Systemic Risks as Cascading Process in a Networked Society

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Outline

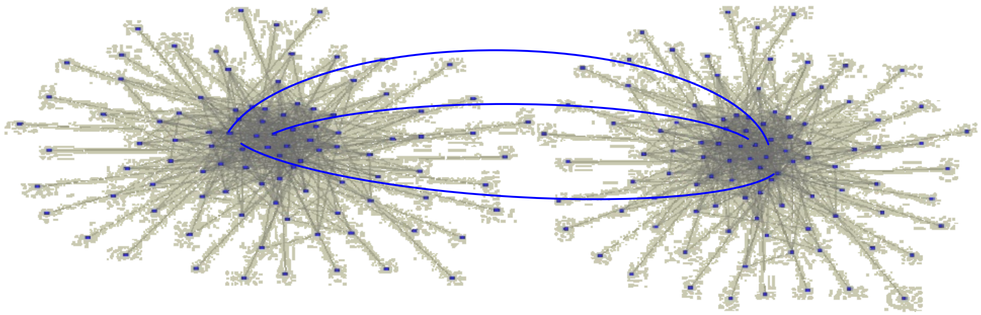
- **Concept of Systemic Risks**
- **Study on Diffusion and Contagion**
- **Financial Networks as Complex Adaptive Systems**
- **Risk Propagation Models**

  : An Epidemic Spread Model

  : A Threshold Model

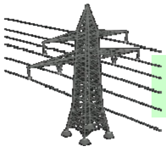
  : A Compound Model of Endogenous and Exogenous Risks

Interconnected Societies



<Many social-economic networks show cascading effects>

Internet



Power grids

Internet congestion collapse

↓

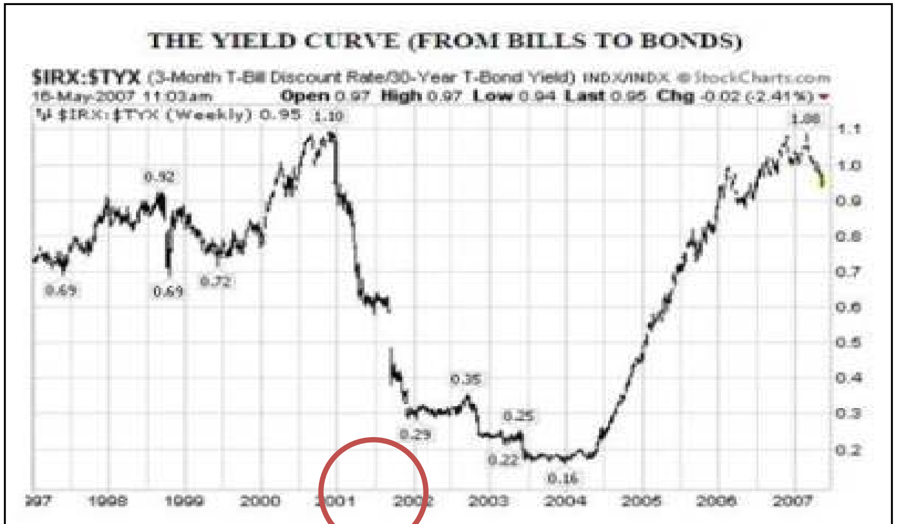
Drop in speed of a factor 100

initial disturbance in some area

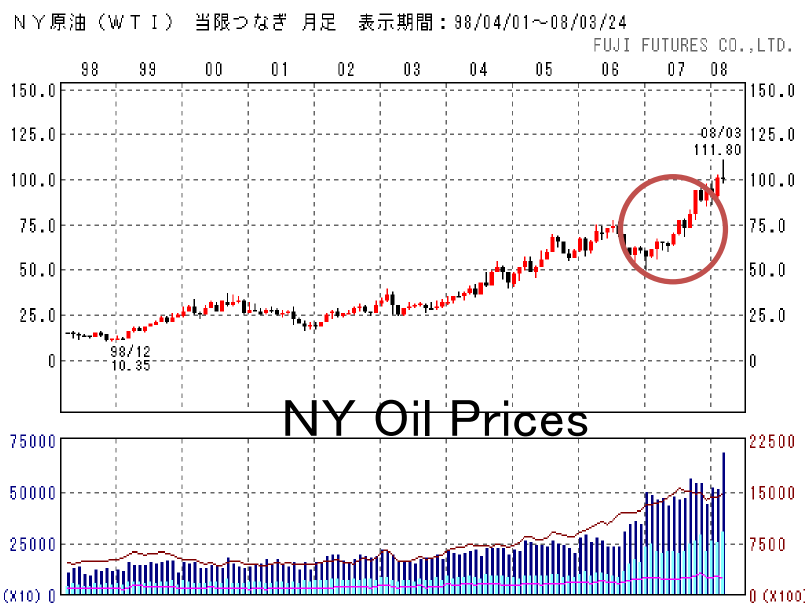
↓

Largest blackout

Cascade in Economics



NY Stock Prices



NY Oil Prices

Cascade phenomena

Systemic Risk

– A systemic risk is a risk that something bad happens in the system that is a good deal bigger and worse than the failure of any single node or subsystem.

– A systemic risk is a phase transition from one equilibrium to another, much less optimal equilibrium, characterized by self-reinforcing feedback mechanisms (vicious circle).

Systemic Risk and Diffusion

- Concepts of “diffusion” arise quite generally in biological and social sciences
  - Diffusion of innovations
  - Rumor spreading
  - Spread of infectious disease
  - Emergence of collective belief
  - Transmission of financial distress

- We would like to understand in what sense these kinds of “diffusion” are the same and how they are different from “systemic risk”.

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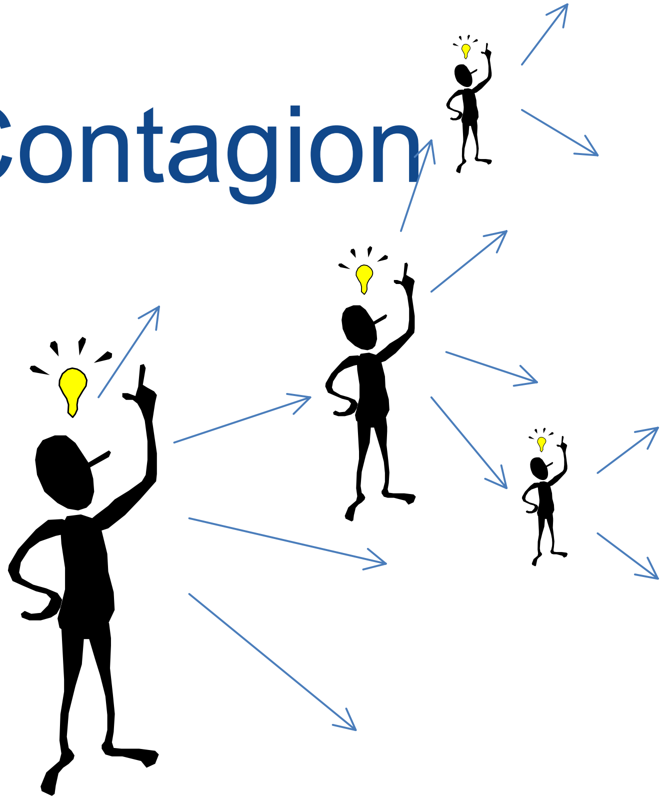
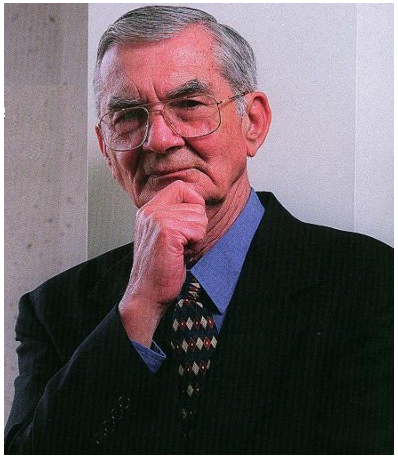
Diffusion and Contagion



- Diffusion
  - Messages of twitter
  - Innovation
  - New products
- Effects of diffusion
  - Make a trend in society
  - Motivate an action
- Basic question
  - Why is there a long lag time between an innovation's first appearance and the time when a substantial number of people have adopt it.

Diffusion: A Bass Model

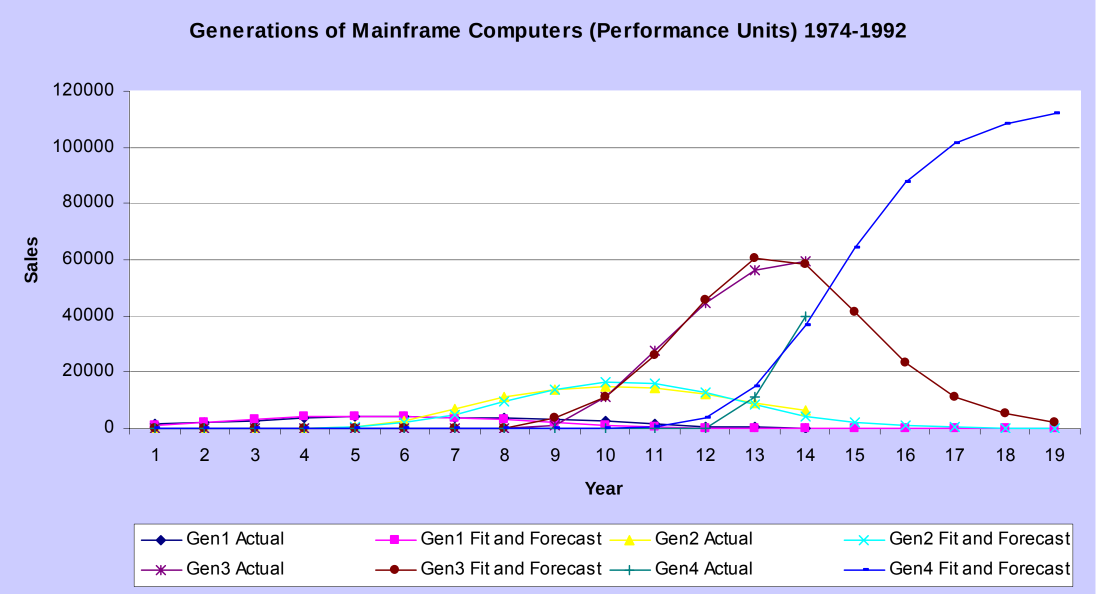


- f(t)=(p+qF(t))[1-F(t)]: The mean-field model
- f(t): the rate of the adoption (growth rate)
- F(t): cumulative proportion of adoption
- p=coefficient of innovation
- q=coefficient of imitation f(t) = [p+qF(t)] [(1-F(t)]

external effect (advertising)

individuals who already adopted

individuals who are unaware



Special Cases:

q=0: Exponential Distribution

p=0: Logistic Distribution,

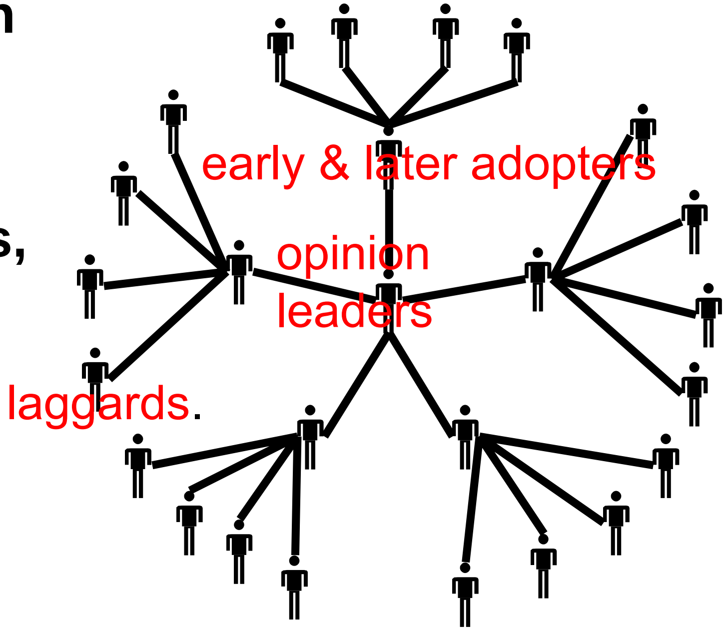
Diffusion: A Network-based Model

(M. Roger, 1995)

Two steps in the transmission of information (media → influentials → others)

Two kinds of individuals :
average individuals : most of the population
influentials individuals : opinion leaders

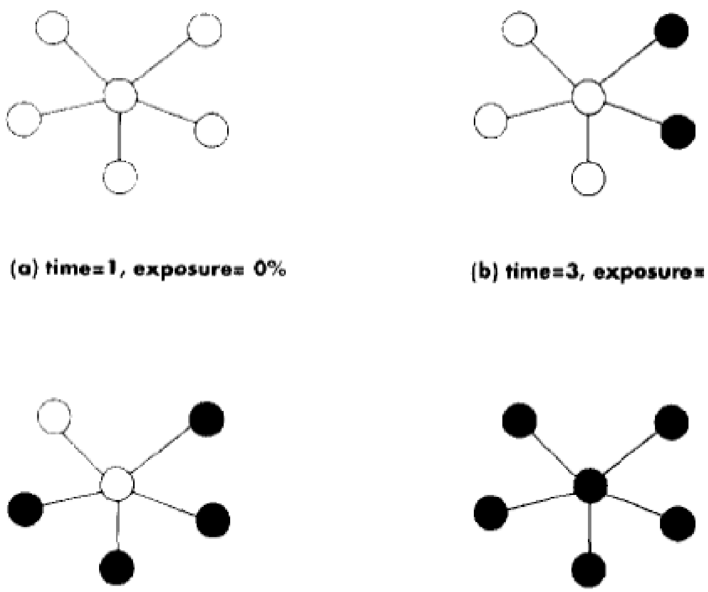
Transferring new knowledge from creators to users involves their network connections, which diffuse information in two-step flows from opinion leaders to early & later adopters, then to laggards.



Diffusion: A Threshold Model

Tom Valente's (1996)

Network threshold diffusion model involves micro-macro effects & non-adopters' influence on adopter decisions. It assumes "behavioral contagion through direct network ties"



(a) time=1, exposure= 0%

(b) time=3, exposure= 40%

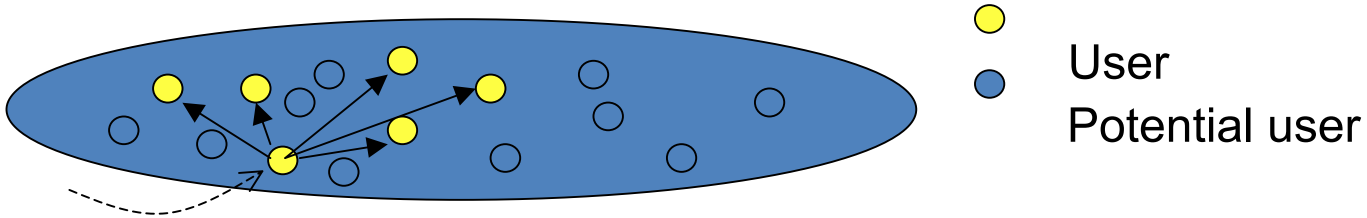
(c) time=5, exposure= 80%

(d) time=8, exposure= 100%

Diffusion: A Model of Network Externality

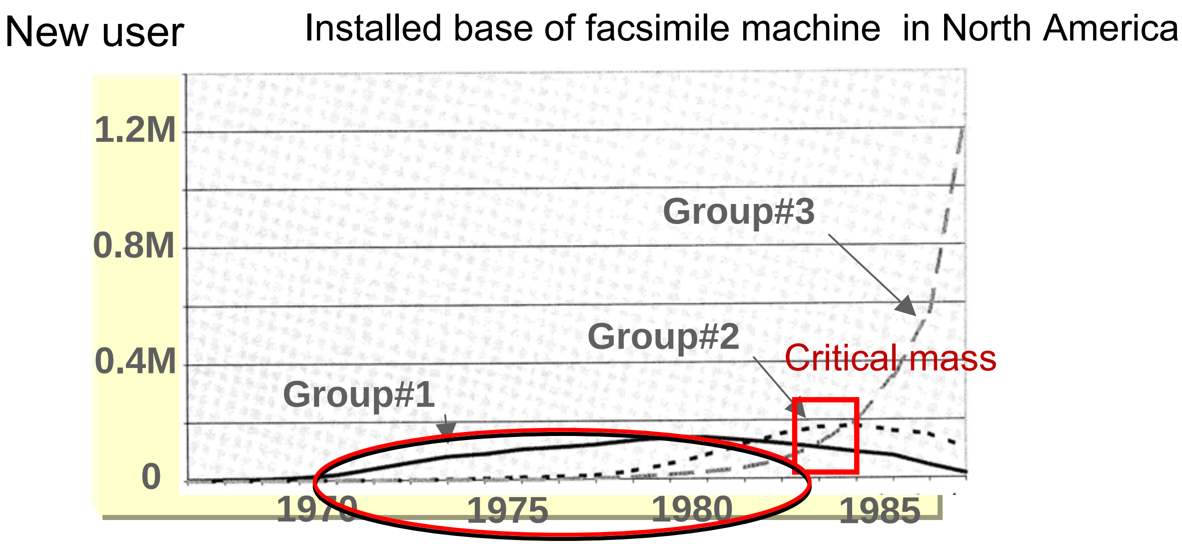
positive network externality

More usage of a product by any user increases the product's value for other users



- High-tech and IT products, systems, and services have this property.
  - Critical mass is important.

    If the initial adoption rate reaches to critical mass, it diffuses massively



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Global Financial Network

Modern financial systems exhibit a high degree of interdependence, with connections between financial institutions

Chart 1: Global Financial Network: 1985

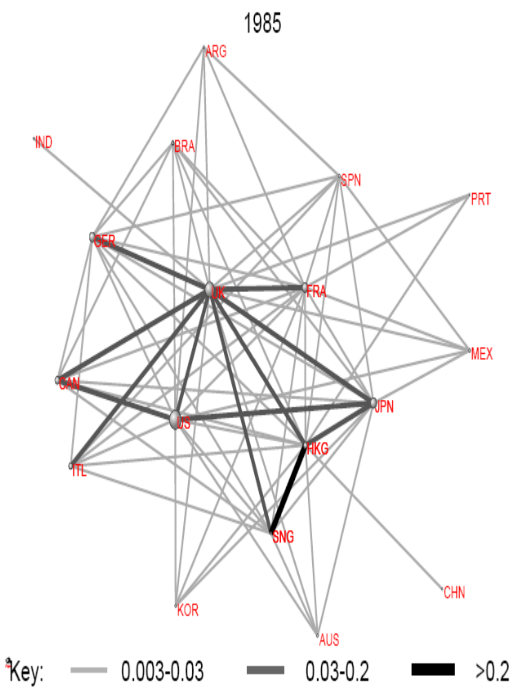


Chart 2: Global Financial Network: 1995

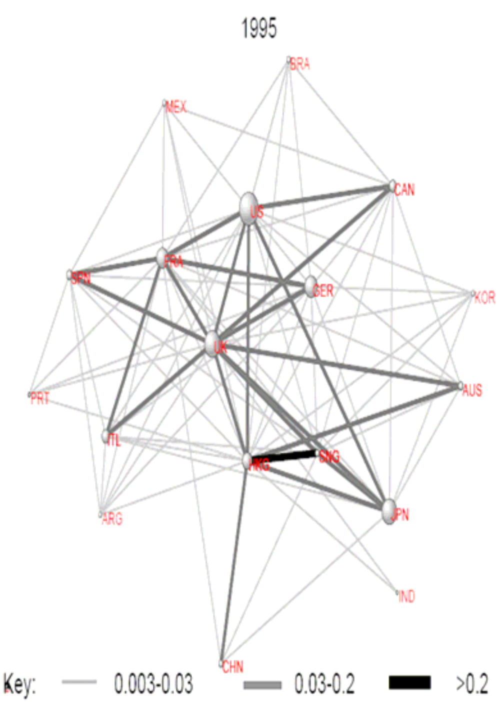
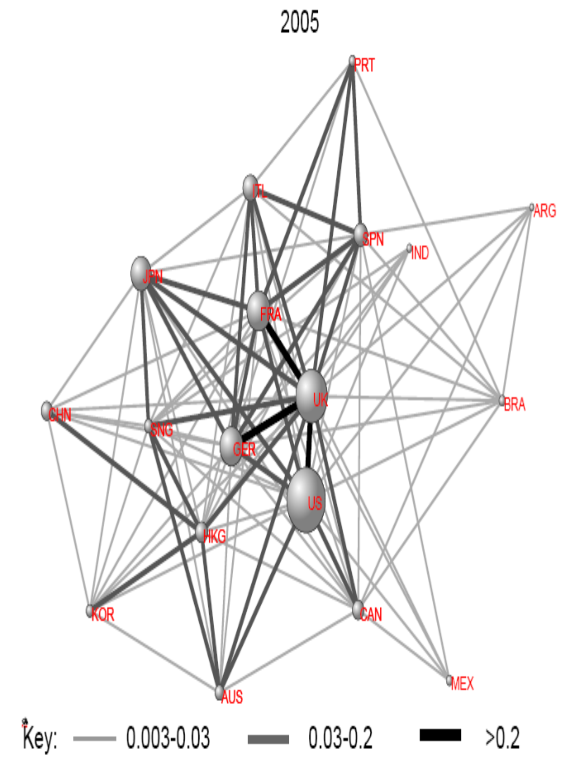


Chart 3: Global Financial Network: 2005



Rethinking the Financial Network

A. Haldane (Bank of England ,2009)

- Financial system = complex adaptive network
- Complex adaptive networks extensively explored in physics, biology, engineering, epidemiology, philosophy, etc
- What lessons can we learn to improve network robustness?

Connectivity and Stability

- Three well-established network properties
  a) “Robust-yet-fragile” networks – tipping points
  b) “Long-tailed” networks – hub and spokes
  c) “Small world” networks – long hops
- International financial networks have developed all three characteristics
- In combination, generates a “robust-yet-fragile” and “small world” financial system
- Mirrors events during the present crisis

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Diffusion on Network

	Maximize Spread	Minimize Spread
Public Health	Selecting peer health advocates for **diffusing** safe practices (e.g. bleaching) and material	Who to **immunize or quarantine** in order to slow spread of infectious disease
Criminal Justice	Who to "turn", feed false information to, or surveil	Who to **arrest or discredit to disrupt** criminal networks
Management	Select employees for intervention prior to change initiative	Where is an organization most vulnerable to turnover?

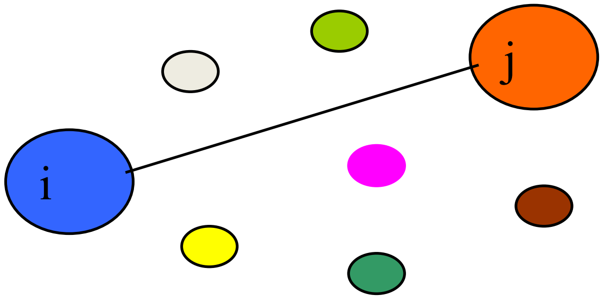
(Borgatti, 1997)

Epidemic Diffusion

The SIR model

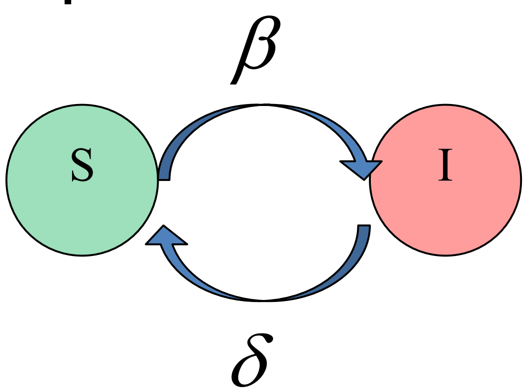
- Consider a fixed population of size N
- Each individual is in one of three states:
  - susceptible (S), Infected (I), recover (R)

$$S \xrightarrow[\alpha]{} I \xrightarrow[\beta]{} R$$



SIS Model

- Susceptible-Infected-Susceptible
  - People are divided into two categories.

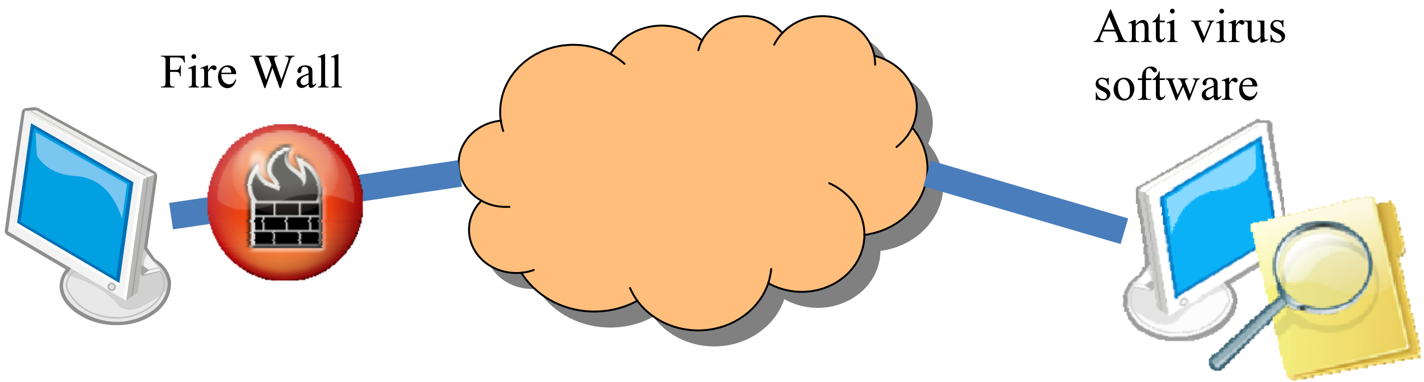


[*Kermack and Mckendrick, 1927*]

$$\frac{dS(t)}{dt} = -\beta SI + \delta I$$

$$\frac{dI(t)}{d(t)} = \beta SI - \delta I$$

Computer Network Security



- Until now
  - End point defense
- From now
  - Network-wide defense
    - Against worm propagation

Diffusion on Networks

- Diffusion depends on network topology.

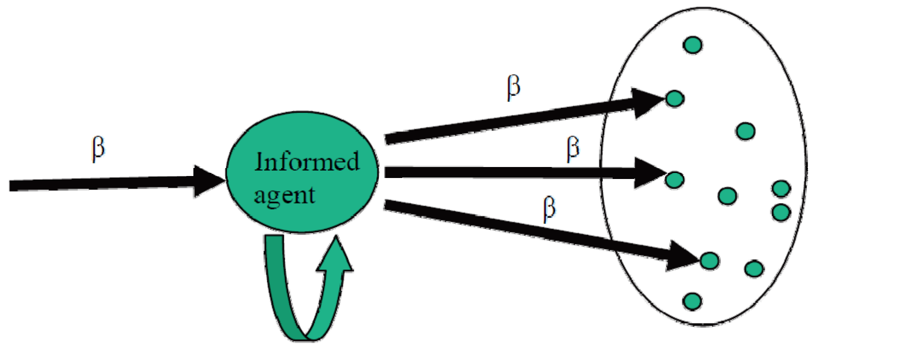
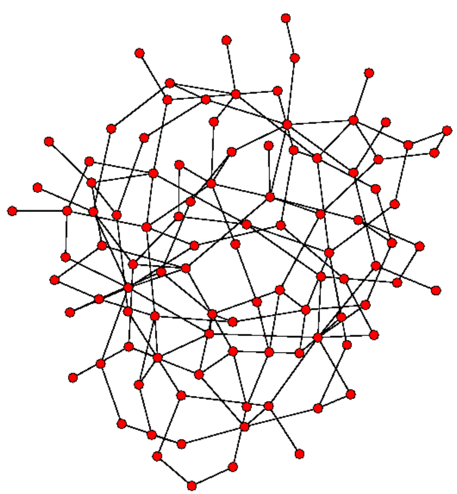


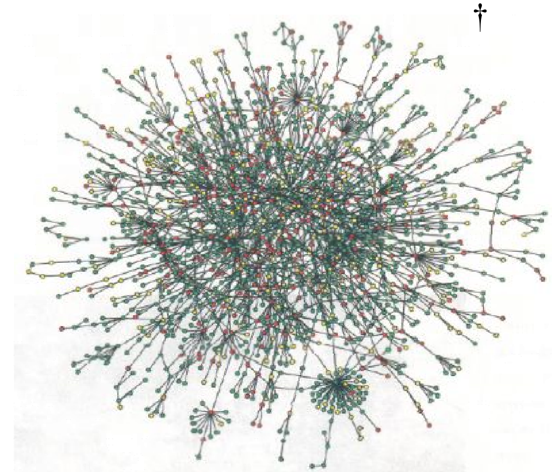
Fig. 1 Information spread with forgetting

Random network



Scale free network

†



Epidemic Dynamics

- The expected state of the system at time t is given by $\overline{\mathbf{v}^{\mathrm{t}}} = \left(\alpha\mathbf{A} + (1-\beta)\mathbf{I}\right)\overline{\mathbf{v}^{\mathrm{t-1}}}$
- As t → ∞

$$\text{if } \lambda_1\left(\alpha\mathbf{A} + (1-\beta)\mathbf{I}\right) < 1 \Leftrightarrow \lambda_1(\mathbf{A}) < \beta/\alpha, \text{ then } \overline{\mathbf{v}^{\mathrm{t}}} \to 0$$

- the probability that all copies die converges to 1

$$\text{if } \lambda_1\left(\alpha\mathbf{A} + (1-\beta)\mathbf{I}\right) = 1 \Leftrightarrow \lambda_1(\mathbf{A}) = \beta/\alpha, \text{ then } \overline{\mathbf{v}^{\mathrm{t}}} \to \mathbf{c}$$

- the probability that all copies die converges to 1

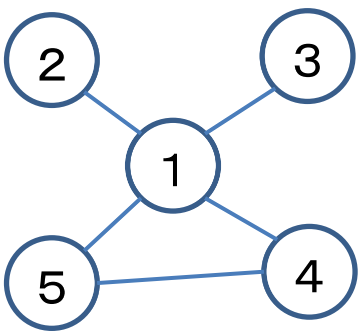
$$\text{if } \lambda_1\left(\alpha\mathbf{A} + (1-\beta)\mathbf{I}\right) > 1 \Leftrightarrow \lambda_1(\mathbf{A}) > \beta/\alpha, \text{ then } \overline{\mathbf{v}^{\mathrm{t}}} \to \infty$$

- the probability that all copies die converges to a constant < 1

$\lambda_1(\mathbf{A})$ The largest eigenvalue of the adjacent matrix A

Threshold of Epidemic Spread (1)

- Epidemic threshold of SIS model can be written as $\tau = \dfrac{1}{\lambda_1(A)}$
  where λ _max(A) is maximum eigenvalue of adjacency matrix of network [*Wang, 2008*]



Network

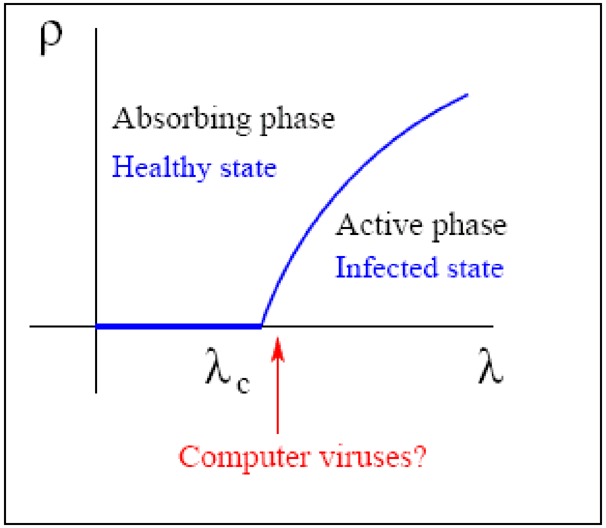
$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Adjacency matrix

$$\tau = \frac{1}{\lambda_1(A)} = \frac{\beta}{\delta}$$

Threshold of Epidemic Spread (2)

- If A is the adjacency matrix of the network, then the virus dies out if $\lambda_c = 1/\lambda_1(A)$



$$\lambda_1(\mathrm{A}) \leq \beta / \alpha$$

adjacent matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Network Optimization

- Objective function
  - Find network which have lowest number of E by genetic algorithm

$$E = \omega \frac{1}{\lambda_1(A)} + (1-\omega)\frac{<k>}{n-1}$$

  - First term maximize largest eigenvalue
  - Second term minimize average degree

Optimal Diffusion Network

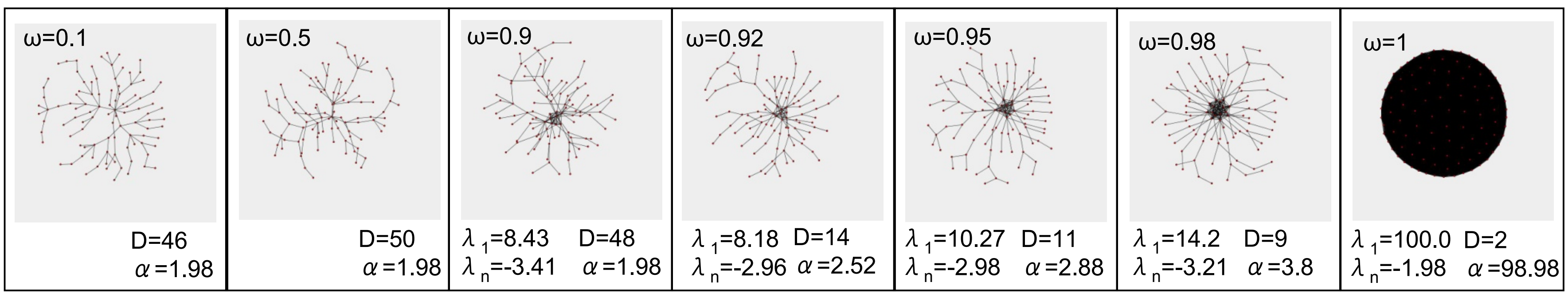
- The largest eigenvalue λ_1
- Object function (minimize) $F = \omega / \lambda_1 + (1-\omega)\alpha$

dense network

sparse network

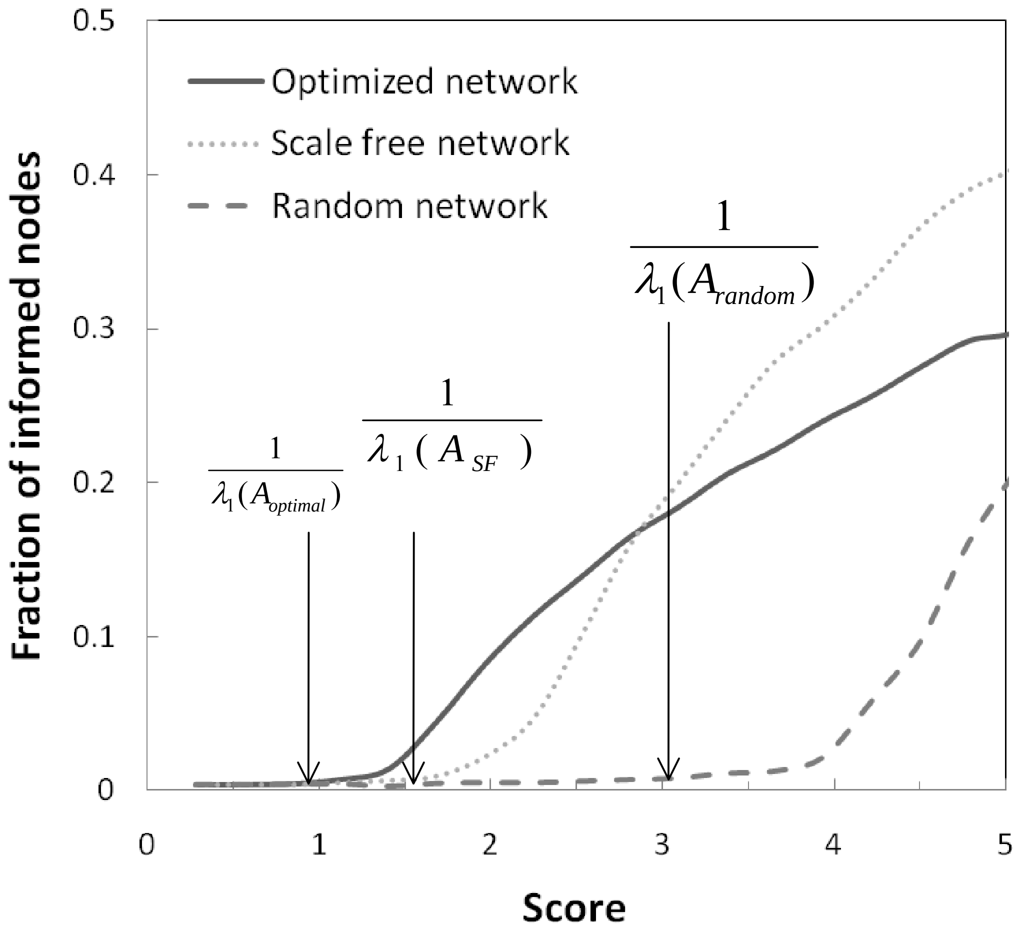
Hub network

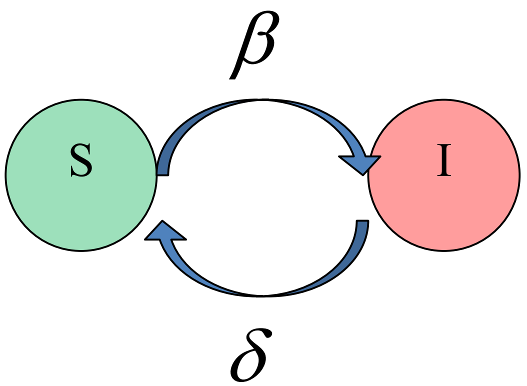
All connected



Scale-free or small-world graphs are not optimal for maximum diffusion

Comparison of Diffusion on Different Networks





$$Score = \frac{\beta}{\delta} \div \frac{1}{\lambda_1(A_{optimal})}$$

Diffusion starts at the early stage on the optimized networks

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Millennium Bridge (1)

- First new Thames crossing for over a hundred years
  - New design, extensive tests, riskless
  - Opened by the Queen on June 10^{th}, 2000
- 

Millennium Bridge (2)

What happened?

- Wobbled violently within moments of bridge opening
- Remain closed for the next 18month

• What endogeneity?

- Pedestrians had some problems

What went wrong?

- An engineering answer
  -Cause: horizontal vibrations at 1hertz
  -Walking pace: 2steps per second: 2hertz
  -Producing 1 hertz horizonal force

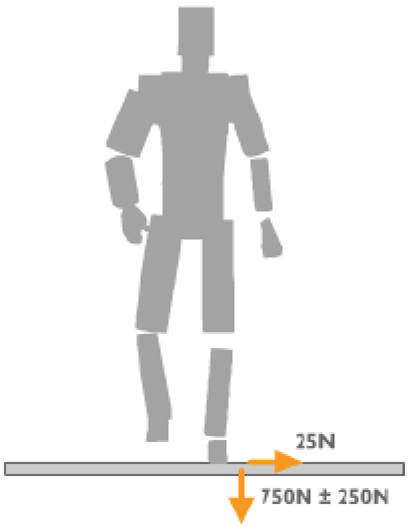
- Why should it matter?
  - People's sway to the left and right cancel out each other
  - Only a problem when people walked in step
  - Probability of a thousand people walking at random ending up walking exactly in step?
  - Close to zero if individual steps are independent events.

Diagnosis (1)

•Trouble was at 1hertz (one complete cycle per second)

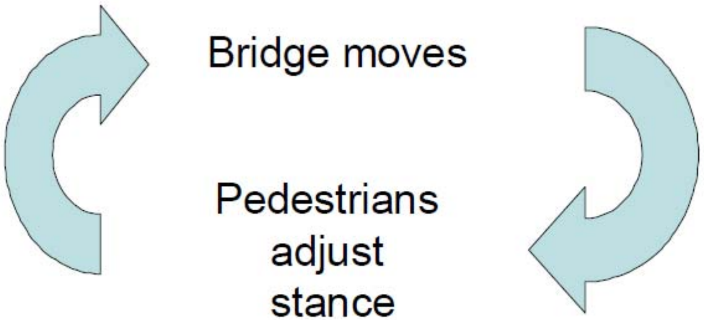
•Walking pace is approximately two steps per second (2hertz)

• Although most force exerts down when walking, there is small sideways force every two steps .

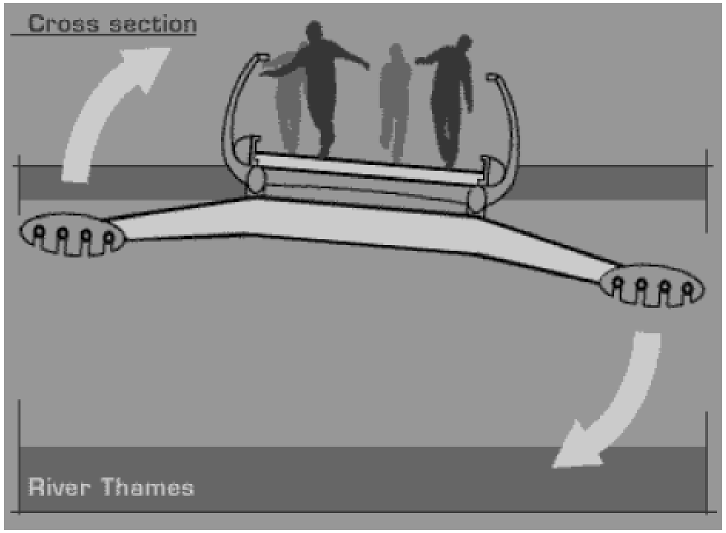


H. Shin (2008)

Diagnosis (2)



H. Shin (2008)



Endogenous Risk

- What endogeneity?
  - Pedestrians had some problems

- Endogenous risk refers to the risk from shocks generated and amplified within the system (feedback e·ects)

- It stands in contrast to exogenous risk, which refer to the risk from shocks that originate from outside the system (natural disaster such as earthquakes, heavy loads)

Endogenous Risk vs. Exogenous Risk

Exogenous risk: shocks that arrive from outside the system

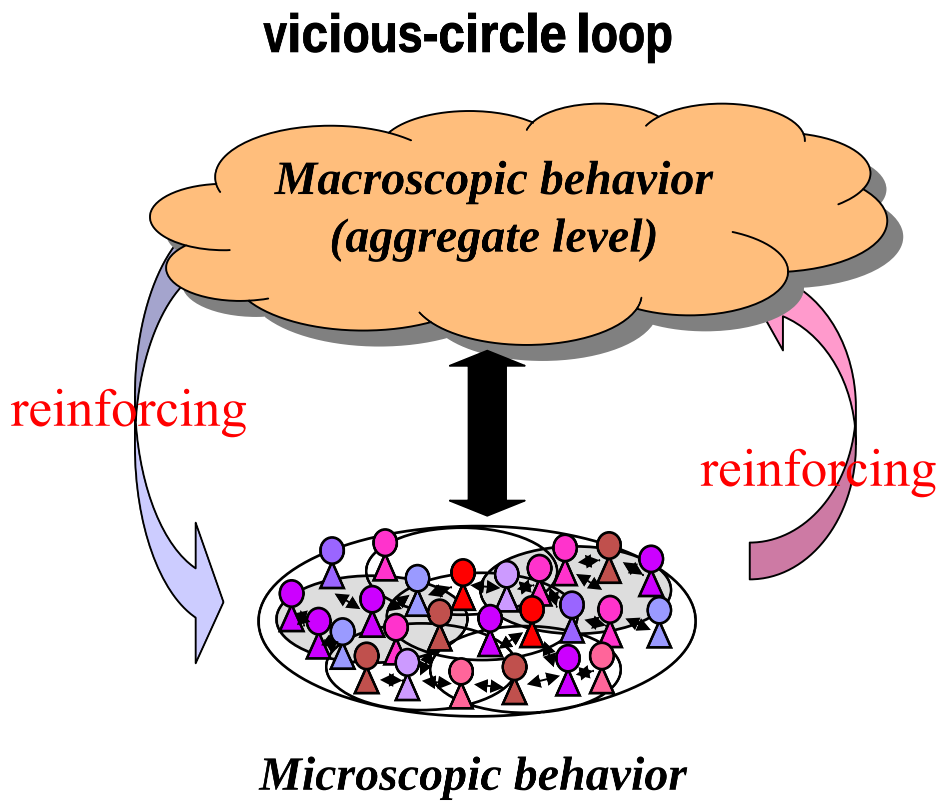
Endogenous risk: the risk from shocks that are generated and amplified within the system

Exogenous risk: situations where an agent cannot affect outcomes

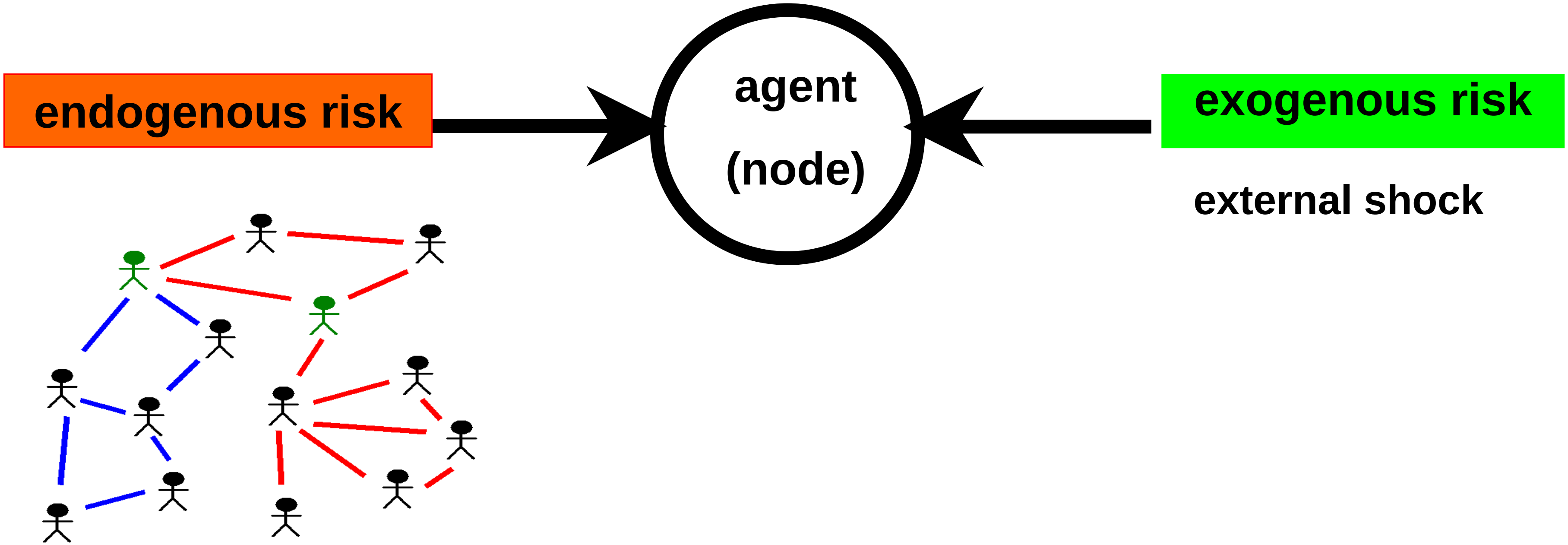
Endogenous risk: situations where the agent affects outcomes

Self-Reinforcing with a Micro-Macro Loop

- The vicious-circle loop might be understood as a self-reinforcing process with a "micro-macro loop".

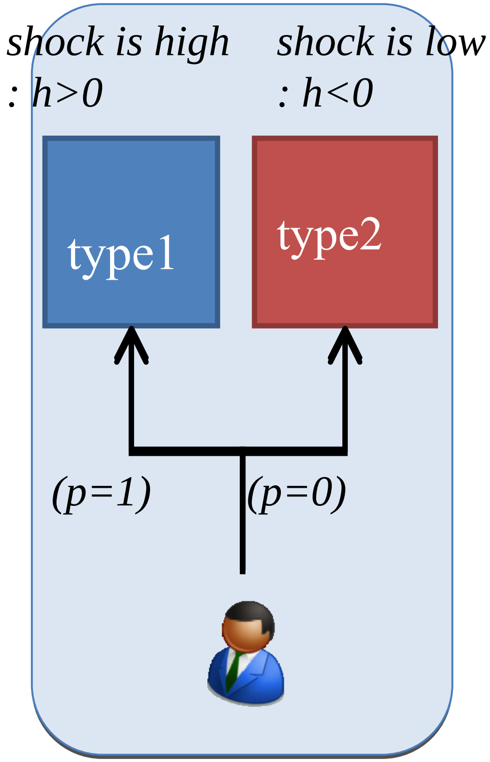


A Compound Model



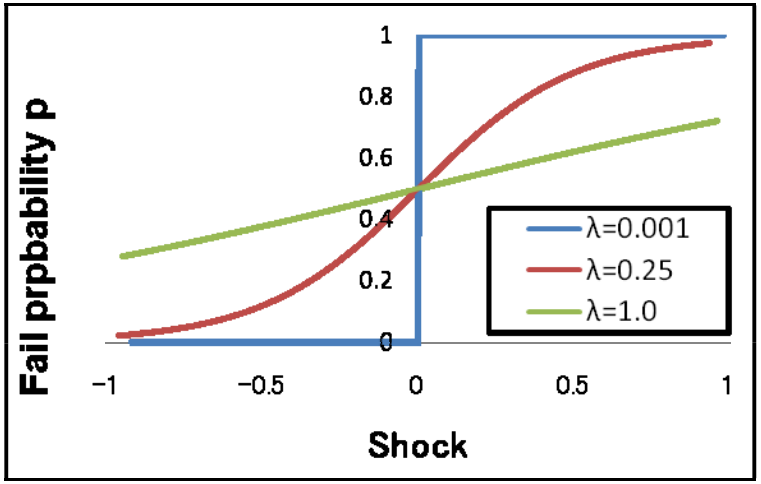
Two Types of Agents

We consider heterogenous agents with respect to external shock
: very sensitive agents (type 1) and insensitive agents (type 2)



p: fail probability due to external shock

$$p = \frac{1}{1 + e^{-\frac{Shock}{\lambda}}}$$

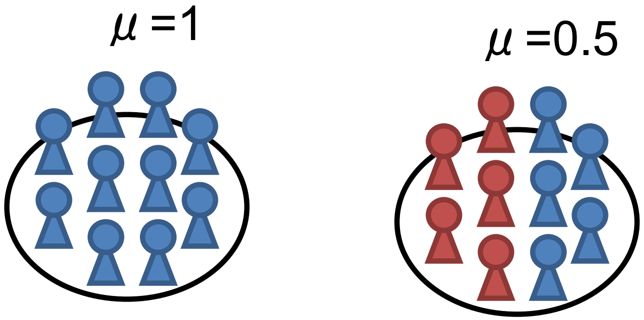


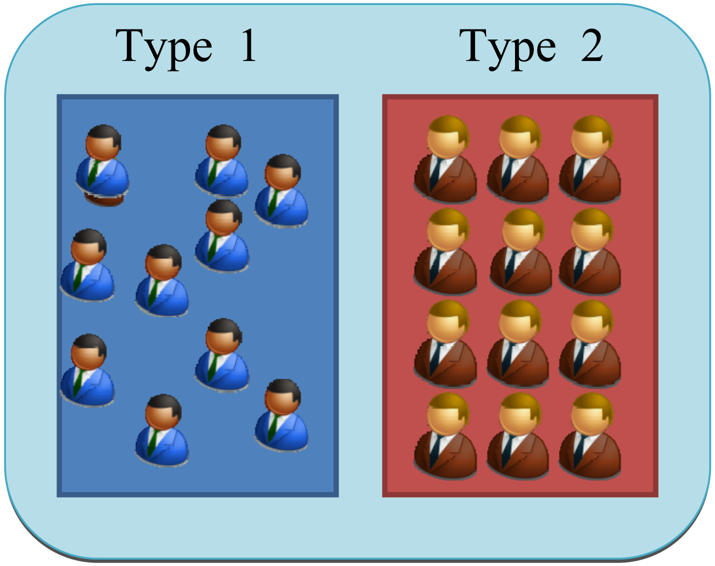
Exposure to shock

A Collection of Agents

Type- 1: agents who face exogenous risk (external shock)

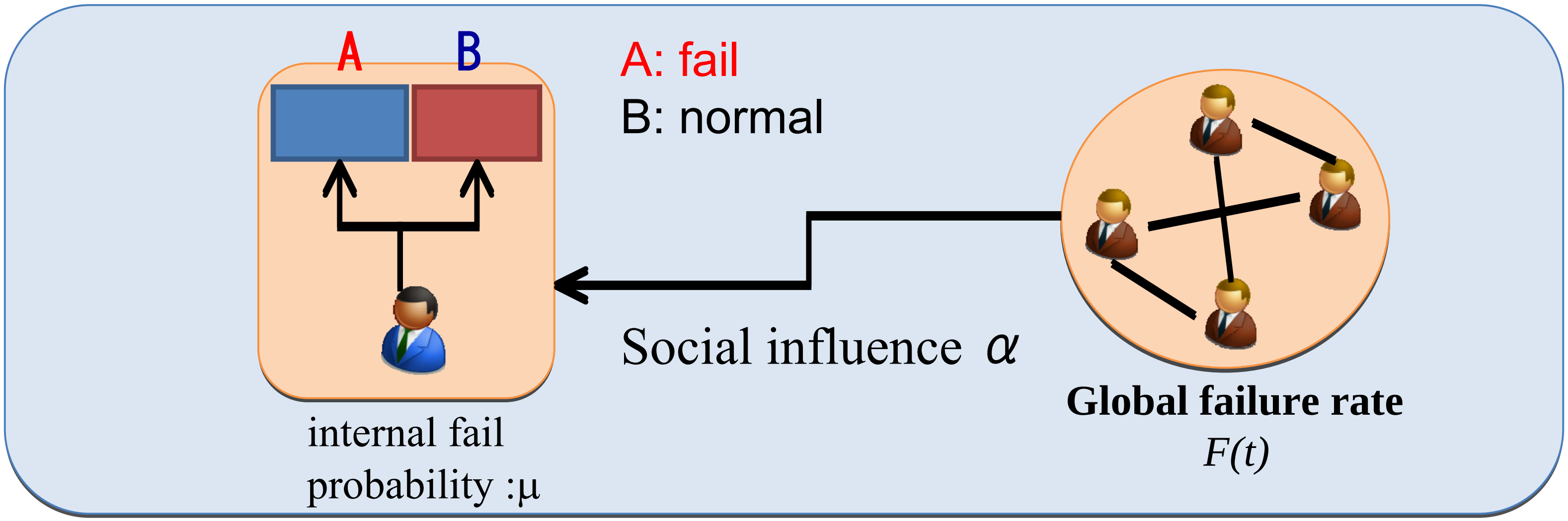
Type- 2 agent who are not influenced by exogenous risk





μ : the ratio of type-1 agents in the population

State Transition under Social Influence



Fail probability of an agent at time t

$$q(t+1) = (1-\alpha)p + \alpha S(t) \qquad S(t) = A(t)/\{A(t)+B(t)\}$$

internal fail **collective fail**

F(t): the proportion of failed agents

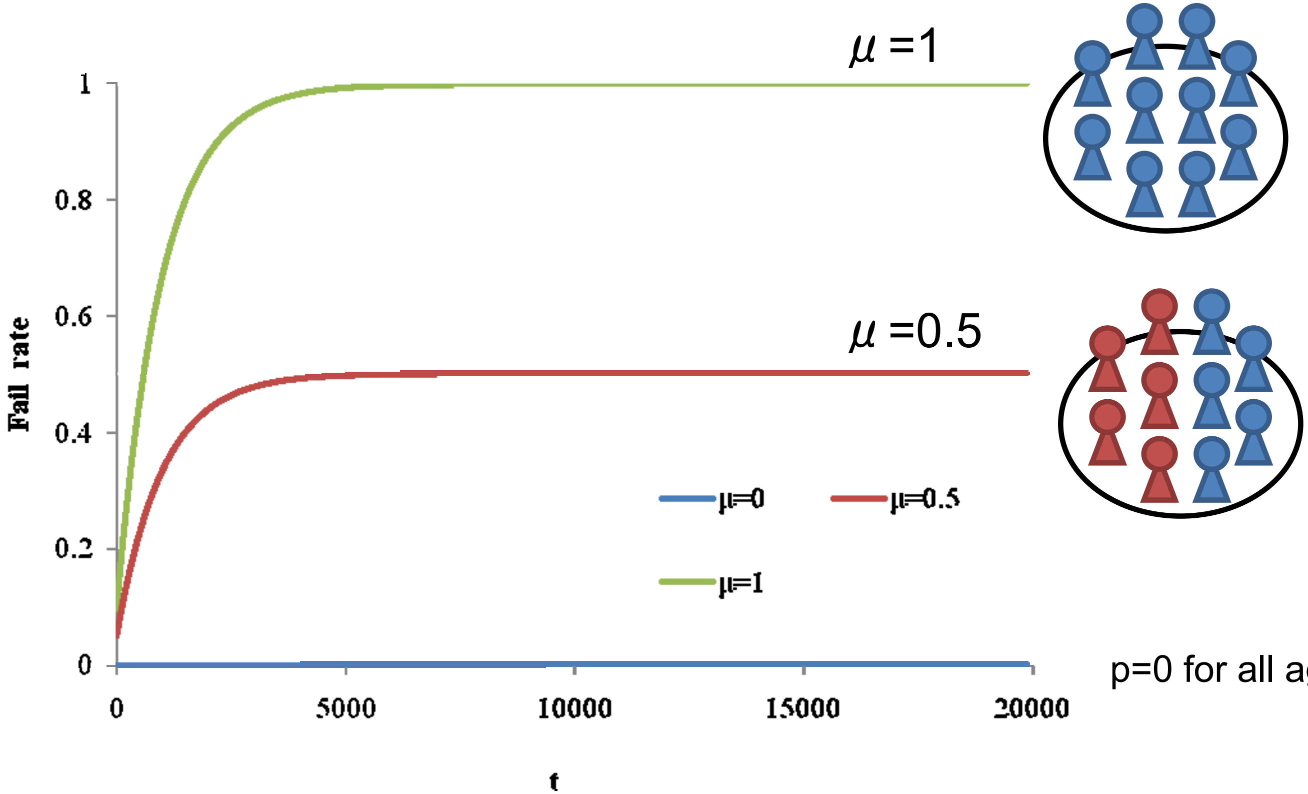
$\alpha \in [0, 1]$: the weight between endogenous and exogenous factors

Experiment 1: Only external risk

$$p(t+1) = (1-\alpha)p + \alpha F(t)$$

α = 0: no endogenous risk

μ : the ratio of type- 1 agents who are sensitive to exogenous risk

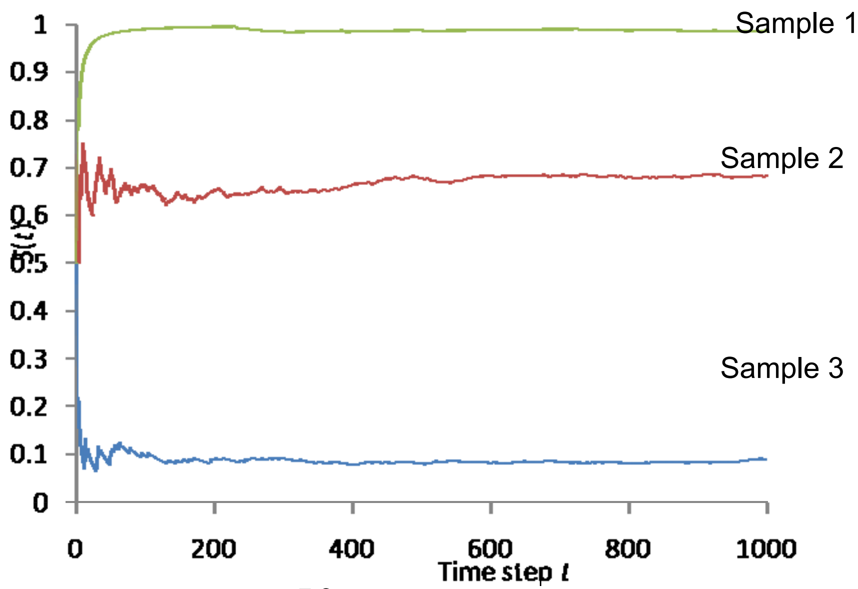


p=1 for all agents

p=1 for a half of agents

p=0 for all agents

Experiment 2: Only endogenous risk

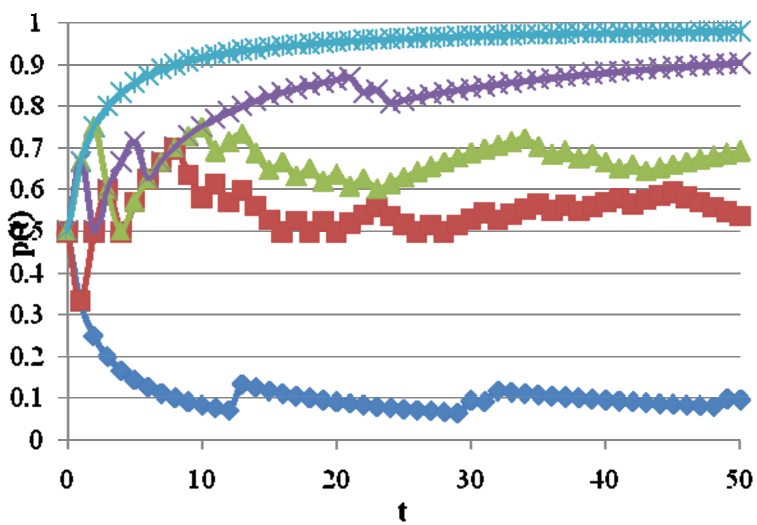


$$p(t+1) = (1-\alpha)p + \alpha F(t)$$

α = 1: only internal influence

Martingale Property

Early stages up to t=50

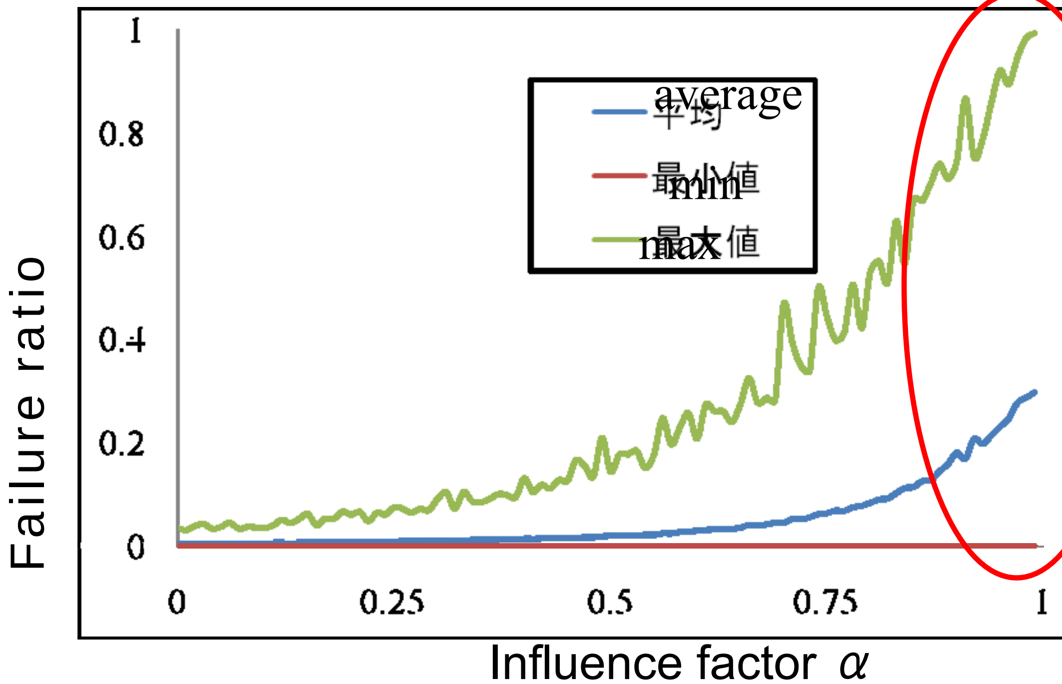


Experiment 3: compound of exogenous and endogenous risks

$$q(t+1) = (1-\alpha) + \alpha S(t)$$

μ =0

p=0 for all agents

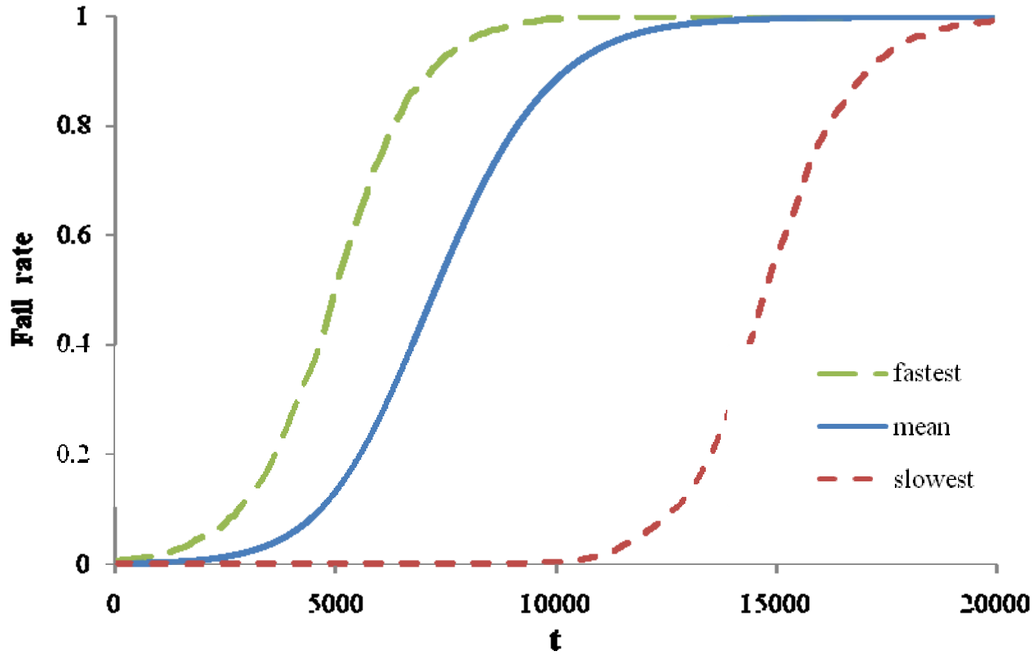


- α is large: harder to predict the size of cascade failure
- α is small: no cascade failure

Experiment 4: repeated exposure to endogeous risk

α = 1: internal risk only

$$q(t+1) = (1-\alpha)p + \alpha F(t)$$

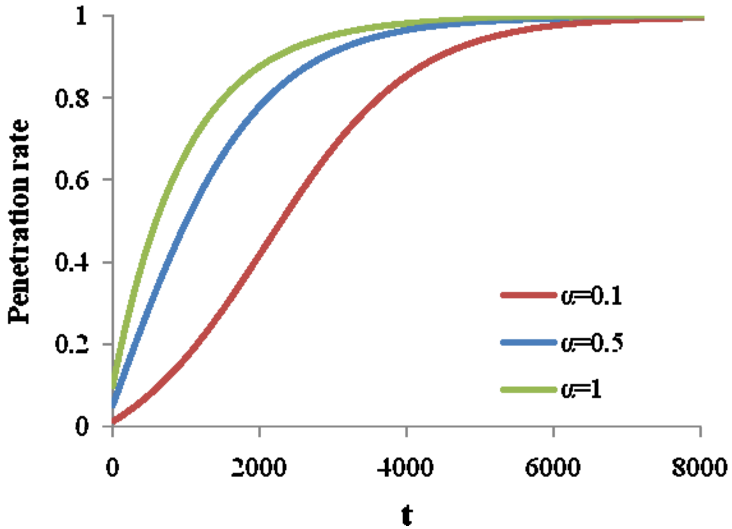


t=10,000 each agents faces 10 times

Diffusion process becomes a S-shape function

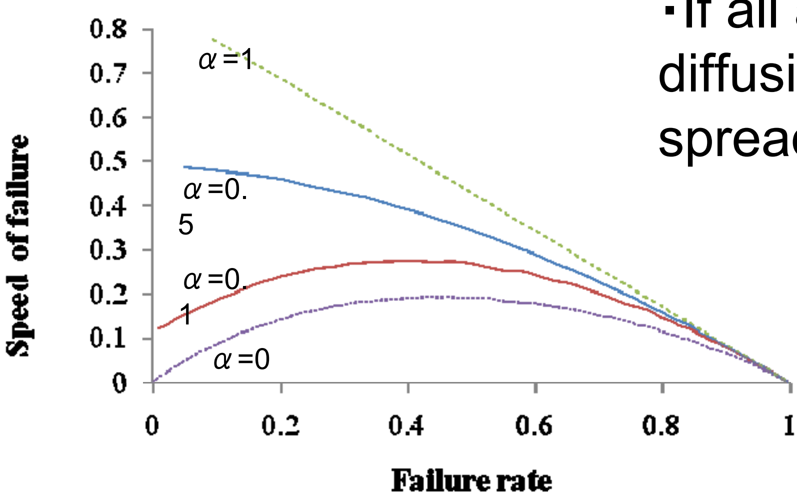
Experiment 5-1: repeated exposure to both exogenous and endogenous risk

μ =1



$$q(t+1) = (1-\alpha)p + \alpha F(t)$$

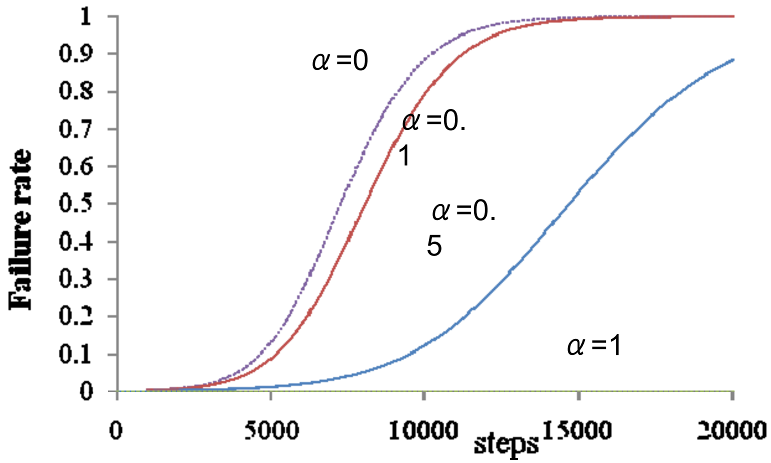
μ =1 p=1 for all agents



- If all agents expose to external shock, the diffusion process is fast and the speed of the spread is faster at the early stages if α is large

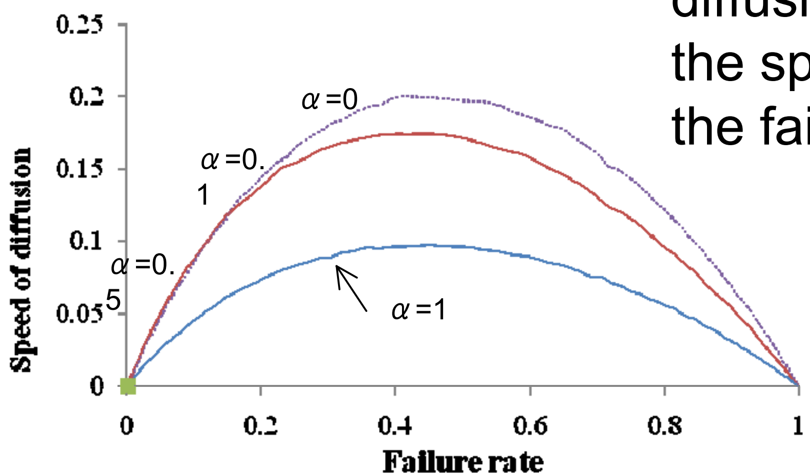
Experiment 5-2: repeated exposure to both exogenous and endogenous risk

$\mu = 0$



$$q(t+1) = (1-\alpha)p + \alpha F(t)$$

$\mu = 0$ p=0 for all agents

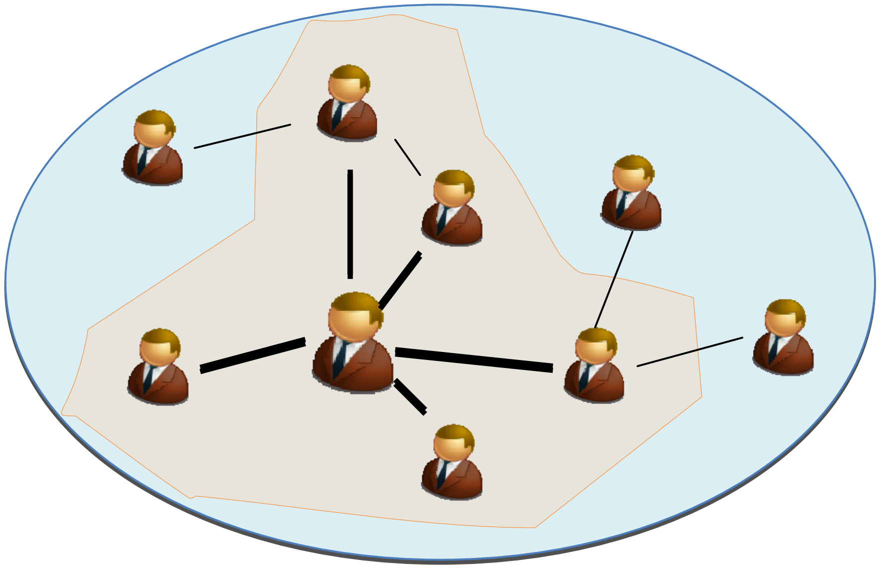


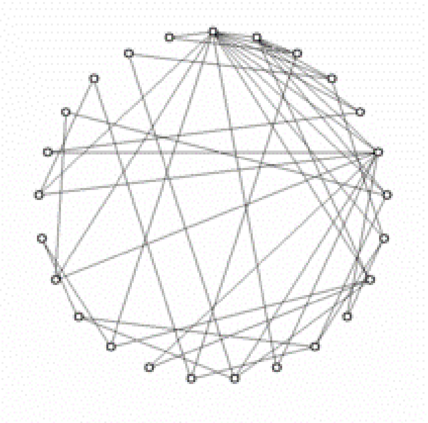
- If no agent exposes to external shock, the diffusion process is very slow the speed of the spread is slow then it accelerate when the failure rate is high

Systemic Risk on Networks (1)

- Regular networks with the same degree:

- Scale-free networks with the average degree: <d>=d

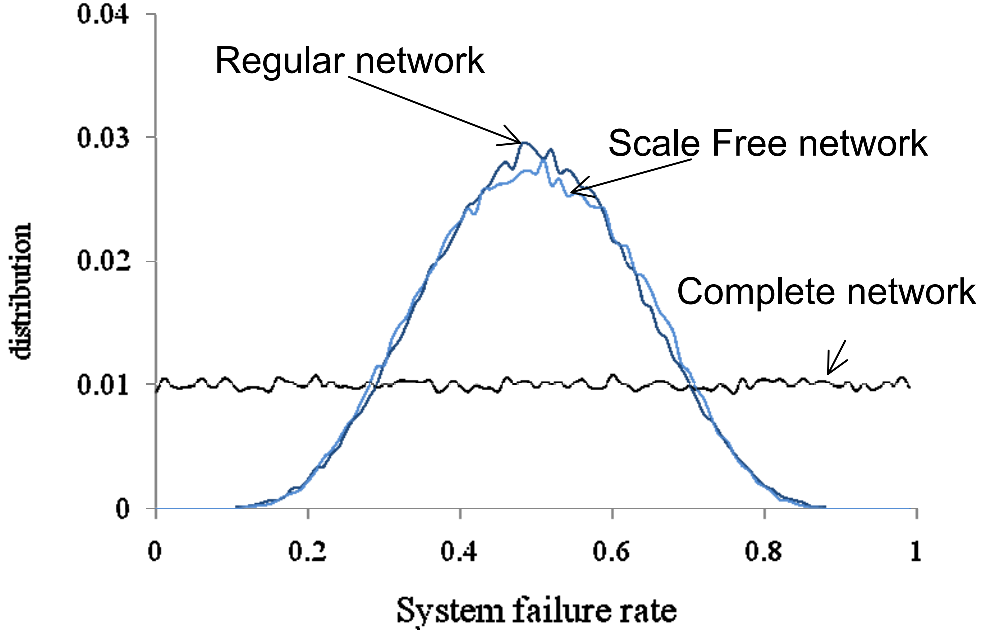




Systemic Risk on Networks (2)

Fail probability: Agents fail only by receiving external influence

$$\boxed{q(t+1) = S(t)}$$



- Too connected networks are difficult to predict

Conclusion (1)

We would like to understand in what sense these kinds of "diffusion" are the same and how they are different from "systemic risk".

- Concepts of "diffusion" arise quite generally in social sciences
  - Diffusion of innovations
  - Spread of infectious disease

- Concept of "systemic risk" arises quite recently
  - Emergence of collective belief
  - Transmission of financial distress

Conclusion (2)

- **Micro-macro link: Individual decisions are influenced by the adoption behavior of the social system.**
- **Martingale property that makes the diffusion (transmission process) to be unpredictable.**

Future works: Further investigation the compound effects of exogenous events and endogenous events.