Agent-based Modeling for the Study of Diffusion Dynamics

Akira Namatame
National Defense Academy of Japan
www.nda.ac.jp/~nama

Interdisciplinary Study of Diffusion Process

- Concepts of “diffusion and contagion”arise quite generally in biological and social sciences
  - Spread of infectious disease
  - Diffusion of innovations
  - Rumor spreading
  - Transmission of financial distress
  - Growth of cultural fads
  - Emergence of collective beliefs
- We would like to understand in what sense these different kinds of diffusion are the same and how they are different

Diffusion and Contagion

- **Question 1: What conditions trigger the decision to adopt something?**
- **Question 2: Are individuals more influenced by their beliefs or are they more influenced by the adoption behavior of their partners or the social trends?**

Which items to buy?

No purchase

Slow Pace of Fast Change

- **Question 3: Why the markets occasionally accept innovations rather slowly compared with the superior technological advances of the innovation?**

  "The slow pace of the fast change" (B. Chakravorti, 2003)

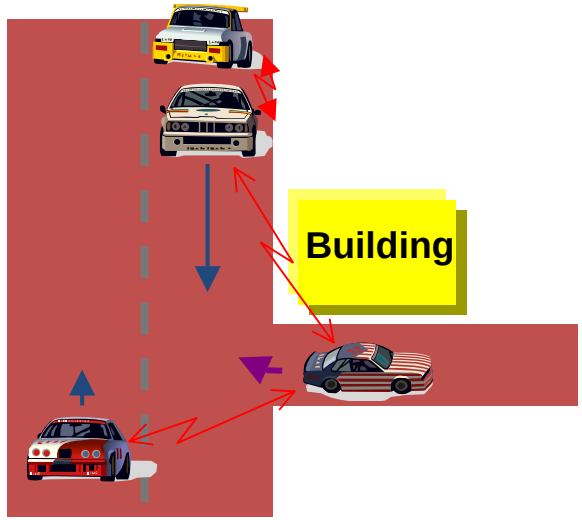
- Social benefits can be obtained only after full popularization
- Many IT technology developments and trials

Innovations

: Electronic money

: Car communication: safety at intersection, coordinate driving

E-money



Car communication

Outline

- **A Survey on the Study of Diffusion Dynamics**
  : Literatures on Contagion and Innovation
- **Sequential Decisions with Social Influence**
  : Agent Model Description
  : Simulation Results on Diffusion Patterns
- **Diffusion Dynamics on Networks**
- **Evolutionary Design of Optimal Diffusion Networks**

Historical Data: Diffusion of Innovation 1

USA

It takes a dozen of years to diffusion of household goods!!

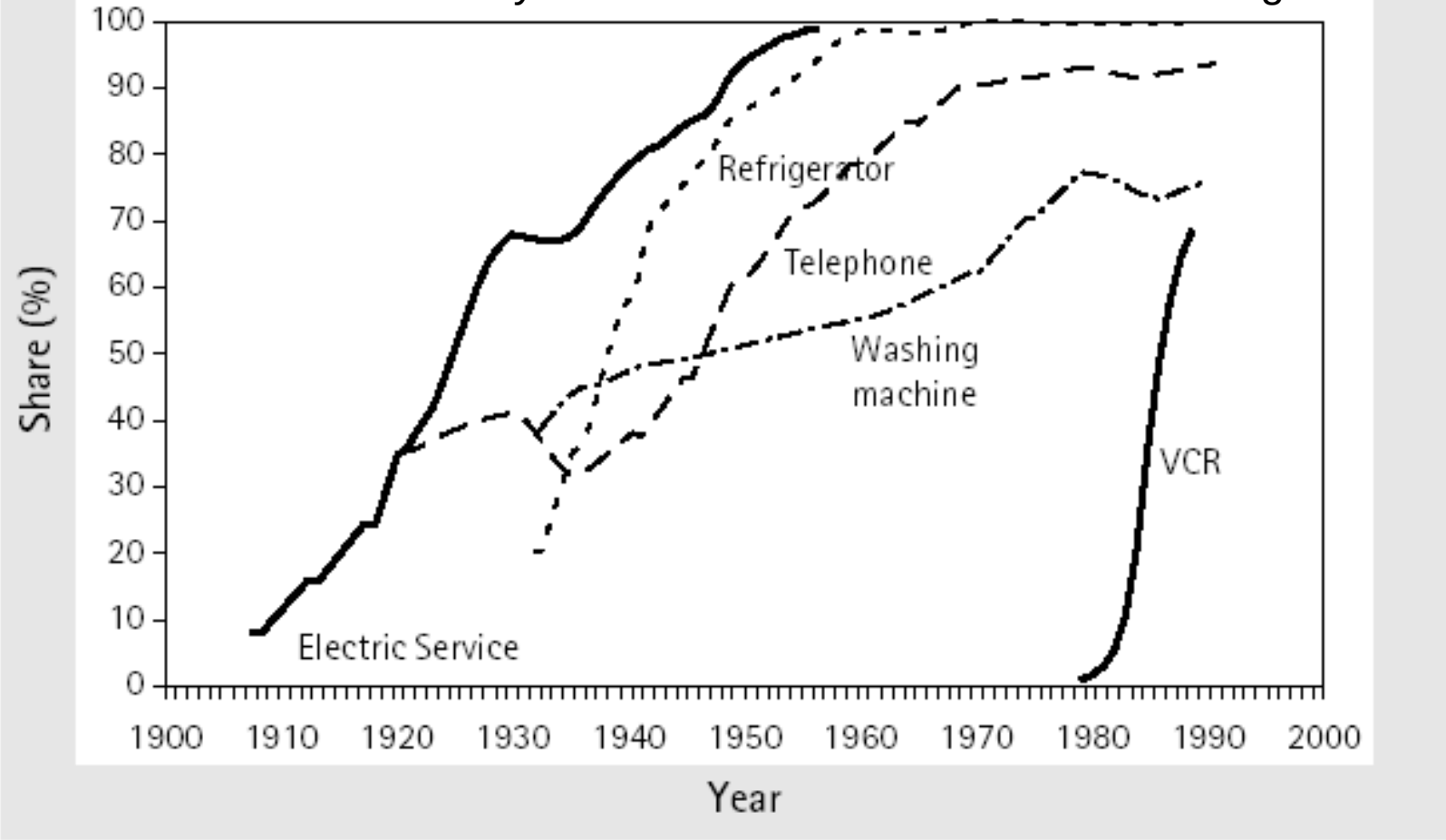


Figure 17.2 Diffusion of major innovations in the United States

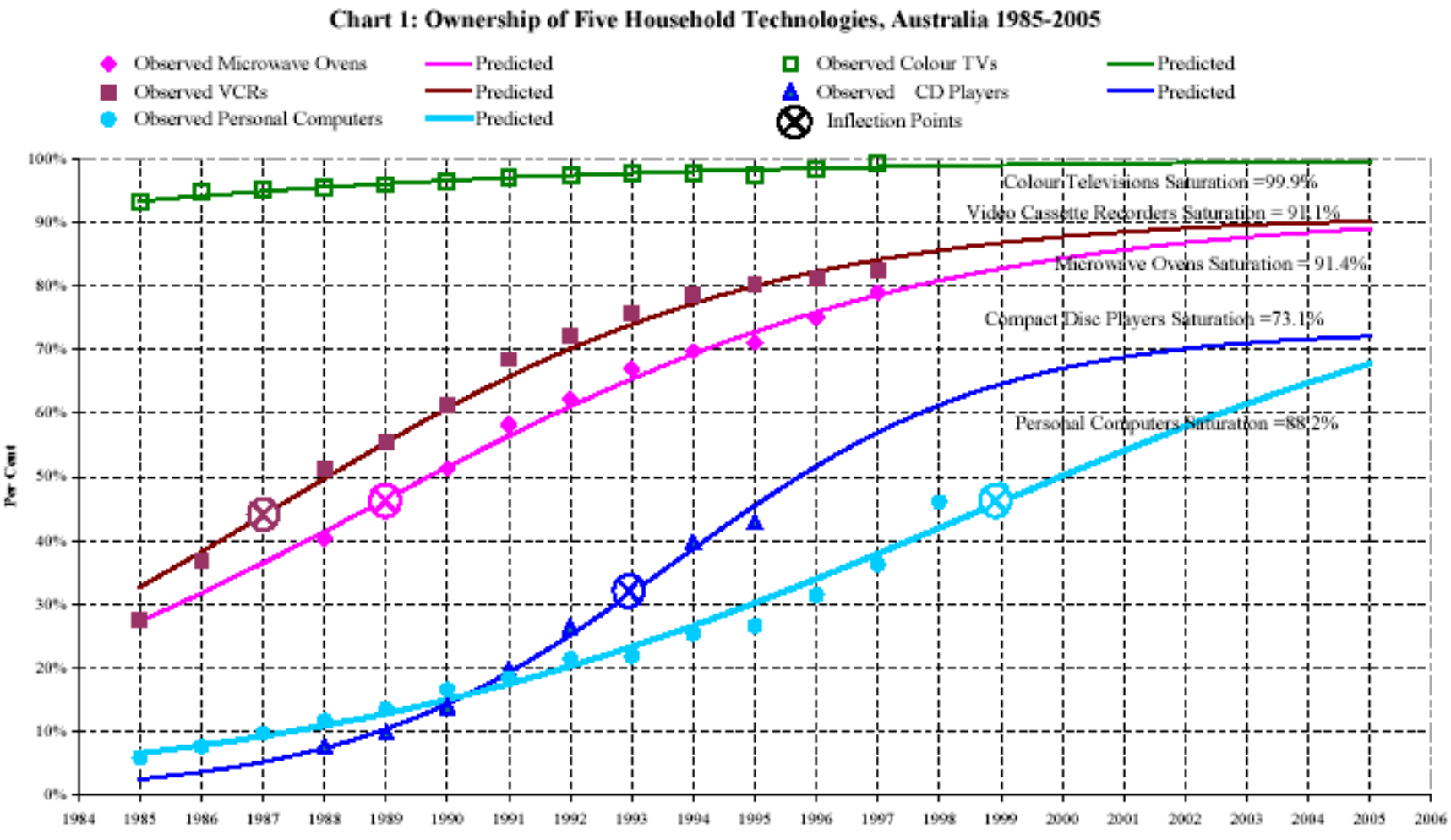
Source: Dallas Federal Reserve Bank.

SOURCE: Bronwyn H. Hall. 2004. "Innovation and Diffusion." Oxford Handbook of Innovation, Oxford University.

Historical Data: Diffusion of Innovation 2

Australia

Diffusion processes are very slow than we expect

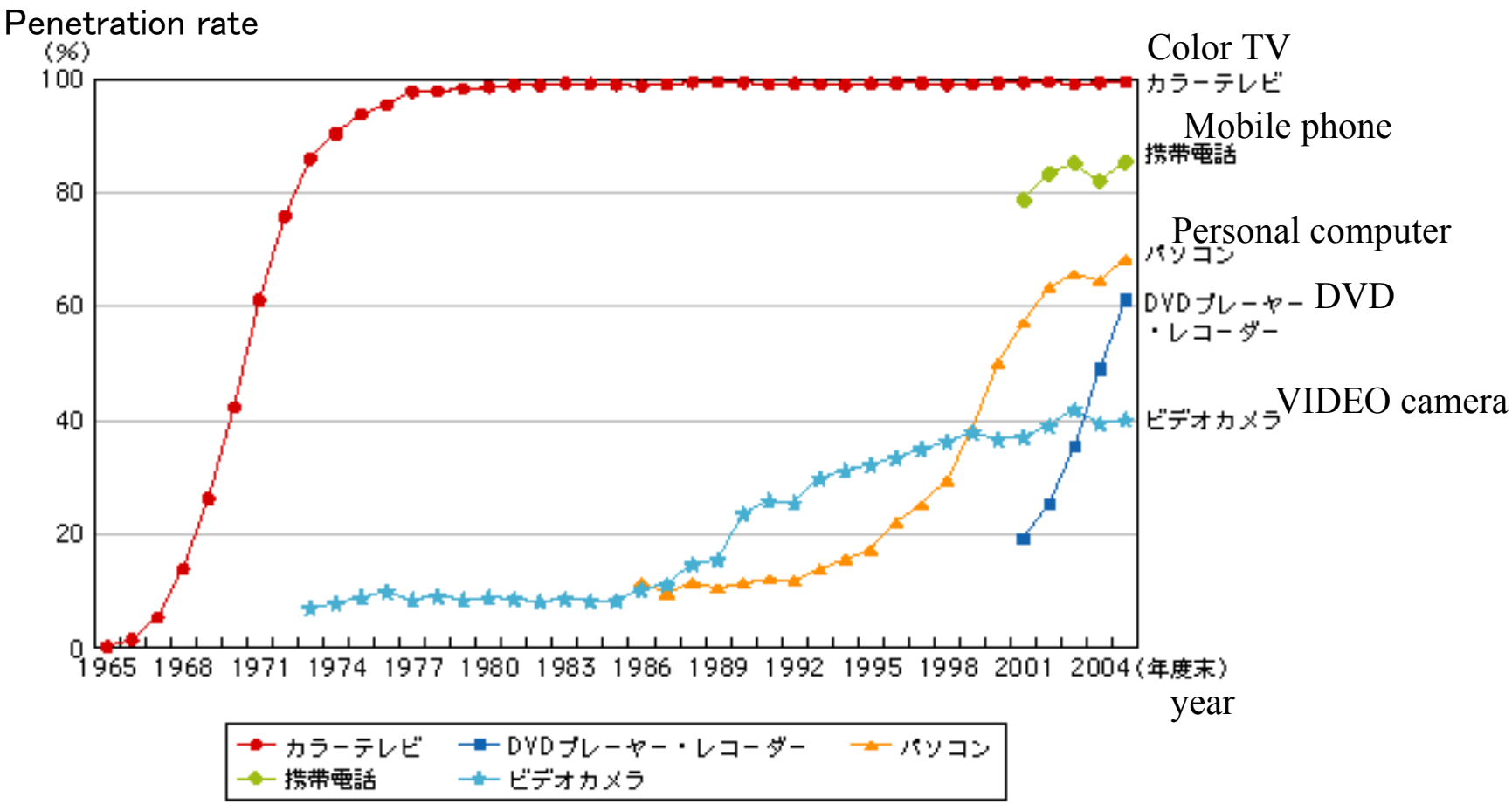


SOURCE: D.S. Ironmonger, C.W. Lloyd-Smith and F. Soupourmas. "New Products of the 80s and 90s: The Diffusion of Household Technology in the Decade 1985-1995." University of Melbourne.

Historical Data: Diffusion of Innovation 3

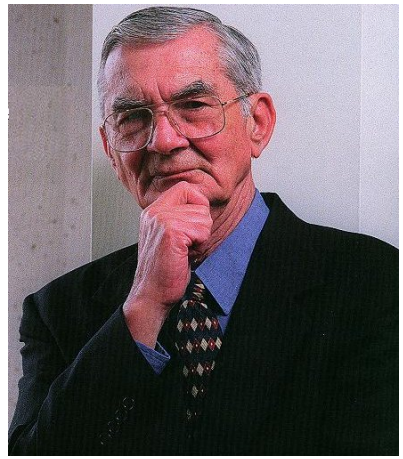
Japan

Diffusion curves have different shapes



内閣府経済社会総合研究所「消費動向調査」により作成

Innovation Diffusion: Bass Model

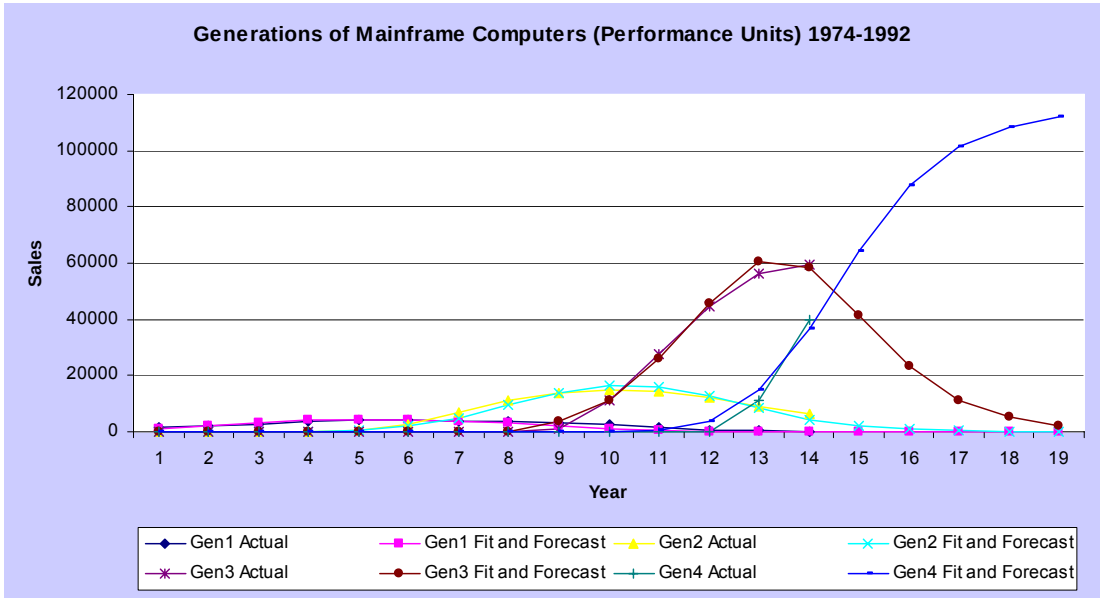


- $f(t)=(p+qF(t))[1-F(t)]$: Hazard Model
- $f(t)$: the rate of the adoption (growth rate)
- $F(t)$: cumulative proportion of adoption
- p=coefficient of innovation
- q=coefficient of imitation $f(t) = [p+qF(t)]\,[(1-F(t)]$

external effect (advertising)

individuals who already adopted

individuals who are unaware



Special Cases:

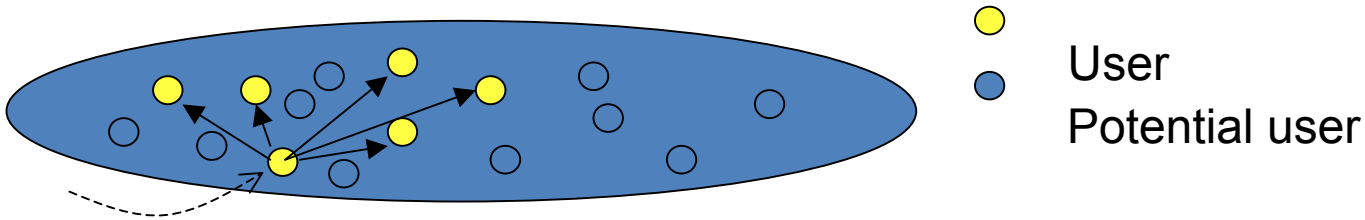
$q=0$: Exponential Distribution

$p=0$: Logistic Distribution,

A Rohlfs' Diffusion Model

Network effect model

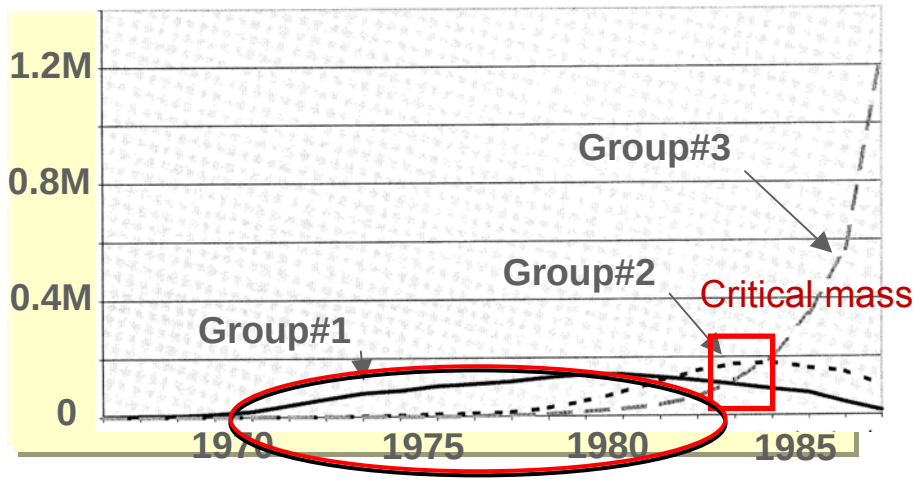
More usage of a product by any user increases the product's value for other users



- High-tech and IT products, systems, and services have this property.
  - Critical mass is important.

    If the diffusion reaches that point , diffuses massively

New user Installed base of facsimile machine in North America



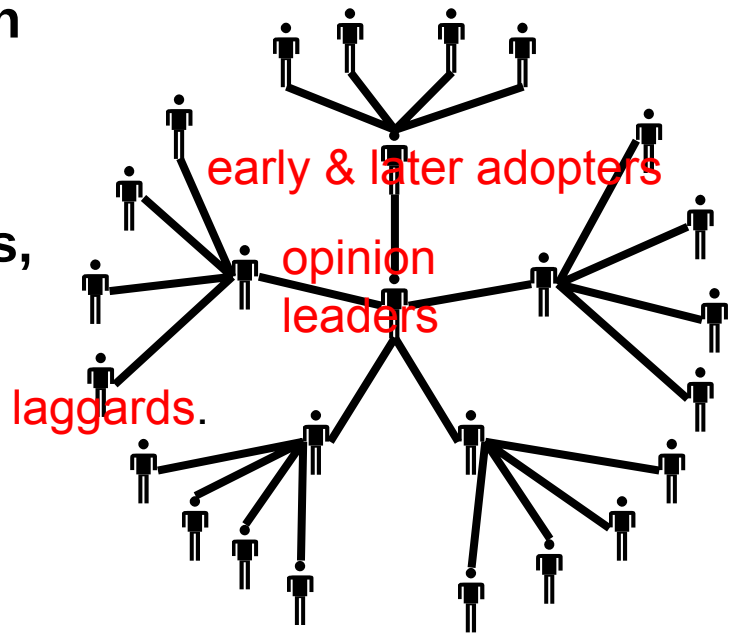
Innovation Diffusion via Networks

(M. Roger, 1995)

Two steps in the transmission of information (media → influentials → others)

Two kinds of individuals :
average individuals : most of the population
influentials individuals : opinion leaders

Transferring new knowledge from creators to users involves their network connections, which diffuse information in two-step flows from opinion leaders to early & later adopters, then to laggards.



Consumer Classification in Diffusion Process

Bass model and S-shape function:

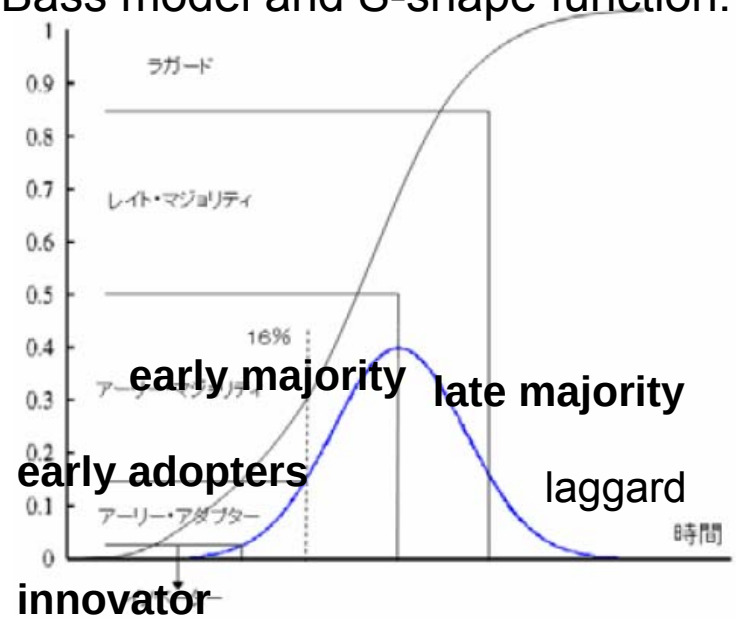


図1　ベルカーブとS字カーブ

Penetration rate	Types of consumers
2.5%	innovator
16%	early adopters
50%	early majority
84%	late majority
100%	laggard

Innovators are the first 2.5 percent of the individuals in a system to adopt an innovation and play a gate keeping role in the flow of new ideas into a system.

Early adopters are the next 13.5 percent of the individuals in a system to adopt an innovation.

Early majority is the next 34 percent of the individuals in a system to adopt an innovation.

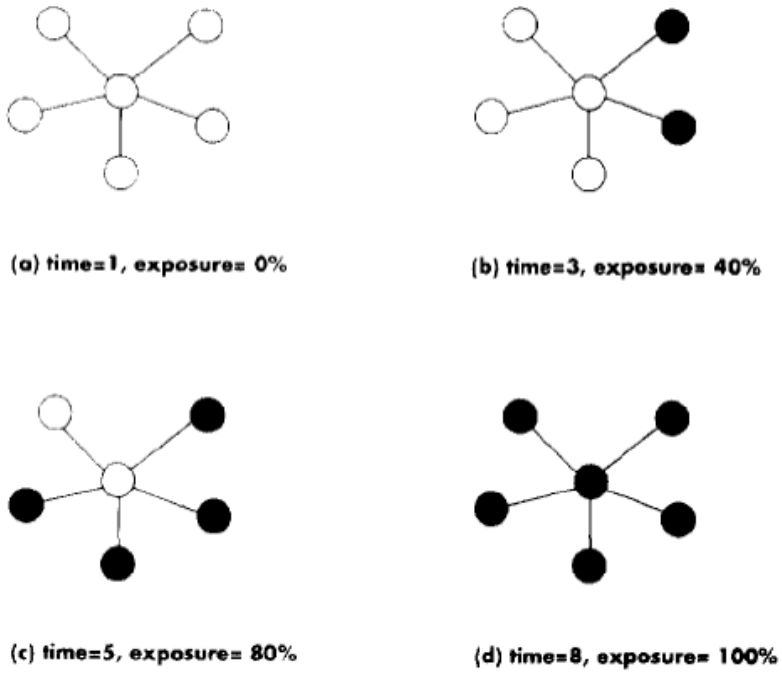
Late majority is the next 34 percent of the individuals in a system to adopt an innovation.

Laggards are the last 16 percent of the individuals in a system to adopt an innovation.

A Network Threshold Model

Tom Valente's (1996)

Network threshold diffusion model involves micro-macro effects & non-adopters' influence on adopter decisions. It assumes "behavioral contagion through direct network ties"

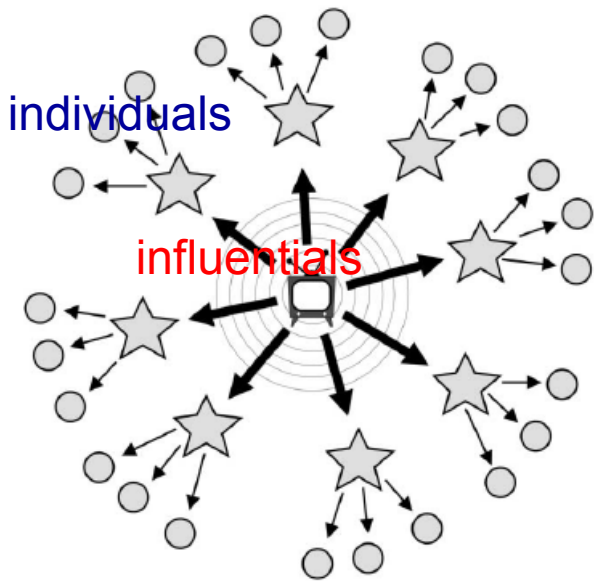


Two-step flow vs. Mutual influence

Two steps in the transmission of information

- Describe the flow of information from media to population and the formation of public opinion.
- (media → influentials → others)

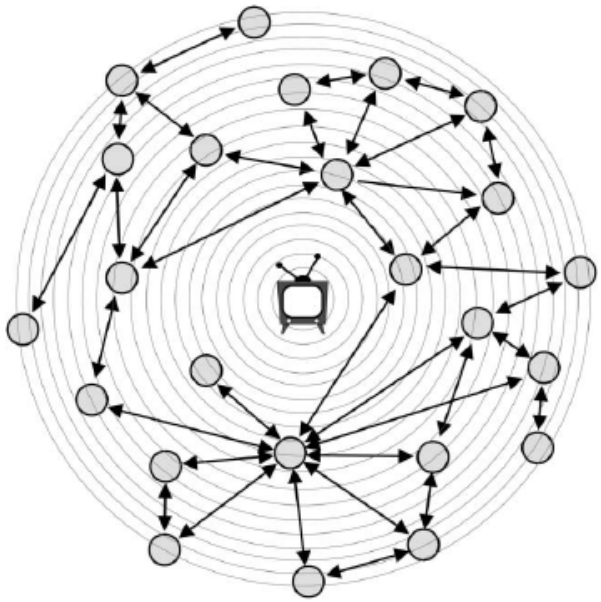
SCHEMATIC OF THE TWO-STEP FLOW MODEL OF INFLUENCE



Mutual influence network model

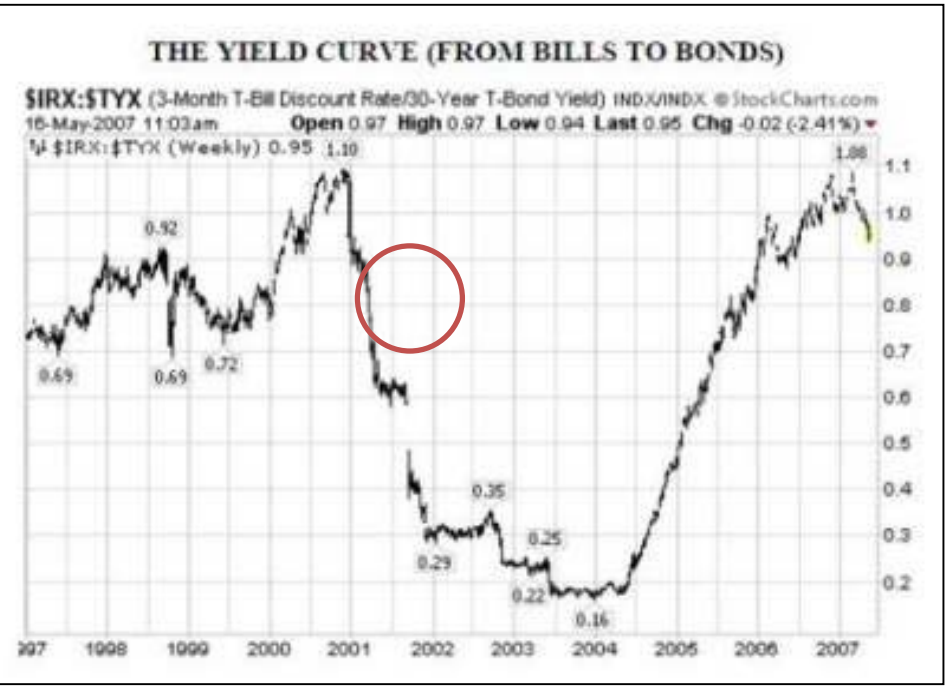
Average individuals become much more influent than influentials.

SCHEMATIC OF NETWORK MODEL OF INFLUENCE

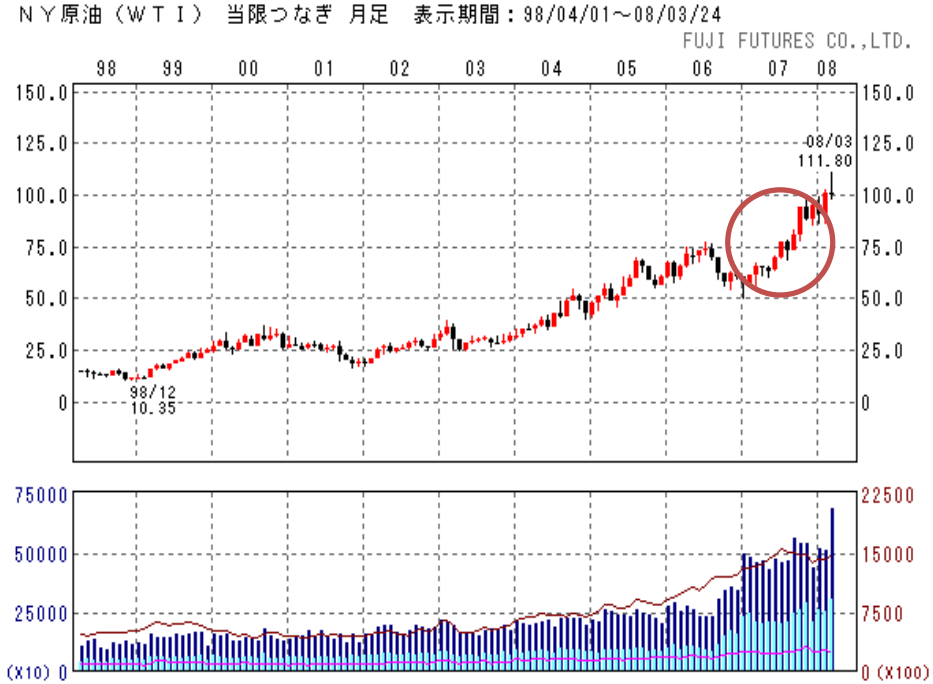


D. Watts (2007): Influentials, networks and public opinion formation, Journal of consumer research

Cascade in Economics



NY Stock Prices



NY Oil Prices

Definition of Cascade

Bikhchandani, Hirshleifer & Welch.
" A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades," J. of Political Economy 100:5,992-1026. (1998)

Definition: We say that there is cascade or herding if after some period T, everyone makes the same choice

Cascade :

: Customers are uncertain about the quality of a product
: Signals that other customers think the product is good; this makes it more likely customers will buy
: This trend produce a positive reinforcing cycle (cascade)

Mechanism behind cascade behavior
Individual rationality:
Bayesian rational learning

Price and Externality

Question 1: What conditions or behaviors trigger the decision to adopt something? price
Question 2: Are individuals more influenced by their beliefs or are they more influenced by the adoption behavior of their partner and the social trends? network externality

Economists' point of view: price and externalities

Which items to buy?

Binary Choices with Externalies

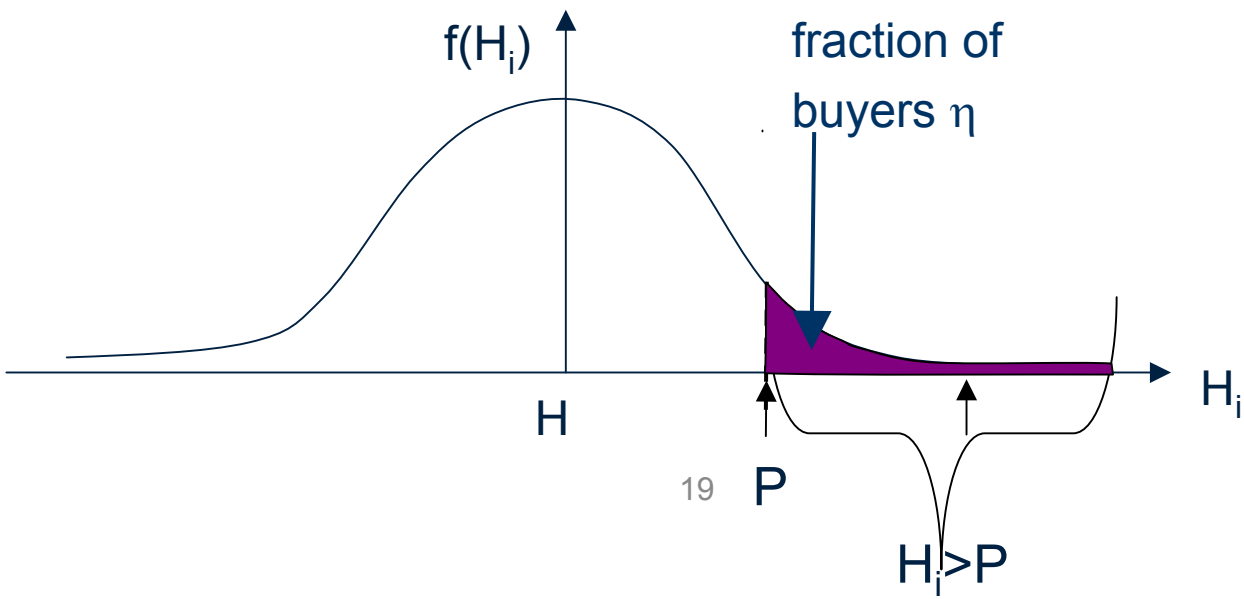
- A general model (D. Phan, 2004)
  - N potential customers (i=1,2,…,N)
  - a single good at an exogenous price, P
  - choice: to buy or not $\omega_i = 1$ $\omega_i = 0$
- idiosyncratic willingness to pay (IWP) or reservation price of agent i: H_i
  - distribution $f(H_i)$ of mean H and variance σ
- externalities:
  - the utility of buying the good is proportional to the number of buyers
  - Reservation price of agent i = H_i +J x fraction of buyers

$$\frac{\#\ \text{buyers}}{N} \equiv \eta$$

Analysis of Collective Decision

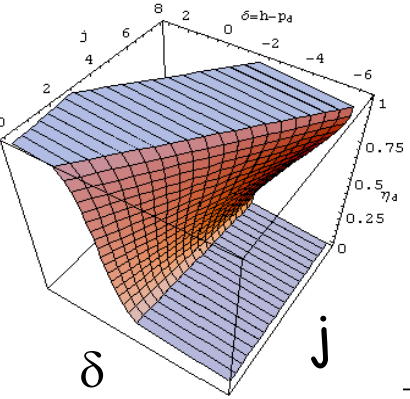
- individuals maximize their surpluses: $S_i \equiv H_i + d\eta - P$
  - buy if : $H_i + d\eta > P$
  - do not buy if: $H_i + d\eta < P$
- buyers: fraction of agents $\eta = \int_P^\infty f(H_i)\,dH_i$

$$f(H_i) = \delta(H_i - H) \Rightarrow \begin{cases} \text{if } H < P & \Rightarrow \omega_i = 0 \quad \forall i \\ \text{if } H > P & \Rightarrow \omega_i = 1 \quad \forall i \end{cases}$$



Phase Transition in Demand Functions

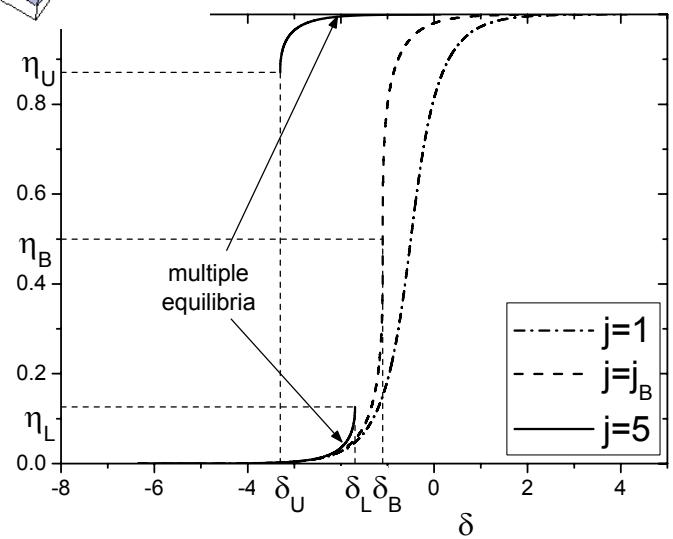
- demand functions with normalized parameters

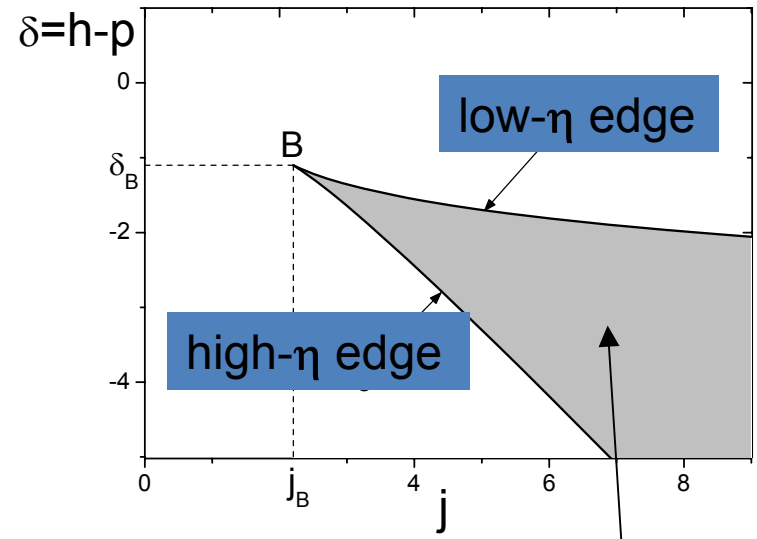


J=0 (no social influence)

$$\delta \equiv \frac{H - P}{\sigma} \; ; \quad j = \frac{J}{\sigma}$$

logistic distribution of the IWP





increasing δ = H-P

demand gaps

Collective Decision on Networks

S. Chen and L. Sun (2006)

$$\mathbf{x}(t+1) = F(H + DA\mathbf{x}(t) - P)$$

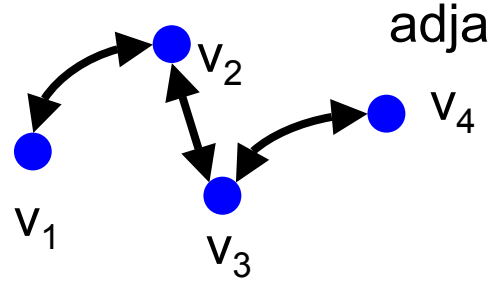
$$f(H_i) \Rightarrow \begin{cases} 0 if \quad H < P \quad \Rightarrow \omega_i = 0 \quad \forall i \\ 1 if \quad H > P \quad \Rightarrow \omega_i = 1 \quad \forall i \end{cases}$$

Adjacent matrix

$$A = \begin{pmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{pmatrix} \qquad D = \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{pmatrix}$$

$a_{ij} =$ 1: i and j are connected
0: not connected

$$H = \begin{pmatrix} H_1 \\ \vdots \\ H_n \end{pmatrix} \qquad P = \begin{pmatrix} p_1 \\ \vdots \\ p_n \end{pmatrix}$$



adjacent matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Consumer Networks and Demand Functions

S. Chen and L. Sun (2006)

Penetration rates under different consumer networks

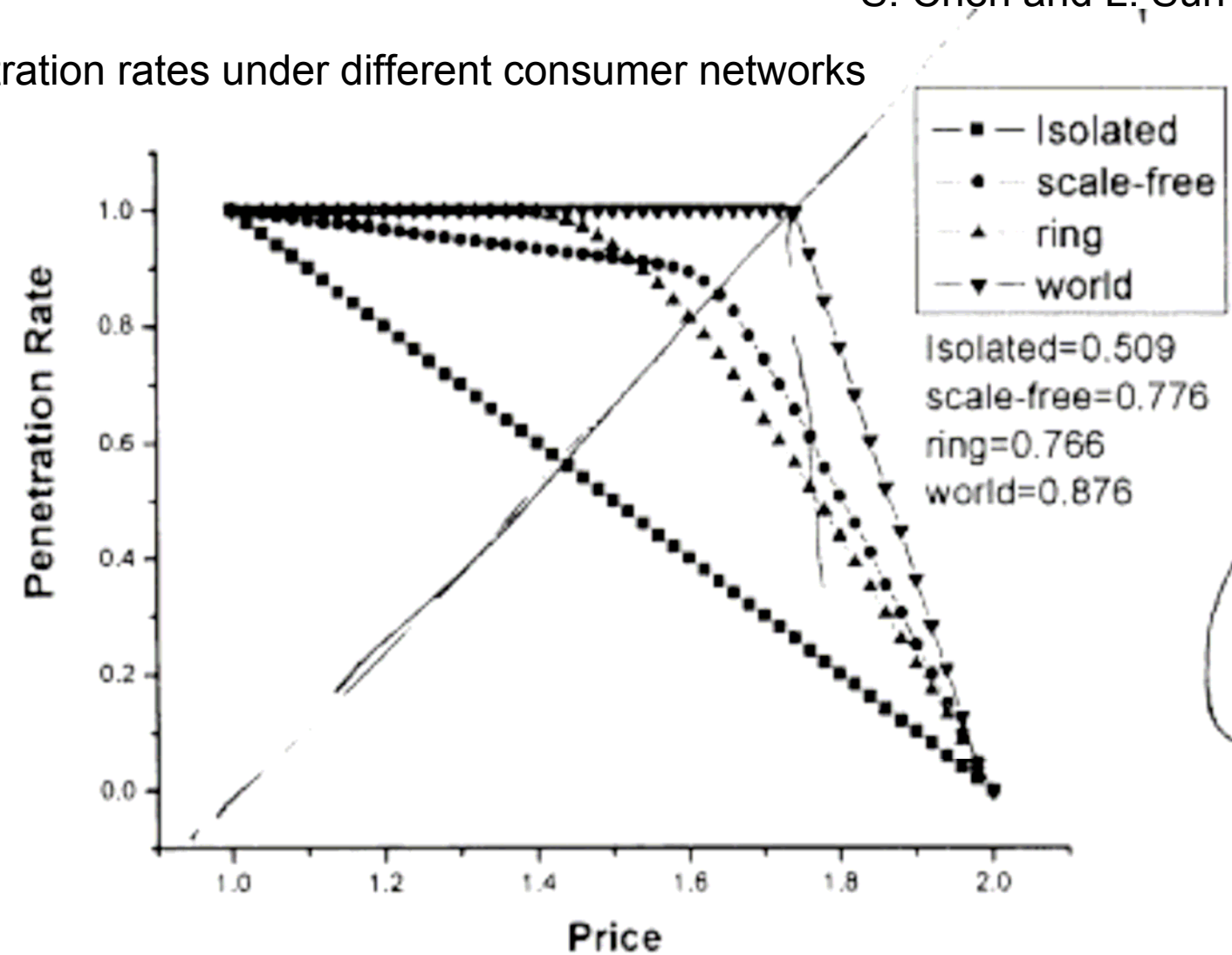
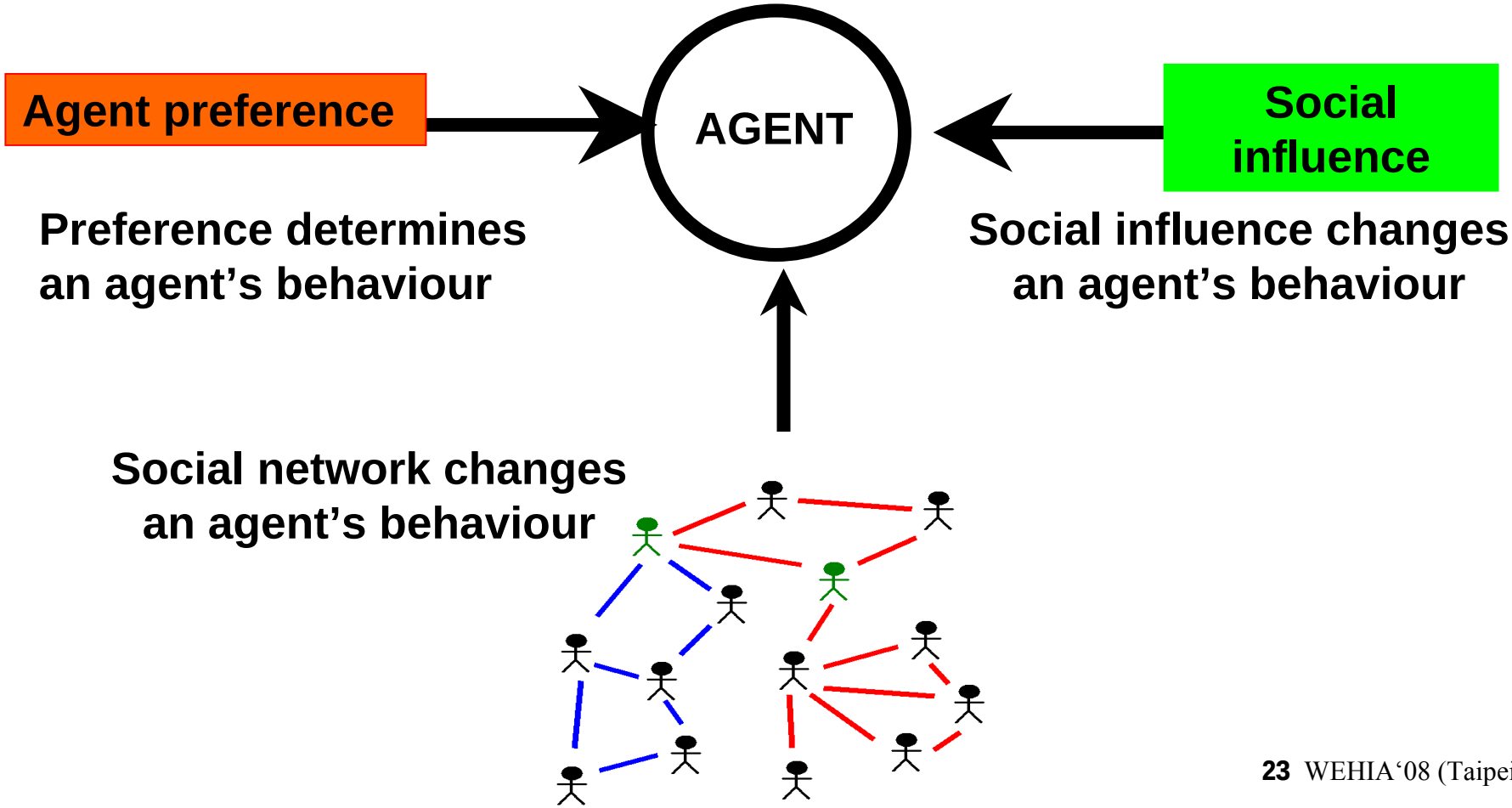


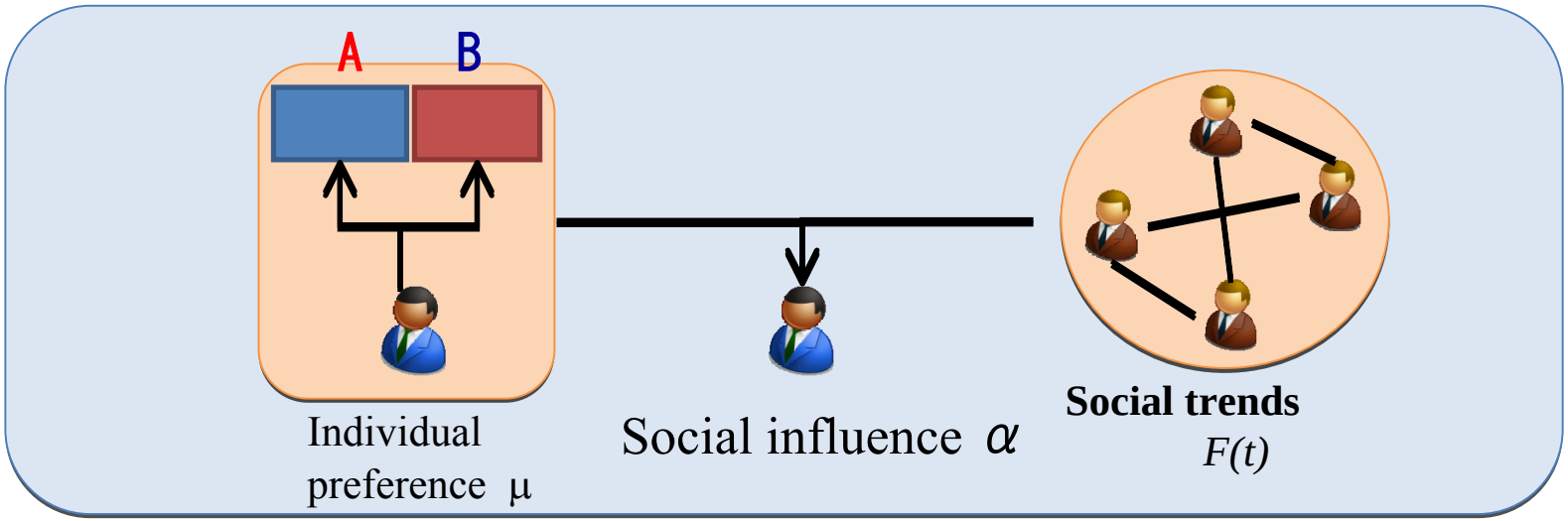
Fig. 1. Demand Curve in Various Networks

A Unified Agent Model

(1) Preference heterogeneity
(2) Social influence
(3) Social Network: Degree heterogeneity



Agent Decision with Social Influence



Probability of choosing A at time t

$$p(t+1) = (1-\alpha)\mu + \alpha F(t)$$

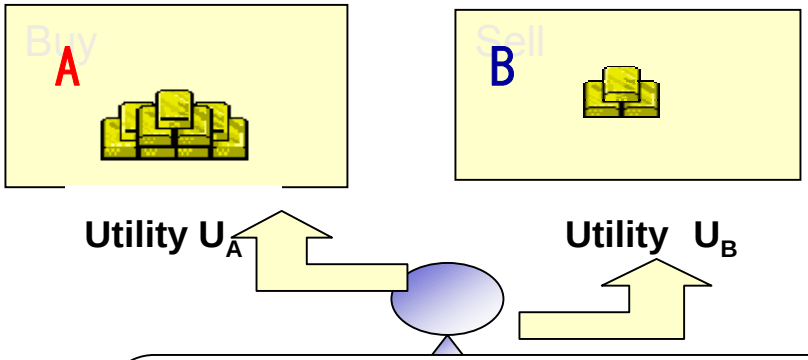
preference **social trend**

$$F(t) = A(t)/\{A(t)+B(t)\}$$

F(t): the proportion of the agents to choose A

$\alpha \in [0, 1]$: social influence factor

Heterogeneity in Agent Preference



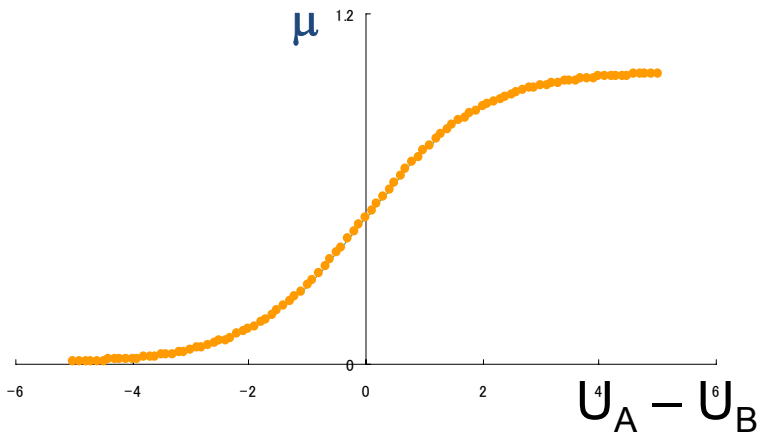
- **Preference of A**

$$\mu = 1 / (1+\exp(-(U_1 - U_2)))$$

- **Preference of B**

$$1 - \mu = 1 / (1+\exp(-(U_2 - U_1)))$$

Heterogeneity in binary choice: Logit model



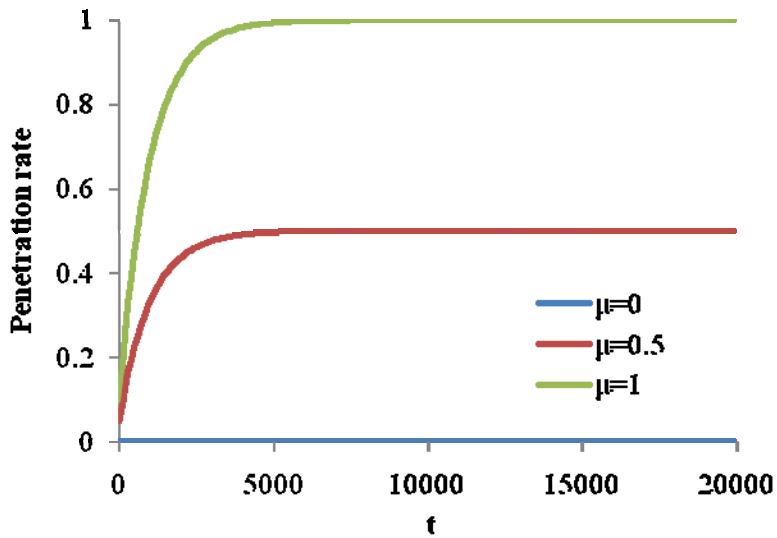
Probability to buy: p_1

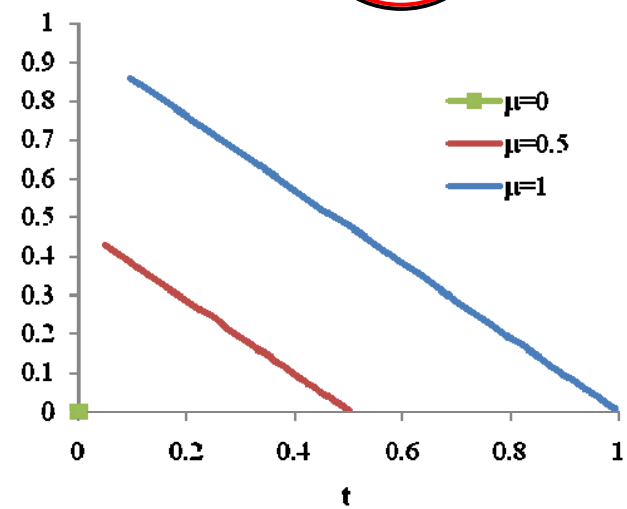
Experiment 1: without social influence

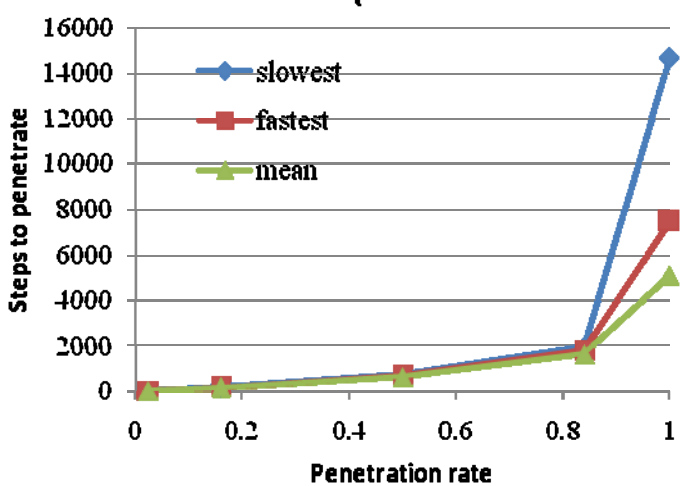
α = 0: no social influence

μ : the ratio of agents to have preference of buying

$$p(t+1) = (1-\alpha)\mu + \alpha F(t)$$







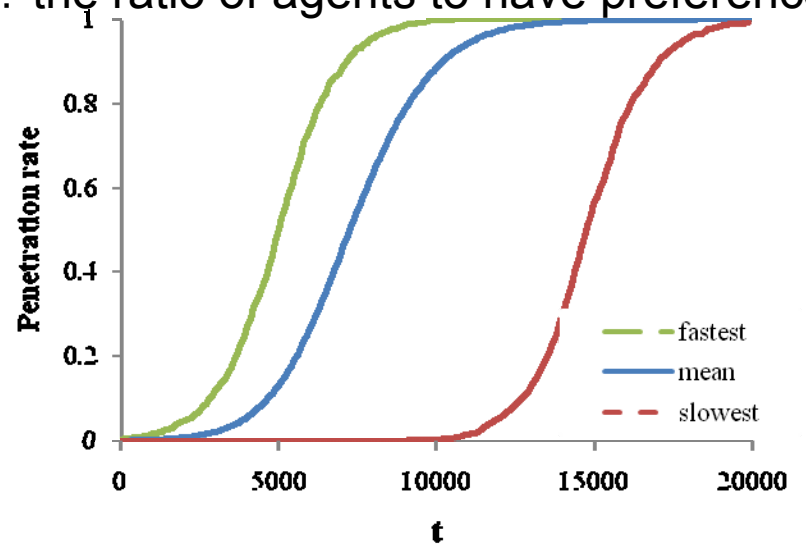
- Agents who have preference
- Change of penetration ratio linearly declines

Experiment 2: strong social influence (1)

α = 1: full social influence

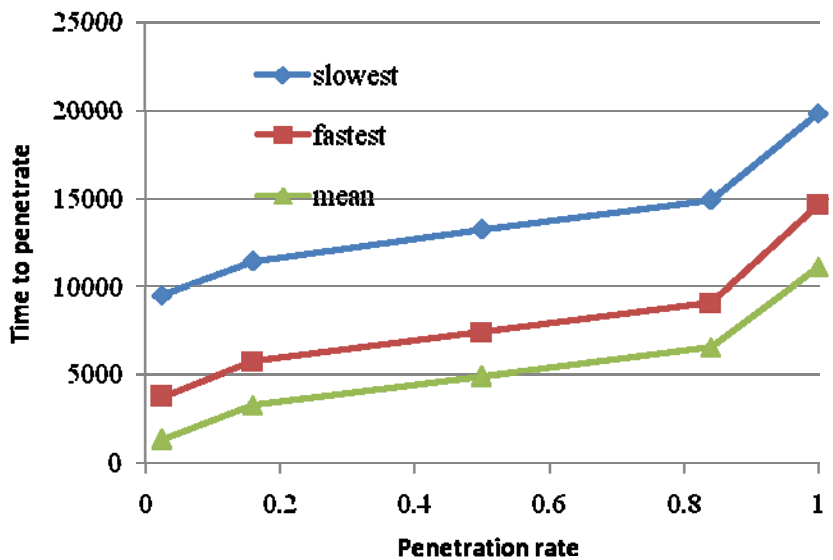
μ : the ratio of agents to have preference of buying

$$p(t+1) = (1-\alpha)\mu + \alpha F(t)$$



Diffusion follows S-shape process

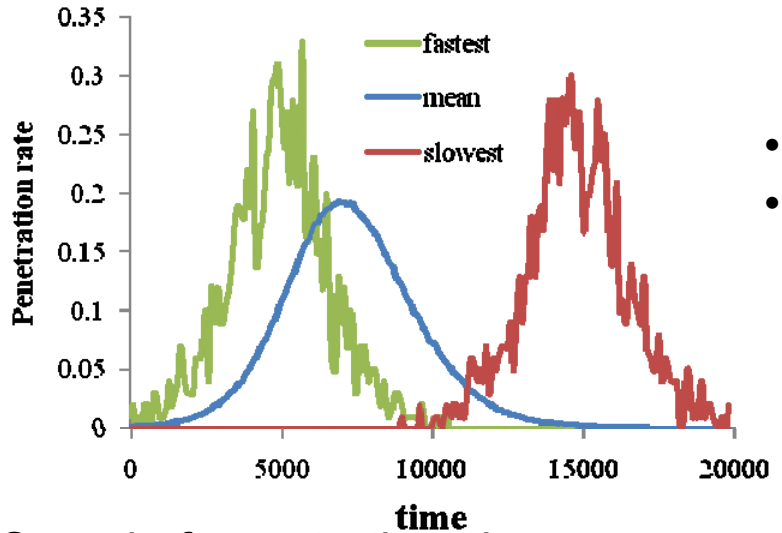
When agents make decisions with strong social influence, the aggregate behavior follows S-shape function:



- Diffusion depend on the success at the initial stage

Experiment 2: strong social influence (2)

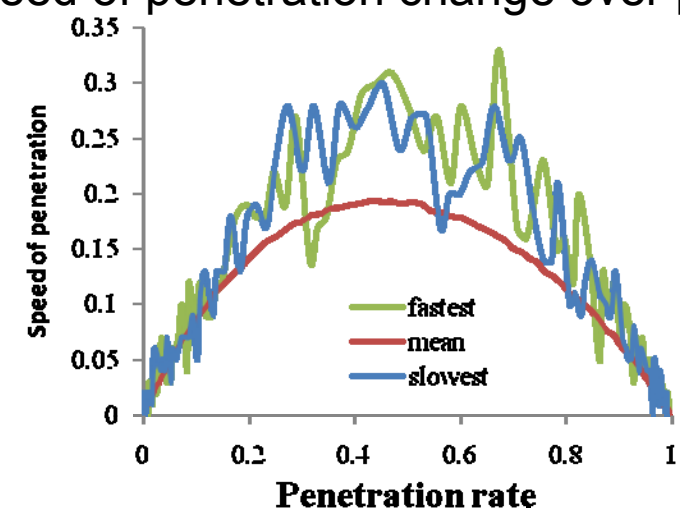
Speed of penetration rate over time



•Change of penetration ratio over time

- r(t)=(p(t+1)-p(t)

Speed of penetration change over pnetration status



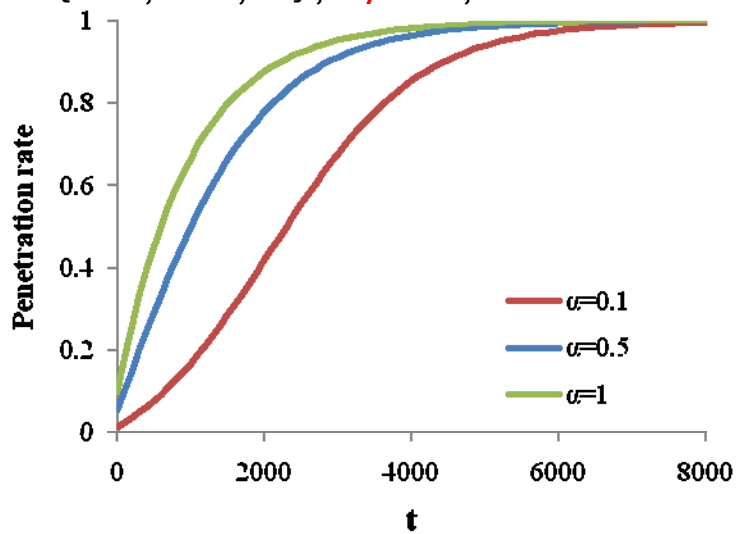
•Change of penetration ratio over penetration ratio

- r(t)=(p(t+1)-p(t)

Experiment 3: some social influence

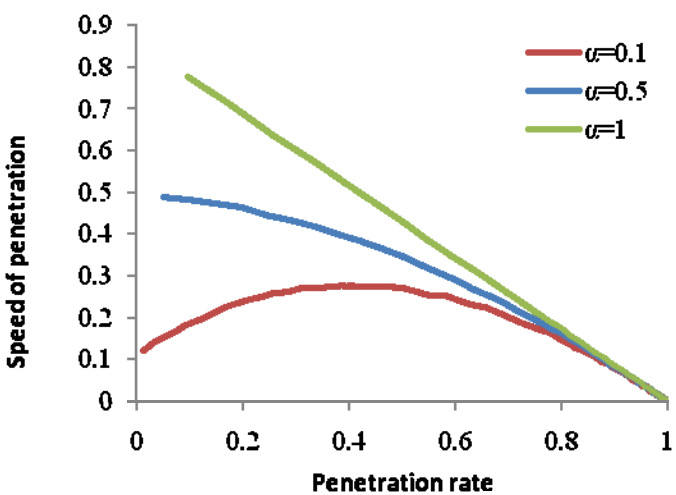
$(0 < \alpha < 1)$

α = {0.1, 0.5, 1}, μ =1,



$$p(t+1) = (1-\alpha)\mu + \alpha F(t)$$

- When consumers care about the social trend, the diffusion process becomes slow and it follows the S-function



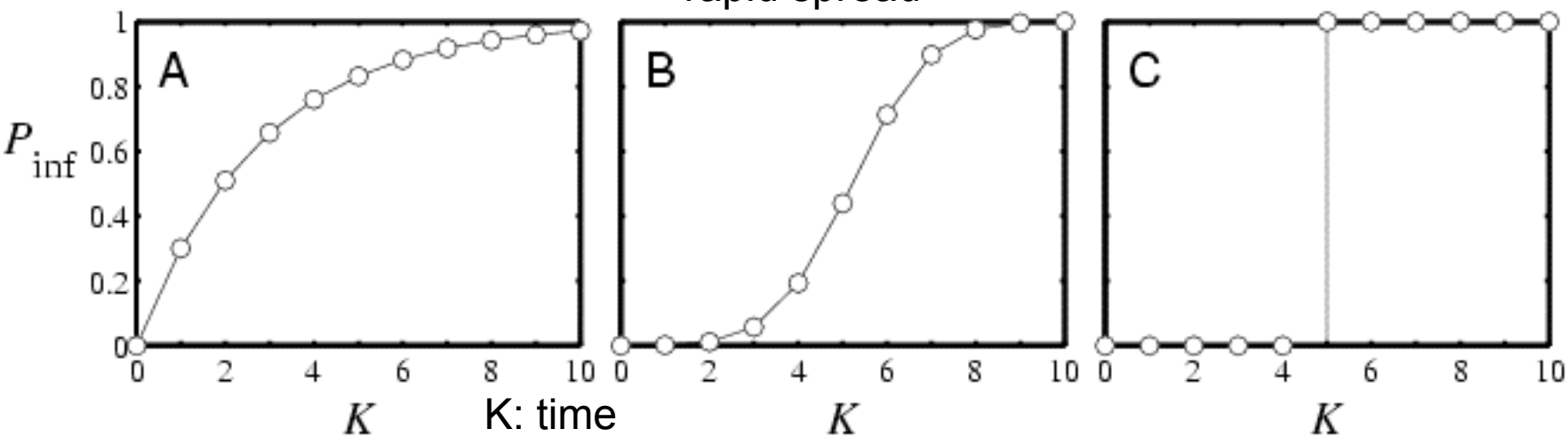
Summary:
Three Types of Diffusion Patterns

Macroscopic phenomena

Burst diffusion

Slow start then rapid spread

Diffusion with critical mass



K: time

Individual characteristic

very low social influence

$\alpha = 0$

mild social influence

$\alpha = 0.5$

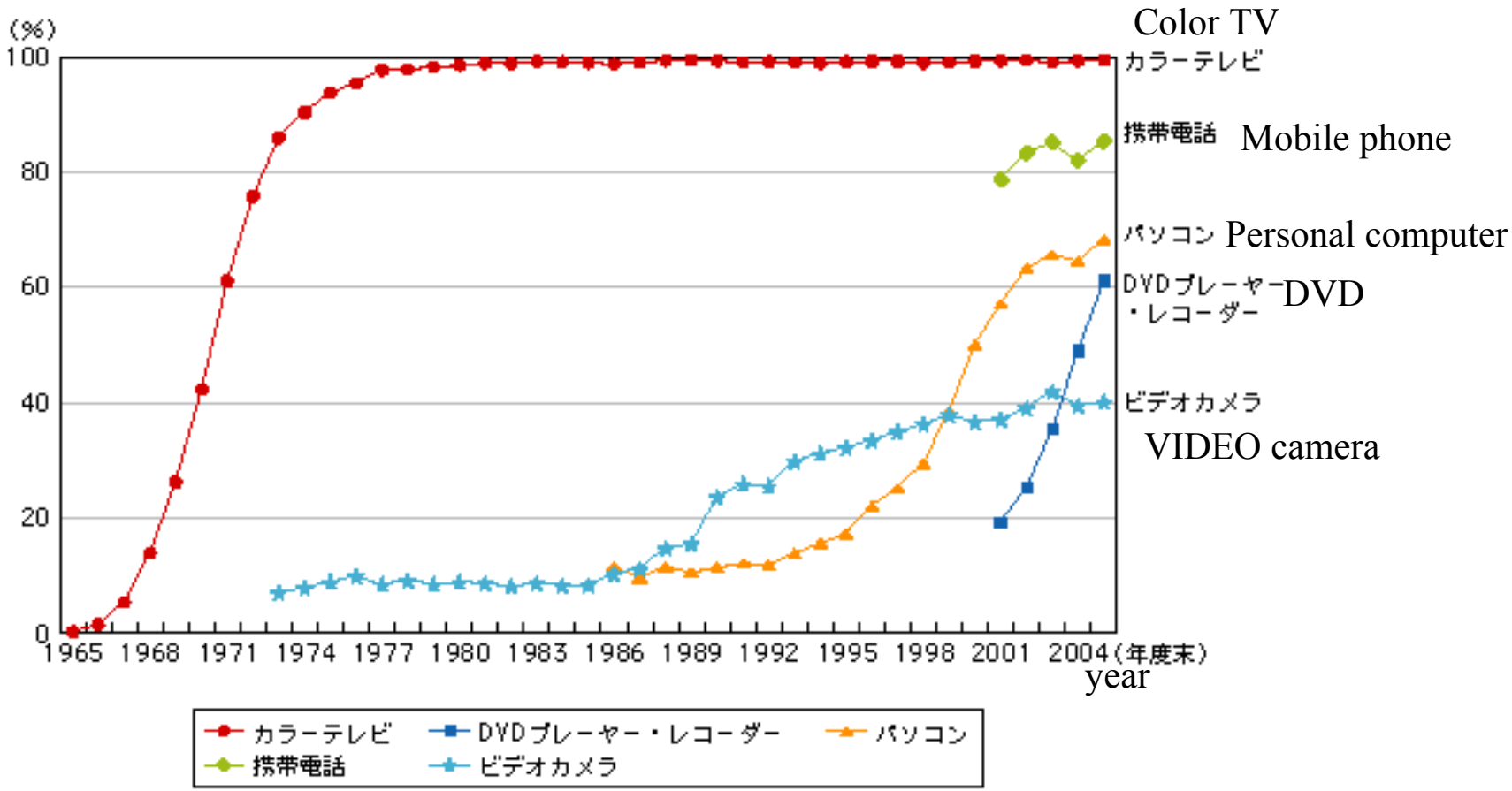
very strong social influence

$\alpha = 1$

$$p(t+1) = (1-\alpha)\mu + \alpha F(t)$$

Product Classification: Strong or Weak Social Influence

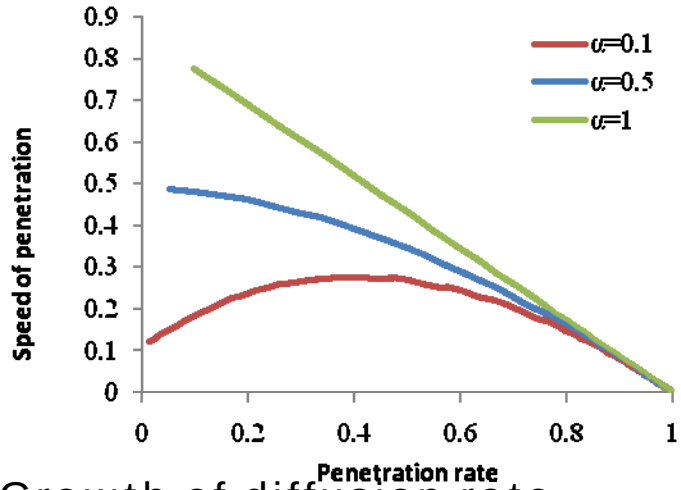
Japan



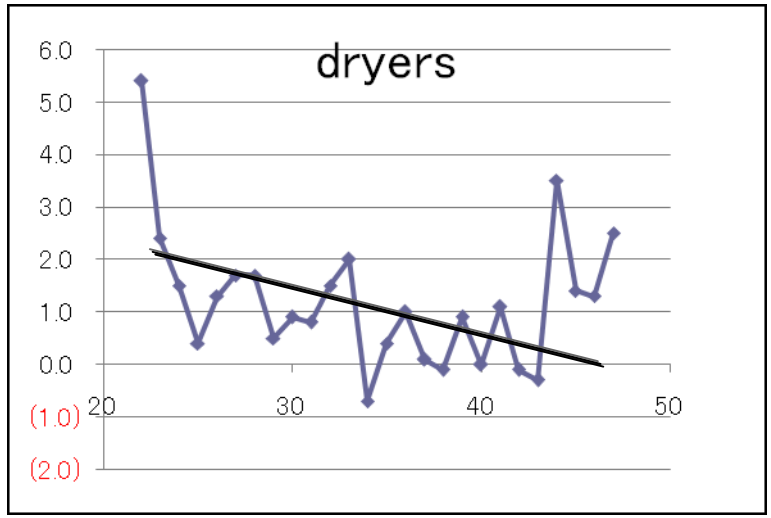
内閣府経済社会総合研究所「消費動向調査」により作成

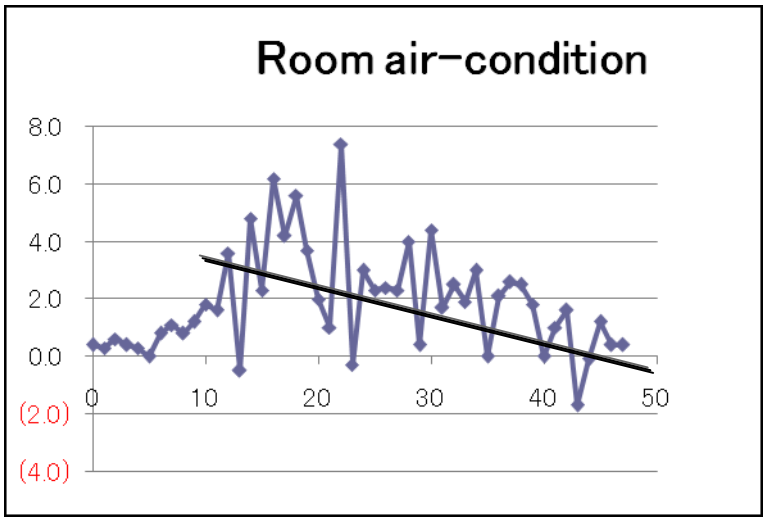
Products: Weak Social Influence

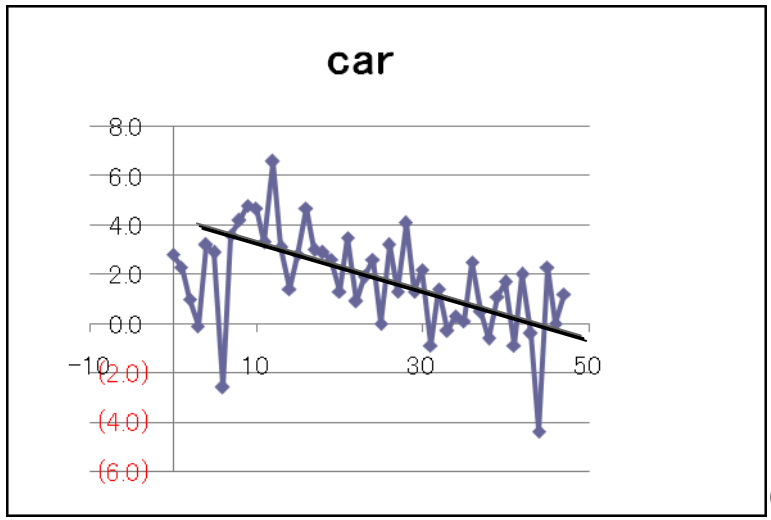
X-axis : year from 1960 to 2008
Y-axis : p(t+1)-p(t)



Growth of diffusion rate

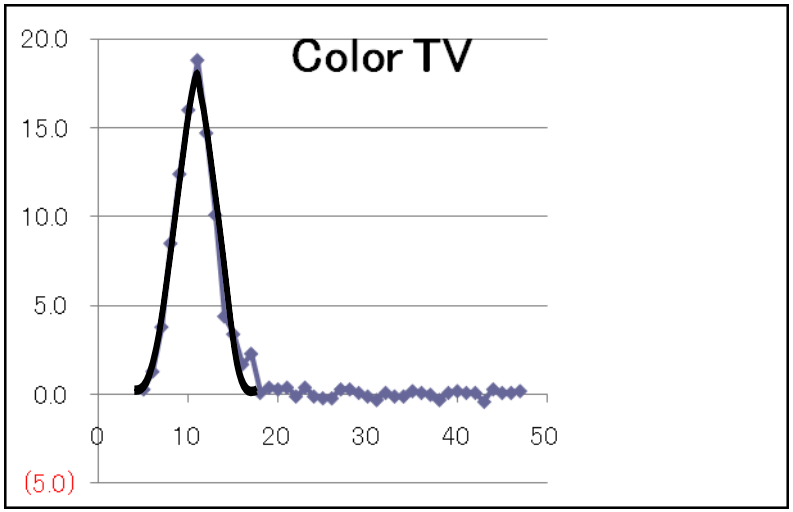


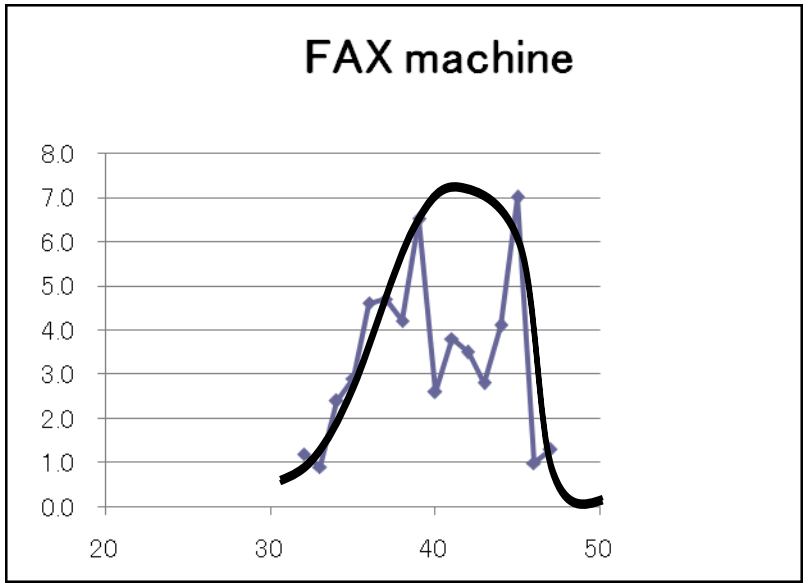




Products: Strong Social Influence

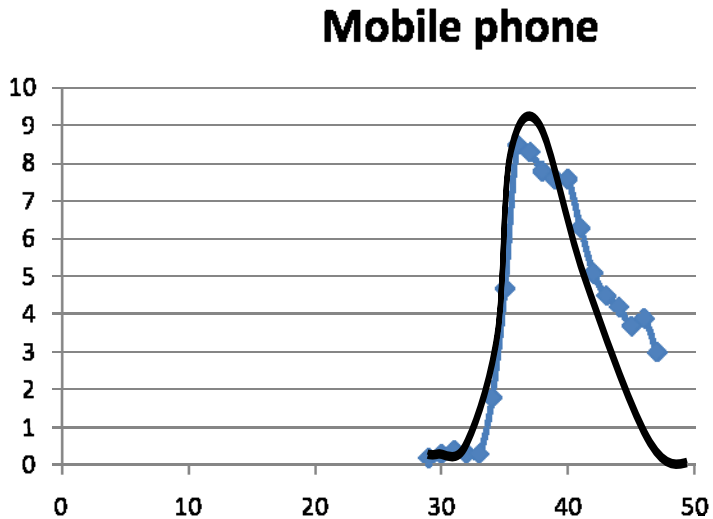
Growth of diffusion rate

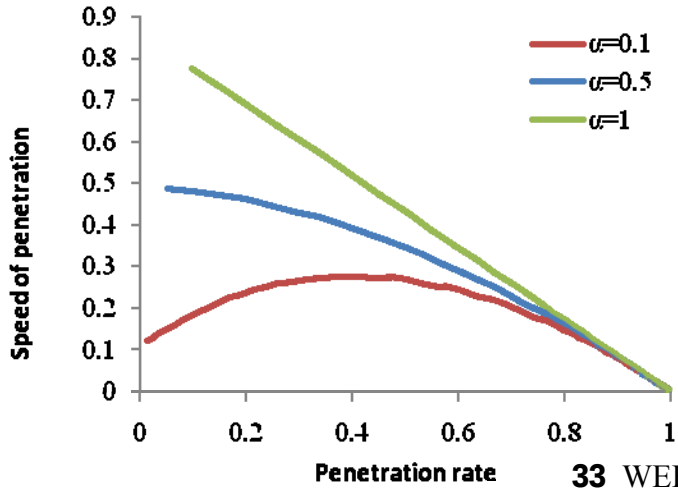




X-axis : year from 1960 to 2007

Y-axis : p(t+1)-p(t)

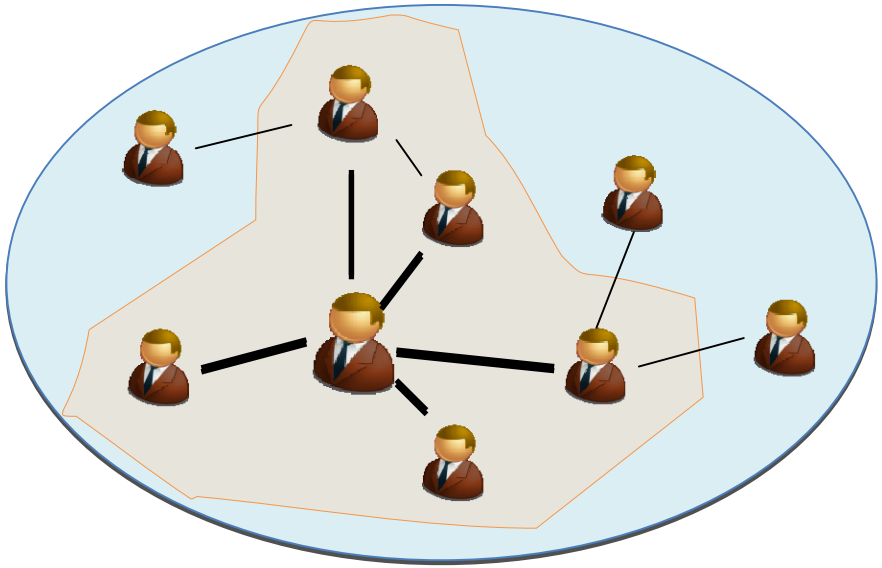


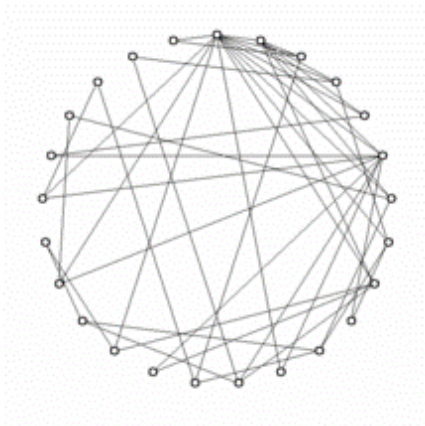


Social Influence via Networks

- Regular networks with the same degree:

- Scale-free networks with the average degree: <d>=d

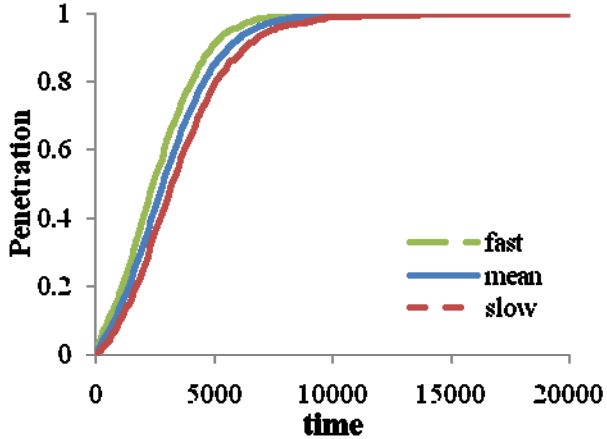




Experiment 4: Social Influence via Networks

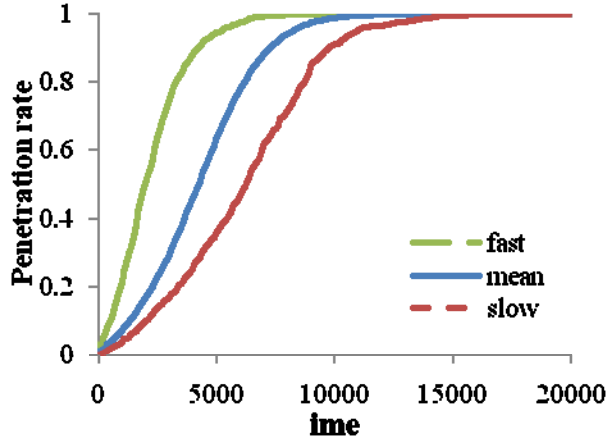
α =1: all agents receive strong social influences

(a) regular network



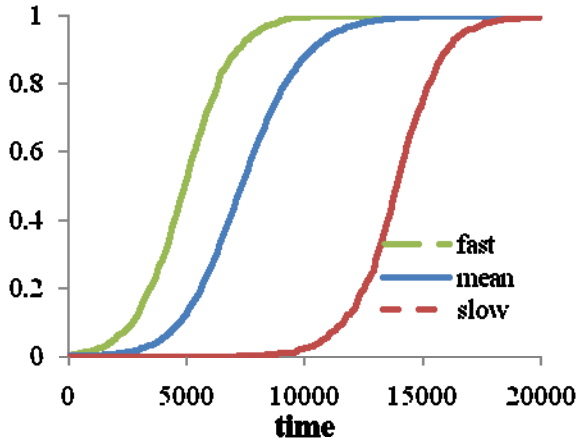
Burst diffusion

(b)scale Free network



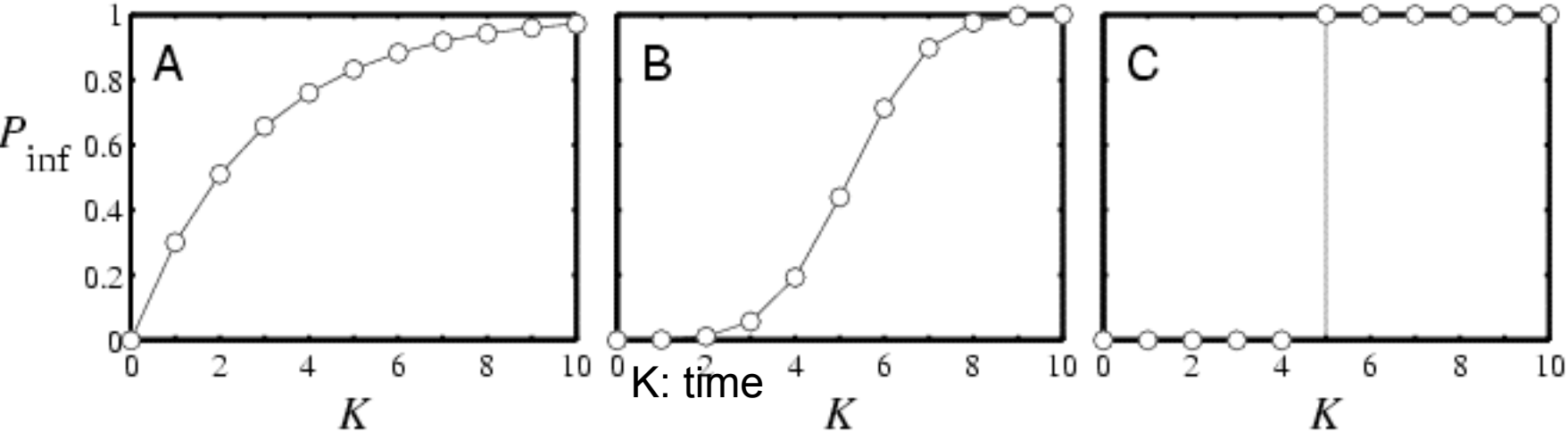
Slow start then rapid spread

(c) complete network



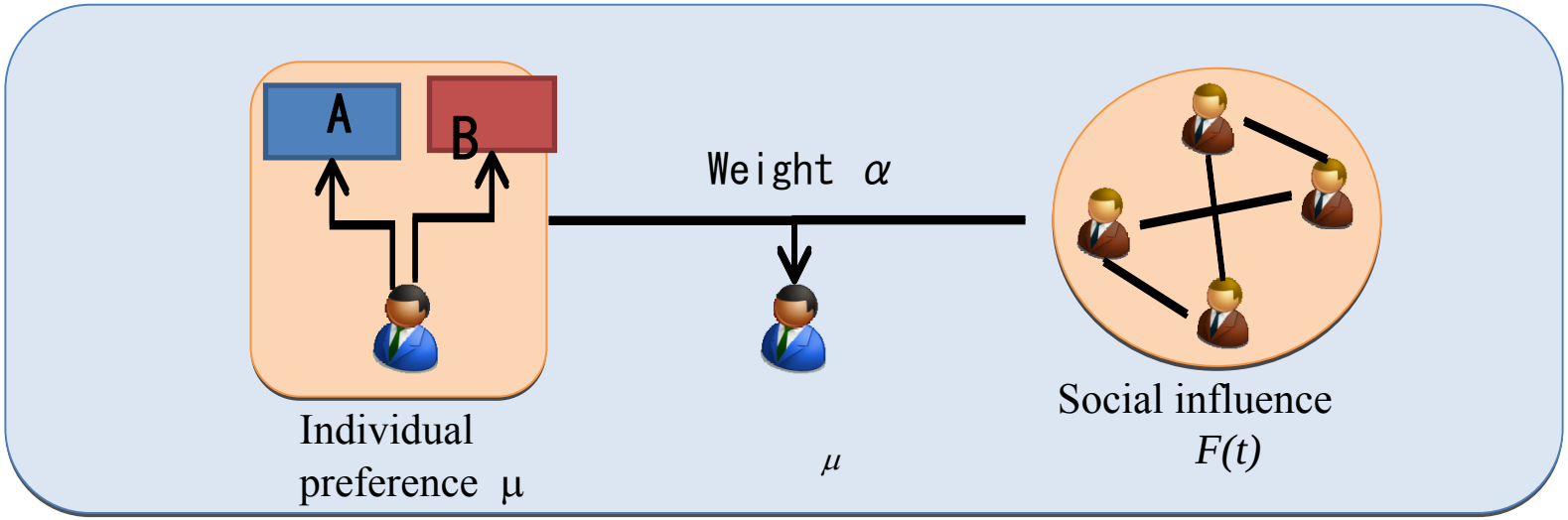
Diffusion with critical mass

Macroscopic diffusion patterns



K: time

The Effects of Heterogeneity



Probability of choosing A at time t

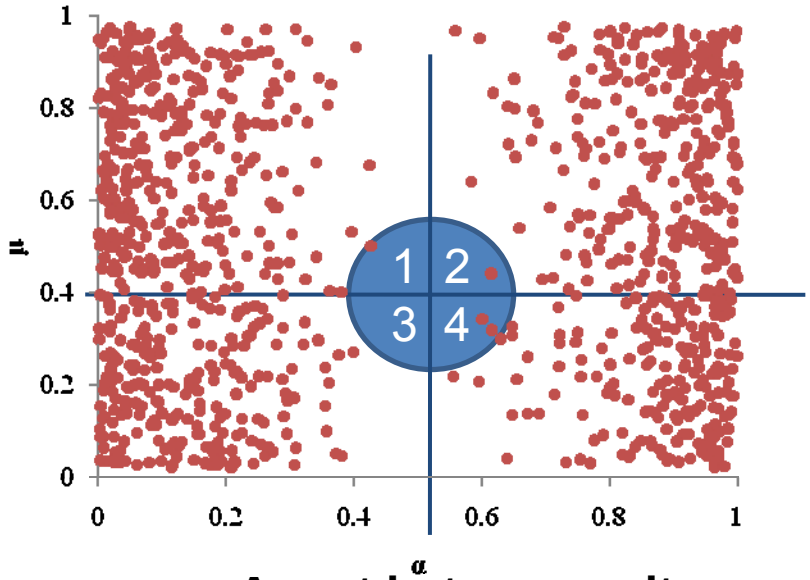
$$p(t+1) = (1-\alpha)\mu + \alpha F(t)$$

Heterogeneity in preference μ

Heterogeneity in social influence $\alpha \in [0, 1]$

What Agent Types Diffuse Most?

$$p(t+1) = (1-\alpha)\mu + \alpha F(t)$$



Agent heterogeneity

Consumer types

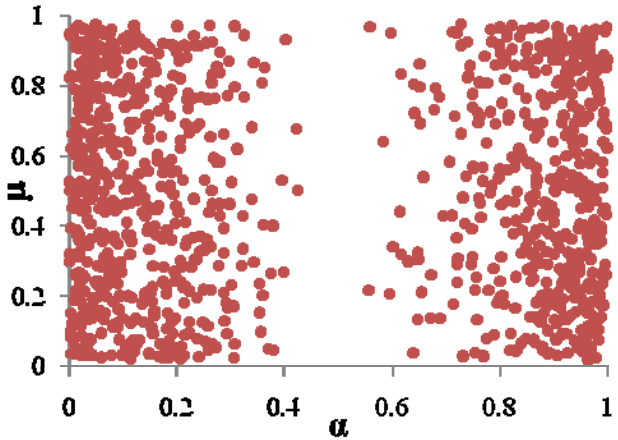
Penetration rate	Types of consumers
2.5%	innovator
16%	early adopters
50%	early majority
84%	late majority
100%	laggard

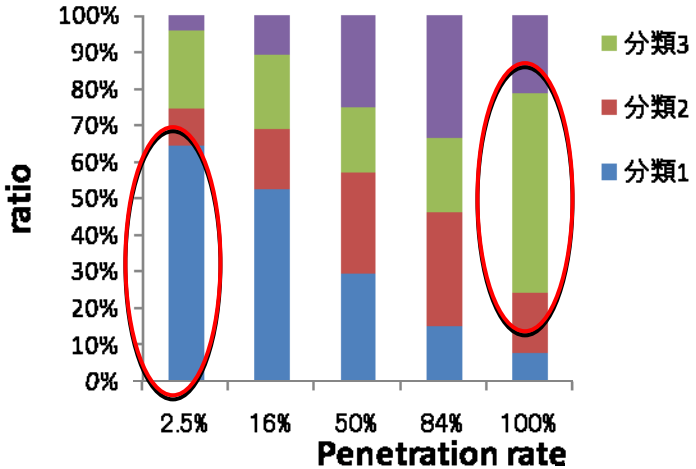
Type	Social influence	preference
1	strong	strong
2	weak	strong
3	strong	weak
4	weak	weak

Experiment 4: What Agent Types Diffuse Most?(1)

$<\mu>=0.5$, $<\alpha>=0.5$

(1) extreme case





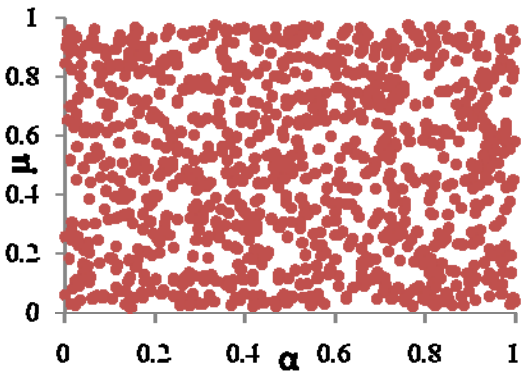
Type	Social influence	preference
1	strong	strong
2	weak	strong
3	strong	weak
4	weak	weak

- Type 1 diffuses most at the beginning
- The next stage is type 2 and 4
- Type 3 is laggard

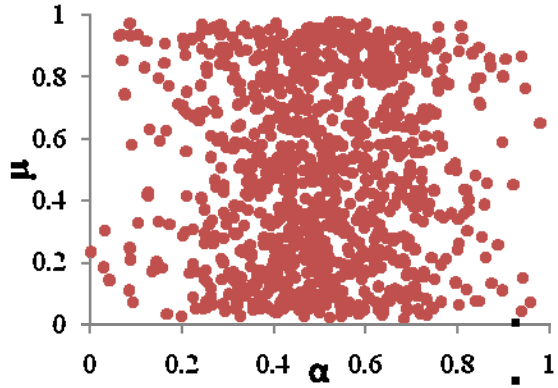
Experiment 4: What Agent Types Diffuse Most?(2)

α:social influence level
μ : preference

Uniformly distributed

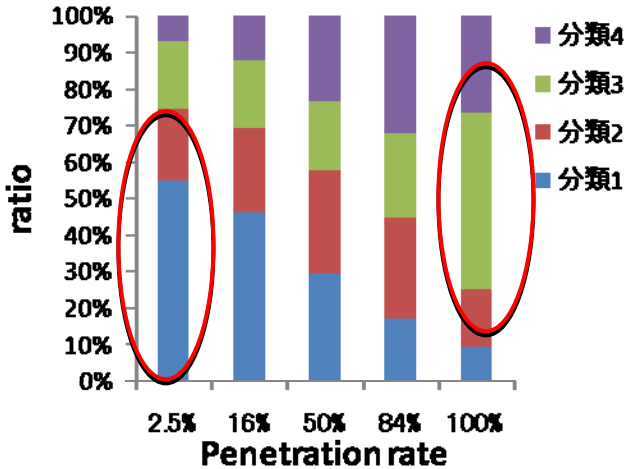


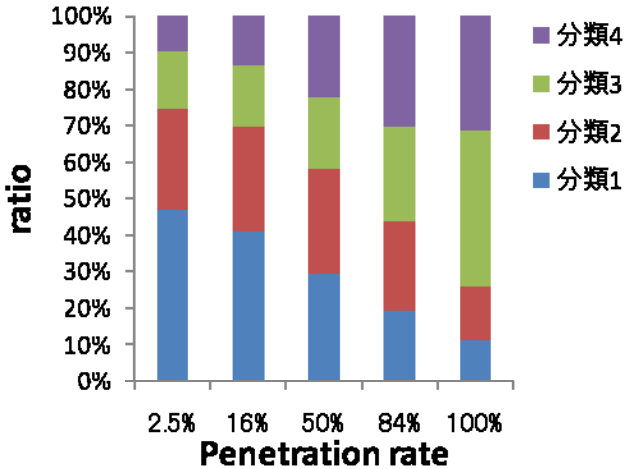
Normally distributed



Type	Social influence	preference
1	strong	strong
2	weak	strong
3	strong	weak
4	weak	weak

- Type 1 diffuses most at the beginnir
- The next stage is type 2 and 4
- Type 3 is laggard

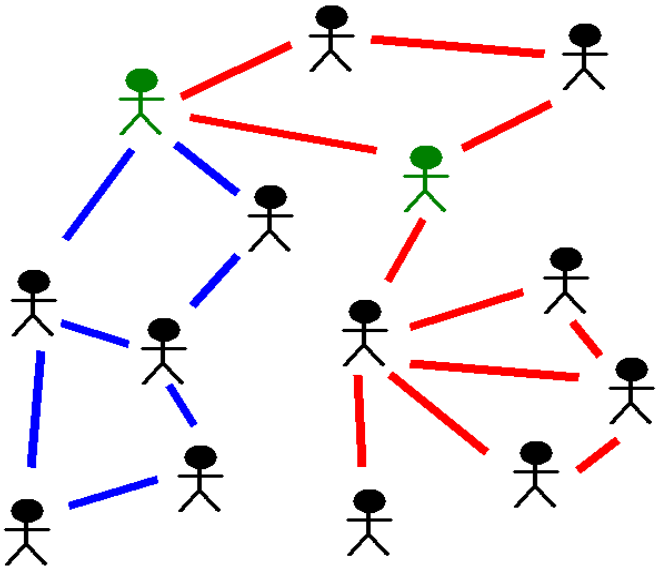




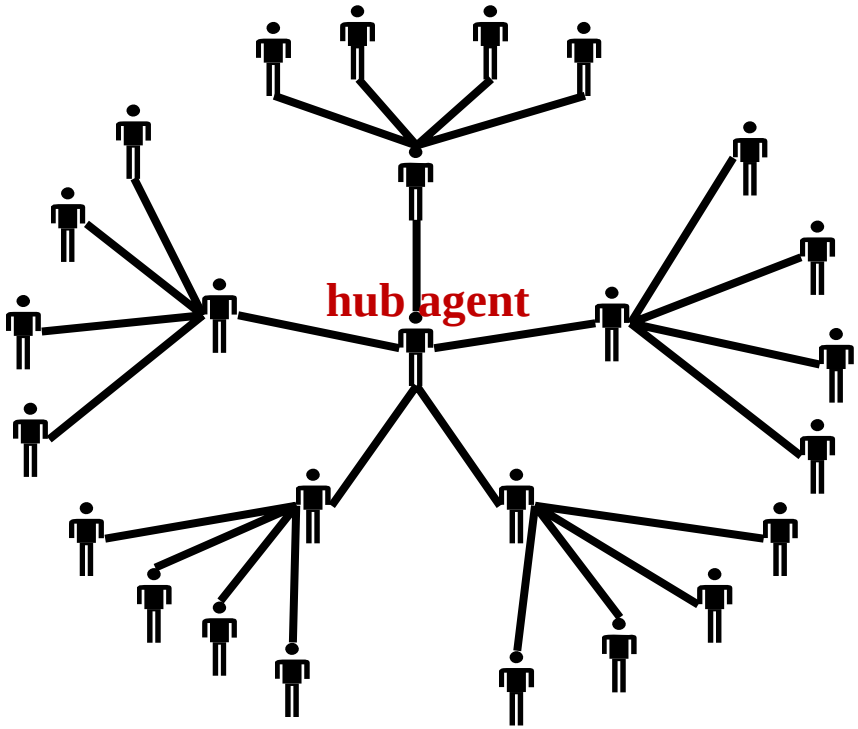
Design of Optimal Diffusion Networks

Peer influence creates consensus

within ***small*** social groups



Local peer influence networks



Scale-free networks

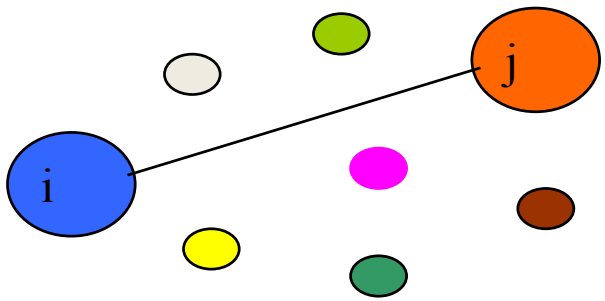
Impact of opinion leaders may be large

Epidemic Diffusion Process

The SIR model

- Consider a fixed population of size N
- Each individual is in one of three states:
  - Unaware (S), aware (I), buy or lose interest (R)

$$S \xrightarrow[\alpha]{} I \xrightarrow[\beta]{} R$$



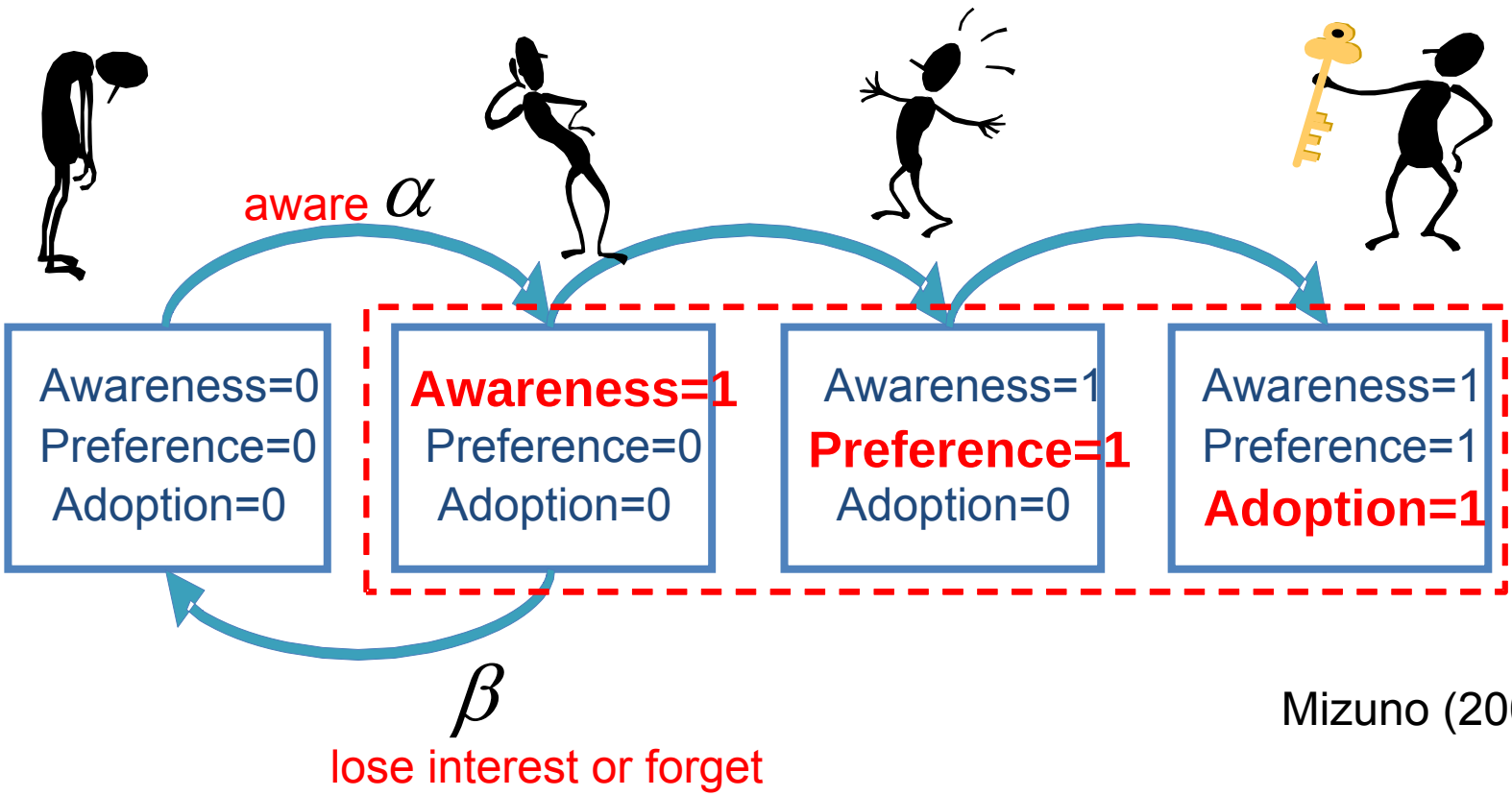
adjacent matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Agent State Transitions

Each individual is in one of three states:

- Susceptible (S) (unaware, also inactive, non-adopter)
- Infected (I) (aware, also active, informed, adopter)
- Removed (R) (lose interest or forget)



Mizuno (2008)

Analysis

- The expected state of the system at time t is given by $\overline{\mathbf{v}^{t}} = \left(\alpha\mathbf{A} + (1-\beta)\mathbf{I}\right)\overline{\mathbf{v}^{t-1}}$
- As t → ∞

$$\text{if} \quad \lambda_1\left(\alpha\mathbf{A} + (1-\beta)\mathbf{I}\right) < 1 \Leftrightarrow \lambda_1(\mathbf{A}) < \beta/\alpha, \quad \text{then } \overline{\mathbf{v}^{t}} \to 0$$

  - the probability that all copies die converges to 1

$$\text{if} \quad \lambda_1\left(\alpha\mathbf{A} + (1-\beta)\mathbf{I}\right) = 1 \Leftrightarrow \lambda_1(\mathbf{A}) = \beta/\alpha, \quad \text{then } \overline{\mathbf{v}^{t}} \to \mathbf{c}$$

  - the probability that all copies die converges to 1

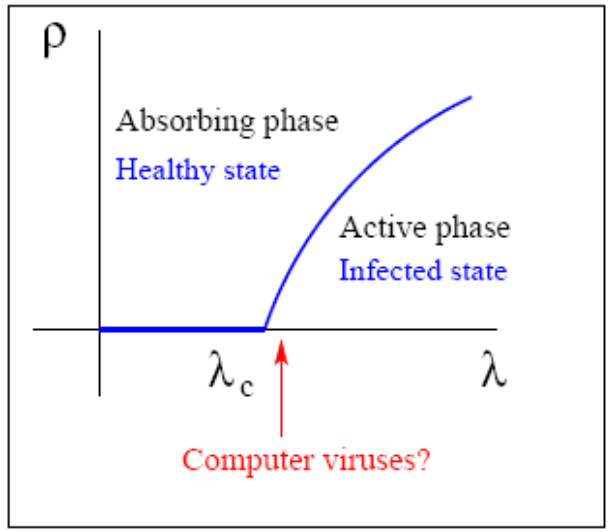
$$\text{if} \quad \lambda_1\left(\alpha\mathbf{A} + (1-\beta)\mathbf{I}\right) > 1 \Leftrightarrow \lambda_1(\mathbf{A}) = \beta/\alpha, \quad \text{then } \overline{\mathbf{v}^{t}} \to \infty$$

  - the probability that all copies die converges to a constant < 1

$\lambda_1(\mathbf{A})$ The largest eigenvalue of the adjacent matrix A

An eigenvalue point of view

- If A is the adjacency matrix of the network, then the virus dies out if $\lambda_c = 1/\lambda_1(A)$



$$\lambda_1(A) \leq \beta / \alpha$$

Optimal Network for Maximal Diffusion: N=100

- The largest eigenvalue λ_1 $\qquad \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$
- Link density $\alpha = L/Lmax$
- Object function (minimize)

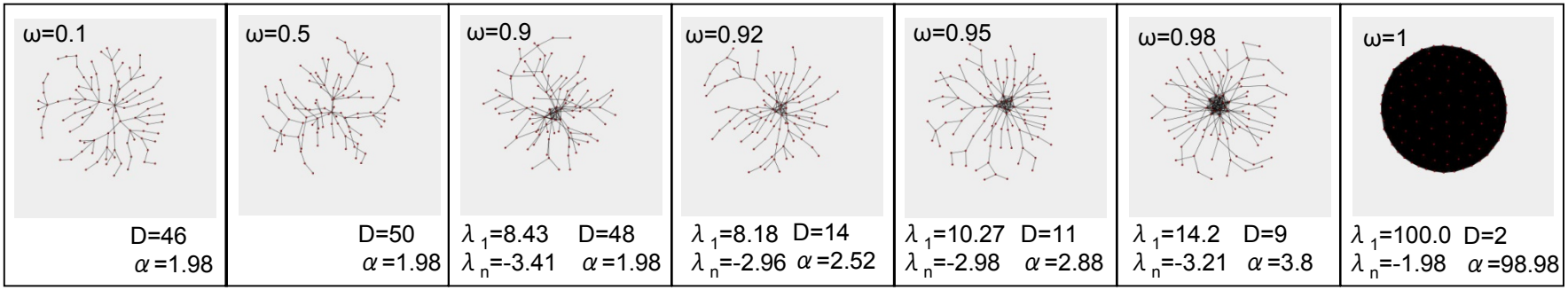
$$F = \omega / \lambda_1 + (1-\omega)\alpha$$

sparse network

Hub network

dense network

All connected



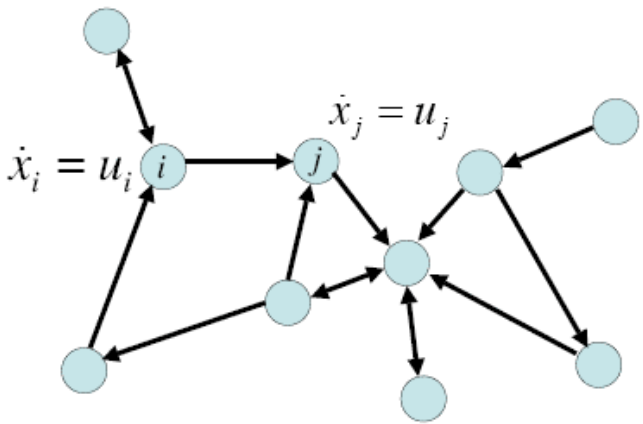
Scale-free or small-world graphs are not optimal for maximum diffusion

Consensus and Synchronization

- "Consensus" means to reach an agreement regarding a certain quantity of interest that depends on the state of all nodes (subsystems).
- *More specific, a consensus algorithm is a rule that results in the convergence of the states of all network nodes to a common value.*

$$x_i = x_j = \ldots = x_{consensus}$$

[01]: Olfati-Saber 2007



Source: Olfati-Saber 2007 [C1]

Collective Decision on Networks

S. Chen and L. Sun (2006)

$$\mathbf{x}(t+1) = F(H + DA\mathbf{x}(t) - P)$$

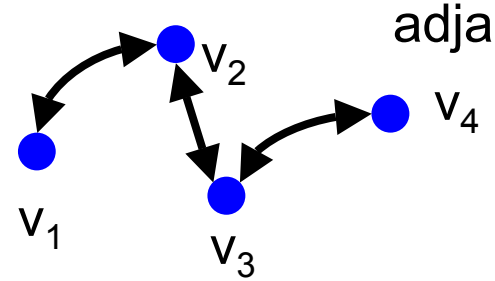
$$f(H_i) \Rightarrow \begin{cases} 0 if \quad H < P \quad \Rightarrow \omega_i = 0 \quad \forall i \\ 1 if \quad H > P \quad \Rightarrow \omega_i = 1 \quad \forall i \end{cases}$$

Adjacent matrix

$$A = \begin{pmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{pmatrix} \qquad D = \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{pmatrix}$$

$a_{ij} =$ 1: i and j are connected
0: not connected

$$H = \begin{pmatrix} H_1 \\ \vdots \\ H_n \end{pmatrix} \qquad P = \begin{pmatrix} p_1 \\ \vdots \\ p_n \end{pmatrix}$$



adjacent matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Laplacian Matrix

- L=D−A : Symmetric Matrix
  - The eigenvalues λ_i :

$$0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N \leq 2k_{\max}$$

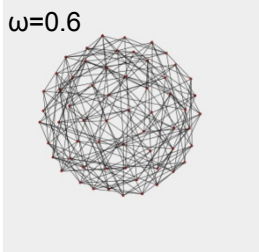
  - Algebraic connectivity λ_N / λ_2 should be minimized

Optimal Network for Maximal Diffusion with Consensus

Object function

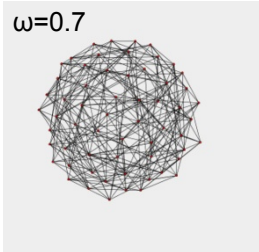
$$E(\omega) = \omega \cdot \frac{\lambda_n}{\lambda_2} + (1-\omega)\cdot\alpha$$

ω=0.6



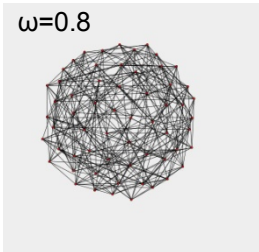
λ_2=3.4 α=6.9
λ_n=11.46 P=0.47
λ_n/λ_2=3.35

ω=0.7



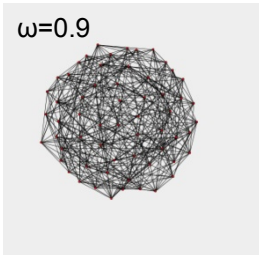
λ_2=4.64 α=8.3
λ_n=13.35 P=0.51
λ_n/λ_2=2.87

ω=0.8

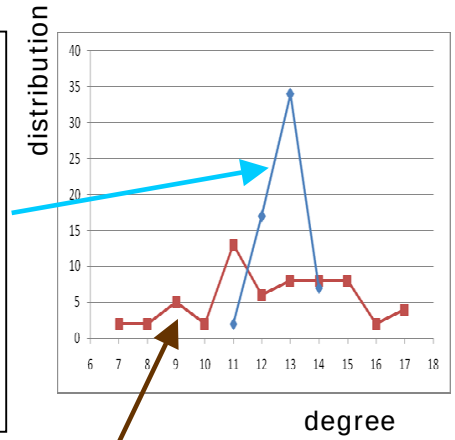


λ_2=6.17 α=10.2
λ_n=15.63 P=0.50
λ_n/λ_2=2.53

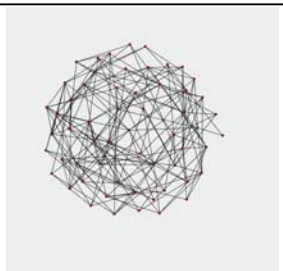
ω=0.9



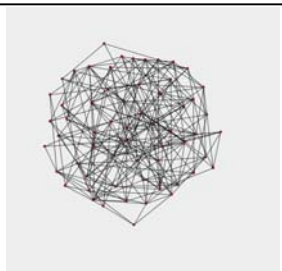
λ_2=8.34 α=12.8
λ_n=18.57 P=0.31
λ_n/λ_2=2.23



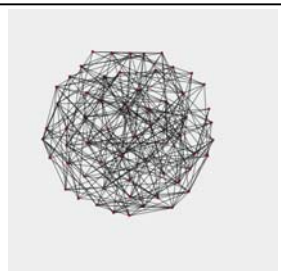
Random network



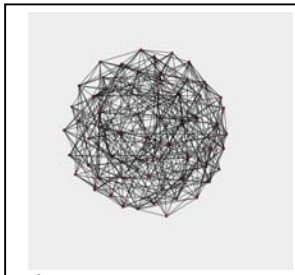
λ_2=1.58 α=6.9
λ_n=13.89 P=1.37
λ_n/λ_2=8.75



λ_2=2.08 α=8.3
λ_n=15.86 P=1.99
λ_n/λ_2=7.6



λ_2=3.87 α=10.2
λ_n=19.15 P=1.77
λ_n/λ_2=4.95

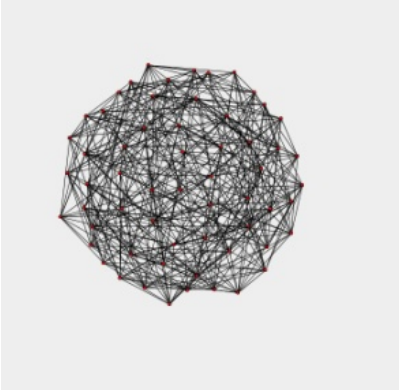


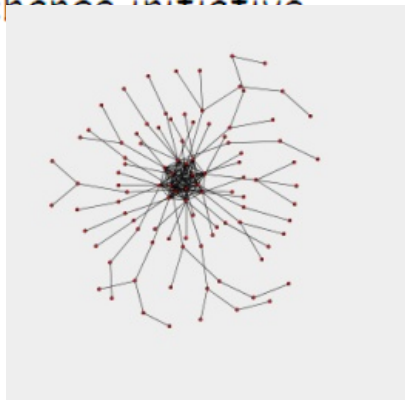
λ_2=5.53 α=12.4
λ_n=21.02 P=0.95
λ_n/λ_2=3.8

Optimal Network for Diffusion

	Maximize Spread	Minimize Spread
Public Health	Selecting peer health advocates for **diffusing** safe practices (e.g. bleaching) and material	Who to **immunize or quarantine** in order to slow spread of infectious disease
Criminal Justice	Who to "turn", feed false information to, or surveil	Who to **arrest or discredit to disrupt** criminal networks
Management	Select employees for intervention prior to change initiative	Where is an organization most vulnerable to turnover?

Consensus formation





Conclusion

<Cascade in Contagion and Innovation>

- **Individual decisions are influenced are influenced the adoption behavior of the social system.**
- **Martingale property makes the diffusion process to be unpredictable.**
- **Diffusion of innovation process that requires persuasion and consensus among consumers becomes very slow since most social influence networks are asymmetric**

Future works: How can we optimize the diffusion process with martingale property?