Symmetric Coordination of Heterogeneous Agents

Saori Iwanaga (Nonmember) & Akira Namatame (Member)
Dept. of Computer Science,
National Defense Academy,
Yokosuka, 239-8686, JAPAN,
E-mail: {g38042, nama}@nda.ac.jp

Abstract
Large-scale effects of locally interacting agents are called emergent properties of the system. Emergent properties are often surprising because it can be hard to anticipate the full consequences of even simple forms of interaction. In this paper we address the following questions: how do heterogeneous agents generate emergent coordination, and how do they manage and self-organize macroscopic orders from bottom up without any central authority? These questions will depend crucially on how they interact and adapt their behavior. Agents myopically evolve their behavior based on the threshold rule, which is given as the function of the collective behavior and their idiosyncratic utilities. We obtain the micro-macro dynamics that relate the aggregate behavior with the underlying individual behavior. We show that agents' rational behavior reflecting their micromotives combined with the behavior of others produce stable orders, and sometimes unanticipated cyclic behavior. We consider the roles of particular types of agents, conformists, and nonconformists to manage emergent collective behavior. As a specific example, we address an emergent and evolutionary approach for designing the efficient network management policies.

Keywords: heterogeneity, symmetric coordination, asymmetric coordination, emergence, evolutionary dynamics

1 Introduction
As many tasks in information processing grow in complexity, the strong growing interests have been given toward heterogeneous agents [1][18]. To take advantage of the growing interests for multi-agents, the methodologies must be explored to process which beyond each agent's capability and knowledge. For instance, the integration of heterogeneous viewpoints becomes important [14]. The design of efficient collective action also becomes crucial. The group can offer something not available in the individuals. In collective activity, two types of activities may be necessary: each agent behaves as a member an organization, while at the same time, it behaves independently by adjusting its view and action. For the individual, it can learn to improve its problem-solving skills based on its own observation and experiences. At the cooperative stage, they can put forward their learnt knowledge for consideration by others. Therefore, cooperative work, if it is by a team of engineers, or by a collection of agents, requires efficient coordination, communication, knowledge sharing. In order to satisfy these requirements, it may need to create dynamic and highly interconnected networks among them [3][20].

In this paper we consider the coordination problems of more loosely connected agents, where they behave to satisfy their own goal, and they may coordinate with others in order to improve it. There are many situations where interacting agents can benefit from coordinating their action. Coordination implies that increased effort by some agents leads the remaining agents to follow suit, which gives rise multiplier effects. We call this type of coordination as symmetric coordination. Coordination is also necessary to ensure that their individual actions are carried out with little conflicts, and we distinguish this type of coordination by referring asymmetric coordination.

We illustrate some examples of symmetric and asymmetric coordination. Consider the situation in which two agents must independently decide where to locate or which route to use. They receive the utility if and only if they choose the same location (symmetric coordination). They gain utility if and if only if they choose the different route (asymmetric coordination). For coordinating situations, there are multiple equilibria, and then a coordination failure, resulting in inefficiency, can easily arise. Coordination failures may also result from their independent inductive processes [7].

In this paper, we study the problem of asymmetric coordination and obtain some conditions for emergence of coordination among heterogeneous agents. Our focus is the theoretical basis for coordination problems in strategic situations and the likely outcome. We explore the mechanism in which heterogeneous agents may be stuck at an inefficient equilibrium. While all agents understand the that the outcome is inefficient, each acting independently is power less to manage this collective activity about what to
do and also how to decide.

We also address the following question: how do the heterogeneous micro-worlds of individuals generate the global macroscopic orders of the whole? The large-scale effects of locally interacting agents are called emergent properties of the system [10]. Emergent properties are often surprising because it can be hard to anticipate the full consequences of even simple forms of interaction. We aim at discovering fundamental local or micro mechanisms that are sufficient to generate efficient coordination without any central authority. This type of coordination may be referred as coordination emerged from the bottom up.

2 Symmetric and Asymmetric Coordination

The question of how it is possible for a group of independent individuals achieve both their own goal and a common goal has been addressed in many fields. By a common goal we mean a goal that is achievable by a group it requires cooperation. The key element that distinguishes a common goal from an individual goal is that it and which requires collective action among them. Collective action problems, however, pose some difficult problems, and it requires cooperation to overcome them. Coordination is different concept from cooperation, which does not assume the common goal shared by among agents. Coordination is necessary to achieve each agent’s independent goal efficiently.

We distinguish two types of coordination, symmetric coordination, and asymmetric coordination. We emphasize their essential differences by illustrating specific examples. There are many situations where interacting individuals or agents can benefit from coordinating their interdependent action and taking the same action. We define this type of coordination as symmetric coordination. Examples where symmetric coordination is important include trade alliance, the choice of compatible technologies or conventions such as the choice of a software or language. A convention, for instance, is a pattern of behavior that is customary expected. Everyone conforms, everyone expects others to conform, and everyone wants to conform given that everyone else conforms. For each role in such symmetric situation, there is a customary and expected behavior, and everyone prefers to follow the behavior expected of him provided that others follow the behavior expected of them.

For economists, the concepts of the networks refer both the structures of agents' interaction and to the economic property of positive externalities [5]. Networks as an interaction structure derive from the observation that many aggregate phenomena depend on the way in which economic agents interact. If the number of agents joining the networks increase, each one receives more benefit. So networks can be viewed both a set of links that build the interaction among agents, and they adopt a similar behavior for different purposes or motives. Agents try to establish links in order to achieve higher payoffs. Most often, yet not necessarily, it is assumed that the more agents join the network, the higher the payoff they receive, a property referred to as positive externality. Perhaps an essential point to put forward is that networks induce a special interdependency and a specific heterogeneity, which can affect network structure and aggregate phenomena in ways that are out of reach without them. To capture the intuition of this, it is enough to accept an agent's rational decision, depends on the agents it is directly linked with. This interdependence with heterogeneity in decision may in turn influence the evolution of the networks. As a result, networks can allow a rise in the number of emergent phenomena.

As a specific example of symmetric coordination, we consider a collection of agents that face the binary decision problem such that an agent wants to add in the social network only if the majority joins the network. Add the network may be given by threshold. In each time period, each agent is informed the proportion of the agents who add in the networks. Given this knowledge, each agent decides whether to add. The model thus predicts how collective action emerges and grows, as agents know more about each other in time. The most basic and common mechanism for collective action is that agents first get aggregate information about other agents’ actions, and then each agent decide whether to add or not. The main point is that an agent’s decision depends on what it knows about others, and the social network carries information, not social influence itself. Initially, each agent knows only its own threshold. After one period of time, it finds out the aggregate action through the social network. At each period of time, each agent decides whether to add or sever the network given the knowledge it has of the aggregate. An agent thinks strategically, knowing that everyone else is also making a rational decision given their own information. As time progress, and as each agent knows more about others, and then the network becomes to be larger.

There is another type of a coordination problem in which we have to utilize different methodologies. The EL Farol bar problem and its variants may provide a clean and simple example of asymmetric coordination problems [2][6]. Brian Arthur used a very simple yet interesting problem to illustrate effective uses of inductive reasoning of heterogeneous agents. There is a bar called El Farol in
downtown Santa Fe. There are $N$ agents interested in going to the bar each night. All agents have identical preferences. Each of them will enjoy the night at El Farol very much if there are no more than $m$ agents in the bar; however, each of them will suffer miserably if there are more than $m$ agents. In Arthur’s example, the total number of agents is $N = 100$, and the threshold number is $m = 60$. The only information available to agents is the number of visitors to the bar in previous nights.

What makes this problem particularly interesting is that it is impossible for each agent to be perfectly rational, in the sense of correctly predicting the attendance on any given night. This is because if most agents predict that the attendance will be low (and therefore decide to attend), the attendance will actually high, while if they predict the attendance will be high (and therefore decide not to attend) the attendance will be low. Arthur investigated the number of agents attending the bar over time by using a diverse population of simple rule based agents. One interesting result it obtained is that over time, the average attendance of the bar is about 60. Arthur examined the number of agents attending the bar over time by using a diverse population of simple rule based agents. One interesting result it obtained is that over time, the average attendance of the bar is about 60. Arthur examined the dynamic driving force behind this equilibrium. Agents make their choices by predicting ahead of time whether the attendance on the current night will exceed the capability and then take the appropriate course of action.

How exactly does an agent’s utility depend on the number of total participants? In other words, where does an agent's threshold come from? An agent’s threshold is best thought of as a reduced form parameter of a possibly complicated idiosyncratic decision process. For example, we participate if the expected benefits to us outweigh the expected. Specific features of coordination problems are that agent care about what others do, thus making each person’s decision is not completely independent. Each agent wanting to participate only if others do not, for example wanting to go to join only when many others do or go to the bar only it is not crowded.

There are strong interests in many disciplines to answer the following questions: how do individuals with heterogeneous micro-motives or micro-worlds generate self-organized global macroscopic orders or regularities [10][16]. However, there has been no natural methodology for systematically studying the dynamics of highly heterogeneous populations of agents. Evolutionary approaches are promising to study heterogeneity in agents. The literature on evolutionary models, however, has not considered heterogeneity of preferences or payoffs. Some models treat heterogeneous adaptive processes with the assumption of homogeneous payoff [8][10]. The growing literature on evolutionary models also treats agents [19] as automata, merely responding to changing environments without deliberating about individuals’ decisions. Within the scope of our model, we treat models in which agents make deliberate decisions by applying rational procedures, which also guide their reasoning.

We consider strategic coordination problems among $N$ heterogeneous agents $G = \{A;1 \leq i \leq N\}$. The best-developed formal account of rational decisions in interdependent environments is given by game theory [7][8]. We formulate a finite population game in which a pair of randomly chosen agents is matched within a period to play a game as shown in Fig.1. Each agent has two strategies, and the benefit of each strategy depends on how the other agent chooses its strategy as shown in Table 1. We assume that there are infinite numbers of random matches within each time period, and at any given moment, each agent is given the opportunity to observe the exact proportions of agents of choosing these strategies. Each agent is not assumed to be knowledgeable enough to correctly anticipate the other agents’ choices. An agent only needs to know about the collective behavior of the population. Then actions of a large number of agents can be set into the analysis of the following two-person game: Each agent with idiosyncratic payoff or utility plays against all others took collectively as shown in Fig. 2.

3 Formulation of Coordination Problems as $2 \times 2$ Heterogeneous Games

In our model, agents are assumed to have idiosyncratic utility function and local rules of interaction. They are rational in the sense that they maximize their exogenous utility or interest. To that extent our model can be characterized as methodologically individualists [15]. However, our model differs from the individualist camp insofar as we claim the collective orders have feedback effects in the agent organization, altering the behavior of individuals.
The choice of the majority | $S_1$ | $S_2$
---|---|---
$p(t)$ | $U_i^1$ | $U_i^3$
$(1 - p(t))$ | $U_i^2$ | $U_i^4$

Table 1: The payoff matrix

Fig. 2: The aggregated a $2 \times 2$ game between an agent and the collectivity

The proportion of agents having chosen $S_1$ at time $t$ is denoted by $p(t)$ ($0 \leq p(t) \leq 1$). Agents are assumed to be rational – that is, given their goals and preferences, and their perception of their situations, they act so as to maximize their utility. Each agent can take the best response against all the others taken collectively by calculating its expected payoff. Agent $A_i$ needs to be able to compare the expected utilities with $S_1$ and $S_2$, which are obtained as:

$$U_i(S_1) = p(t)U_i^1 + (1 - p(t))U_i^2$$

$$U_i(S_2) = p(t)U_i^3 + (1 - p(t))U_i^4$$

Agent $A_i$ chooses $S_1$ if

$$p(t)U_i^1 + (1 - p(t))U_i^2 \geq p(t)U_i^3 + (1 - p(t))U_i^4$$

or $S_2$ if

$$p(t)U_i^1 + (1 - p(t))U_i^2 < p(t)U_i^3 + (1 - p(t))U_i^4$$

The rational decision of agent $A_i$ is then described as the following two cases depending on its idiosyncratic utilities $U_i^1, U_i^2, U_i^3$ and $U_i^4$:

**Case 1:** $U_i^1 + U_i^2 - U_i^3 - U_i^4 > 0$

(i) $p(t) \geq (U_i^4 - U_i^3) / (U_i^4 + U_i^3 - U_i^2 - U_i^1) = \theta_i$

: chooses $S_1$

(ii) $p(t) < (U_i^4 - U_i^3) / (U_i^4 + U_i^3 - U_i^2 - U_i^1) = \theta_i$

: chooses $S_2$

**Case 2:** $U_i^1 + U_i^2 - U_i^3 - U_i^4 < 0$

(i) $p(t) \leq \theta_i$ : chooses $S_1$

(ii) $p(t) > \theta_i$ : chooses $S_2$

We define the right-hand side of (3.4) $\theta_i$ as threshold of agent $A_i$. The crucial point for dealing with heterogeneity among agents is threshold. A threshold model takes the elements of collective behavior which game theory handles with difficulty. This is possible because the two-dimensional payoff matrix can be replaced by one-dimensional threshold, one for each agent, which makes it easy to aggregate for the whole population. The payoff matrix of game theory allows us to investigate, for any particular agent, which outcome maximizes its utility and whether outcomes are Pareto optimal. However, threshold analysis does not permit this. When an individual is activated because its threshold is exceeding, it acts so as to maximize its utility under existing conditions. The resulting equilibrium may or may not maximize anyone’s overall utility [9].

4 Competitive Routing Problems

In this section we describe the dynamics of the populations in which each agent adapts its rational decision to the collectivity. Each agent is assumed to follow simple local rules. These rules are derivable from the rational calculation based upon costs and benefits, or forward-looking strategic analysis. Interdependent situations, in which an agent’s decision depends on the decisions of other agents, are the ones that usually don’t permit any simple summation or extrapolation to the aggregates. To make that connection we usually have to look at the system of interactions between individuals, which is also treated as the relation between individuals and the collectivity.

As a specific example, we consider a competitive routing problem in networks, in which the paths from sources to destination have to be established by multiple agents. For example, in the context of traffic networks, agents have to determine their route independently. In telecommunication networks, they have to decide on what fraction of their traffic to send on each link of the network. Consider two parallel routes A and B as shown in Fig. 3. There are $N$ agents that have to travel using one of the route A or B. They receive the utility, which is also determined what the majority does as shown in the payoff matrix in Table 2. In Table 3, the parameter $\alpha_i$ represents agent $A_i$’s relative preference over $S_1$. If $\alpha_i > 0$, agent $A_i$ personally prefers to use route A, and if $\alpha_i < 0$, agent $A_i$ prefers the route B. The absolute value of $\alpha_i$ represents the strength of preference. The parameter $\beta_i$ represents the preference to the consistency with the majority. An interesting problem is then under what circumstances will a collection of agents realizes some particular stable situations, and whether they satisfy the conditions of social efficiency?

Fig. 3 A competitive routing problem
Threshold of agent $A_i$ in (3.5) is given as follows:

$$\theta_i = -\frac{\alpha_i + \beta_i}{(\alpha_i - \alpha, \alpha - \beta, \beta_i)} = (1 + \frac{\alpha_i}{\beta_i}) / 2$$

(4.1)

The aggregate information $p(t)$, the current status of the collective decision, provides a significant effect on agents' rational decisions. A rational decision rule of agent $A_i$ is then given as:

(I) $p(t) \leq \theta_i$; takes route A ($S_1$)

(II) $p(t) > \theta_i$; takes route B ($S_2$)  

(4.2)

The proportion of agents with threshold $\theta_i$ in (3.5) is given as follows:

$$\theta = -\frac{\alpha_i + \beta_i}{(\alpha_i - \alpha, \alpha - \beta, \beta_i)} = (1 + \frac{\alpha_i}{\beta_i}) / 2$$

(4.3)

The collective decision at equilibrium is characterized by the fixed point, which is given by

$$p^* = 1 - F(p')$$

(4.4)

As an example, we consider the distribution function of threshold in Fig.5 (a). Its accumulated function is given in Fig.5 (b). In this case, there are two unstable equilibria $E_1$ and $E_2$. Starting from any initial condition, it converges to either $E_1$ or $E_2$ and, once it reaches $E_i$ or $E_2$, then it cycles between $E_1$ and $E_2$ as shown in Fig.5(c). This cyclic phenomena indicates that, if all agents use route A ($p(t) = 1$) at some time $t$, then they use route B at the next time period ($p(t+1) = 0$), and they repeat this inefficient cycle forever.

The number of agents with the same threshold $\theta_i$ is given by $n(\theta_i)$. The proportion of agents with threshold less than $\theta$ is given by

$$F(\theta) = \sum_{\theta_i \leq \theta} n(\theta_i) / N$$

(4.3)

where $N$ represents the number of the total agents. For simplicity we assume that there is no strategic interaction across time; an agent’s decision depends only on current information and not on any previous actions. The dynamics for collective decision of agents can be described as follows: The proportion of agents who use route A ($S_1$) at time $t$ is given as $p(t)$. By definition of the accumulated function $F(\theta)$ in (4.3), and the rational decision rule in (4.2), the proportion of agents who use route A at $t+1$ is given by

$$p(t+1) = 1 - F(p(t))$$

(4.4)

This example has a considerable intuitive appeal since it displays situations where rational individual action, in pursuit of well-defined preferences, lead to undesirable outcomes. One may reasonably wonder why there are not situations where the behavior of agents cannot usefully be summed up and predicted by the proportions of others who engage in one or another of two possible behaviors. At each period of time, each agent decides which strategy to choose given the knowledge of the aggregate behavior of the population. Each agent thinks strategically, knowing that everyone else is also making a rational choice given its own information.

These coordination failures may arise in many situations. They leave open an important question: what are the underlying strategic interactions that lead to successful coordination or unsuccessful coordination failures? We are interested in the bottom-up approach for leading to more
efficient coordination without the power of the central authority.

5 An Emergent Approach for Designing Efficient Network Management Policies

The crucial concept for describing such variation among individuals is that of their threshold. Threshold models choose the elements of collective behavior which game theory handles only with difficulty. On the other hand, the payoff matrix of game theory allows us to investigate, for any particular agent, which outcome maximizes its utility, and whether outcomes satisfy social efficiency. Threshold analysis, however, does not permit this.

In this section, we address the emergent approach for designing network management policies for inducing efficient equilibria. We introduce the different type of an agent, termed as conformist, with the payoff matrix in Table 3 with $\beta_i > 0$. The parameter $\beta_i$ represents the consistency level of each agent’s decision with the majority, and it prefers the majority does if $\beta_i > 0$. In Table 3, if we set $\beta_i < 0$, then it becomes equivalent with Table 2. Such an agent with $\beta_i < 0$, can be termed as a nonconformist, since an agent prefers the choice of the majority does not. All analysis in the previous section was for nonconformists. The rational decision rule for a nonconformist was given by (3.5), and that of a conformist is given by (3.4).

The other

<table>
<thead>
<tr>
<th>Agent $A_i$</th>
<th>$p(t)$</th>
<th>$1 - p(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$\alpha_i + \beta_i$</td>
<td>$\alpha_i$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$0$</td>
<td>$\beta_i$</td>
</tr>
</tbody>
</table>

Table 3. The payoff matrix of a conformist ($\beta_i > 0$) and a nonconformist ($\beta_i < 0$)

Threshold $\theta_i$ of a conformist $A_i$ is obtained as,

$$\theta_i = (\beta_i - \alpha_i) / (\alpha_i + \beta_i + \beta_i - \alpha_i) = (1 - \alpha_i / \beta_i) / 2$$

(5.1)

and the rational decision rule is given as follows:

(I) $p(t) \geq \theta_i$ : takes route A ($S_1$)

(II) $p(t) < \theta_i$ : takes route B ($S_2$)

(5.2)

In section 2, we distinguished two types of coordination problems, symmetric coordination, and asymmetric coordination. With symmetric coordination, each agent expects coordinating effect by taking the same action with the majority. According to the structure of the payoff matrix in Table 3 ($\beta_i > 0$) and the rational decision rule in (5.2), the symmetric coordination can be formulated as the problem of conformists. On the other hand, with asymmetric coordination, each agent can expect reward by choosing the behavior that the majority does not. According to the rational decision rule in (3.5), the asymmetric coordination can be formulated as the coordination problem among nonconformists.

We now consider the multi-stage competitive routing problem as illustrated in Fig.6. Here, we consider the policy to split a collection of agents into small groups at many distributions as shown in Fig.6 (a). At each turning point of the routing, each agent should decide to take the left route ($S_1$) or the right route ($S_2$). If we consider a collection of nonconformists only, as we discuss in the previous section, everybody may take the same route, therefore, all agents arrive at the same distribution as shown in Fig.6 (a). Then, the question is that how a collection of heterogeneous agents can split themselves into many subgroups without any central authority.

![Fig.6 Multi-stage competitive routing problems](image)

We consider a collection of agents of both conformists and non conformists, and the proportion of conformist ($\beta_i > 0$) is $k = N_1 / N$ ($0 < k < 1$), and that of nonconformist ($\beta_i < 0$) is $1 - k = N_2 / N$, where $N = N_1 + N_2$ is the total number of agents. Then the dynamic change of the proportion of $p(t)$ is described as:

$$p(t + 1) = k \times F_1(p(t)) + (1 - k) \times (1 - F_1(p(t)))$$

(5.3)

We now characterize the collective decision of heterogeneous agents by changing the value of $k$. In Fig.7, we show the distribution of threshold of both conformists (Fig.7(a)) and nonconformists (Fig.7(b)). The collective
decision is oscillated between 0.4 and 0.6 with $k=0.3$ (Fig.8 (a)). However with $k=0.5$ (Fig.8 (b)), it converges to 0.5, starting from any initial status. With $k=0.8$ (Fig.8(c)), it converges to either 0.2 or 0.8 depending on the initial conditions.

In Fig.9, we show the different distributions threshold of conformists (Fig.8(a)) and nonconformists (Fig.8(b)). The convergence of the collective decision is shown in Fig.10 with different $k$. With the small proportion of conformists ($k=0.3$) the collective decision is oscillated between 0.2 and 0.8 as shown Fig.10 (a). The oscillation level becomes small by increasing $k$ as shown in Fig.10 (b), and it converges to 0.5 at $k=0.8$ as shown Fig.10(c).

![Fig.7: The distribution function of threshold](image)

(a) conformist           (b) nonconformist

Fig.7: The distribution function of threshold

![Fig.8: The convergence of collective decision with different k](image)

(a) $k=0.3$ (b) $k=0.5$ (c) $k=0.8$

The above evolutionary process of collective action is guided by the self-interest seeking of heterogeneous agents. The mechanism has a strong similarity to the nature of a self-organizing and growing process. The growth starts from the set of the unstructured decision with heterogeneous motivations or interests. However, they are let to self-organize by establishing some stable collective decision as a whole.

The resulting dynamics can be quite complex. The stability of the dynamics is determined the distribution and combination of conformists and no conformists. Nonconformist do the opposite of the majority does. Conformists do what the majority does. Conformists have the feature to accelerate their collective action to converge. On the other hand, nonconformist has the feature to oscillate it. By joining conformists and nonconformists, the property of lock-in of conformists and the property of oscillation of nonconformists are merged and they can produce a stable macro behavior.

In previous works in the area of collective action or coordination behavior, the standard assumption was that agents use the same kind of adaptive rule[12][16]. In this paper, we departed from this assumption by considering a model heterogeneous agent with respect to their payoff structures. We showed how agents interact with the others
and their aggregate. We used the term emergent to denote stable macroscopic patterns arising from the local rules of agents. Form these simulation results, we can induce that knowing preferences, motives, or beliefs of agents can only provide a necessary but not a sufficient condition for the explanation of outcomes of the collective decision. The interaction of many individuals produces some kind of coherent, systematic behavior. Since it emerges from the bottom up, we point it as an example of self-organization. The surprise consists precisely in the emergence of macrostructure from the bottom up, which is from simple local rules that outwardly appear quite remote from the collective phenomena they generate. In short, it is not the emergent macroscopic object per se that is surprising, but the generative sufficiency of the simple local adaptive rules.

6 Conclusion

In this paper, we investigated the collective behavior of heterogeneous agents. Each agent was modeled to be rational in the sense that it sought to optimize its idiosyncratic payoff. We analyzed large-scale and heterogeneous interactions in a finite population of agents in which agents were repeatedly matched within a period to play a game. We explore the relation between the behavior characteristics of the individuals who comprise some aggregate, and characteristics of the aggregate. The motives of individuals can lead to striking and unexpected collective results. We describe the relation between each agent’s rational decision (micro behavior) and the collective decision (macro behavior) as the nonlinear dynamics. We provide a methodology of characterizing these nonlinear dynamics. Understanding how simple local rules give rise to collective structure is a central goal of the sciences of complexity. In the science of complexity, we would call a stable macroscopic or aggregate pattern induced by the local interaction of the agents as an emergent structure. Since it emerge from the bottom up, and we point it as an example of self-organization. In short, it is not the emergent macroscopic object per se that is surprising, but the generative sufficiency of the simple local adaptive rules.

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