Evolutionary Dynamics of Heterogeneous Agents with Local Interactions

Saori Iwanaga[1] & Akira Namatame[1]

[1]Dept. of Computer Science
National Defense Academy. Yokosuka, JAPAN
Phone: +81 468 41-3810 (ext. 2432) Fax: +81 468 44-5911
Email: {g40074, nama}@nda.ac.jp

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ABSTRACT

There are many researches on emergent properties and collective phenomena. The previous researches have mainly focused on the global interaction or random interactions of homogeneous agents. In this paper, we focus on heterogeneity of agents. And we address the following questions: how do heterogeneous agents generate emergent properties? These questions will depend crucially on how they interact and adapt their behavior. Agents myopically evolve their behavior based on their own rules, which is given as the function of their idiosyncratic utilities or of the action of their neighbors. We show that agents' behaviors reflecting their micromotives combined with of others' behaviors produce orders. The heterogeneity of agents is also concern for investigating emergent behavior. We consider the relation between emerged order and the diversity of the population.

Keywords and phrases: global matching, local matching, heterogeneity, emergence, best-response dynamics, mimicry dynamics

1.0 INTRODUCTION

Natural evolution has ceased a multitude of systems in which the actions of simple and interacting components give rise to coordinated global information processing (Sipper 1996). Insect colonies, cellular assemblies, the retina, and the immune system have all been cited as examples of systems in which emergent computation occurs. This term refers to the appearance of global information-processing capabilities that are not explicitly represented in the system's elementary components or in their interconnection. Our interest lies with systems in which many connected processors, with no central control, locally interact to procedure globally coordinated behavior.

We address the following question: how do the heterogeneous micro-macro worlds of agents generate the global macroscopic orders as the whole? The large-scale effects of interacting agents are called emergent properties, which emerges through the interactions among heterogeneous agents (Hofbauer *et al* 1998). Emergent properties are often surprising because it can be hard to anticipate the full consequences of even simple forms of interaction.

To understand the emergence, important concept is micro-macro loop. Micro world means his circumstance, in which an agent can interact. It is small world and each agent has each micro world. We call the agent's behavior as micro behavior. On the other hand, macro world is the world being brought about by the networks of the interaction. And we call the behavior of the population (collective behavior) as macro behavior. And it is the micro-macro loop that is relation between micro behavior and macro behavior. Micro behaviors produce macro behavior. On the other hand, macro behavior effects on micro behaviors. And there is a loop between them, which is shown in Fg.1. When we employ the concept of micro-macro loop, it is necessary to concern the connection among agents and the information of other agents. An agent behaves not only depending his preference, but also the others behavior in his micro world. So, it is important to consider with whom an agent interacts, which means micro world. And it is also important to consider how each agent decides his behavior depending on others behaviors.

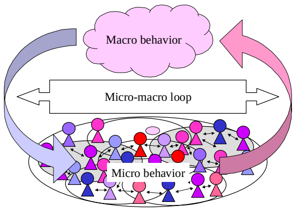


Fig.1: Micro-macro loop

There is a growing literature on the approach of bounded rationality, and the hypotheses employed in these researches reflect the ability of each agent to receive partial information from other agents in the course of their interaction (Rubinstein 1998). Our model can be interpreted in like manner, however, we intend to combine the hypotheses of adaptation and local interactions in modeling evolutionary processes. The first hypothesis reflects limited ability (on the agent's part) to receive, decide, and act upon information they get in the course of interactions. The second interpretation is that agents perform optimization calculations. We formalize these ideas in a model with a finite population of agents in which agents are repeatedly matched within a period to play a game and we consider to describe coordination (or common interest) game. Here are many parameters to be considered such as payoff structure, localization, the shadow of the future, the number of agents and so on. Among these parameters, we examine parameters: payoff structure and localization.

In many applications it is of interest to know which strategies survive in the long run. The evolutionary dynamic model with the assumption of random matching can be analyzed using replicator dynamics. The replicator dynamics is modeled as follows: Each agent is assumed to choose one of the possible pure strategies. The population state, in which each component of the state represents the population share of agents who adapt each possible pure strategy. The replicators are here the pure strategies, and these can be imitated from parents to child. As the population state changes, so do the payoffs to the pure strategies.

The replicator dynamics highlight the role of selection. In this paper, we also provide a new methodology for studying evolutionary games in the large that highlights the concepts both of rational mechanism that provides variety and selective mechanism that favors some varieties over others. From the results of the replicator dynamics, the qualitative dynamic behavior can be characterized by equilibria of the generic 2×2 static games (Weibull 1996). For the coordination game, there are three equilibria, the combinations of pure strategies, and the mixed strategies. The question of which strategy can survive depends on the initial distribution of those pure strategies, and the turning point is given by the mixed strategy. On the other hand, in the evolutionary model with the assumption of local matching, there is still a possibility that some other strategies can survive and many equilibria can coexist. It is also shown that weakly dominated strategies need not be eliminated. Even strongly dominated strategies can survive in certain special cases. Rationality provides variety and a selection favors some varieties over others. The criteria of rationality or selection are the average payoff of the population, which are the absolute criteria. However, in our model, no one needs to calculate the average payoff of the population. The criterion of rationality or selection is a relative one. Each agent compares his reward or neighborhoods' rewards, not to all agents in the population, can survive. This property is also essential to maintain both variety and harmony of the population.

2.0 THE COMPONENTS OF EVOLUTIONARY DYNAMICS

The term evolutionary dynamics often refers to systems that exhibit a time evolution in which the character of the dynamics may change due to internal mechanisms. We focus on several dynamics that may change in time according to rules of interaction of agents with both conscious and imitate decisions. We demonstrate various evolutionary phenomena by studying best-response dynamics and mimicry dynamics.

2.1 Matching Models; Random Matching vs. Local Matching

In order to describe the interactions of agents, we may have two fundamental models, global or random matching and local matching.

The approach of global matching is modeled as follows: in each time period, every agent is assumed to match (interact) with an agent at random from a population as shown in Fig.2 (Iwanaga *et al* 2001). Each agent chooses an optimal strategy based on an information about what all the other agents have done in the past. And each agent can calculate his reward or others' rewards and can play best action in population. An important assumption of the global matching is that they receive knowledge of the current situation through global matching. Then agents gradually learn the strategy in the population. In global matching, a micro world means as big as macro world.

The other model is local matching. In many situations, agents are not assumed to be so knowledgeable as to correctly guess or anticipate the other agent's strategies, or they are less sophisticated in that they do not know how to calculate best replies (Rubinstein 1998). The hypotheses we employ in the local interaction model reflect limited ability (on the agent's part) to receive, decide, and act upon information they get in the course of interactions. Each agent to receive partial information from other agents in the course of their interaction (Huberman 1993). As the local interaction model, we consider the lattice structure in Fig.3, in where each agent interacts with his eight neighbors (micro world). And each agent chooses an optimal strategy based on an information about what other agents have done in the past in his micro world. The consequences of their behaviors may take some time to have an effect on agents with whom they are not directly linked.

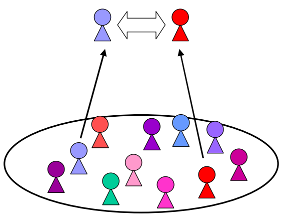


Fig.2: Global matching

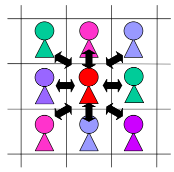


Fig.3: Local matching

2.2 Adaptive Models; Best-response vs. Mimicry

Next, we consider how an agent makes decision and adapts.

Agents may interact with other agents and they receive knowledge of the current strategy share of the population. Each agent chooses an optimal strategy based on an information about what other agents in his micro world have done in the previous period. An agent can calculate his reward and can play best strategy in his micro world. That is Best-response model.

We propose mimicry dynamics in which agents learn the most successful strategy in his micro world, and they co-evolve their strategies over time. Each agent adapts the most successful strategy in his micro world as guided for his own decision.

3.0 ADAPTATION MODELS

In this paper, we adopt the payoff matrix given in Table 1, which describes the outcome of the interaction between them. In this matrix, if both agents coincide same strategy, they can get the payoff $1-\theta_i$ or θ_i (≥ 0), otherwise they receive nothing. It means coordination game.

There are many situations where interacting agents can benefit from coordinating their actions. Coordination usually implies that increased effort by some agents leads the remaining agents to follow suit, which gives rise multiplier effects. Examples where coordination is important include trade alliance, the choice of compatible technologies or conventions such as the choice of a software or language. These situations can be modeled as coordination games; in which agents are expected to select the strategy the majority does (Huberman 1993; Schelling 1978). The traditional game theory, however, is silent on how agents know which equilibrium should be realized if a coordination game has multiple equally plausible equilibria, where these can be Pareto ranked (Arthur 1994; Fogel *et al* 1999). This silence is all the more surprising in games with common interest since one expects that players will coordinate on the Pareto dominant equilibrium (Hansarnyi *et al* 1988). The game theory has been also unsuccessful in explaining how agents should behave in order to improve an equilibrium situation (Fudenberg *et al* 1998).

Table 1: The payoff matrix ($0 \le \theta_i \le 1$)

Choice of agent A_i \ Choice of other Agents	S_1	S_2
S_1	$1-\theta_i$	0
S_2	0	θ_i

3.1 Adaptation with Best-response

The proportion of agents having chosen S_1 at time t is denoted by $p_i(t)$ $(0 \le p_i(t) \le 1)$ in agent A_i 's micro world M_i . (Micro world means macro world (population) in global matching. and it means agent A_i 's 8 neighbors in local matching.) Agents are assumed to be proportional - that is, given their goals and preferences, and their perception of their situations, they act so as to maximize their utility. Each agent can take the best response against all the others taken collectively by calculating its expected payoff. Agent A_i needs to be able to compare the expected utilities with S_1 and S_2 , which are obtained as:

$$\overline{U}_i(S_1) = p_i(t)(1-\theta_i)$$
$$\overline{U}_i(S_2) = (1-p_i(t))\theta_i \ . \tag{3.1}$$

Agent A_i chooses S_1 if

$$\overline{U}_i(S_1) \ge \overline{U}_i(S_1) \tag{3.2}$$

or S_2 if

$$\overline{U}_i(S_1) < \overline{U}_i(S_1) \ . \tag{3.3}$$

The proportional decision of agent A_i is then described as the following two cases depending on its idiosyncratic parameter θ_i .

$$p_i(t) \ge \theta_i : \text{chooses } S_1$$
$$p_i(t) < \theta_i : \text{chooses } S_2 \ . \tag{3.4}$$

That is best response rule. We define the right-hand side of (3.4) θ_i as threshold of agent A_i . The crucial point for dealing with heterogeneity in population is threshold. According to the threshold, agent's behavior is different from each other. Moreover, in local matching, agent differs in the proportion of agents having chosen S_1 among neighborhood $p_i(t)$ by the locations of agents. So, depending on the $p_i(t)$, agent's behavior is different from each other. That is, threshold (payoff structure) of agent effects on micro behavior and macro behavior in global matching. And in local matching, payoff structure and localization of agent effects on micro behavior.

3.2 Adaptation with Mimicry

In this section, we obtain the optimal strategy of imitators. Each agent can know others' utilities in his micro world and imitate from the agents who has received the highest payoff in the previous time. Agent in agent A_i 's micro world M_i is denoted by agent A_j . And we define the utility of agent A_i or A_j as U_i or U_j . The strategy of the agent with the highest payoff at time t is denoted by next functions.

$$\underset{A_j \in M_i}{Max} U_j > U_i : \text{Agent } A_i \text{ adapt agent } A_j \text{ 's strategy}$$

$$\underset{A_j \in M_i}{Max} U_j \le U_i : \text{Agent } A_i \text{ doesn't change his strategy} \tag{3.5}$$

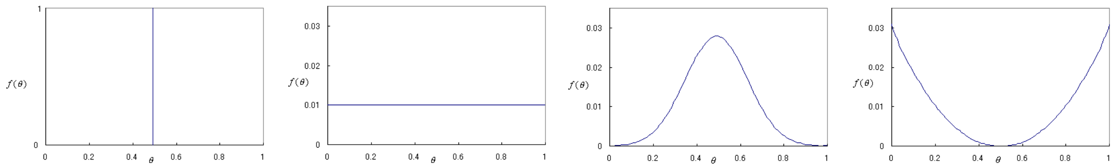
That is mimicry rule. The crucial point for dealing with heterogeneity in population is the payoff parameter. According to payoff parameter, agent's behavior is different from each other. Moreover, in local matching, the micro world is different form each other. So, the strategy that succeeded is different from each other. That is, the payoff structure effects on micro behavior and macro behavior in global matching. And in local matching, the payoff structure and localization of agent effects on micro behavior and macro behavior.

4.0 REPRESENTATION OF HETEROGENETY OF AGENTS

In this section, we dwell on the expression of heterogeneity of agents. In the population, each agent has different payoff parameter θ_i and $1-\theta_i$ in payoff matrix (Table1). So, we characterized the distribution pattern of the payoff parameter as the diversity of the population. We denote the number of agents with the same payoff value θ in the population G (consists of N agents) by $n(\theta)$. And the divided $n(\theta)$ by N is the distribution pattern of payoff parameter $f(\theta)$, which is given by $f(\theta) = n(\theta)/N$.

By the way, we consider the distribution pattern of the payoff parameter here again. If payoff parameter $1-\theta_i$ is greater than θ_i of agent A_i, it means that he prefers S_1. And if $1-\theta_i$ is less than θ_i, he prefers S_2. In short, if payoff parameter of agent A_i is less than 0.5, he prefers S_1. Otherwise, he prefers S_2.

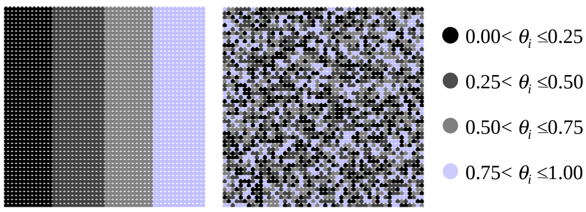
In this paper, we deal with the four populations whose distribution pattern of payoff parameter given as Fig 4(a), 4(b), 4(c) and 4(d). In Case 1, all agents' payoff parameters are 0.5 and this means homogeneous population, where all agents prefer S_1 as much as S_2. And in Case 2-4, each agent has different payoff parameter and those mean heterogeneous populations. And each population has half of agents with $\theta_i < 0.5$ and half of agents with $\theta_i > 0.5$, which means that there are half of agents preferring S_1 and half of agents preferring S_2. And Case 2 shows that agents' payoff parameters are uniformity and the population is most diverse. In Case 3, population is consisting of agent with payoff $0 < \theta < 1$ and there are many agents with payoff 0.5, which are moderate agents and prefer S_1 as much as S_2. And in Case 4, there are many agents with payoff 0 or 1, which strongly prefer S_1 or strongly prefer S_2. In any cases, these average payoff parameters are 0.5.



(a) Case 1 (b) Case 2 (c) Case 3 (d) Case 4

Fig.4: The distribution pattern of payoff parameter

In this paper, we also employ local interaction. So, location of heterogeneous agents in the population is important and we also characterized the configuration of agents as the diversity of the population. Here, we classify the configuration of agents into structural assignment and random assignment. Structural assignment is that agents with similar payoff parameter are located at nearby site. From the viewpoint of the population, it is well ordered. For example, the structural assignment of Case 2 is shown in Fig.5 (a). One the other hand, random assignment is that agents with similar payoff are located at random. And population is disordered. The configuration of Case 2 is shown in Fig.5 (b).



(a) Structural assignment (b) Random assignment

Fig.5: Configuration of agents (Case 2)

5.0 SIMULATION RESULTS

As specific examples of simulation, we arrange agents for an area of 50×50 (2500 agents) with no gap, and four corners and end of an area connect it with an opposite side. We set up 4 cases of population and simulate.

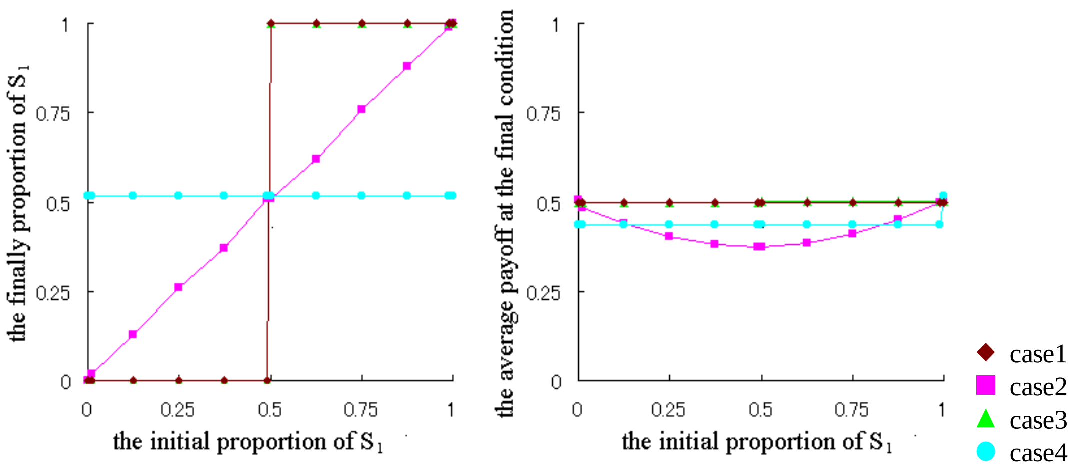
5.1 Evolutionary Dynamics with Best-response

The simulation results of globally best response model in these four populations are shown in Fig.6. In Fig.6 (a), the x -axis represents the proportion of agents to choose S_1 at the initial stage, and the y -axis represents that of the proportion at the final stage. In Case 1 (homogeneous population), depending on the initial proportion of S_1, collective behavior is widely different. Initial condition is less than 0.5, all agents finally choose S_1. However, if it is greater than 0.5, then all agents finally choose S_2 . On the other hand, in diverse populations, in which half of agents prefer S_1 and half of agents prefer S_2 , the collective behaviors depend on the diversity of the populations. In Case 3, collective behavior is same as Case 1, and all agents choose a strategy at the risk of the preferences. But, in Case 2, the initial proportion of S_1 comes to the final proportion of S_1 as it is and collective behavior is different according to the initial condition. And in Case 4, at any initial conditions, the simulation result comes to 0.5 and half of agents choose S_1 and half of agents choose S_2 according to the preferences. To clear on population's satisfaction, we investigated payoff. Fig.6 (b) shows the average payoff at the final condition. In any cases, we found that the average payoffs are almost same 0.5 independent of diversity of the population or initial conditions. Besides, investigating each agent's payoff, all agents get 0.5 in Case 1. But, in Case 2-4, we found there are huge discrepancies among agent.

In local matching, we classified a population as structural and random assignment by the configuration of agents. At first, we simulated random assignment (Fig.7 (a)). In Case 1, depending on the initial proportion of S_1, collective behavior is widely different. Initial condition is less than about 0.375, all agents finally choose S_1. However, if it is greater than about 0.375, then all agents finally choose S_2 . And the turning point is not sharper than that of global matching. And in Case 2-4, the simulation results are nearly same as that of global matching. As for the average payoffs (Fig.7 (b)), average payoffs are almost same 0.5 independent of initial conditions. But, there are huge discrepancies in Case 2-4 among agents.

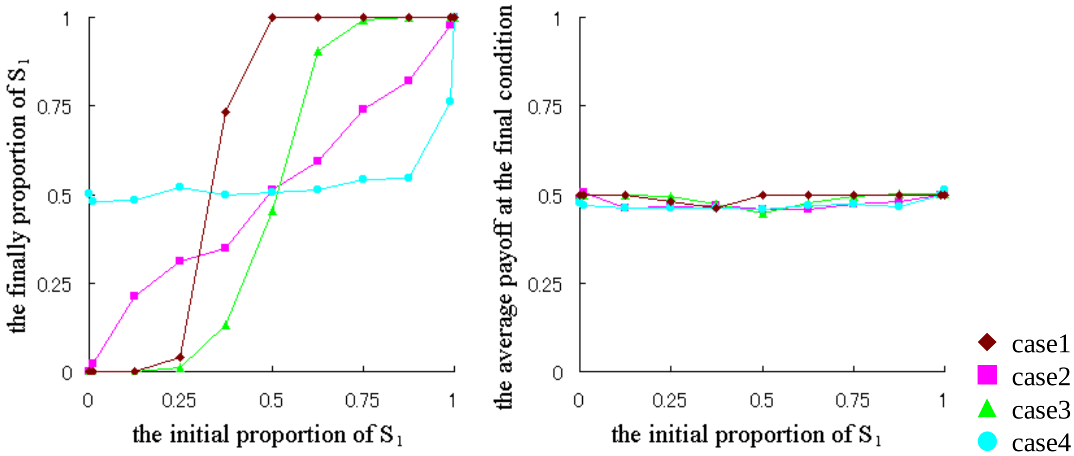
In structural matching (Fig.8 (a)), Case 1 is same as random assignment because all agent have a same threshold 0.5. But, in Case 2-4, at any initial conditions, the simulation result comes to about 0.5 and half of agents choose S_1 and half of agents choose S_2 according to the preferences, which have the similar property in Case 4 in global matching. As for the average payoffs (Fig.8 (b)), average payoff in Case 4 is the highest. Investigating each agent's payoff, all agents get 0.5 in Case 1, and in Case 2-4, the discrepancy among agents are smaller than random matching.

In the globally best response model, the collective behaviors depend on diversity of the population. But, in the locally best response model, we found that the collective behaviors depend on localization of agents. At the random assignment, the collective behaviors depend on diversity of the population. On the other hand, at the structural assignment, there turns up micro-macro loop to reflect their preferences in any populations and each agent gets better payoff.



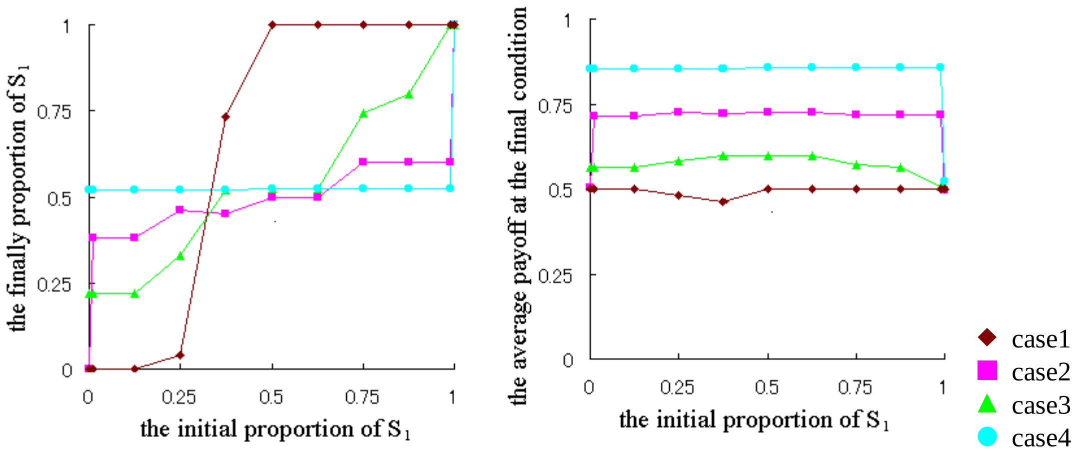
(a) Proportion of agents (b) Average payoff

Fig.6: Simulation results (Global matching)



(a) Proportion of agents (b) Average payoff

Fig.7: Simulation results (Random assignment-Local matching)



(a) Proportion of agents (b) Average payoff

Fig.8: Simulation results (Structural assignment-Local matching)

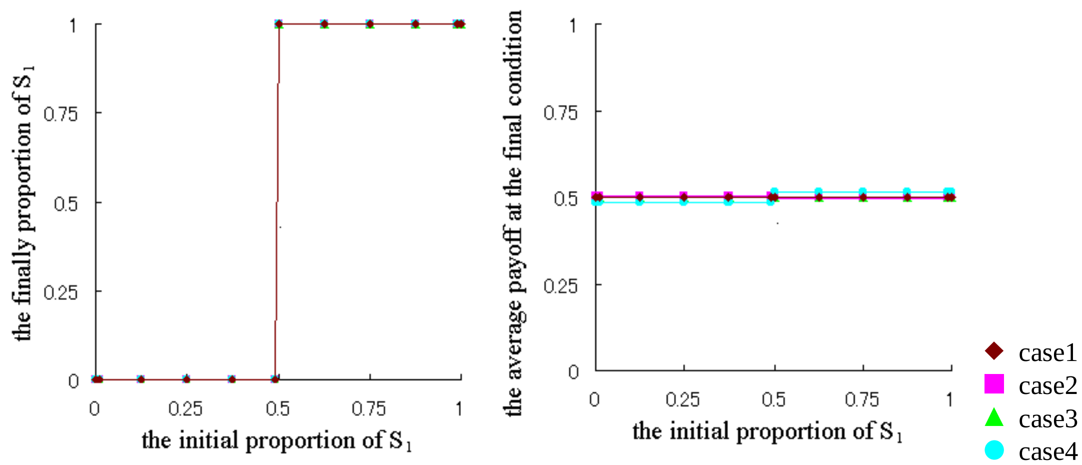
5.2 Evolutionary Dynamics with Mimicry

The simulation results of globally mimicry dynamics in four populations are shown in Fig.9 (a). In any cases, depending on the initial proportion, collective behavior is widely different and the average payoffs are almost same 0.5 independent of initial conditions (Fig.9 (b)). As for each payoff, all agents get 0.5 in Case 1, but in Case 2-4, there are huge discrepancies among agents.

Next, we simulated locally mimicry model. In random assignment (Fig.10 (a)), the simulation results are almost same in spite of the different population. In any cases, depending on the initial condition, collective behavior is widely different. Initial condition is less than about 0.5, all agents finally choose S_1 and if it is greater than 0.5, all agents finally choose S_2. But, the turning point is not sharp. And average payoffs (Fig.10 (b)) are almost same 0.5 independent of initial conditions. But, in Case 2-4, there are huge discrepancies among agents.

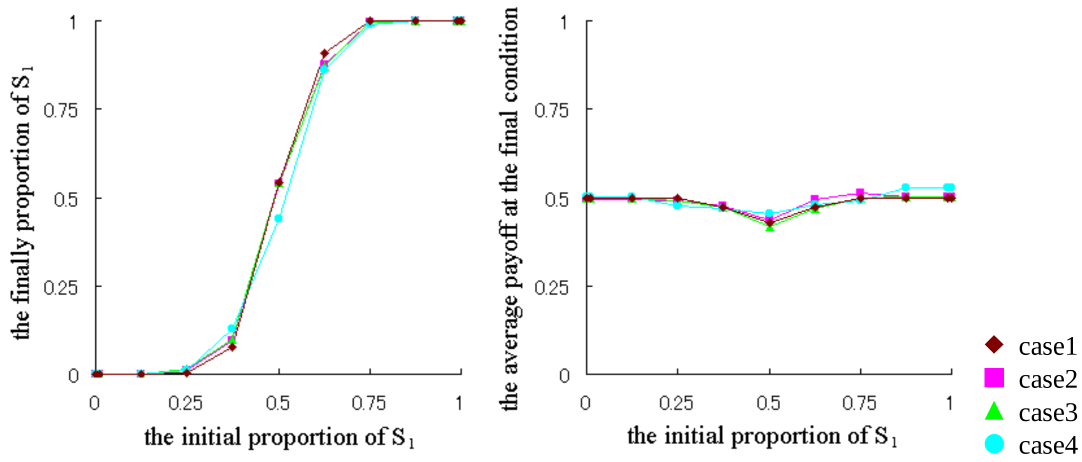
In structural assignment (Fig.11 (a)), Case 1 is same as random assignment. And in Case 2-4, at any initial conditions, the simulation result comes to about 0.5 and half of agents choose S_1 and half of agents choose S_2 according to the preferences. And average payoff (Fig.11 (b)) in Case 4 is the highest. As for each payoff, many agents get about 0.5 in Case 1, but in Case 2-4, the discrepancy among agents are smaller than random matching.

In the globally mimicry model, the collective behaviors don't depend on diversity of the population. But, in the locally mimicry model, we found that the collective behaviors depend on the assignment of the agents. At the random assignment, the collective behaviors don't depend on diversity of the population. On the other hand, at the structural assignment, there turns up micro-macro loop to reflect their preferences in any populations and agents get better payoffs.



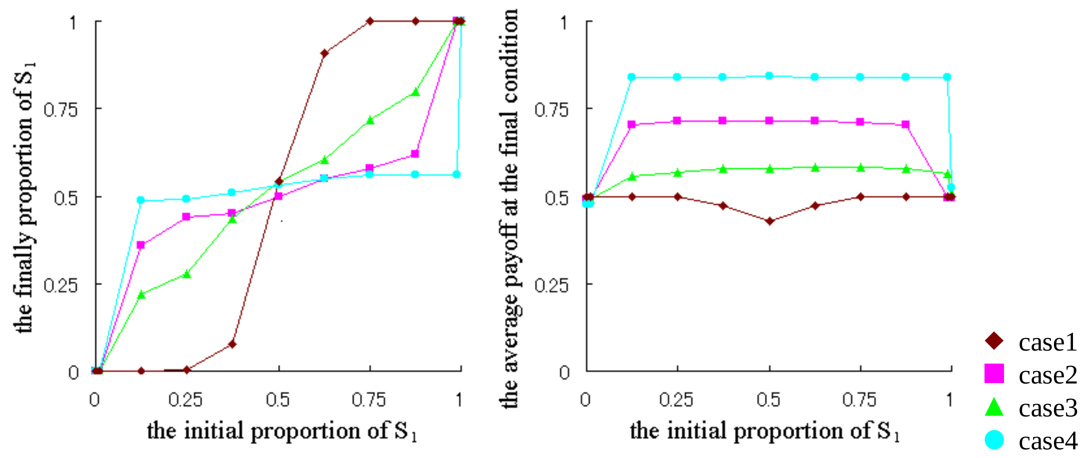
(a) Proportion of agents (b) Average payoff

Fig.9: Simulation results (Global matching)



(a) Proportion of agents (b) Average payoff

Fig.10: Simulation results (Random assignment-Local matching)



(a) Proportion of agents (b) Average payoff

Fig.11: Simulation results (Structural assignment-Local matching)

6.0 CONCLUSION

The evolutionary process of collective behavior is guided by the heterogeneous agents. The mechanism has a strong similarity to the nature of an emergence and growing process. The growth starts from the set of the unstructured decision with heterogeneous motivations or interests. However, they are let to emergence by establishing some stable collective decision as a whole.

In previous works in the area of collective behavior or coordination behavior, the standard assumption was that agents use the same kind of adaptive rule (Fogel 1999; Fudenberg *et al* 1998). In this paper, we departed from this assumption by considering a model heterogeneous agent with respect to their payoff structures. We showed how agents interact with the others and their aggregate. We used the term emergent to denote stable macroscopic patterns arising from the idiosyncratic rules of agents. Form these simulation results, we can induce that knowing preferences, motives, or beliefs of agents can only provide a necessary but not a sufficient condition for the explanation of outcomes of the collective behavior. The interaction of many agents produces some kind of coherent, systematic behavior. The surprise consists precisely in the emergence of macrostructure from the bottom up, which is from simple rules that outwardly appear quite remote from the collective phenomena they generate.

There emerges micro-macro loop to reflect the coordination of among agents at random assignment in any populations. And there emerges micro-macro loop to reflect their preferences at the structural assignment in any populations and agents get better payoffs.

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