Simulation Models on Systemic Risks

Akira Namatame

Dept. of Computer Science,

National Defense Academy , Japan

www.nda.ac.jp/~nama

Background (1)

- Systemic Risk
  - The risk caused cascading default of financial institution

Background (2) :
The systemic risk of financial systems

Between 2000 ~ 2007:
29 banks failed.

Between 2007 ~ present:
469 banks failed.

Publications: Systemic risk to the financial system

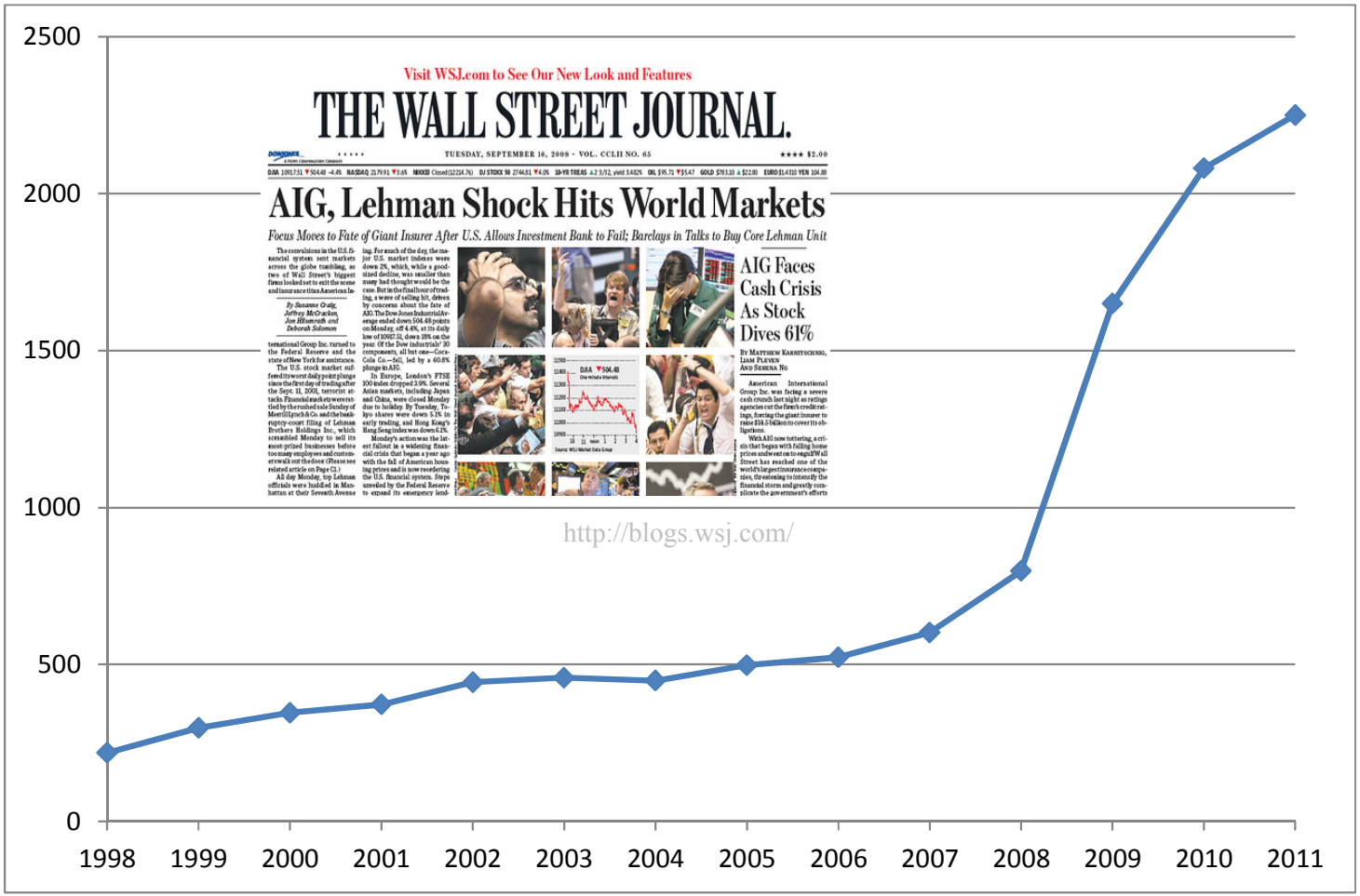
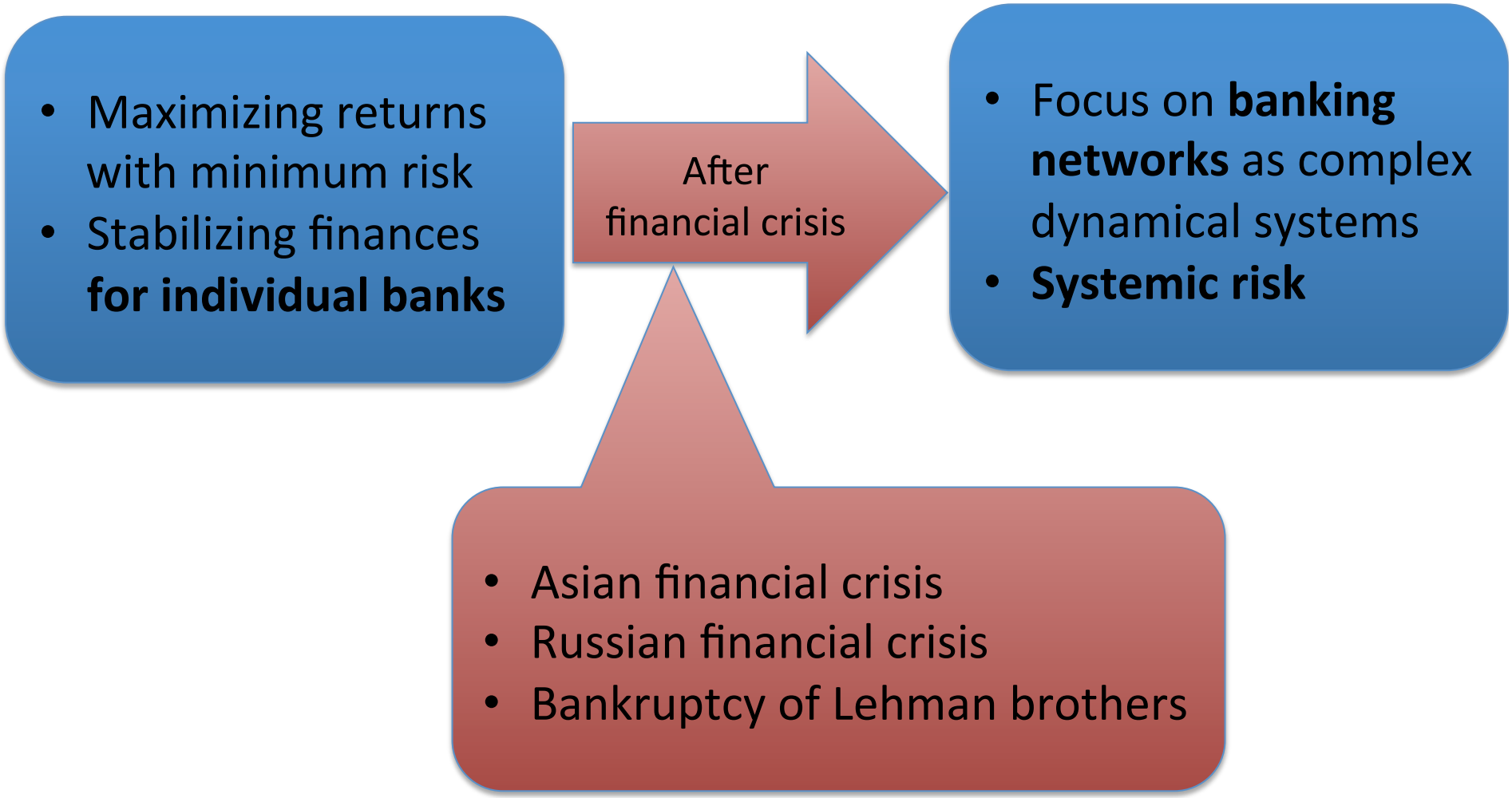


Figure 1. The annual output of publications in the field of systemic risk to the financial system for the period 1998-2011.

Markeloff , R, ”Modeling Systemic Risk to the Financial System” (MITRE, 2012)

Focus of financial study shift



Structure of the Tutorial

"The goal of this tutorial is to understand how simulation models can help us to understand the nature of systemic risks."

More specific topics and questions I intend to address in this tutorial include:

1. Introduction to cascading failures and systemic risks
2. Some fundamental (simplified but useful) models
3. Systemic risk in realistic financial networks
4. How to set up the simulation study on systemic risks?
5. Further extension and conclusion

I. Introduction:

Cascading Failures and Systemic Risks

Failure Cascade in Socio-Economic Networks

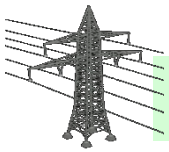
Cascade of failures are common in many socio-economic systems

Internet

Internet congestion collapse

↓

Drop in speed of a factor 100



Power grids

Initial disturbance in some area

↓

Largest blackout

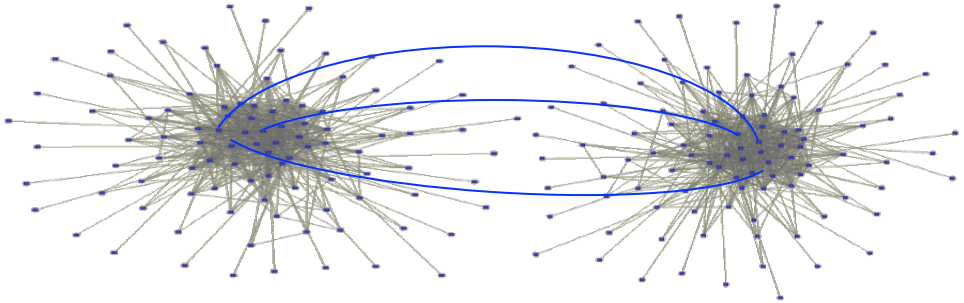
Cascade

Failure Cascade

Systemic risks is also referred to cascade of failures or chains of bad reactions.

- How a failure of the bank induces other failures?

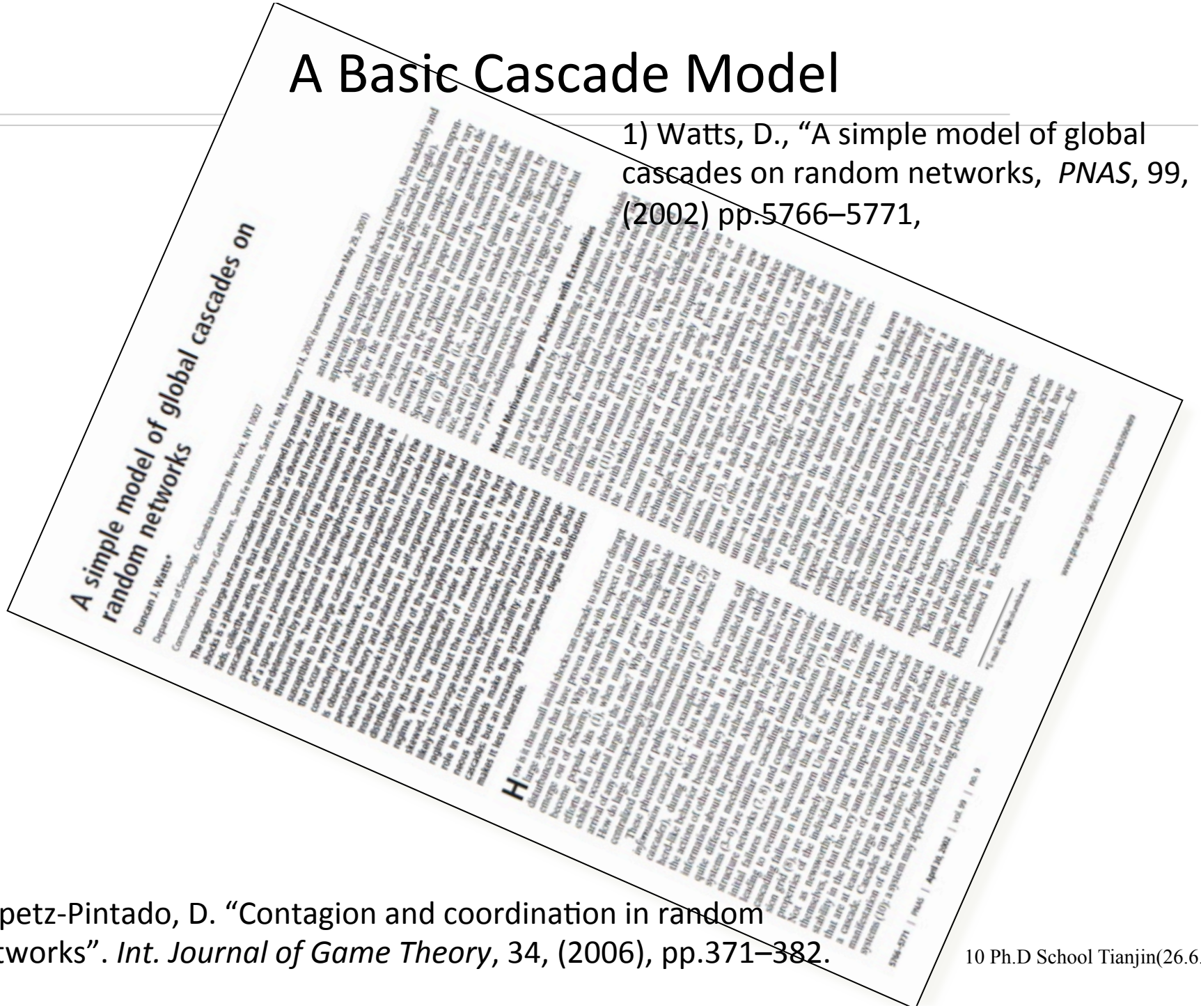
: Insolvency (default) and liquidity shock



Coupled financial networks

Domino effect

A Basic Cascade Model


A simple model of global cascades on random networks

Duncan J. Watts*

Department of Sociology, Columbia University, New York, NY 10027

Communicated by Murray Gell-Mann, Santa Fe Institute, Santa Fe, NM, February 14, 2002 (received for review May 29, 2001)

The origin of large but rare cascades that are triggered by small initial shocks is a phenomenon that manifests itself as diversely as cultural fads, collective action, the diffusion of norms and innovations, and cascading failures in infrastructure and organizational networks. This paper presents a possible explanation of this phenomenon in terms of a sparse, random network of interacting agents whose decisions are determined by the actions of their neighbors according to a simple threshold rule. Two regimes are identified in which the network is susceptible to very large cascades—herein called global cascades—that occur very rarely. When cascades are limited in size, their sizes are power law distributed with an exponent that depends on the mean number of neighbors for an agent. These results rest on percolation theory and are derived by using techniques from random graph theory. Simulation study confirms these analytical results. It is shown how heterogeneity in the threshold distribution allows cascades to propagate in otherwise globally stable networks and thereby extends current understanding of cascades in both directed and undirected networks. Finally, the relevance of this work for applied problems is discussed, and it is argued that the most practical advice for reducing the likelihood of large cascades is to identify and remove nodes with low thresholds rather than to focus attention on highly connected nodes.

How is it that small initial shocks can cascade to affect or disrupt large systems that have proven stable with respect to similar disturbances in the past? Why do some books, movies, and albums emerge out of obscurity, and with small marketing budgets, to become popular hits (1), when many *a priori* indistinguishable efforts fail to rise above the noise? Why does the stock market exhibit occasional large fluctuations (2) when most price changes are small and uncorrelated with one another? How do large, geographically extended organizations (3) maintain control of public communication (3)? These phenomena are all examples of what economists call information cascades (ref. 4, but which are herein called simply cascades), during which individuals in a population exhibit herd-like behavior because they are making decisions based on the actions of other individuals rather than relying on their own information about the problem. Although they are ubiquitous in social and economic systems (5–9) are similar to cascading failures in physical systems (10), a system may appear stable for long periods of time

and withstand many external shocks (robust), then suddenly and apparently inexplicably exhibit a large cascade (fragile). Although the specific origin of a cascade is idiosyncratic, cascades in many systems can be explained in terms of the connectivity of the network by which influence is transmitted between individuals. Specifically, this paper addresses the set of conditions under which an initial shock to a single node or small group of nodes can trigger a cascade that affects a significant fraction of the network as a whole. Simulation results indicate that (i) global cascades occur rarely relative to the number of cascades that are initiated, but (ii) when they occur, they affect a large fraction of the network, and (iii) global cascades occur relatively frequently for a wide range of parameter values. Finally, it is shown that the most connected nodes are far more likely than average nodes to trigger cascades, but not to the second order, where the distribution of network neighbors is highly skewed, it is found that the most connected nodes are far more likely than average nodes to trigger cascades, but not to the second regime. Finally, it is shown that heterogeneity plays an ambiguous role in determining a system's stability: increasingly heterogeneous thresholds make the system more vulnerable to global cascades; but an increasingly heterogeneous degree distribution makes it less vulnerable.

Model Motivation: Binary Decisions with Externalities

This model is motivated by considering a population of individuals each of whom must decide between two alternative actions, where decisions depend explicitly on the actions of other members of the population. In social and economic systems, decision makers often pay attention to each other because they have limited information about the problem itself or limited ability to process it. Even when we have access to potential information, such as when we rely on the recommendations of friends, colleagues, or advisors, to other decision makers have an incentive to make some of it, hence, again we rely on the advice of others. And in other problems (3) or social dilemmas (13), an individual's payoff is an explicit function of the actions of others. To take an extreme example, the decision to adopt a new technology (14), the utility of a single additional unit—a fax machine for example—may depend on the number of units that have already been sold. In all these problems, therefore, regardless of the details, individual decision makers have an incentive to pay attention to the decisions of others.

In economic terms, this entire class of problems is known generically as binary decision with externalities (6). As simplistic as it appears, a binary decision with externalities is relevant to surprisingly complex problems. To take an extreme example, the creation of a political coalition or an international treaty is empirically a complex, multifaceted process with many potential outcomes that one the coalition exists or the treaty has been drafted, the decision of whether or not to join is essentially a binary one. Similar reasoning applies to a firm's choice between two neighborhood restaurants—the factors involved in the decision may be many, but the decision itself can be regarded as binary.

Both the detailed mechanisms involved in binary decision problems, and also the origins of the externalities can vary widely across specific problems. Nevertheless, in many applications that have been examined in the economics and sociology literature—for

(1) Watts, D., "A simple model of global cascades on random networks, *PNAS*, 99, (2002) pp.5766–5771,

(2) Lopetz-Pintado, D. "Contagion and coordination in random networks". *Int. Journal of Game Theory*, 34, (2006), pp.371–382.

A Threshold Model

: Each node takes the binary state: x=0 (normal) or x=1 (fail)

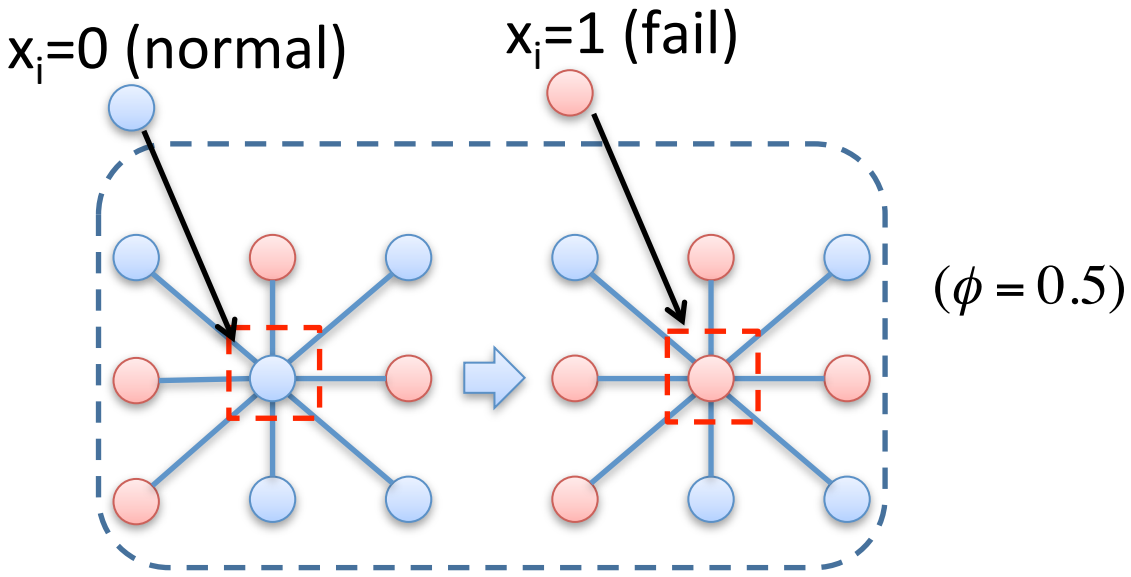
: The normal node shifts to failure if more neighboring nodes fail

$If\ \ p_i \equiv \sum_{j \in N_i} x_j / k_i \geq \phi$, then node I shifts from x_i=0 (normal) to x_i=1 (fail)

ϕ : threshold of a node

k_i : the degree (the neighbors) of node i

p_i : the ratio of the failed neighbors of node i



Cascade Condition (1)

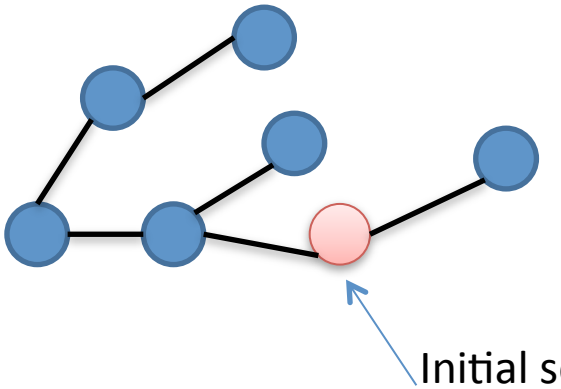
Threshold rule:

$If \sum_{j \in N_i} x_j / k_i \geq \phi$, then node i shifts from the state: x_i=0 (normal) to x_i=1 (fail)

Under this threshold rule, the initial failure can grow if one of its immediate nodes has a threshold such that ： $\phi < 1/K$

We concentrate on two quantities:

- The probability that a cascade (a large-scale failure) will be triggered by a single initial seed
- The expected size of cascade once it is triggered



Cascade Condition (2)

p(k): The degree distribution of a network

There exits a critical threshold of the state x=1, ϕ^*, such that contagion occurs if $\phi < \phi^*$.

The critical threshold ϕ^* is obtained by solving

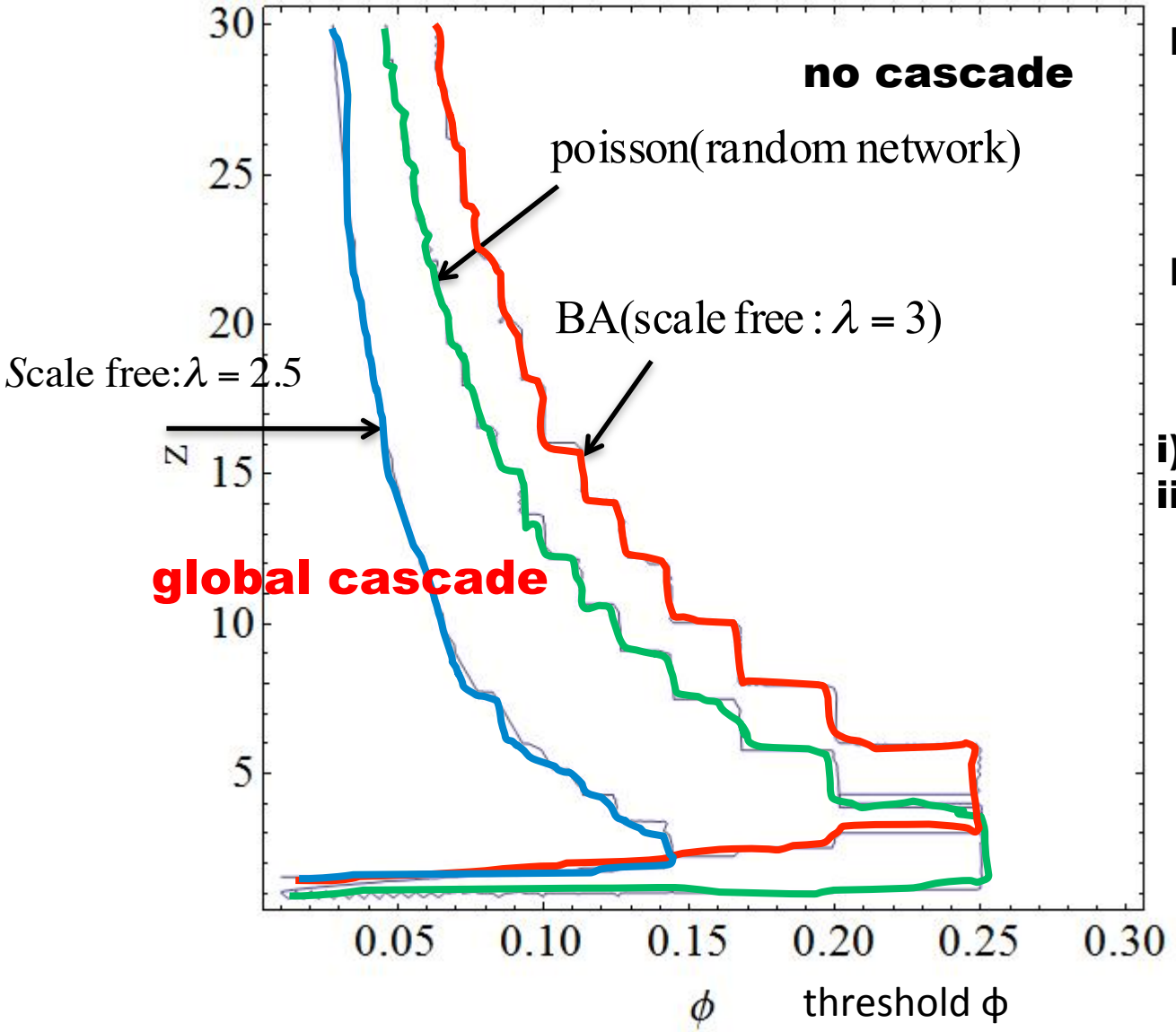
$$\phi^* = \arg\min_{\phi} \frac{1}{z} \sum_{k=1}^{[1/\phi]} k^2 p(k) \qquad (z \equiv \sum_k kp(k))$$

subject to $$\frac{1}{z} \sum_{k=1}^{[1/\phi]} k^2 p(k) \geq 1$$

The Cascade Window

:The cascade widow depends on the network topology.

z: average degree



Poisson distribution (random network)

$$p(k) = e^{-z} z^k / k!$$

Power law distribution

$$p(k) = Ck^{-\lambda}$$

i)BA(Barabasi-Albert) $\lambda = 3$

ii)Scale Free $\lambda = 2.5$

II. Fundamental Models of Systemic Risks

Fundamental model 1: “Eisenberg-Noe Model”

Eisenberg, L, Noe,.H., (2001). “Systemic risk in financial systems”. Management Science , 47 (2), 236–249.

Important Questions

1) Will the failure of a financial institution trigger the subsequent failure of others? This is perhaps the most important question financial supervisors have to answer when faced with an institution in distress.

2) There has been relatively little empirical work in systemic risk. In part this is because most institutions whose failure could give rise to contagion are rescued before they collapse

3) Economists studying contagion have therefore resorted to simulation methods to test whether, given a given a particular set of exposures, failures could have knock-on effects.

Modeling of Financial Network (1)

- Node: Bank

- Links: Interbank liabilities

Modeling of Financial Network (2)

- directed links

Bank B owes 100 M $ to Bank A.

The default of Bank B makes the loss for Bank A.

Bilateral Exposures and Interbank Markets in a Matrix Form

The interbank market can be represented as a NxN matrix, X.

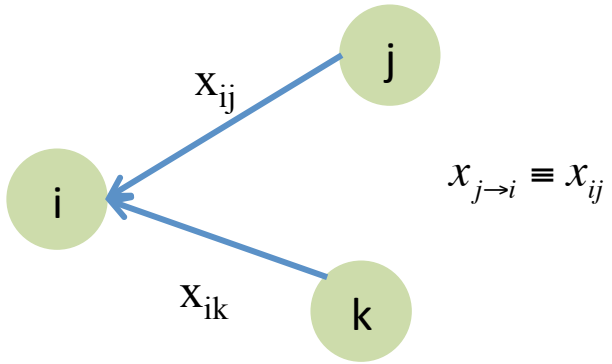
$$X = \begin{bmatrix} 0 & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN} \end{bmatrix}$$

Asset of bank i

$$\sum_{j=1}^{N} x_{ij} = a_i$$

Liability of bank j

$$\sum_{i=1}^{N} x_{ij} = l_j$$



Modeling of Systemic Risk

Eisenberg and Noe (2001) propose the contagion (default) model with clearing mechanism accounts for interbank linkages as follows:

(1) N financial institutions

- Net worth z_i: Bank i's realized value of assets minus liabilities excluding inter-banking relationships.
- x_{ji}: Bank i's debt obligation toward Bank j
- Bank i's total liabilities : $\sum_{j=1}^{N} x_{ji} = l_i$

(2) Default if $z_i + \sum_{j=1}^{N} x_{ij} - \sum_{j=1}^{N} x_{ji} < 0 \quad for\ some\ i$

(3) Propagate through a clearing mechanism

- Bank i pays a fraction θ_i of its liability in case of default
- Compute the joint determination of $\boldsymbol{\theta}=(\theta_i)$
- Without any default, $\mathbf{1}=(1,1..,1)$ is the clearing vector.

How to Estimate Precise Bilateral Exposures?

The matrix of bilateral exposures

$$X = \begin{vmatrix} 0 & 222.91 & 138.37 & 129.28 & 109.64 & 105.29 & \dots \\ 221.42 & 0 & 124.15 & 116.34 & 104.96 & 100.80 & \dots \\ 126.66 & 122.08 & 0 & 70.80 & 60.04 & 57.66 & \dots \\ 118.78 & 114.48 & 71.07 & 0 & 56.31 & 54.07 & \dots \\ 105.10 & 101.29 & 62.88 & 58.74 & 0 & 47.84 & \dots \\ 95.87 & 92.40 & 57.36 & 53.58 & 45.44 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{vmatrix}$$

- No matter how sophisticated, simulations of contagion are only as good as the data they are based on.
- Information on bilateral exposures in the interbank market is scarce and often of limited quality. Individual exposures are therefore intrinsically unobservable for anybody except by the immediate counterparties.
- In the absence of direct information on bilateral lending, the elements of X can be estimated from banks' balance sheets using maximum entropy methods .

Estimation of Bilateral Exposures

- The data provide instrument-specific total holdings of assets and liabilities by each sector
- Can use information on the relative distribution of the sum elements $a_{i,k}$ and $l_{j,k}$ to estimate the individual elements $X_{i,j}$ for each instrument category => provides the who-to-whom links

$$X_k = \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN} \end{bmatrix} \qquad \sum_{j=1}^{N} x_{ij} = a_{i,k}$$

$$\sum_{i=1}^{N} x_{ij} = l_{j,k}$$

Estimation of Bilateral Exposures with the ME (maximum entropy)

$$\boldsymbol{X} = \begin{array}{c|c|c} & \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & & \vdots & & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & & \vdots & & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN} \end{bmatrix} & \begin{matrix} \Sigma_j \\ a_1 \\ \vdots \\ a_i \\ \vdots \\ a_N \end{matrix} \\ \hline \begin{matrix} \Sigma_i \\ \\ \end{matrix} & \begin{matrix} l_1 & \cdots & l_j & \cdots & l_N \end{matrix} & \end{array}$$

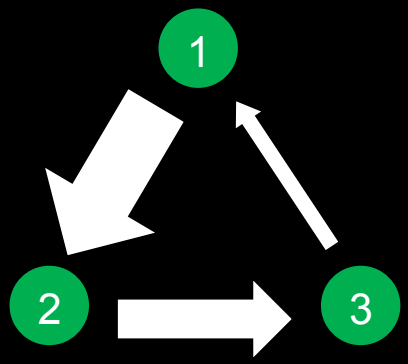
$$a_i = \sum_j x_{ij} \quad \text{and} \quad l_j = \sum_i x_{ij} \ ,$$

With independent assumption between asset a and loan l, we can estimate

$$x_{ij} = a_i l_j \ .$$

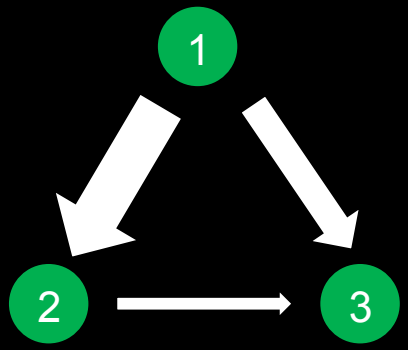
"Estimating Bilateral Exposures in the German Interbank Market" C. Upper, A. Worms (2002)

Small Example



Actual matrix

	bk 1	bk 2	bk 3	
bank 1	0	0.6	0	0.6
bank 2	0	0	0.3	0.3
bank 3	0.1	0	0	0.1
	0.1	0.6	0.3	1



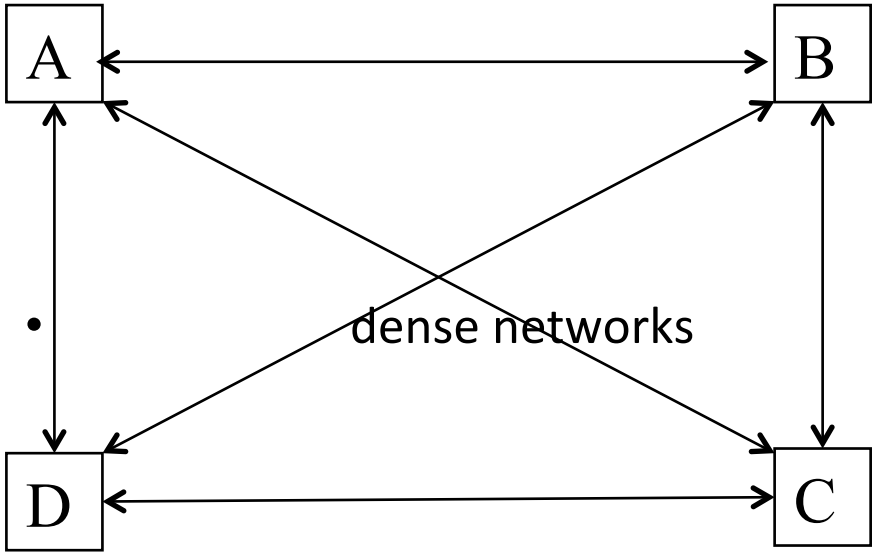
ME's estimate from total exposures

	bk 1	bk 2	bk 3	
bank 1	0.06	0.36	0.18	0.6
bank 2	0.03	0.18	0.09	0.3
bank 3	0.01	0.06	0.03	0.1
	0.1	0.6	0.3	1

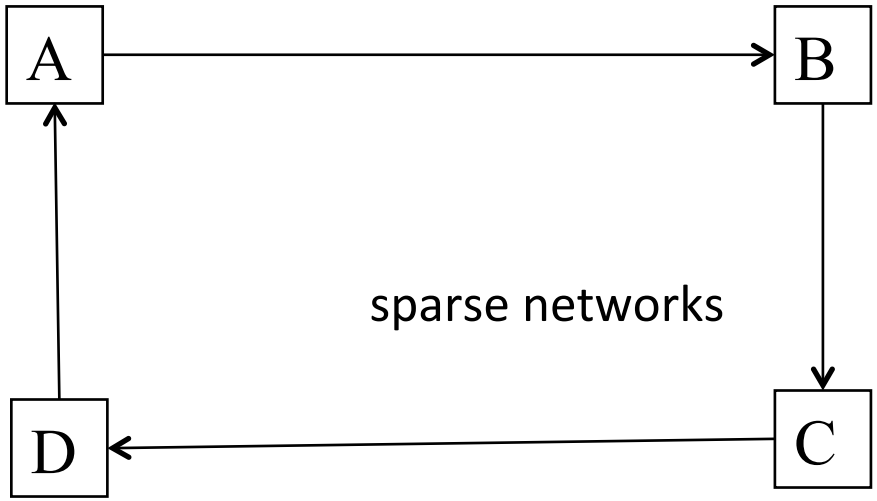
Basic Features of Financial Networks

: increasing complexity, : decreasing diversity

- Dense networks; each bank can make deposits in every other bank.
- Sparse networks: links are limited



- dense networks



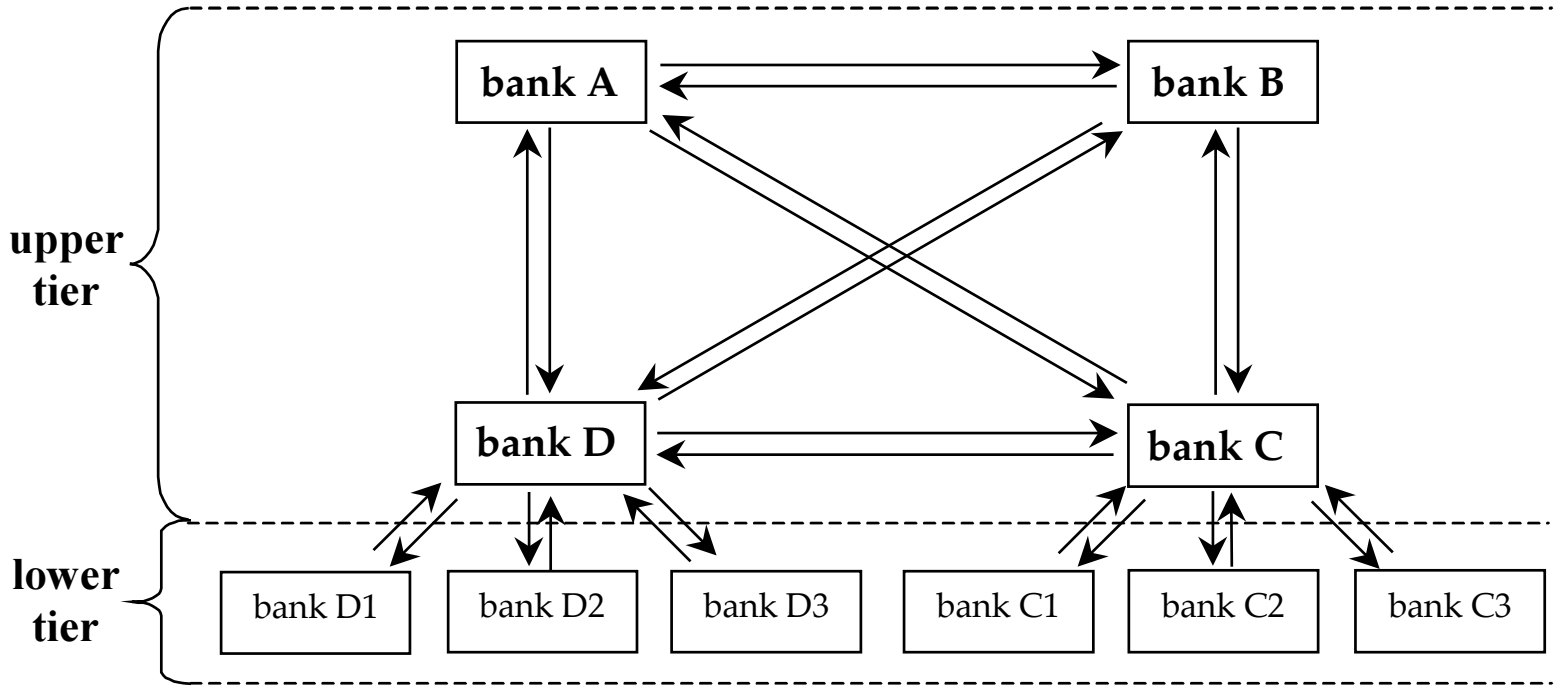
sparse networks

- A small aggregate shock can lead the whole financial system to collapse in sparse networks even though with complete markets there would be no contagion

Allen, F., Gale, D., (2000). “Financial contagion”. Journal of Political Economy” 108 (1), 1–33.

Basic Features of German Interbank Market

Two-tier structure of the German interbank market



"Estimating Bilateral Exposures in the German Interbank Market" C. Upper, A. Worms (2002)

Simulation Models of Systemic Risk

Upper (2011) reviewed 15 recent studies of specific financial system, and identifies two major shortcomings:

1) An exaggerated focus on scenarios involving idiosyncratic failure of a single bank, rather than a market shock;
2) More important is the absence of “behavioral foundations” that preclude different channels for contagion. x_{ij}

:The past studies assume “banks sit tight as problems of their counterparties mount”.
:We have seen that “asset hoarding” and “selling assets” are both rational responses that make systemic risk higher.

Upper, C. (2011) "Simulation Methods to Assess the Danger of Contagion in Interbank Markets", *Journal of Financial Stability*, 7(3): 111-125.

Limitations

The models proposed by Allen and Gale (2000) and Eisenberg, L, Noe,.H., (2001) have initiated the study on systemic risks, but it is not clear to what extent these can be extrapolated to more complex networks observed in the real world.

Therefore, researchers have turned to simulations to study contagion in more complex systems.

Structure of the Tutorial

"The goal of this tutorial is to understand how simulation models can help us to understand the nature of systemic risks."

More specific topics and questions I intend to address in this tutorial include:

1. Introduction to cascading failures and systemic risks
2. Some fundamental (simplified but useful) models
3. Systemic risk in realistic financial networks
4. How to set up the simulation study on systemic risks?
5. Further extension and conclusion

II. Four Fundamental Models

Fundamental Model 2: "GK Model"

"Contagion in Financial Networks"
Gai, P and Kapadia,S. Proc. R. Soc. Interface, 466, 2010

- This is a very basic model of contagion through shocks to the asset side of the balance sheet.
- It is designed to extend the Watts's cascade model as directed-network version.

The assets of bank

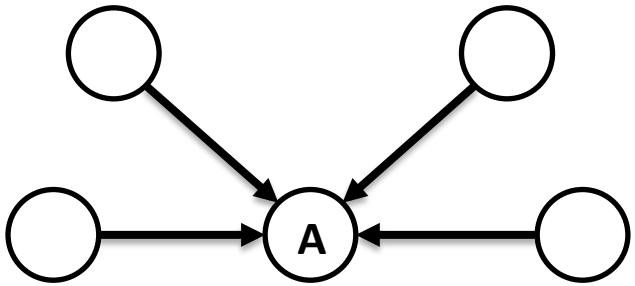
- The total assets of each bank consists of interbank assets, and illiquid external assets.

$$\text{Total assets of bank i} = A_i^M \text{(Interbank assets)} + A_i^M \text{(Illiquid external assets)}$$

$$A^{IB}_{\text{Bank A}} = 500\text{M\$}$$

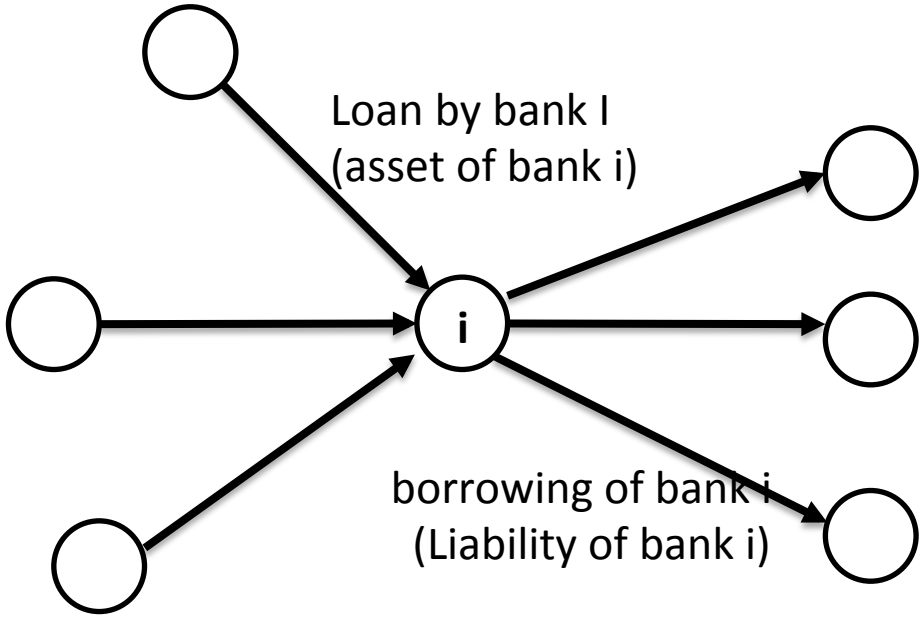
The total interbank asset position of every bank is evenly distributed over each of its incoming links.

=



Each directed link represents the liability of 125M$.

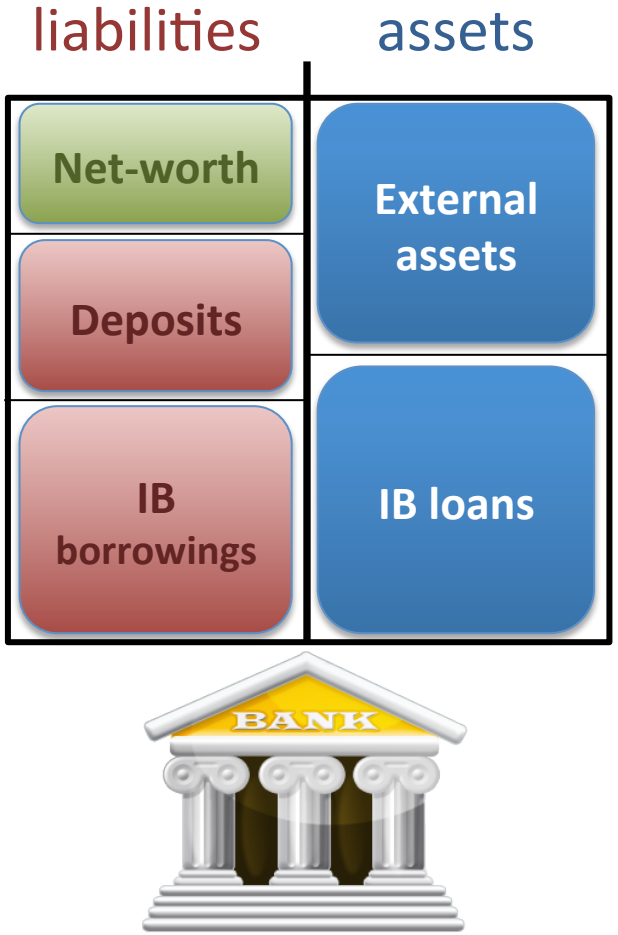
Skeleton Structure of the Network
Location of Bank i



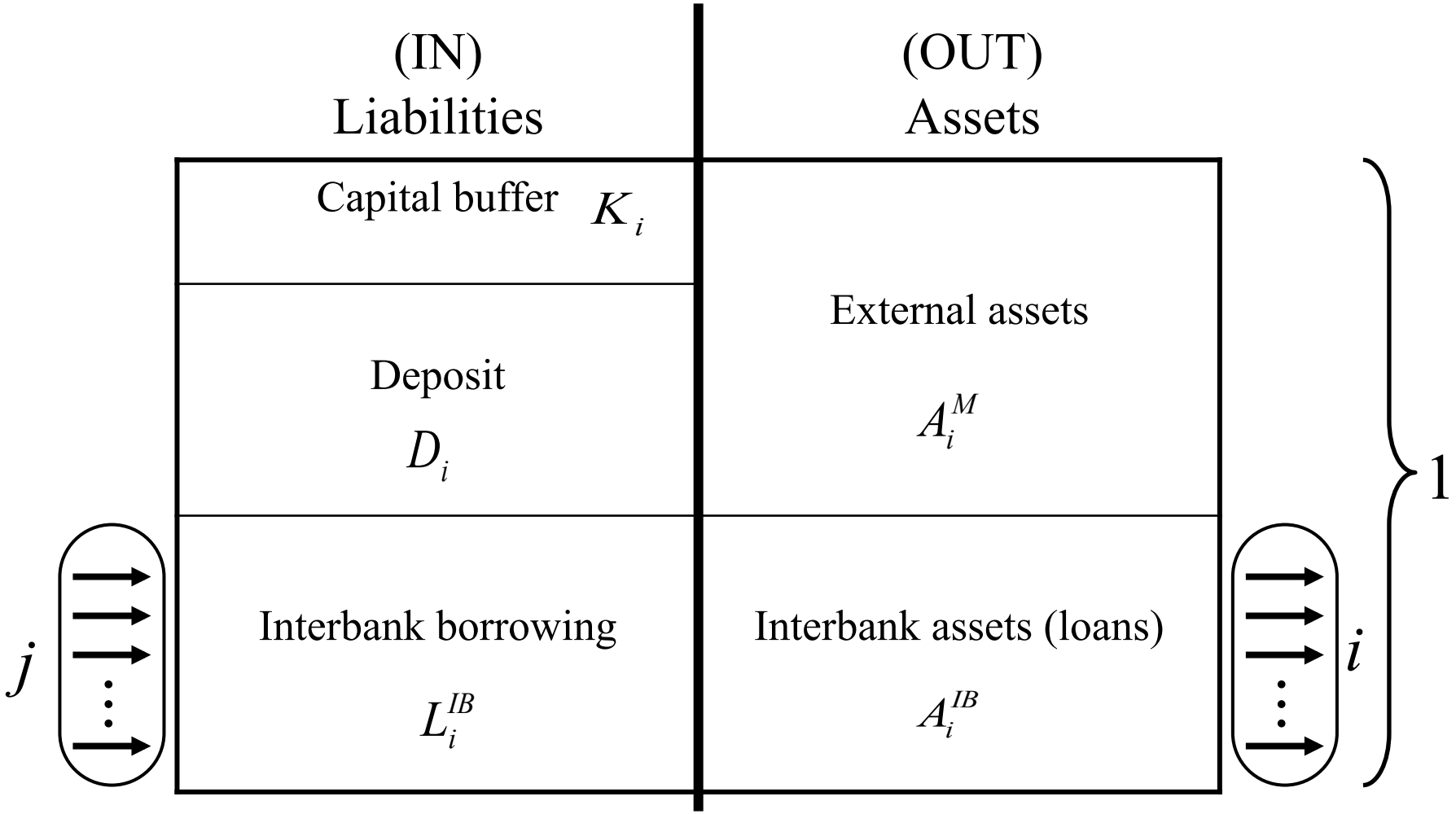
Debtors of bank i

Creditors of bank i

A Stylized Balance Sheet Model



Schematic model for a node with interbank network (Balance sheet)



q : resale price of the illiquid asset

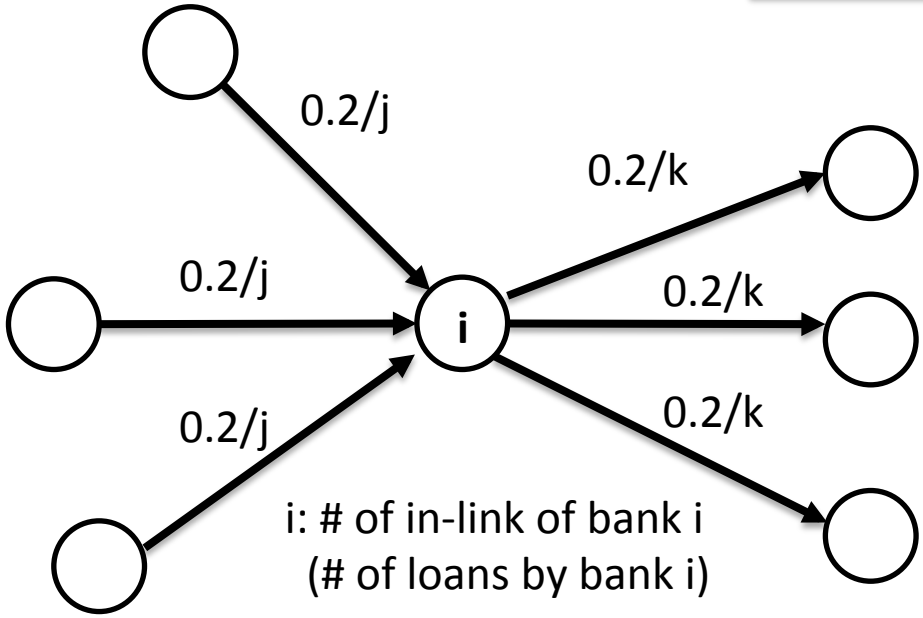
K_i : net-worth (capital buffer)

$$K_i = A_i^M + A_i^{IB} - L_i^{IB} - D_i$$

Financial Network: Interbank Loan to Each Bank

Total asset:1

IB loan(A^{IB})=0.2



j: #of debtors of bank i

(# of in-link of bank i)

k: #of creditors of bank i

(# of out-link of bank i)

Solvency Condition (1)

$$(1\text{-p})\,A_i^{IB} + qA_i^{M} - L_i^{IB} - D_i > 0$$

A_i^{IB} : Intterbank asset (loans)

L_i^{IB} : Interbank liabilities

D_i : Customer deposit

p: the fraction of banks with obligations to bank i have defaulted

q : the resale price of the illiquid asset.

Binary rule of each node i

Bank i suvives if: $(1\text{-p})\,A_i^{IB} + qA_i^{M} - L_i^{IB} - D_i > 0$

Bank i defaults if: $(1\text{-p})\,A_i^{IB} + qA_i^{M} - L_i^{IB} - D_i < 0$

Solvency Condition (2)

Default condition: $(1\text{-p})\,A_i^{IB} + qA_i^M - L_i^{IB} - D_i < 0$

$$\frac{A_i^{IB} + qA_i^M - L_i^{IB} - D_i}{A_i^{IB}} < p$$

$$\frac{(A_i^{IB} + A_i^M - L_i^{IB} - D_i) - (1-q)A_i^M}{A_i^{IB}} < p$$

$$\frac{K_i - (1-q)A_i^M}{A_i^{IB}} < p \qquad (K_i = A_i^{IB} + A_i^M - L_i^{IB} - D_i : \text{capital buffer})$$

Threshold rule

$$p > \frac{K_i}{A_i^{IB}} \equiv \phi \quad (\text{ if we set } \; q = 1)$$: Bank i defaults

p: the fraction of default banks

$\phi = K_i / A^{IB}$:threshold of bank i

(K_i: the capital buffer, A^{IB}: Interbank loan)

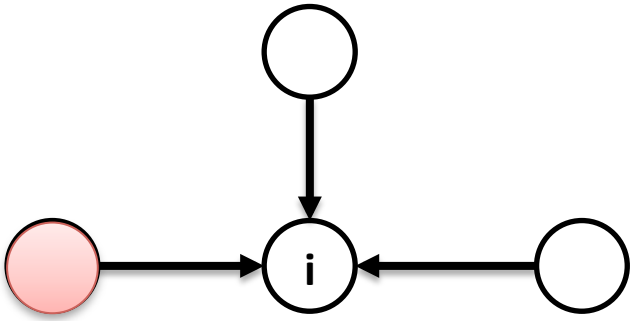
Example: A vulnerable node

Vulnerable nodes that are susceptible to the default of a neighboring node

$$\frac{K_i}{A_i^{IB}} = \phi < p = \frac{1}{d_{in}}$$

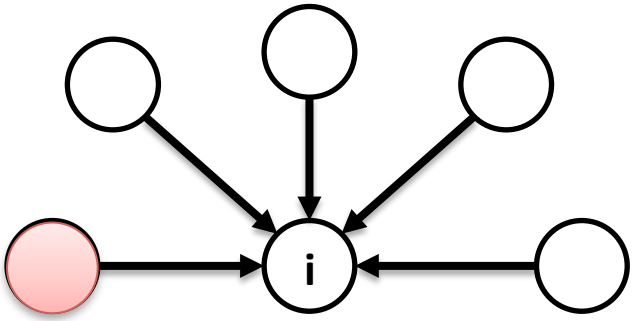
In the case of $\frac{K_i}{A_i^{M}} = \phi = 0.25$

<default>



$$\frac{K_i}{A_i^{IB}} < p = \frac{1}{d_{in}} = \frac{1}{3}$$

<survive>



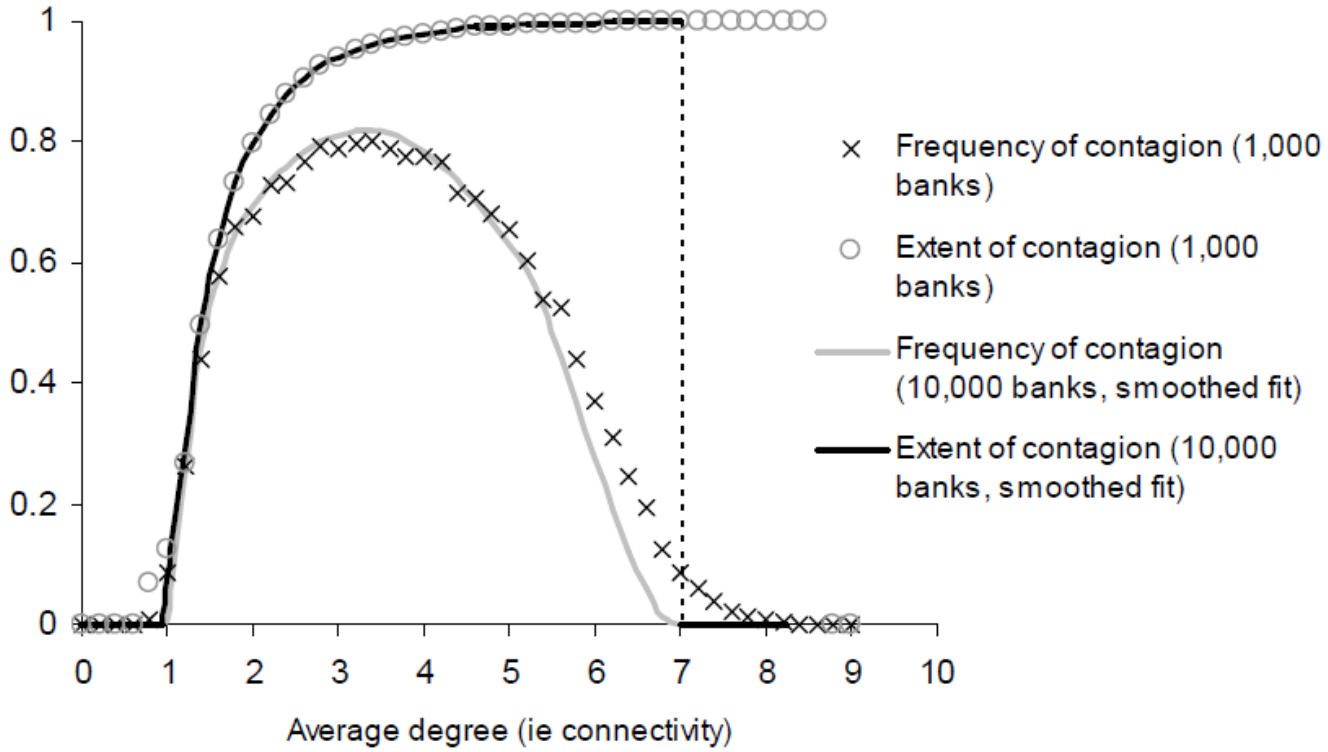
$$\frac{K_i}{A_i^{IB}} > p = \frac{1}{d_{in}} = \frac{1}{5}$$

Simulation Setting

- Network: Random network (ER model)
- # of Banks (nodes): N=1,000
- Average degree z: z=1～10
- Initial seed: 1 (initially one bank defaults)
- $\frac{K_i}{A_i^{IB} + A_i^M} = \frac{A_i^{IB} + qA_i^M - L_i^{IB} - D_i}{A_i^{IB} + A_i^M} = 0.04$ q=1
- $A_i^{IB} = 0.2x, \quad A_i^M = 0.8x, \; x = A_i^{IB} + A_i^M$
- $\frac{K_i}{A_i^{IB}} = \frac{0.04(A_i^{IB} + A_i^M)}{A_i^{IB}} = \frac{0.04(A_i^{IB} + 4A_i^{IB})}{A_i^{IB}} = \frac{0.2A_i^{IB}}{A_i^{IB}} = 0.2$

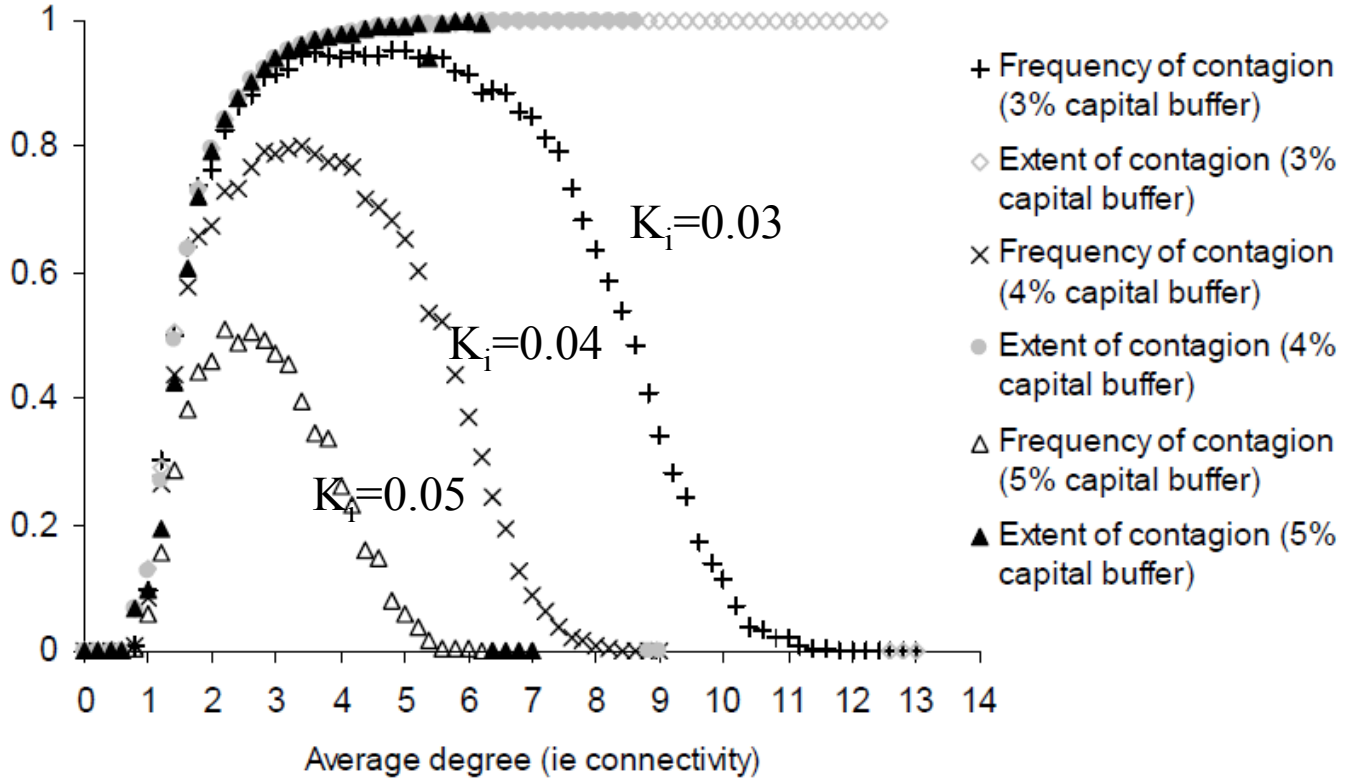
Simulation Results (1)
(Benchmark and analytical solutions compared)

Chart 4: Benchmark and analytical solutions compared



Simulation results (2)
<Effects of increasing K_i : the capital buffer>

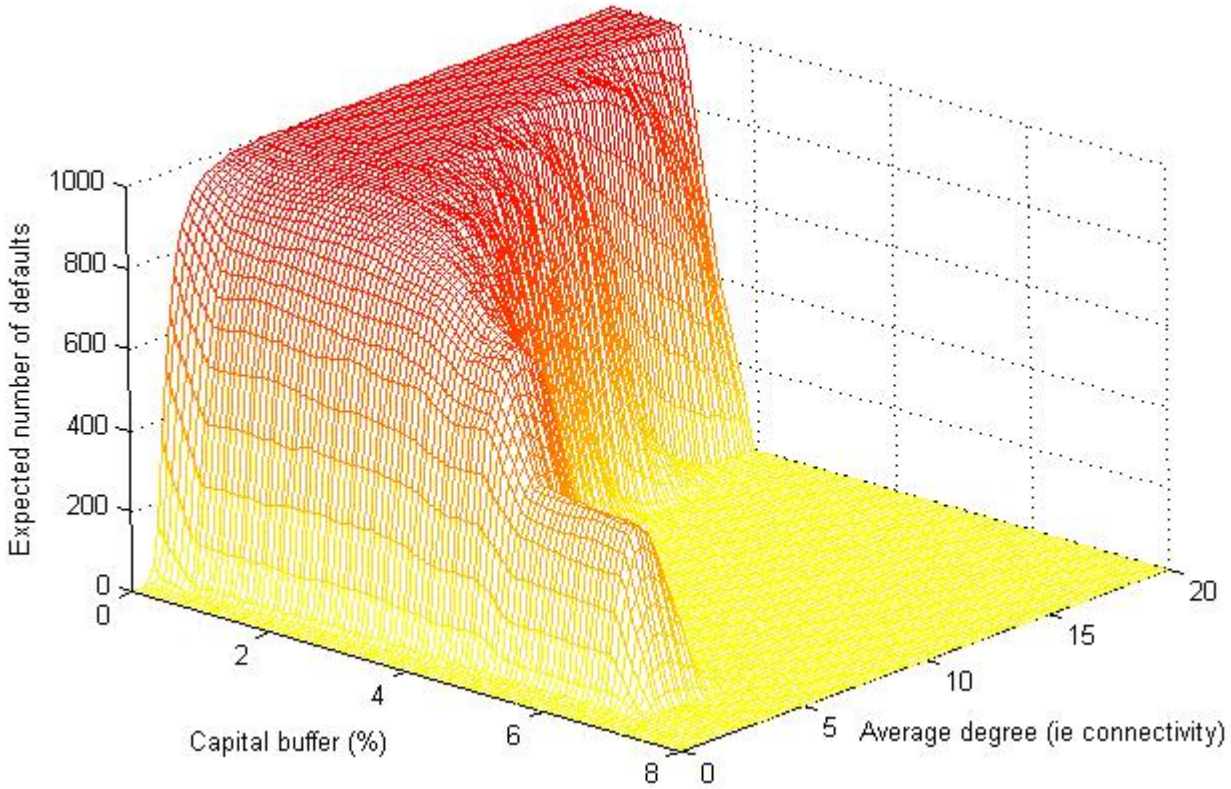
Chart 5: Varying the capital buffer



$$\phi = \frac{K_i}{A_i^{IB}} = \frac{0.03(A_i^{IB} + A_i^{M})}{A_i^{IB}} = \frac{0.03 \times 5A_i^{IB}}{A_i^{IB}} = 0.15 \qquad \phi = \frac{K_i}{A_i^{IB}} = \frac{0.05(A_i^{IB} + A_i^{M})}{A_i^{IB}} = \frac{0.05 \times 5A_i^{IB}}{A_i^{IB}} = 0.25$$

Simulation Results (3)

[# of defaults, capital buffer and average degree]



Conclusion

1. Gai et al. develop a model of contagion in financial networks in an era of rapid financial globalization.
2. This is a very basic model of contagion through shocks to the asset side of the balance sheet. It is designed to mimic the general features of the Watts's model.
3. Their results show financial systems have a "robust-yet-fragile tendency":

Extension

Gai et al. extend this framework to a new cascade problem:

: the hoarding of assets undertaken by stressed banks,

: the transmission of liquidity shocks to the liabilities of its counterparties.

Reference

"Complexity, Concentration and Contagion"
P. Gai, A. Haldane, S. Kapadia, Journal of Monetary Economics, 58, 2011.

II. Fundamental Models

Fundamental Model 3: "NYYA Model"

"Network models and financial stability"

Nier, E., Yang, J., Yorulmazer, T., and Alentorn, A

(1) J. Economic Dynamics and Control, 31, 2007.

(2) Bank of England Working Paper No. 346 , March, 2008

Their model is a so called "toy model" with a small number of banks (N = 25). But their paper was so influential in beginning "network theoretical studies of systemic risk".

: Interbank networks: Random network;

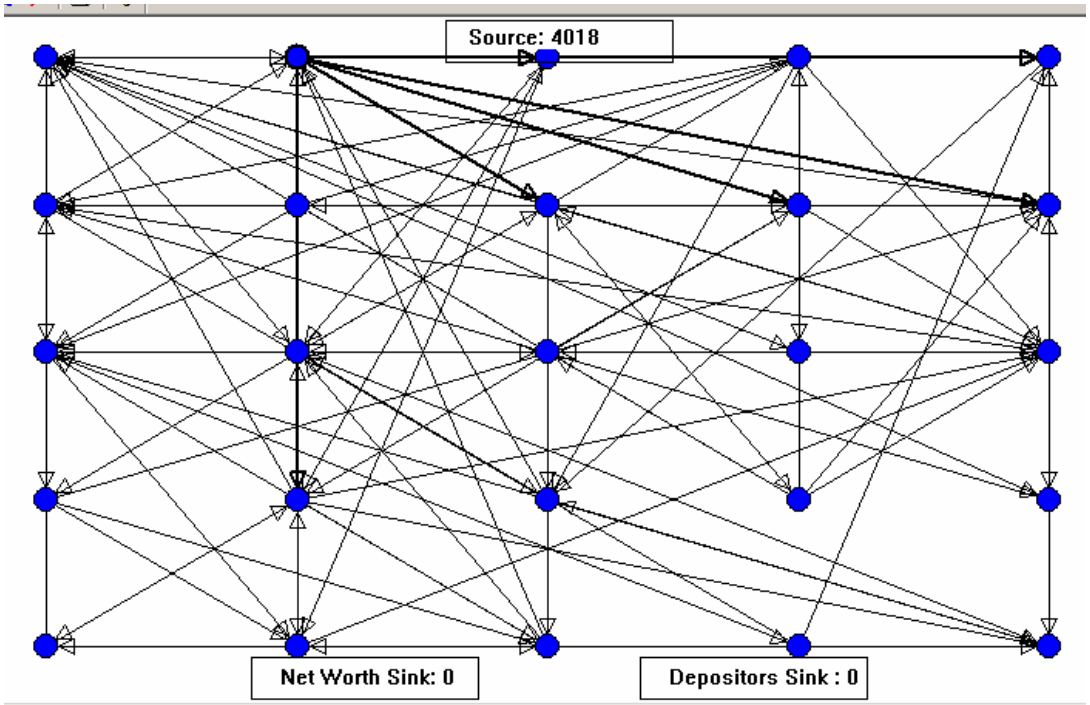
: Assume partial recovery after default

A Basic Model

- N(=25): Number of nodes
- p_{ij} = Probability that bank i has lent to another bank j, which is assumed to be equal across all pairs (i, j):

  p_{ij} ($=p$ = 0.2).
- $z^{+}(z^{-})$: a number of incoming (outgoing) links per node.

Flow of debt (losses)



Individual Bank Assets

- a_i = bank i's assets
- e_i = external assets (investors' borrowing)
- i_i = interbank assets (other banks' borrowing)

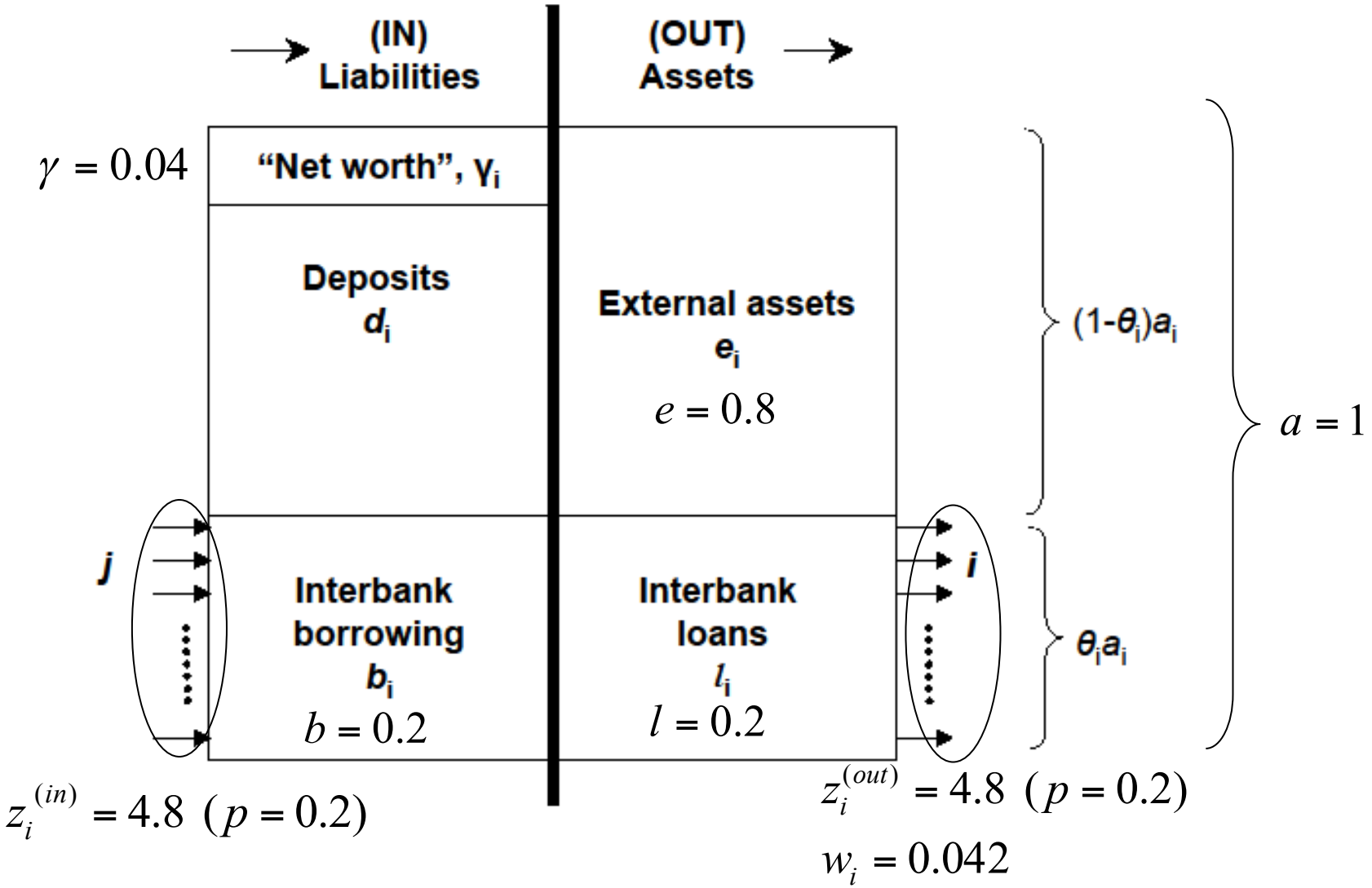
$$a_i = e_i + i_i \quad \text{where } i = 1, \ldots N$$

Individual Bank Liability

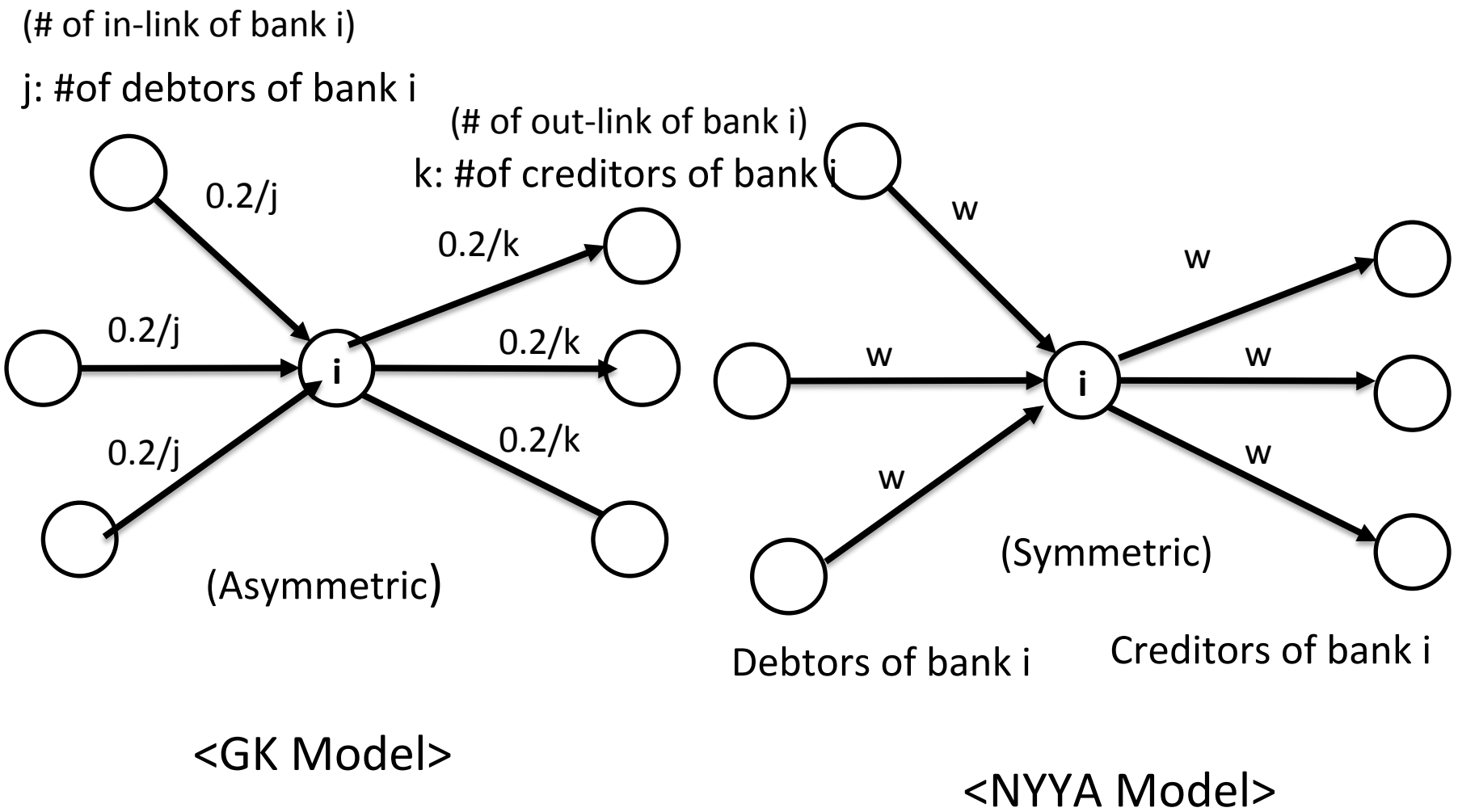
- l_i = bank i's liability
- c_i = net worth of bank i $(c_i = \gamma a_i)$ a_i = bank i's assets
- d_i = customer deposits
- b_i = interbank borrowing

$$l_i = c_i + d_i + b_i \quad \text{where} \quad i = 1, \ldots N$$

Schematic model for a 'node' in the interbank network



Interbank Loans in Models



<GK Model>

<NYYA Model>

Priority of Loss Absorption

Loss is absorbed by the following order:

1. Net worth c_i
2. Interbank liabilities b_i
3. Deposit d_i

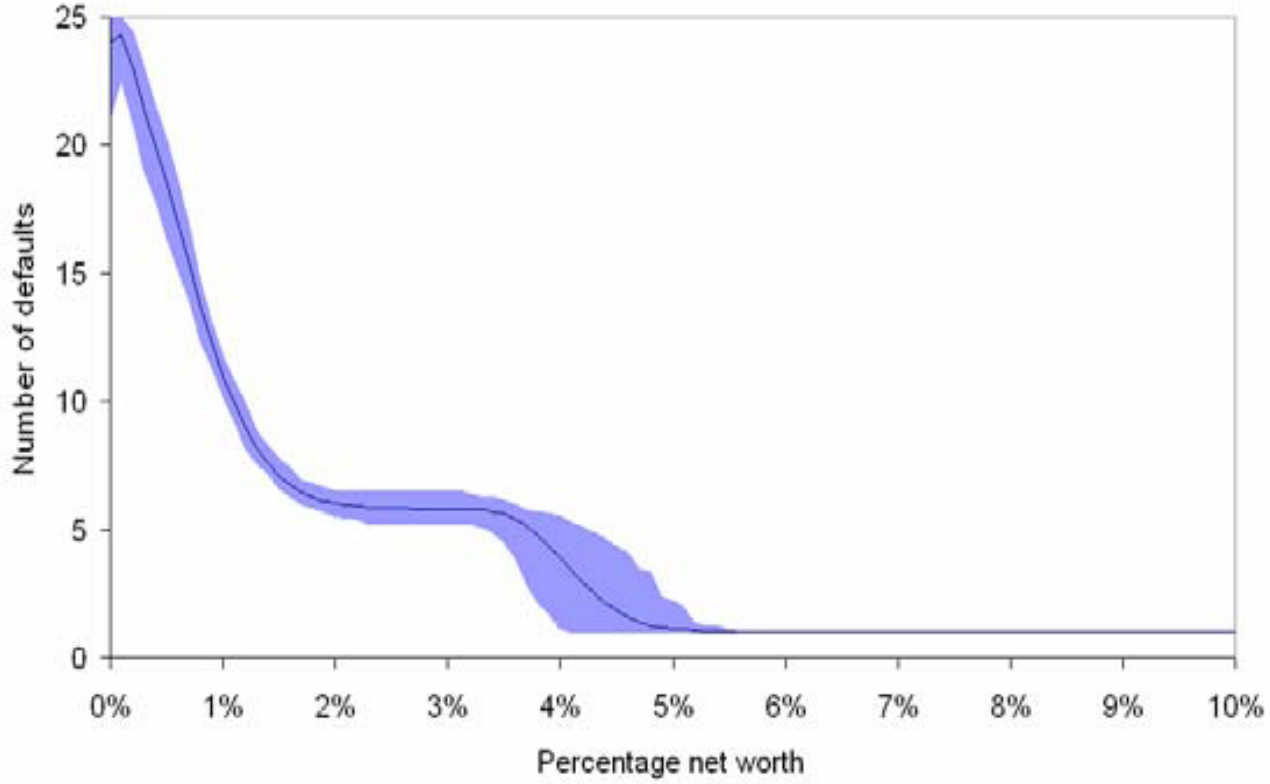
Let s_i be the size of the initial shock.

: If $s_i > c_i$, the bank defaults.

: If $(s_i - c_i) < b_i$ then shock is absorbed by the interbank market, and the residual loss $(s_i - c_i)$ is transmitted to creditor banks.

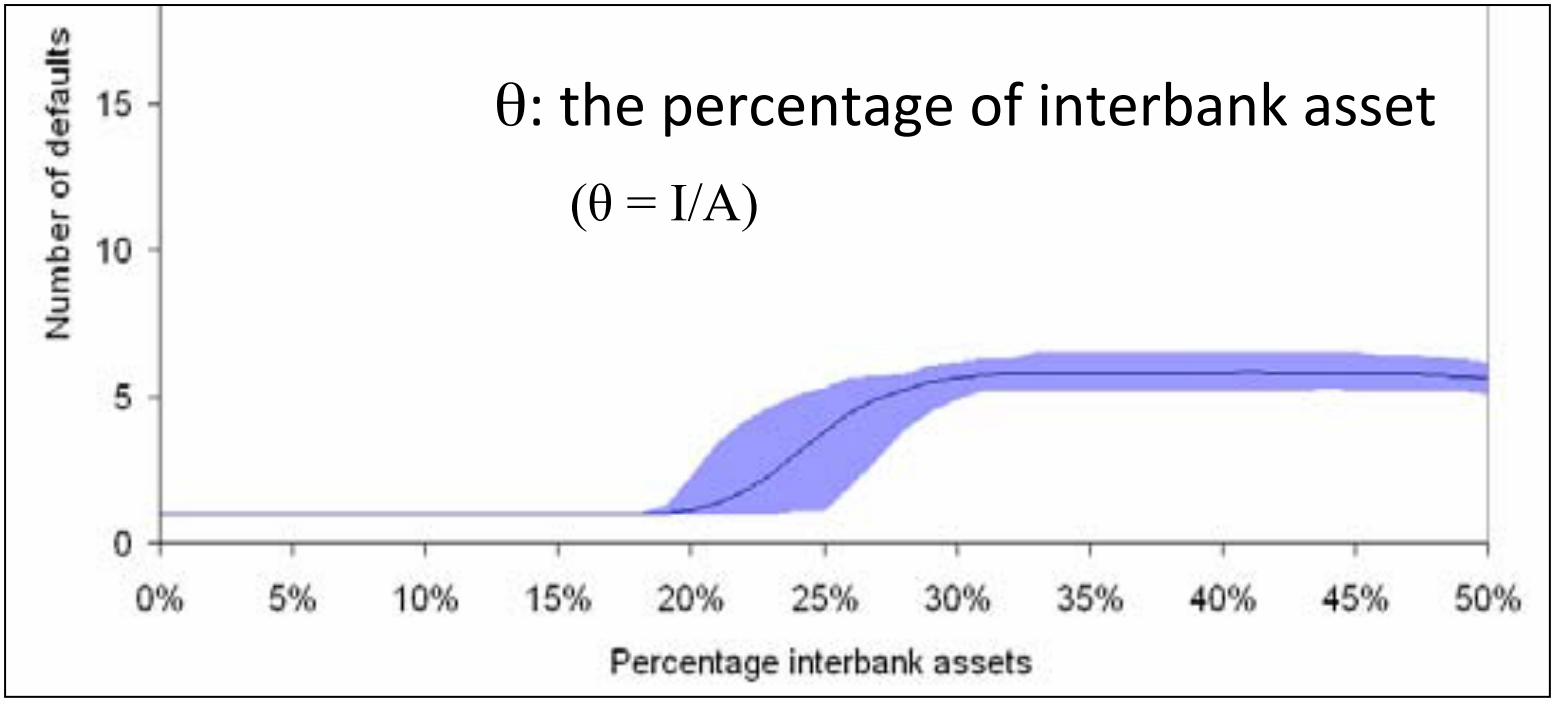
: If $(s_i - c_i) > b_i$, then all of the residual cannot be transmitted to creditor banks. In this case depositors receive a loss of $(s_i - c_i - b_i)$.

Contagion and Net-worth(Capitalization)



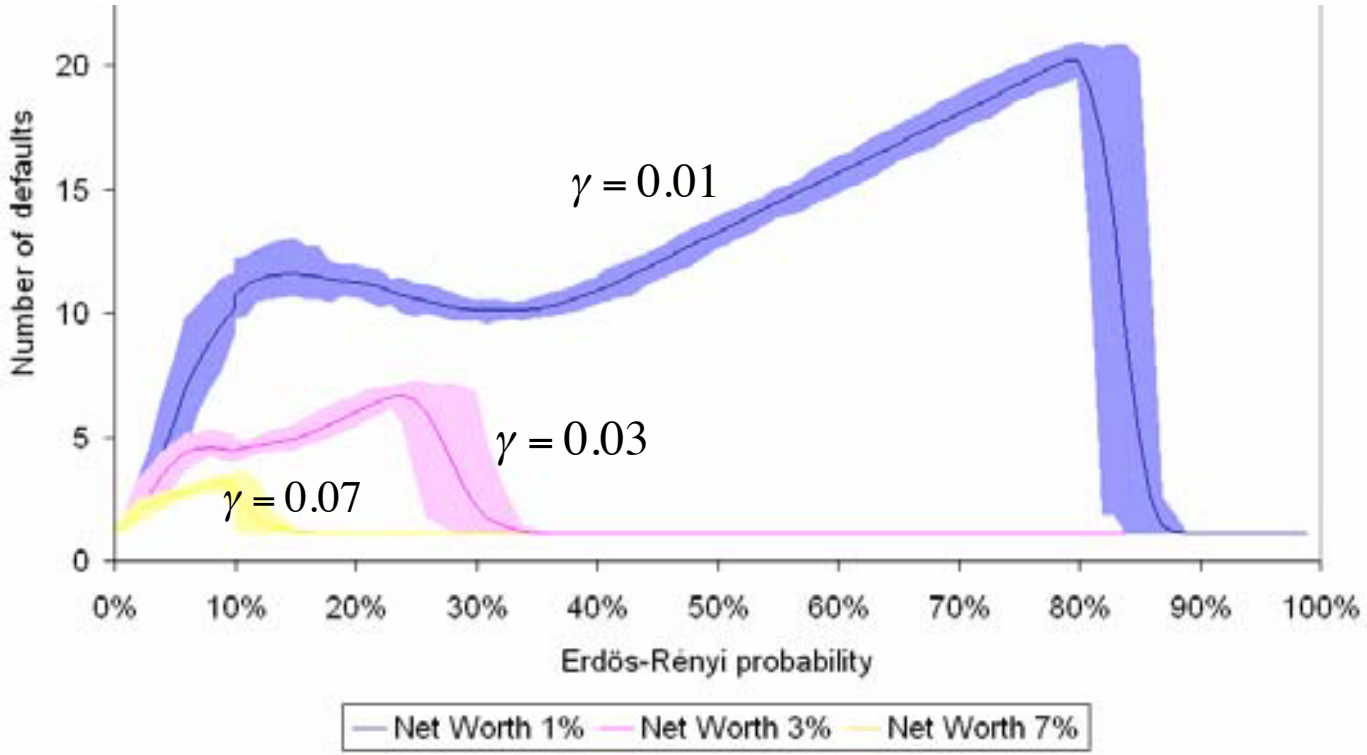
- The number of defaults stays constant for the range 1% to 4% of net worth, because the net worth of banks for a third round defaults remains big enough to absorb the shock received.
- However, if the level of net worth falls below 1%, systemic risks are triggered and the number of defaults increases sharply.

Contagion and Interbank Exposures



- Starts in creasing of default at θ=18%, but stops increasing at θ=30%.
- Increase in the size of the interbank asset θ results in an increase in overall net worth and dampens bank defaults.

Contagion and Interbank Connectivity



- If net-worth is small (γ=1%) the banking system is fragile and even more so when connectivity is high (shocks are transmitted).
- If net-worth is large (γ=7%), the system is robust even if the connectivity is high (shocks are absorbed).

Nier's model was the first simulation study on systemic risk on the small size of financial networks (N=25).

Main findings:

- Contagion decreases in net worth, and this effect is non-linear.
- Contagion increases in the size of interbank liabilities.
- Contagion is a non-monotonic function of the number of interbank connections

Nie's work can be extended to a large sizes of networks including scale-free networks.

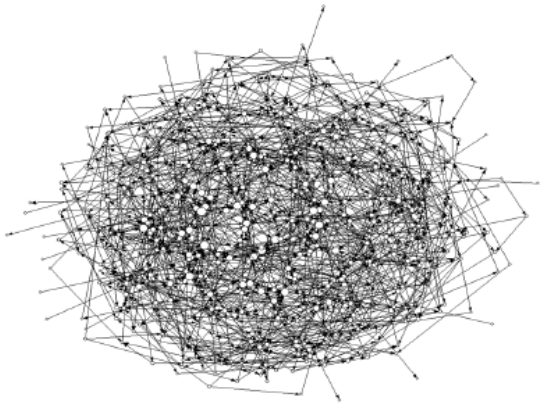
Large Interbank Networks

"Capital surcharge on banks and risk of knock-on defaults" (2011)
by Yoshiharu Maeno (NEC)

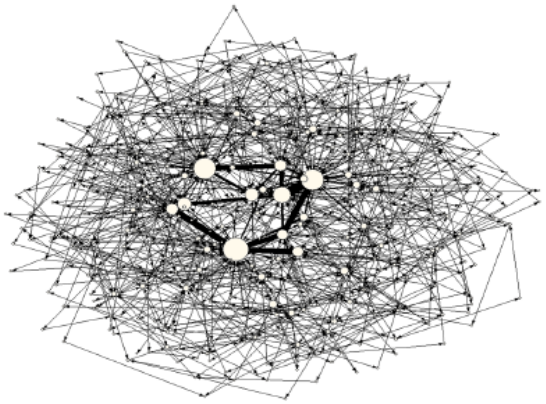
Nodes: N=500,

The connection probability: p=0.005 (sparse network)
p=0.05 (dense network)

- Random network (ER)
- Correlation factor: ρ=0

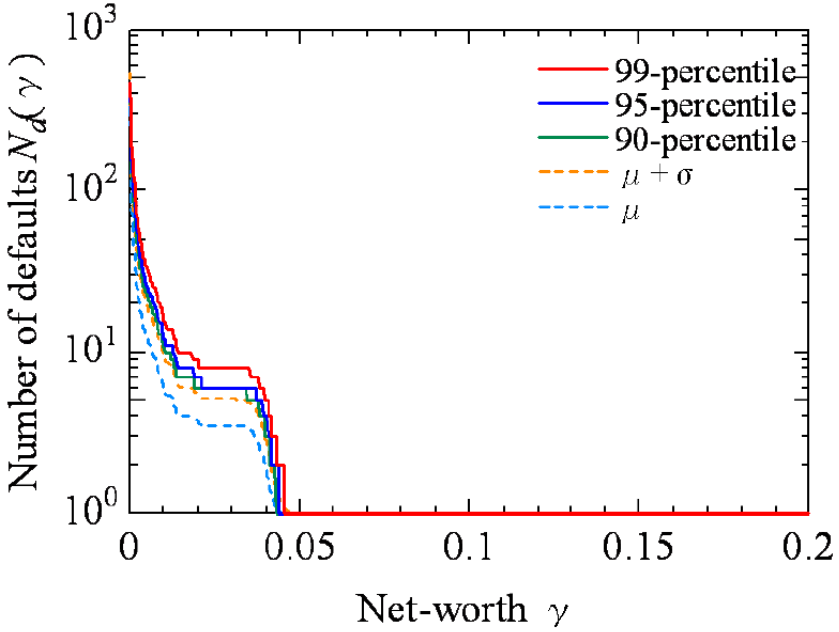


- SF (BA model)
- Correlation factor: ρ=2

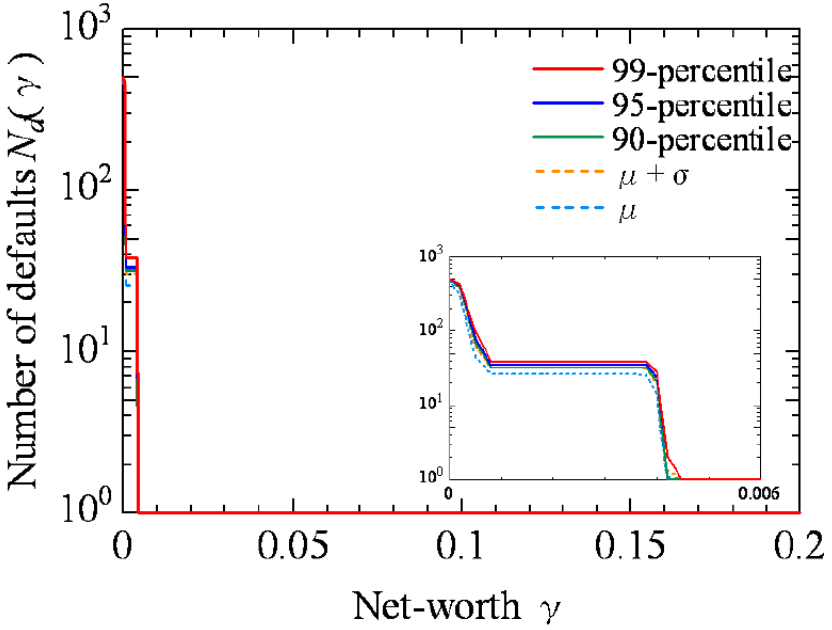


He generates 10^5 different networks of ER and SF and
run many simulations and compare the sizes of the defaults banks.

Default Banks: Random Network



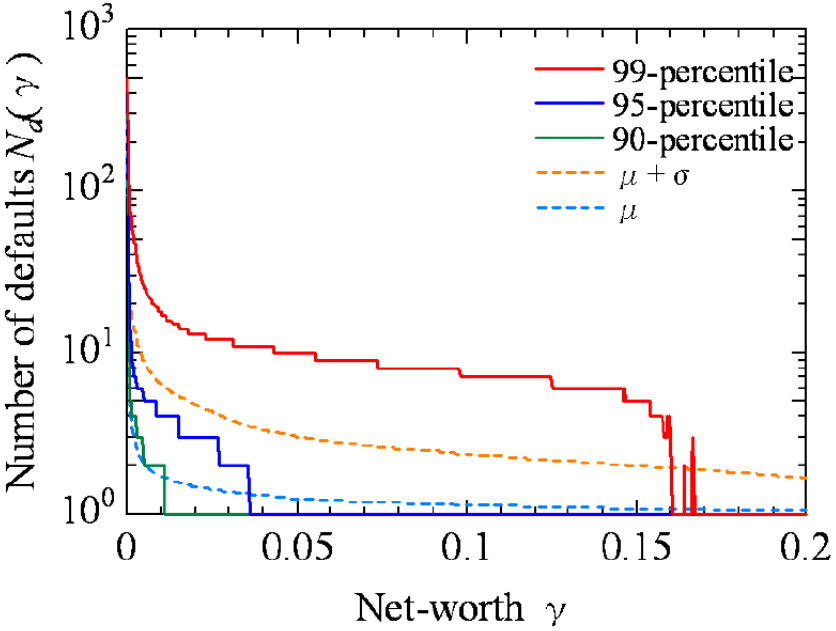
θ=0.1, p=0.005(sparse network)



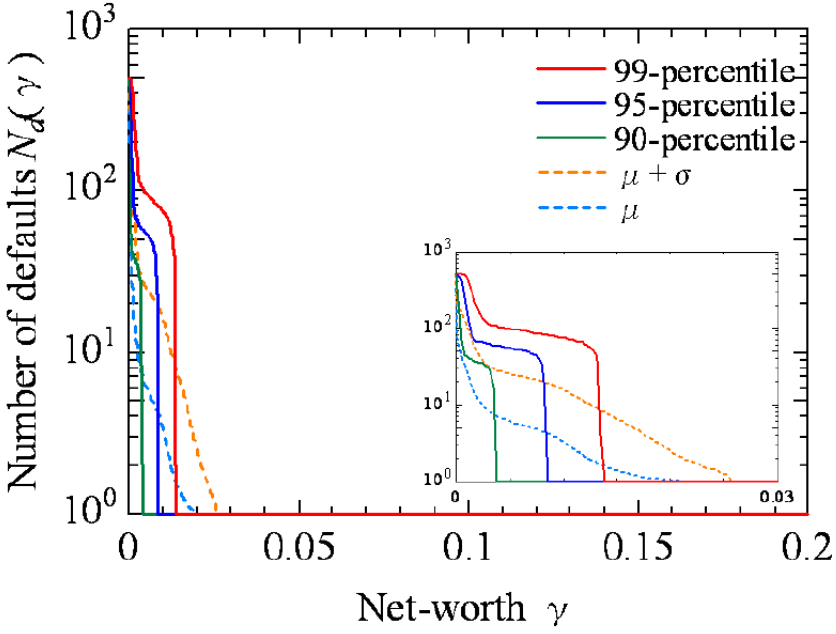
θ=0.1, p=0.05(dense network)

- By increasing the connectivity (p=0.005⇒0.05) the number of default banks decreases.

Bank Default: Scale-free Networks



θ=0.1 , p=0.005



θ=0.1 , p=0.05

- The lines in colors show the average and 90-99 percentiles of defaults
- By increasing the connectivity (p=0.005 to 0.05), the bank defaults decrease drastically.
- Even if the net-worth (bank capital) is small (it should be greater than γ=0.02) no bank defaults with high connectivity, p=0.05.

II. Four Fundamental Models

Fundamental Model 4: “May Model”

“Systemic risk: the dynamics of model banking system”

May, R., and Arinaminpathy, N

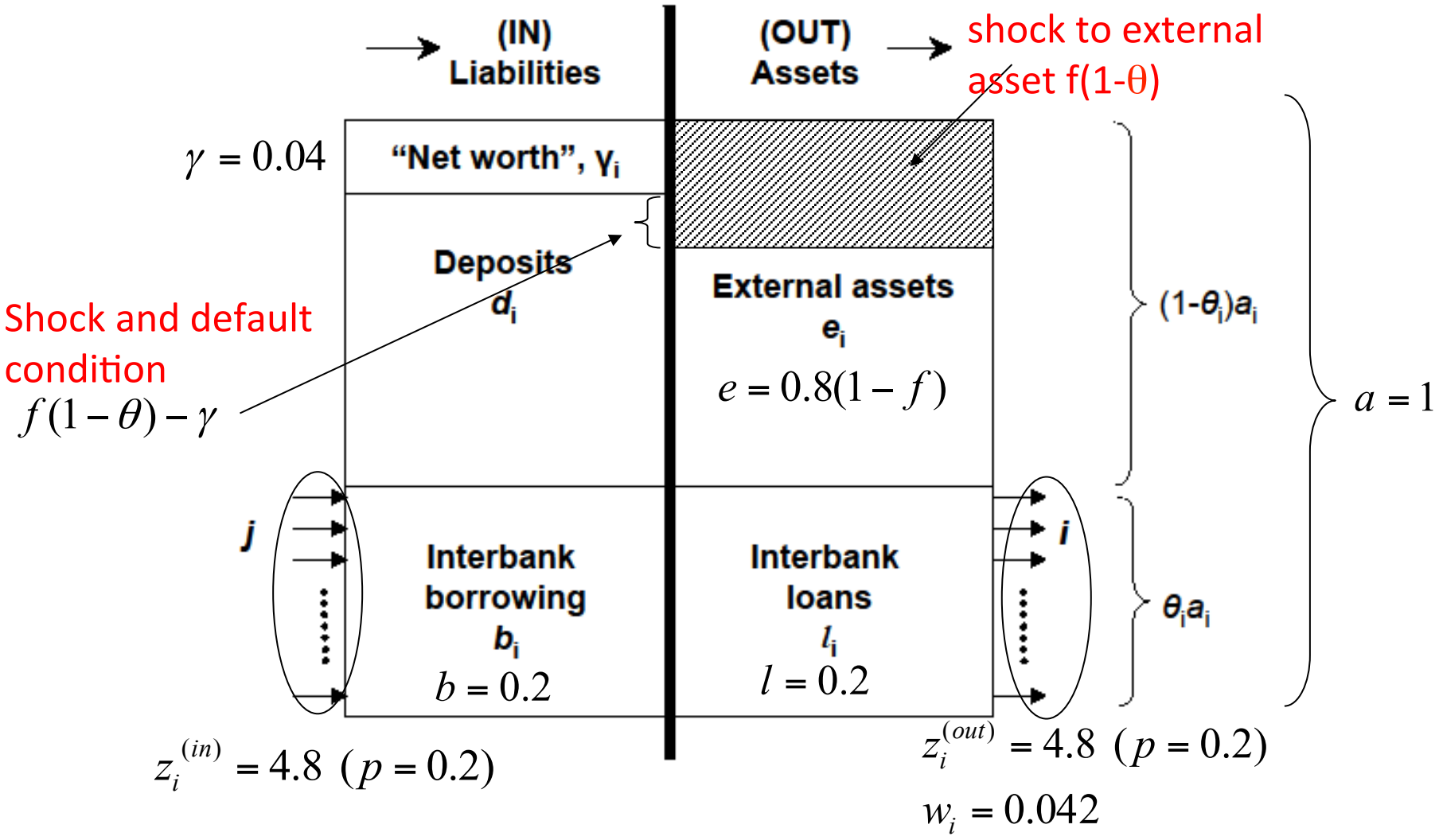
J. R. Soc. Interface, (2010) Vol.7, pp.823-838

1) Gai et al. and Nier et al. run Monte Carlo simulations varying one parameter at a time away from their benchmark values.
2) They present no analytical results.
3) May et al. shows the analytical results which give the theoretical foundation to the work by Nier el al. without simulation.
4) This model applies the mean-field analysis.

- Each bank has an identical balance sheet.
- Each bank is identical in terms of its number of outgoing and incoming links.

Balance Sheet of Each Bank

Schematic model for a 'node' in the interbank network



Initiating Shock Propagation : Phase 1

1) All banks have the same value of assets, a=1.
2) A shock initially hitting any single bank, with the shock taking the form of wiping out a fraction *f* of its assets, a=1.
3) We define that this is the "phase 1 shock", which is .

$$S(\mathrm{I}) = f(1-\theta)$$

The bank fails if $S(\mathrm{I}) > \gamma$

Shock Propagation : Phase 2

Phase I default by single bank results in each of its *z* creditor banks experiencing a phase II shock. Its magnitude is follow.

$$s(\mathrm{II}) = \frac{[\theta, s(\mathrm{I}) - \gamma]_{MIN}}{z}$$

$$\text{if} \quad s(\mathrm{II}) > \gamma$$

the bank will fail, and genarate phase II failures

$[a, b]_{MIN}$ means that we take the lesser of the two quantities a and b.

Phase I and Phase II Failure: Summary

1) Phase I: bank failure if

$$f(1-\theta)-\gamma > 0$$

2) Phase II: banks failure if

$$[\theta, f(1-\theta)-\gamma]_{MIN} > z\gamma$$

$$\theta = f(1-\theta)-\gamma$$

By defining $\theta_c = (f-\gamma)/(1+f)$

$$(1+f)\theta = f-\gamma$$

$$\text{if } \theta < \theta_c \quad \theta > z\gamma$$

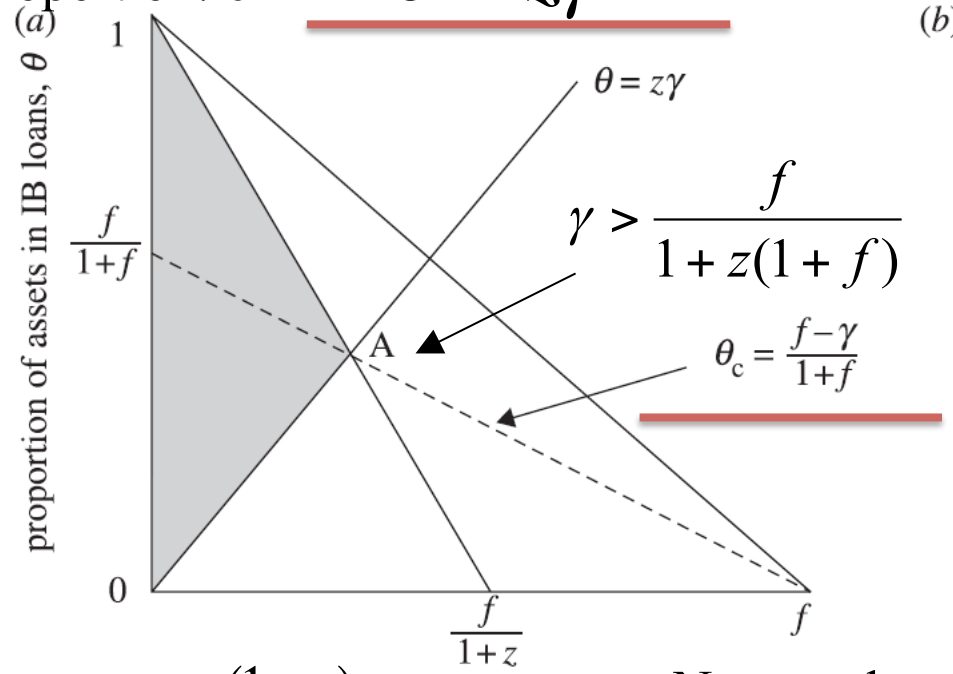
$$\text{if } \theta > \theta_c \quad \theta < 1-\frac{\gamma(1+z)}{f}$$

$$f(1-\theta)-\gamma = z\gamma$$

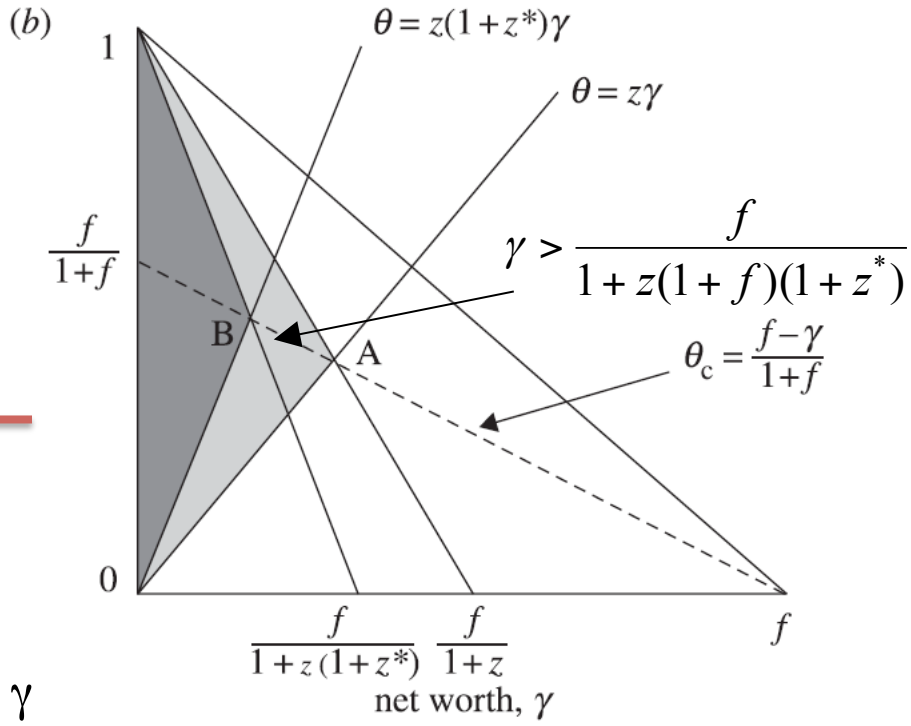
Illustration of Phase I and Phase II Failure

IB loan proportion: θ

$\theta > z\gamma$



Net-worth: γ

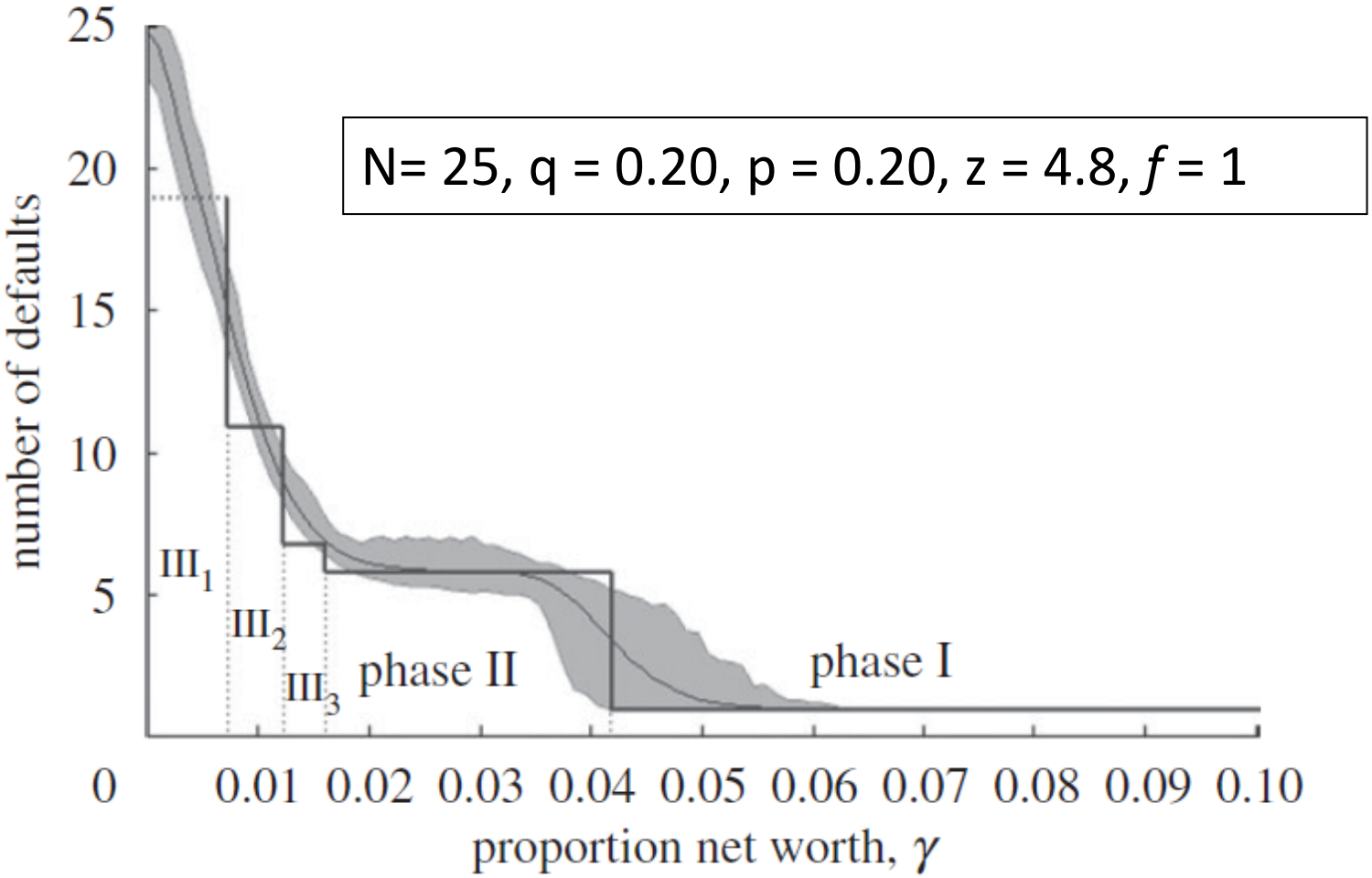


$\theta < 1 - \frac{\gamma(1+z)}{f}$

Light-shaded area : the region in which phase II shocks cause failure

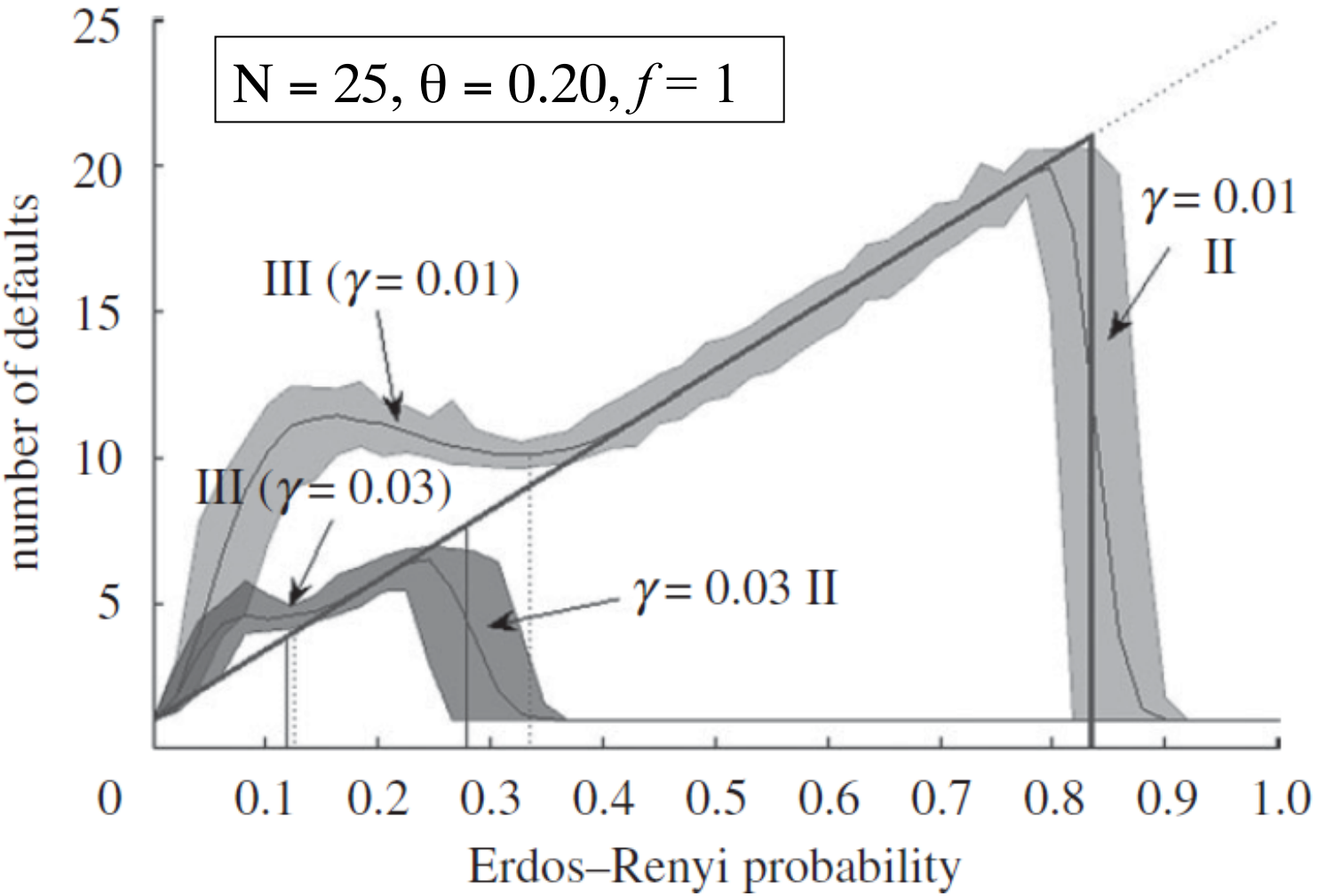
Dark-shaded area : the region in which phase III shocks cause failure

Comparison with the Results of Nier et al.(1)



(Number of banks failing as a function of percentage net worth).

Comparison with the Results of Nier et al.(2)



Number of banks failing as a function of the IB connectivity, p.

Conclusions

1. Shocks propagated by defaulting on IB loans tend to attenuate, by a factor of order *z*, in each successive phase.
2. 'Zone of instability' with respect to IB loan shocks is maximized by having θ (the ratio of interbank loans to the total assets) not too big or small.
3. Robustness - large capital reserves allow greater robustness of both individual banks and of the system.
4. System stability - bigger banks should hold their ratio of capital reserves to total assets at least as high as smaller banks.

– However, in practice, the contrary is observed.

Structure of the Tutorial

"The goal of this tutorial is to understand how simulation models can help us to understand the nature of systemic risks."

More specific topics and questions I intend to address in this tutorial include:

1. Introduction to cascading failures and systemic risks
2. Some fundamental (simplified but useful) models
3. Systemic risk in realistic financial networks
4. How to set up the simulation study on systemic risks?
5. Further extension and conclusion

Realistic Financial Networks

1) The stability of interbank markets does not directly depend on the bank size.
2) Rather it is determined by the magnitude of its interbank liabilities, its exposure to other banks, its capital buffer (net-worth), and more crucially on its position in the interbank network.

Therefore, we need to study using more realistic financial networks.

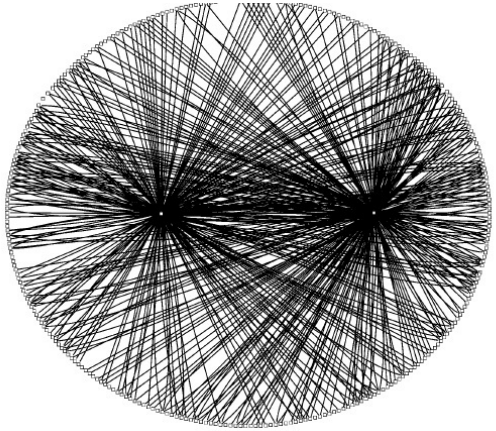
What is known about financial networks?

- Contagion models have mostly been studied on Erdos-Renyi random graphs or scale-free networks, while we know from emperical studies that real financial networks are much more complex.

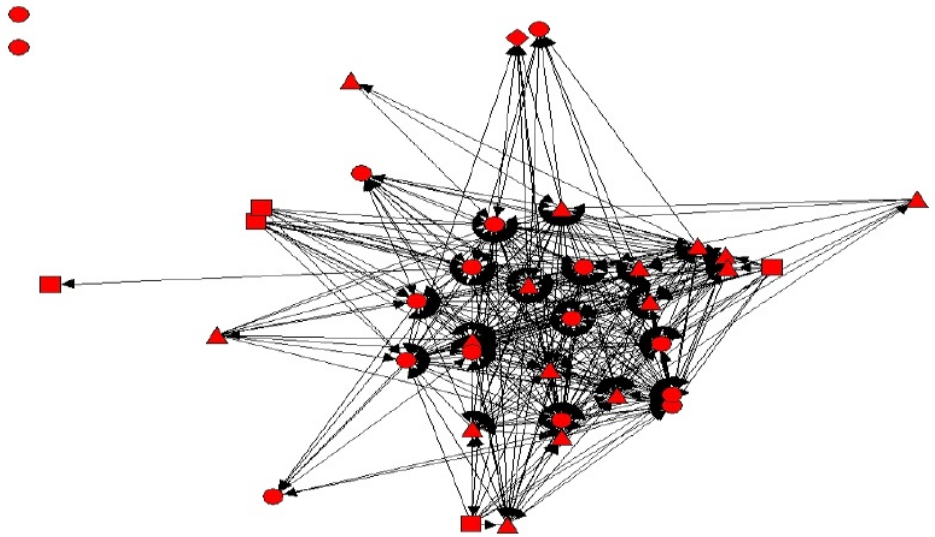
  - G. Iori,, G. De Masi, O. V. Precup, G. Gabbi, G. Caldarelli, A network analysis of the Italian overnight money market, JEBO 32 (2008).
  - M. Boss, H. Elsinger, M. Summer, S. Thurner, The Network Topology of the Interbank Market (Austria), arXiv:cond-mat/0309582v1 (2003)

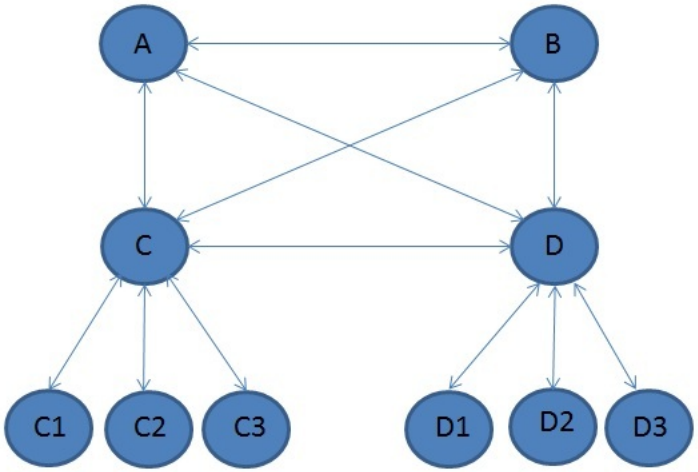
Network Structures of Real-world Banking Systems (1)

Switzerland: sparse and centralized structure (Muller, 2006)



Finland: three-tier structure (Toivanen, 2009)



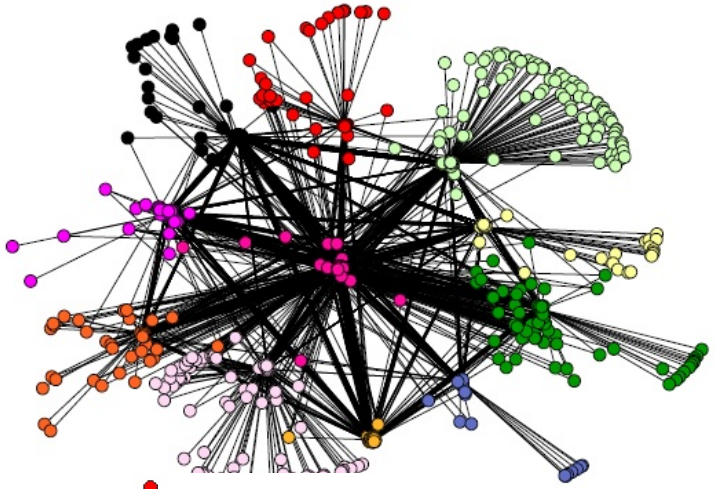


Germany: two-tier structure (Upper, 2004)

Network Structures of Real-world Banking Systems (2)

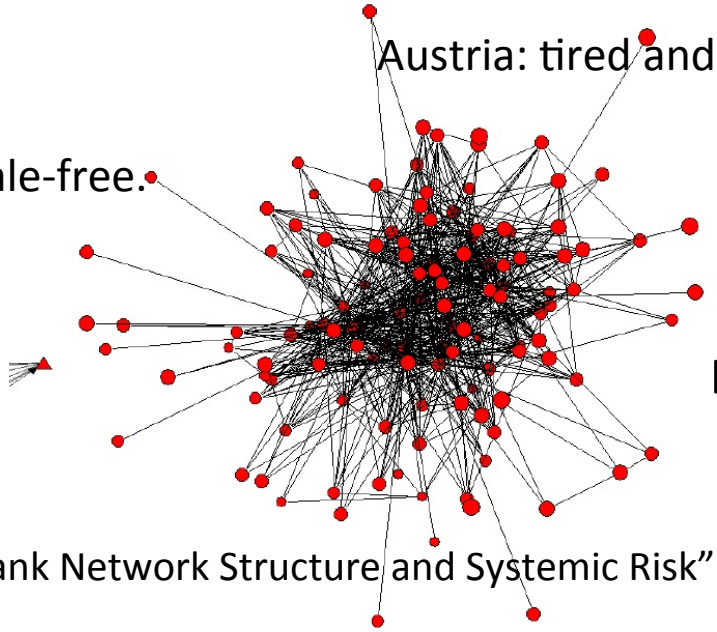
"The network topology of the Interbank market", Boss et al, Quantatitive Finance,4(2006).

Interbank networks are multi-tiered and community structure



Austria: tired and community structure

Interbank networks are roughly scale-free.

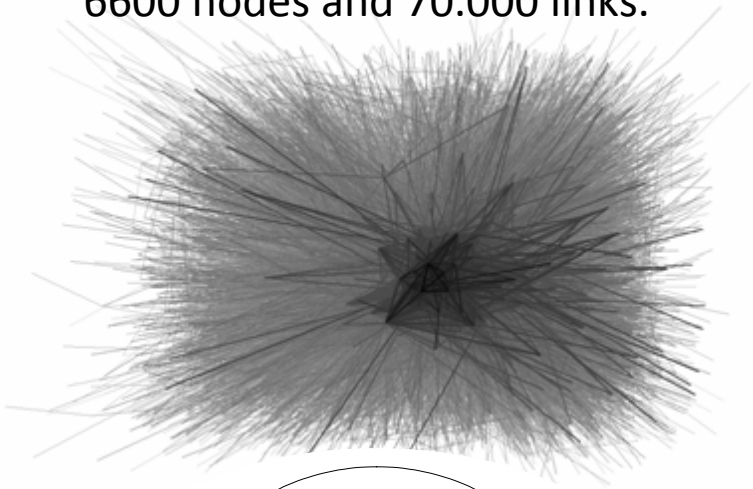


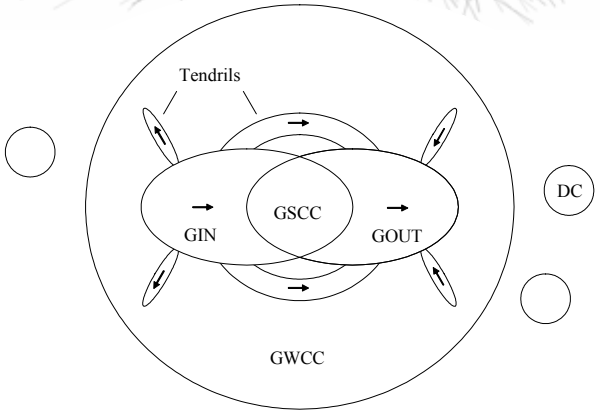
Brazil: scale-free

Santos, E, Cont.R, "The Brazilian Interbank Network Structure and Systemic Risk" Central Bank of Brazil working paper, 2010)

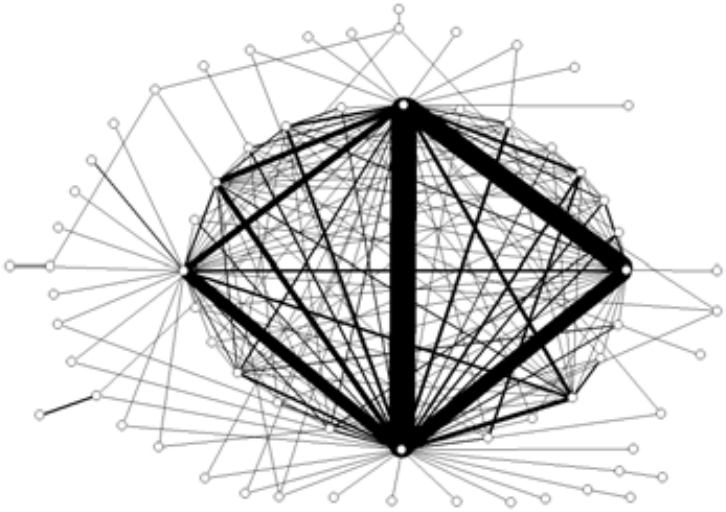
Federal Reserve Wire Network: existence of cores

Fedwire interbank payment network.
6600 nodes and 70.000 links.





GSCC , the undirected links that comprise 75 percent of the value transferred in Fedwire. This network consists of only 66 nodes and 181 links. The prominent feature of this network is that 25 nodes form a densely connected sub-graph



GSCC: Core of Fedwire Interbank Payment Network

GWCC = giant weakly connected component, DC = disconnected component,
GSCC = giant strongly connected component, GIN = giant in component,
GOUT = giant out component.

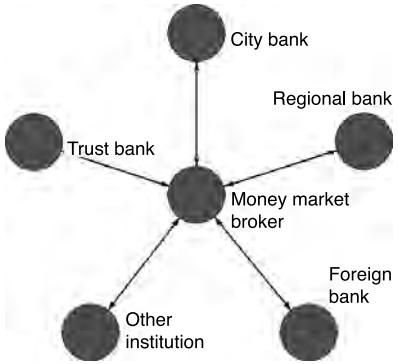
"The topology of interbank payment flows", Soramaki, K, M.L. Bech, J. Arnold, R.J. Glass and W.E. Beyeler, Physica A, Vol. 379, pp 317-333, 2007.

Interbank Transaction in Japan

"The Transaction Network in Japan's Interbank Money Markets" Imakubo, K, et al. Monetary and Economic Studies Vol.28 (2010)

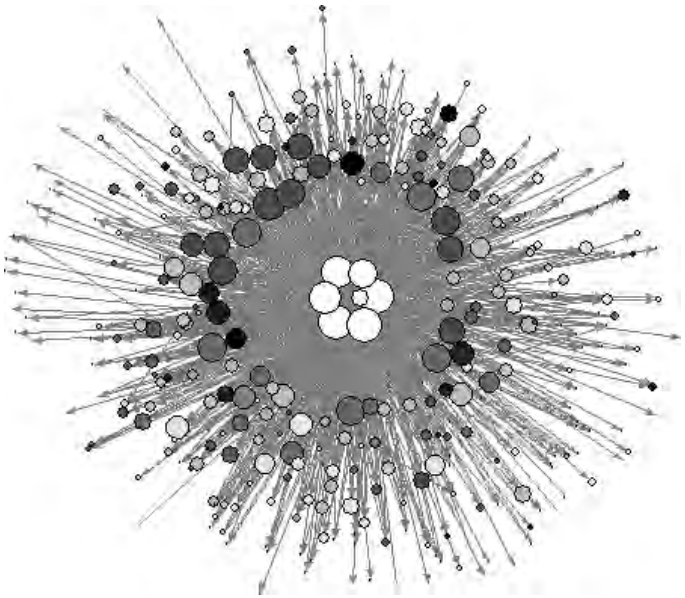
Payments	Receipts (¥ trillions)													
	Major banks	Trust banks	Regional banks	Regional banks II	Credit unions	Securities companies	Foreign banks	Foreign securities companies	Foreign trust banks	Money brokers	Asset management trusts	Affiliated financial institutions	Public financial institutions	Total
Major banks	0	0	2	0	0	1	0	2	0	72	0	0	0	78
Trust banks	9	0	0	0	0	2	0	0	0	272	0	0	0	283
Regional banks	0	0	0	0	0	0	0	0	0	80	0	0	0	81
Regional banks II	0	0	0	0	0	0	0	0	0	23	0	0	0	23
Credit unions	0	0	0	0	0	0	0	0	0	5	0	1	0	6
Securities companies	0	1	0	0	0	0	0	0	0	26	1	0	0	29
Foreign banks	0	0	0	0	0	0	0	6	0	28	0	0	0	35
Foreign securities companies	0	0	0	0	0	0	1	0	0	3	0	0	0	4
Foreign trust banks	0	0	0	0	0	0	0	0	0	5	0	0	0	5
Money brokers	437	79	63	14	2	13	60	4	2	0	12	10	2	698
Securities financial companies	0	0	0	0	0	0	0	0	0	1	0	0	0	1
Affiliated financial institutions	2	0	0	0	1	0	0	0	0	23	0	0	0	27
Public financial institutions	0	0	0	0	0	0	0	0	0	7	0	0	0	8
Total	450	82	65	14	3	15	62	13	2	548	14	12	2	1,281

Star-Shaped Network

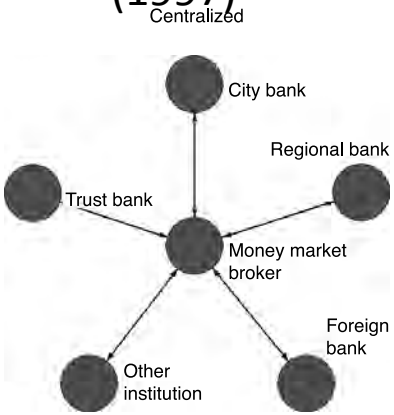


Interbank Transaction Network in Japan (cont.)

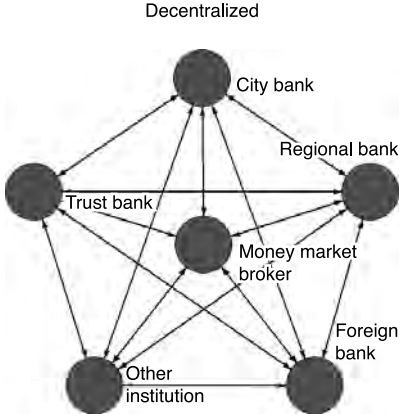
December 1997



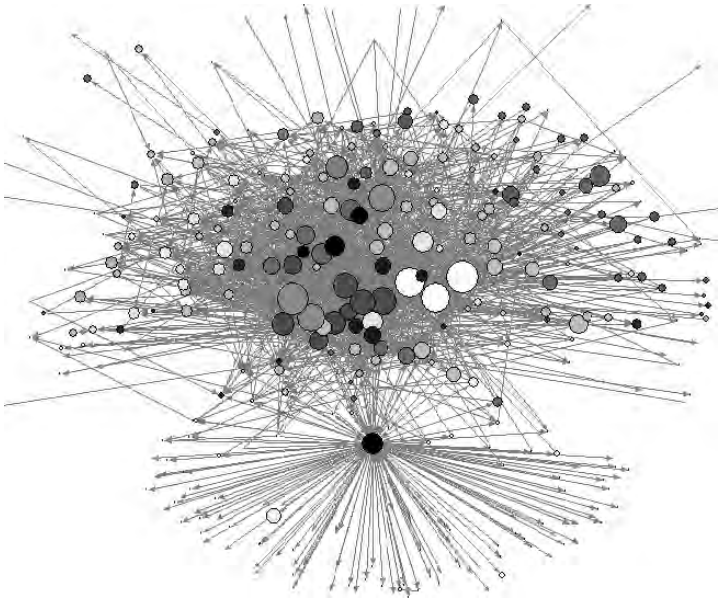
Single core(centralized) (1997)



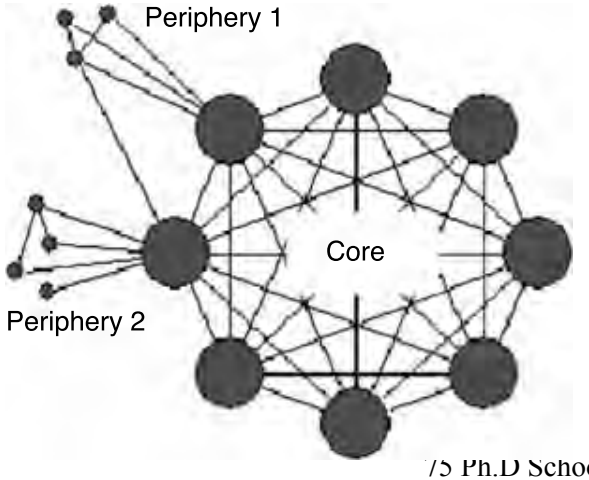
Multi- core (decentralized, 2005)



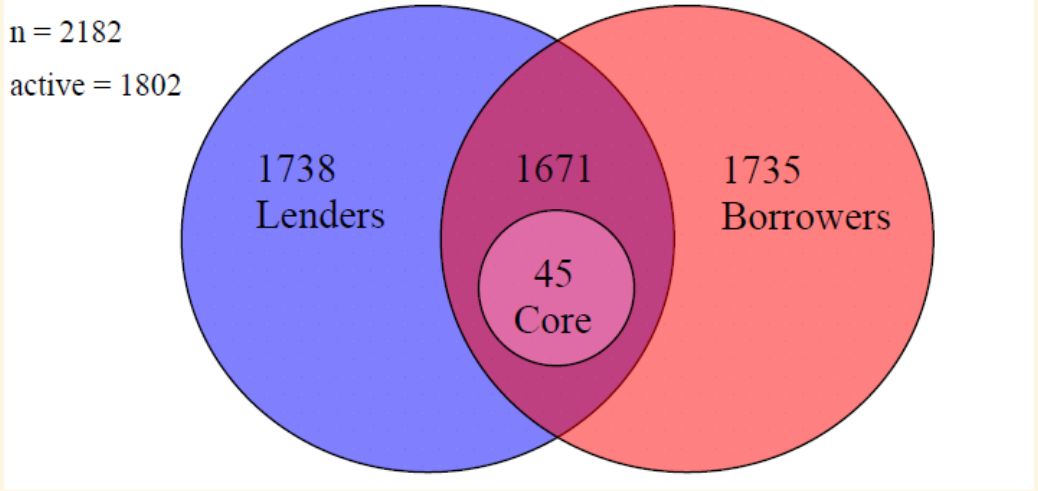
December 2005

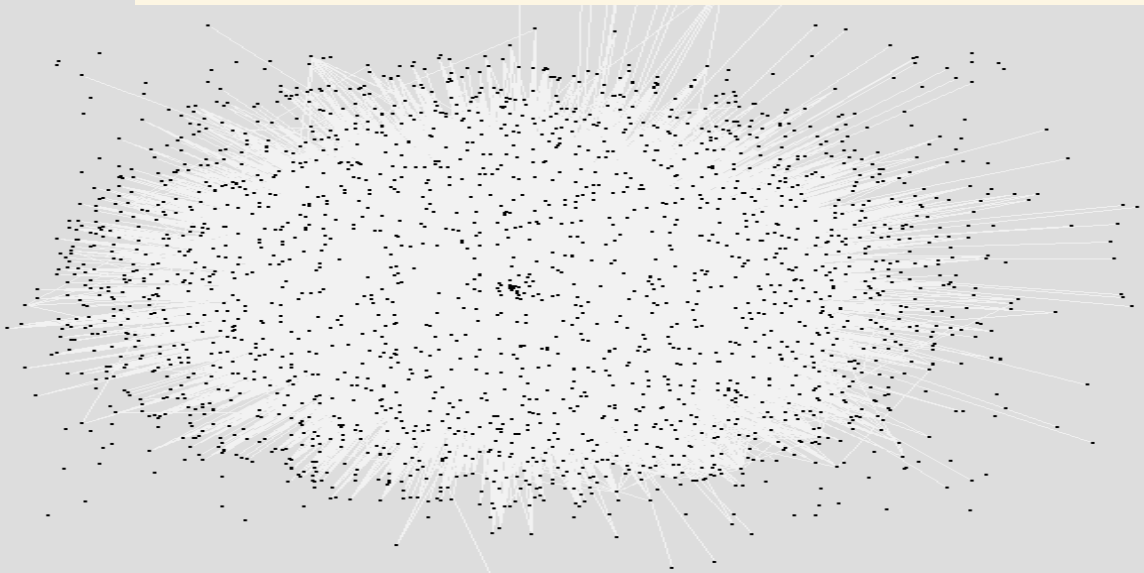


Two-Tier Structure with Core-Periphery (2005)



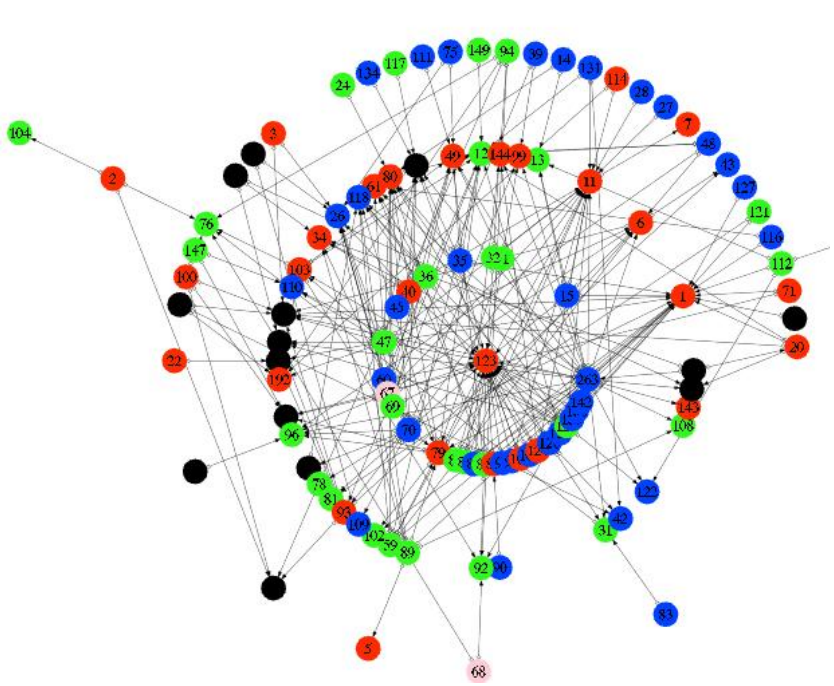
German Banking System: core-periphery

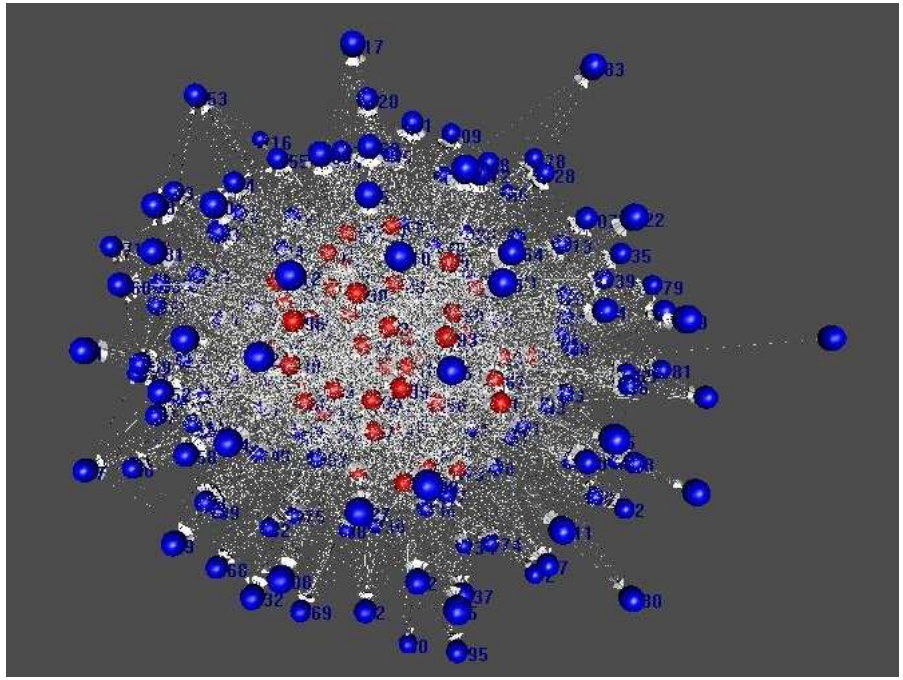




"Interbank tiering and money center banks" Craig, B, Peter,G. BIS working paper, (2010)

Italian overnight money market Core-periphery, Multi-tier Network





"A network analysis of the Italian overnight money market", Iori, et al. Journal of Economic Dynamics and Control, vol. 32(2008)

"Core-Periphery Structure in the Overnight Money Market"

Fricke, D., Lux, T., Kiel Working Paper,（2012）

Core-periphery networks – Definition

"Models of core-periphery structure", Borgatti , S, Everett, (1999).

- In a core-periphery network the nodes can be partitioned into two subsets:
  - **Core** – every two nodes are connected.
  - **Periphery** – every two nodes are disconnected.
  - **No restriction** on the links between the core and the periphery.

Identification of a core-periphery structure in financial networks

"Interbank tiering and money center banks" Craig, B, Peter,G. BIS working paper, (2010)
"Core-Periphery Structure in the Overnight Money Market"
Fricke, D., Lux, T., Kiel University Working Paper, (2012)

The global financial networks

- Minoiu, et al. construct a global banking network using cross-border banking data for 184 countries for the period 1978-2007.
- Hale constructs a global banking network of 7938 banking institutions from 141 countries using interbank lending data from a database of international syndicated bank loans.

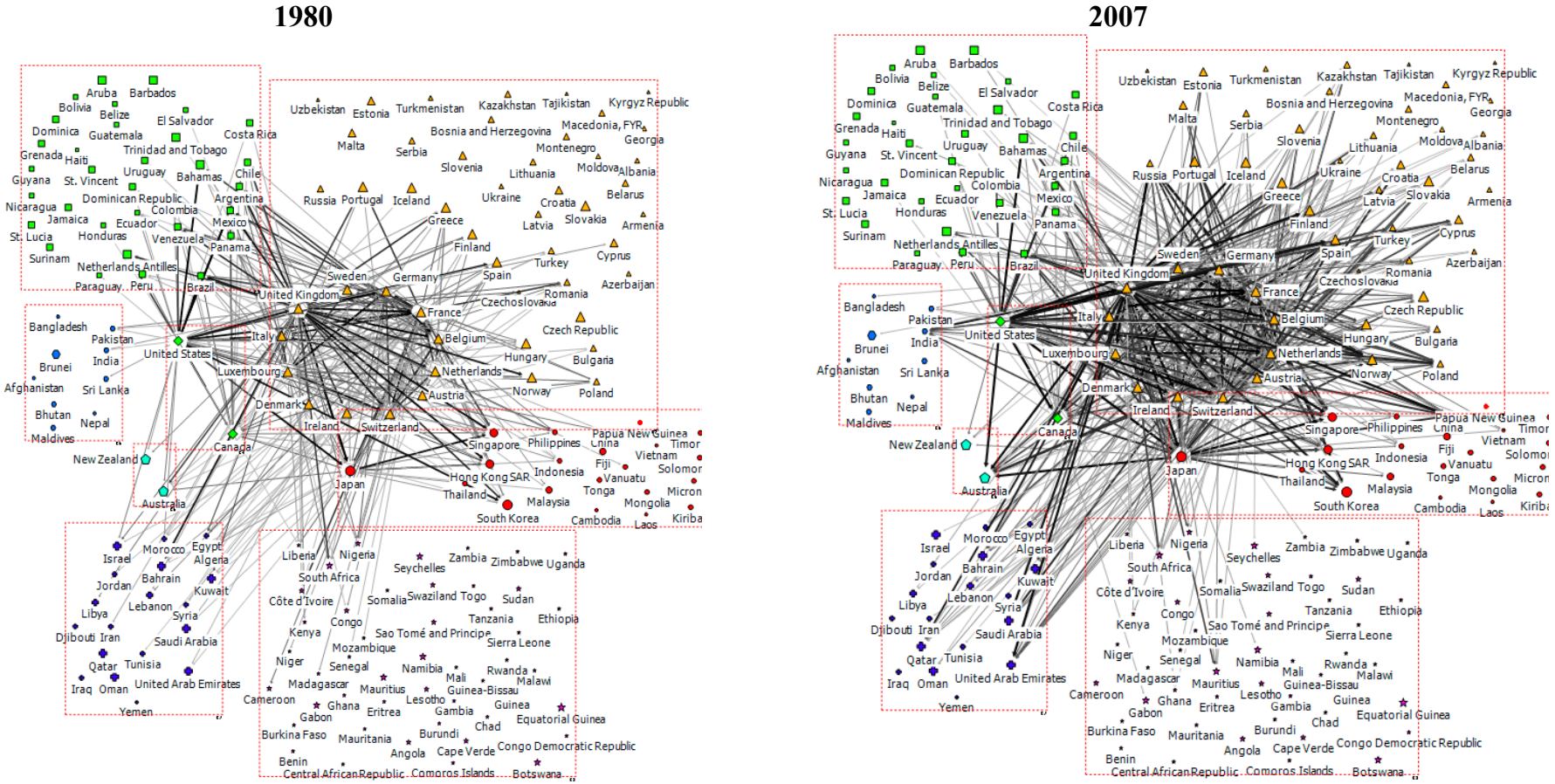
: This is the first study in which bank-level network is analyzed in a global scale.

"A network analysis of global banking", Minoiu, IMF working paper, (2012)

"Bank Relationships, Business Cycles, and Financial Crises", Hale, Federal Reserve Bank, (2012)

Network View of Cross-border Banking (1980 and 2007)

Minoiu et al. (2012) calculate network statistics with the goal of understanding how the flow of global capital changes over time.



"A network analysis of global banking", Minoiu, IMF working paper, 2012)

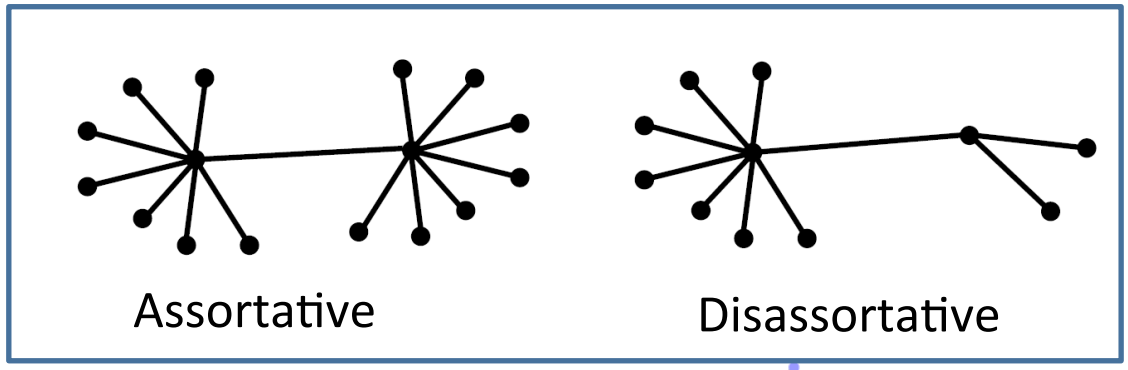
Network Characteristics of Cross-border Banking

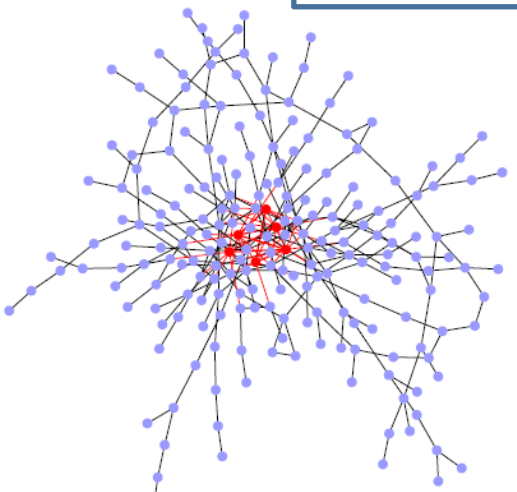
- They divide their network into two parts: the core and periphery.
- The core consists of 15 countries with advanced economies. The periphery comprises an additional 169 emerging and developing countries for which only data on borrowing is available.

- They observe that the flow of capital within the core network is roughly ten times the flow from the core to the periphery.

- Global network is analyzed at a country level, not a bank level.
- There is evidence that countries in which banks were more connected to other banks through syndicated loan market experienced a lesser impact of the global financial crisis

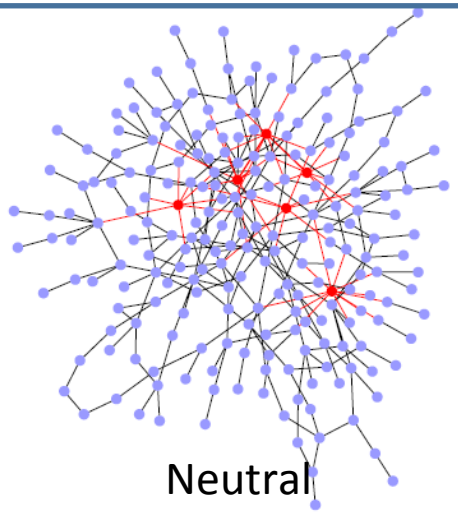
Mixing Pattern in Networks





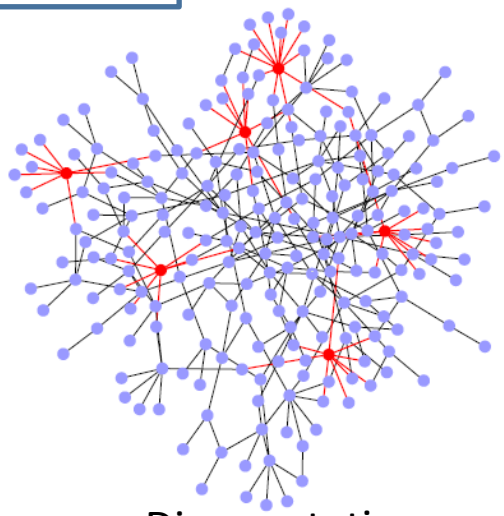
Assortative

Hubs show a tendency to link each other



Neutral
(unassortative)

Nodes connect to each other with the expected random probabilities



Disassortative

Hubs tend to avoid linking to each other

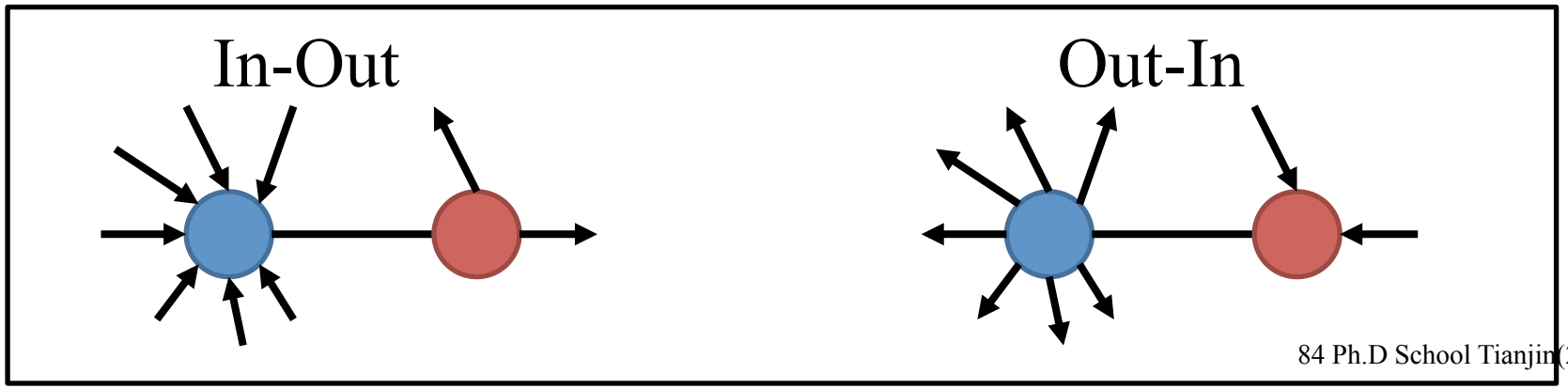
Mixing pattern in real networks

	network	type	size n	assortativity r	error σ_r	ref.
social	physics coauthorship	undirected	52 909	0.363	0.002	a
	biology coauthorship	undirected	1 520 251	0.127	0.0004	a
	mathematics coauthorship	undirected	253 339	0.120	0.002	b
	film actor collaborations	undirected	449 913	0.208	0.0002	c
	company directors	undirected	7 673	0.276	0.004	d
	student relationships	undirected	573	−0.029	0.037	e
	email address books	directed	16 881	0.092	0.004	f
technological	power grid	undirected	4 941	−0.003	0.013	g
	Internet	undirected	10 697	−0.189	0.002	h
	World-Wide Web	directed	269 504	−0.067	0.0002	i
	software dependencies	directed	3 162	−0.016	0.020	j
biological	protein interactions	undirected	2 115	−0.156	0.010	k
	metabolic network	undirected	765	−0.240	0.007	l
	neural network	directed	307	−0.226	0.016	m
	marine food web	directed	134	−0.263	0.037	n
	freshwater food web	directed	92	−0.326	0.031	o

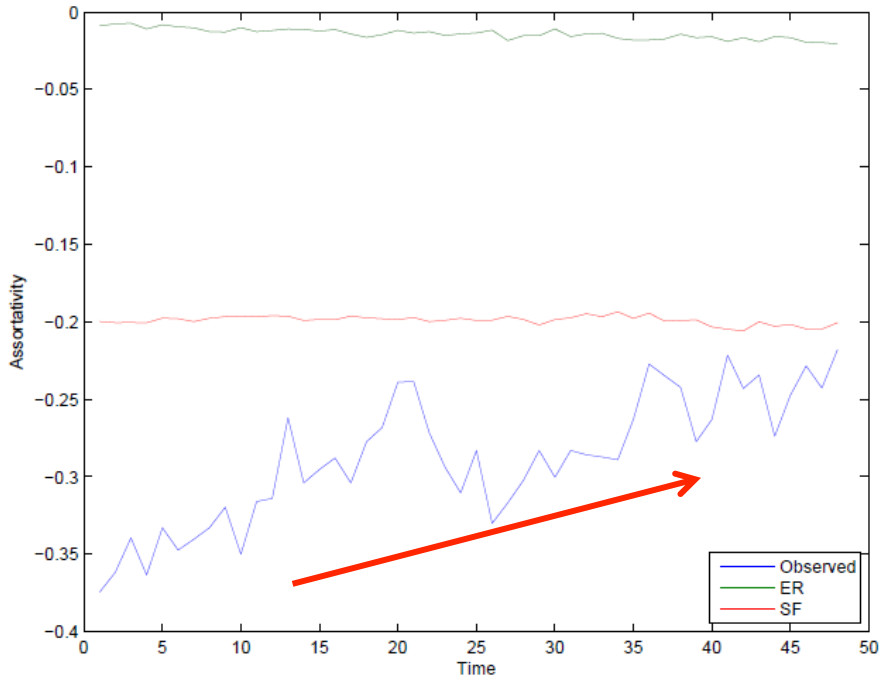
- Social networks: Assortative
- Technological & Biological networks: Disassortative
- Economic networks share features of both depending on the context

Assortativity in the interbank market

- Interbank networks: disassortative
  - Overall: high-degree banks tend to connect to low-degree banks and vice versa
  - Directed case: banks with high-in degree tend to have links to banks with a small-out degree and vice versa

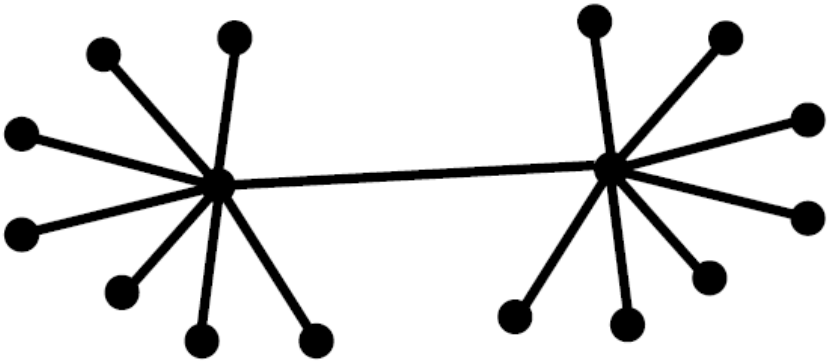


Assortativity in the Italian interbank market

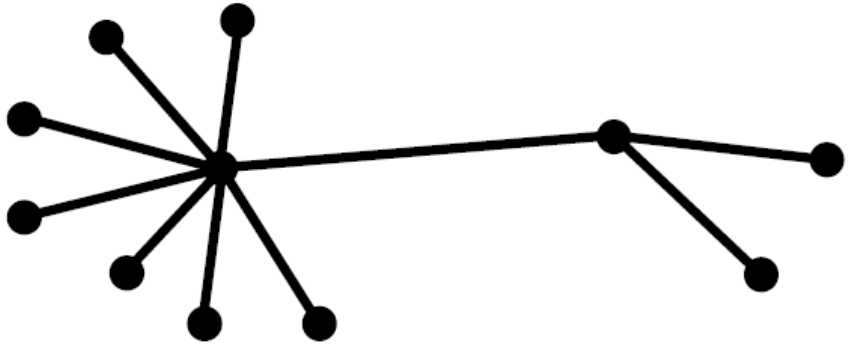


D. Fricke, K. Finger, T. Lux, "On Assortative and Disassortative Mixing in Scale-Free Networks, The Case of Interbank Creadit Networks" Working Paper, Kiel Institute, (2013)

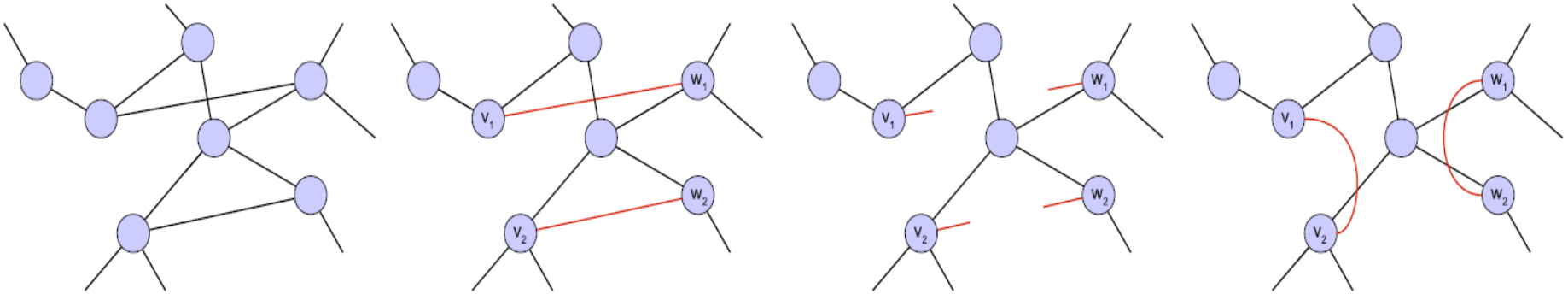
Mixing pattern in networks



Assortative

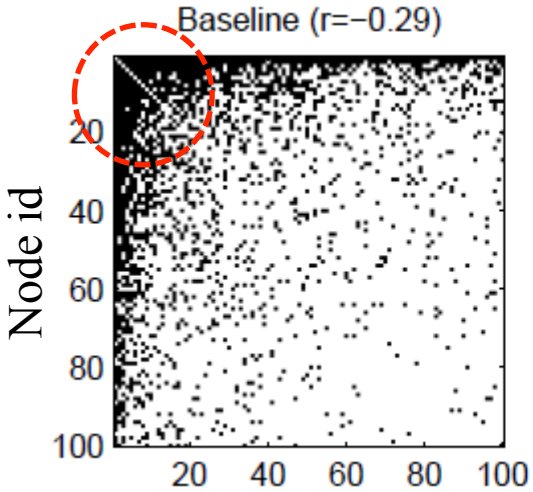


Disassortative

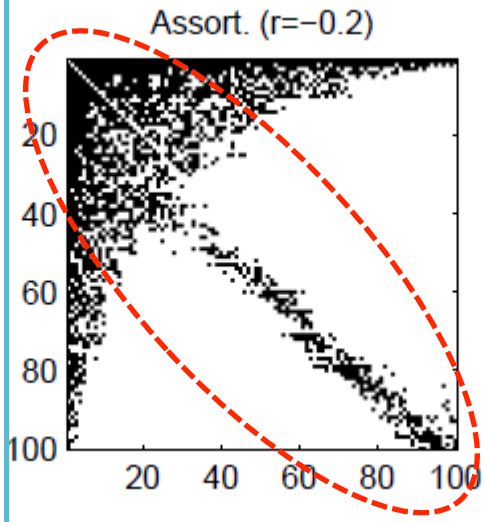


Degree preserving rewiring mechanism

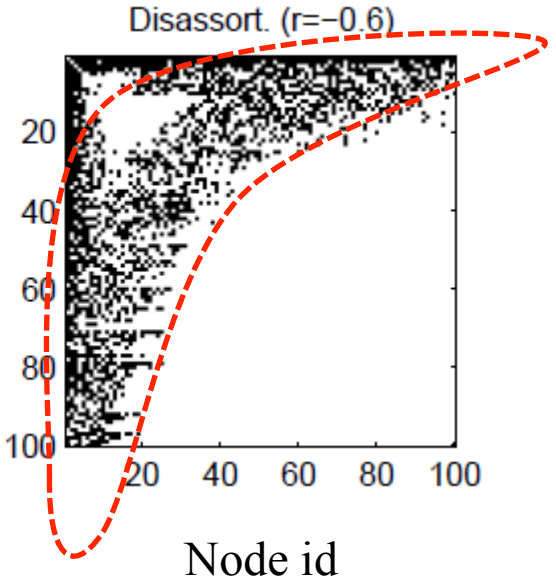
Node-Node Correlation in Scale-free networks, (N=100)



One case



Most assortative case
(but disassortative)



Most disassortative case

Using Goh et al.(2001) algorithm

Using rewiring algorithm by Xulvi et al.(2004)

Further References

- Baral, P, edro Fique, J “Estimation of Bilateral Exposures - A Copula Approach”

  :This paper provides the unified analytical approach by comparing

  the models by Gai et al. and Nier et al.
- Musmeci,N, Battiston, S, etc, “Bootstrapping Topological Properties and Systemic Risk of Complex Networks Using the Fitness Model” J. Stat Phys (2013) 151:720–734
- :This paper provides the unified analytical approach by comp