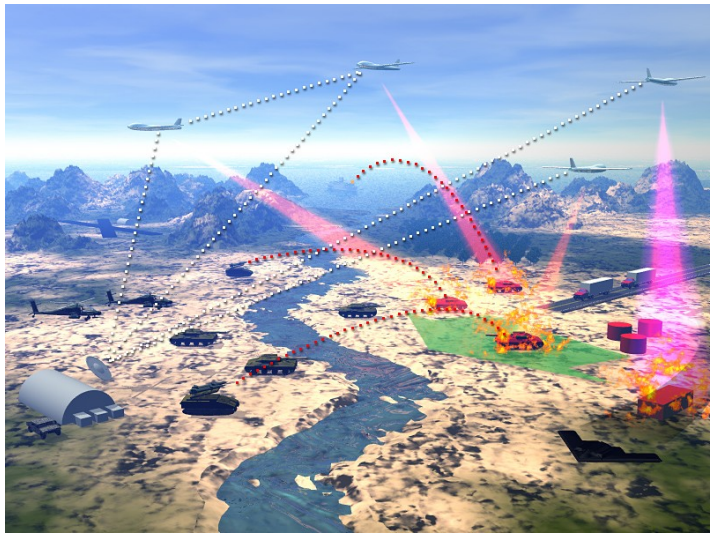
In-Car Navigation Systems

- Smart intersections
- Smart highways
- Smart ‘skyways’
- Smart dispatch of urban emergency vehicles
- Smart routing of people and vehicles

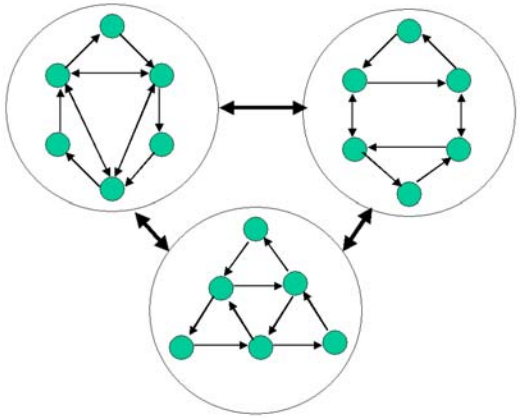
- **Smart agents often change their behaviors by getting information on the aggregate using ICT**

- **Congestion controls in a networked world should receive much attention**

Network (Graph) based Coordination and Controls

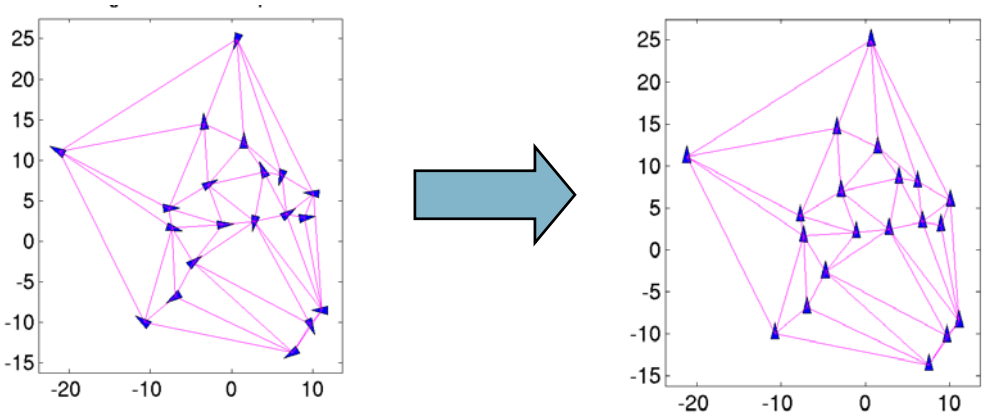


Formation control

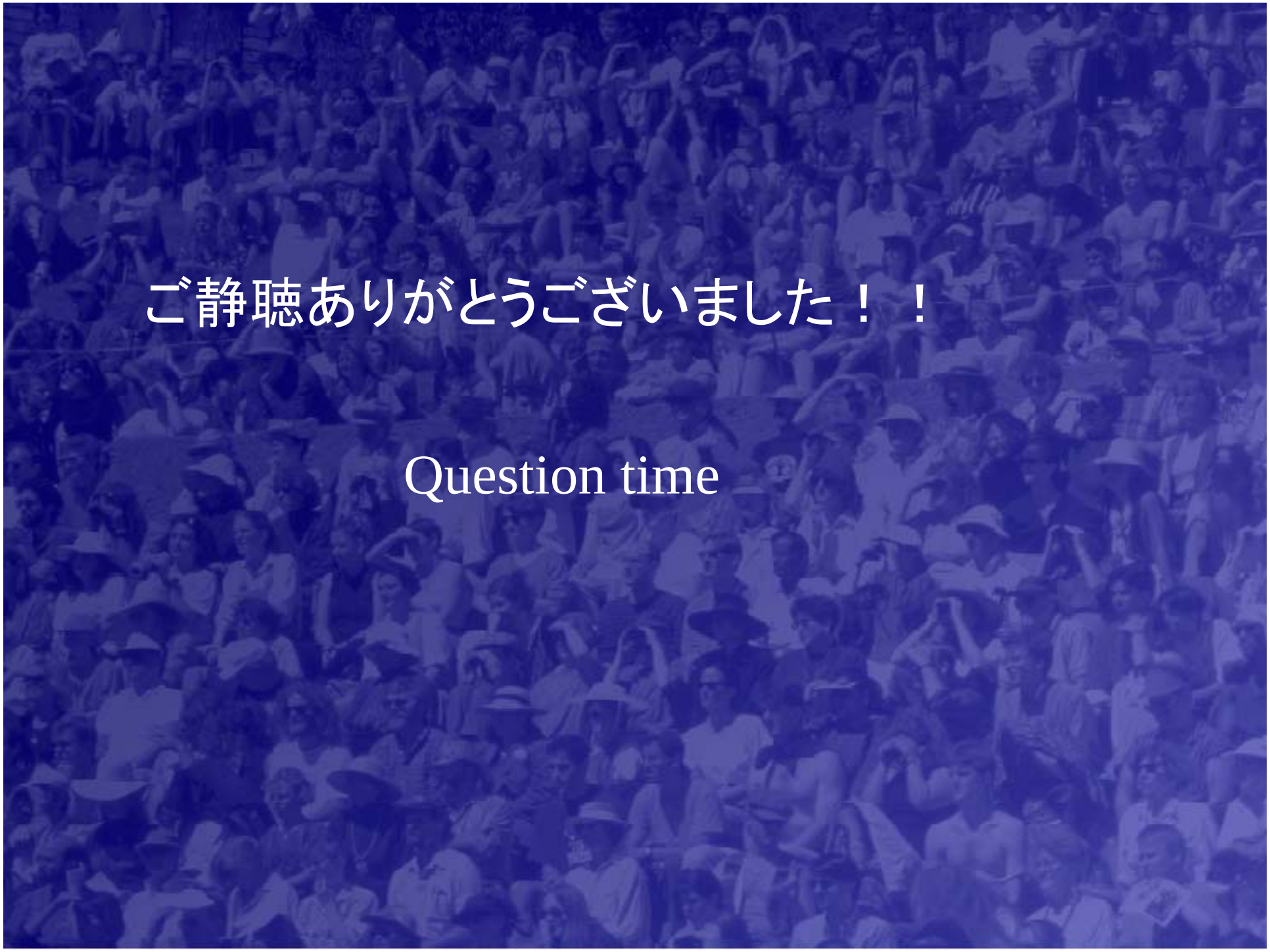


Formation switching using balanced graphs

Control of leader-based networks



Alignment for Collections of Vehicles via Leader



ご静聴ありがとうございました！！

Question time