CHAPTER 7

Selective Interaction with Reinforcing Preference

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Abstract

The performance of a collective system crucially depends on the way they interact as well as how they adapt to others. There are two closely related issues concerning a collective of interacting agents, the forward and inverse problems. The forward problem is to investigate what a collective of interacting agents determines as a complex emergent behavior. The inverse problem is to investigate how the private utility functions of agents can be modified so that their self-interested behavior collectively gives rise to a desired outcome. This paper examines the effects of the combined model of the partner choice and preference reinforcing in order to solve the inverse problem. Agents choose their partners and decide on a mode of behavior when interacting with selected partners. Two types of meta-agents are considered: conformists who behave based on the logic of majority, and nonconformists who behave based on the logic of minority. It is shown that a collection of conformists and nonconformists having the identical preference initially evolves into a collection of heterogeneous agents with diverse preferences and that they achieve both an efficient and equitable outcome.

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1 1. Introduction

3 The following question is often addressed: how does a collection of interacting
 5 agents generate global macroscopic orders as a whole? An interesting aspect of
 7 interacting agents is emergent property. Natural evolution has created a mul-
 9 titude of systems in which the actions of interacting agents give rise to coor-
 11 dinated global information processing. Insect colonies, cellular assemblies, the
 13 retina, and the immune system have been often cited as examples of having
 15 emergent properties. Emergent property is surprising because it can be hard to
 17 anticipate the full consequences of even simple forms of interaction. Emergence
 also refers to the appearance of global information-processing capabilities that
 are not explicitly represented in the systems' elementary components or in their
 interconnection. However, there is no presumption that a population of inter-
 acting agents in an imperfect world leads to a collectively satisfactory result.
 How well agents do for it in adapting to their environment is not the same thing
 as how satisfactory is an environment they collectively create. While all agents
 understand the outcome is inefficient, acting independently is powerless to
 manage an efficient collective outcome.

19 In order to lead collective behavior to be a desirable one, we need to consider
 two different levels: the microscopic level, where the decisions of the individual
 21 agents occur and collective behavior can be observed. We also have to specify
 how agents interact, respond, adapt, or learn from each other. However, to
 23 understand the role of the link between these two levels also remains one chal-
 lenge (Huberman and Glance, 1993; Helbing, 1995).

25 In examining collective effects, we shall draw heavily on the individual de-
 cisions. It might be argued that understanding how individuals make decisions is
 27 sufficient to understand collective behavior. Many researchers have pointed out
 that equilibrium theory does not resolve the question of how people behave in a
 29 particular interdependent decision situation. It is often argued, "it is hard to see
 what can advance the discussion short of assembling a collection of people,
 31 putting them in the situation of interest, and observing what they do" (Bala and
 Goyal, 1998, 2000; Banerjee, 1999). People have preferences, pursuing goals, and
 33 they behave in a way that we might call "purposive". We metaphorically ascribe
 motives to behavior because something behaves as if it were oriented toward a
 35 goal. But the purposes or objectives often relate directly to those of others.
 Therefore their behaviors are also constrained by other people who are pursuing
 37 their own purposes.

Schelling (1978) characterized contingent behavior – behavior that depends
 39 on what others are doing. What makes contingent behavior interesting and
 difficult is that the entire aggregate outcome is what has to be evaluated, not
 41 merely how each person does within the constraints of her own environment.
 Although individual decision is important to understand, it is not sufficient to
 43 describe how a collection of agents arrives at specific decisions. We attempt to
 probe deeper understanding of the issue by specifying how they interact with

each other. The greatest promise lies in analysis of situations where many agents behave in ways contingent on one another, and these situations are central in theoretical analysis of linking micro to macro levels of collective decision. The overall collective performance depends crucially on the type of interaction as well as on the heterogeneity of agent preferences. An externality occurs if one cares about others' decisions, and their decisions also affect her own decision. An agent's outcome, whichever way he makes his choice, also depends on the number of agents who choose one way or the other. An interesting problem is then under what circumstances will a collection of agents realize some particular stable situations, and whether they satisfy the conditions of efficiency (Iwanaga and Namatame, 2001, 2002).

There are two closely related issues concerning a collective of interacting agents, the forward and inverse problems (Tumer and Wolpert, 2004). An agent behaves based not only on her preference but also on others' actions. It is also important to consider with whom an agent interacts and how each agent decides her action depending on others' actions. Agents are selfish in the sense that they only do what they want to do and what they think according to their own best interests, their motivations. They necessarily have different sets of goals, motivations, cognitive states by virtue of their different histories, the different resources they use, or different settings they participate in, and so on.

2. The inverse problem

Agents myopically adapt their behavior based on their idiosyncratic rule to others' behaviors. In the simplest form of our model, agents are born with fixed preferences. They are also assumed to be rational in the sense that they select their strategy in order to maximize their endogenous preferences as shown in Figure 1(a). However individuals' behavior should be understood in a social setting. In order to understand their behavior, we must observe them within social and cultural environments. The inverse problem consists then to investigate how their endogenous preferences can be modified through interactions and strategy choices in the past as shown in Figure 1(b).

Figure 1. The forward and inverse problems (a) Preference determines strategy choice (b) Strategy choice reinforces preference

1 It will be nice to motivate the source of learning algorithms by properly
 3 redesigning the payment schemes. One possible approach is to consider a me-
 5 diator who provides the agents with algorithms to use and suggests payments to
 7 be made. This correlation device is not a designer who can enforce behaviors or
 9 payments, and it does not possess any private knowledge or aim to optimize
 11 private payoffs. Game theory is typically concerned with learning of a strategy
 13 (Fudenberg and Levine, 1998). Agents repeatedly play a game, each time ob-
 15 serving their rewards, which reflect their prefixed preference. Typically, these
 17 studies have the goal of showing that some simple update rule leads the agents to
 19 eventually adopt some Nash equilibrium. However, the learning algorithms
 21 themselves are not required to satisfy any rationality requirement; it is what they
 23 converge to, if adopted by all agents that should be in equilibrium.

13 Shoham and Powers (2003) characterized the so-called ‘AI agenda’ asking
 15 what the best learning strategy is for a given agent for a fixed class of the other
 17 agents in the game [19]. It thus retains the design stance of AI, asking how to
 19 design an optimal agent for a given environment. There is another agenda that is
 21 more prescriptive. It asks how agents should learn in the context of other
 23 learners. The recent literature on multi-agent learning also deals extensively with
 25 connections between distributed systems and the design of economic mecha-
 27 nisms. One central issue in that literature is the selection of a solution criterion:
 29 what can we assume about the agents’ rational behavior. Much of the literature
 31 deals with the search for mechanisms where the agents will have dominant
 33 strategies that lead to desired behavior (maximizing social efficiency). However,
 35 it can be shown that such mechanisms rarely exist. One alternative is to consider
 37 mechanisms where there exists an ex post equilibrium: a strategy profile of the
 39 agents in which it is irrational to deviate from each agent’s learning algorithm,
 41 assuming the other agents stick to their strategies, and regardless of the state of
 43 the system [20].

QA :4

QA :5

31 **3. Classification of social interactions**

31 Social interactions pose many coordination problems to individuals. There are
 33 many situations where interacting agents can benefit from coordinating their
 35 action. Examples where coordination is important include trade alliance, the
 37 choice of compatible technologies or conventions such as the choice of a soft-
 39 ware or language. We can classify interaction with externality into two types.
 41 Coordination usually implies that the increased effort by some agents leads the
 43 remaining agents to follow them, which rises a multiplier effect or bandwagon
 effects. These are also characterized as situations where interacting agents can
 benefit if they take the same action. We call this type as social interaction with
 positive externalities. In this case, agents behave based on the logic of majority,
 since agents receive payoffs if they select the same strategy as the majority does.
 These symmetric social interactions are modeled as coordination games in which
 an agent receives a payoff if he selects the same strategy as the others.

On the other hand, in the route selection problem for instance, an agents receives a payoff if he selects the opposite strategy as the majority does. For example, in the context of traffic networks, agents have to determine their routes independently. In telecommunication networks, they have to decide what fraction of their traffic to send on each link of the network. This type of interaction is distinguished by defining as social interaction negative externalities. In this case, agents behave based on the logic of minority, since agents receive payoff if they select the opposite strategy as the majority does (Beckmann *et al.*, 1956).

The El Farol bar problem and its variants minority games provide a clean and simple example of this type of social interaction (Challet and Zhang, 1997). The market entry game is also a stylized representation of a common problem based on the logic of minority: a number of agents have to choose independently whether or not to undertake some activity, such as enter a market, go to a bar, or drive on a road, the utility from which decreasing in the number of the participants (Ochs, 1998; Duffy and Hopkins, 2002). The choice of market entry games for studying coordination is quite natural. When there are too many potential entrants wishing to exploit a new market opportunity, a problem arises regarding how entry may be coordinated. Without coordination, too many firms may decide to enter the market and consequently they sustain losses. Conversely, fully aware of the consequences of excessive entry, firms may be reluctant to enter and exploit the market in the first place. These examples of social interaction typically admit a large number of Nash equilibria. Given this multiplicity of equilibrium outcomes, an obvious question is which type of equilibrium are agents likely to coordinate upon? The coordination failure is attributed to certain features of the payoff functions.

4. Rational decisions of conformists and nonconformists

In most cases, the decision can be thought of as having a positive and negative side – deciding to do a thing or not to do it. We model the problems of individual and collective decisions as follows: each agent has two alternatives, and the costs and benefits of each alternative depend on how many other agents choose the same alternative.

As a specific example, we consider the situation in which each agent in the population of N agents faces the binary decision problem with the following two choices:

S_1 : votes for, S_2 : votes against

One's decision to vote for a particular alternative may depend heavily on how many others decide to do so, partly because of social influence, partly because one does not want to waste her own vote.

In this example, the payoff matrix is given in Table 1. When $\alpha_i > 0$, agent i personally prefers to vote for, and if $\alpha_i < 0$, she prefers to vote against. The

Table 1. The payoff matrix of a conformist

Agent i	The others	
	$p(t)$ S_1	$1-p(t)$ S_2
S_1	$\alpha_i + \beta_i$	α_i
S_2	0	β_i

absolute value of α_i represents the strength of her preference. The parameter $\beta_i (> 0)$ represents the level of consistency level with the others' choices.

Agents myopically change their actions based on their own rules obtained as a function of their idiosyncratic utility and of the actions of their neighbors. Assignment of heterogeneous agents also becomes important. We investigate the relation between the collective behavior and the assignment of heterogeneous agents.

(1) Rational decision of a conformist

We obtain a rational decision rule of a conformist. Let suppose the ratio of agents to choose S_1 is $p(t)$. We define the threshold of conformist i as:

$$\theta_i = (\beta_i - \alpha_i) / (\alpha_i + \beta_i + \alpha_i - \beta_i) = (1 - \alpha_i / \beta_i) / 2 \quad (1)$$

Then the rational decision rule of a conformist is given as:

(i) $p(t) \geq \theta_i$: votes for (S_1)

(ii) $p(t) < \theta_i$: votes against (S_2) (2)

(2) Rational decision of a nonconformist

Nonconformist with the payoff matrix of Table 2 prefers the opposite choice of the majority. The threshold of a nonconformist i is defined as:

$$\theta_i = (1 + \alpha_i / \beta_i) / 2 \quad (3)$$

Then a rational decision rule of a nonconformist is given as:

(i) $p(t) < \theta_i$: votes for (S_1)

(ii) $p(t) \geq \theta_i$: votes against (S_2) (4)

The strategic decisions of a conformist and a nonconformist can be also modeled with the payoff matrix in Table 3. If $\beta_i > 0$, agent i is a conformist, and if $\beta_i < 0$, she is a nonconformist. From the rational decision rules in (3.2) and (3.4), the agents with heterogeneous payoffs are classified into the following four types.

Table 2. The payoff matrix of a nonconformist

Agent i	The others	
	$p(t)$ S_1	$1-p(t)$ S_2
S_1	α_i	$\alpha_i + \beta_i$
S_2	β_i	0

Table 3. The payoff matrix of a conformist ($\beta_i > 0$) and nonconformist ($\beta_i < 0$)

Agent A_i	The other agents	
	$p(t)$ S_1	$1-p(t)$ S_2
S_1	$\alpha_i + \beta_i$	α_i
S_2	0	β_i

(1) Type 1: hardcore S_1

If agent i is a conformist with the payoff parameters $0 \leq \beta_i \leq \alpha_i$ or she is a nonconformist with $-\alpha_i \leq \beta_i \leq 0$, she always chooses S_1 without regard to the others' decisions. In these cases, the strategy S_1 becomes a dominant strategy. This type of an agent is a hardcore of the S_1 chooser.

(2) Type 2: hardcore S_2

If agent i 's payoff parameters satisfy $0 < \beta_i < -\alpha_i$ when she is a conformist, or $\alpha_i < \beta_i < 0$ when she is a nonconformist, she always chooses S_2 without regard to the others' decisions. In this case, the strategy S_2 becomes a dominant strategy. This type of an agent is regarded as a hardcore of the S_2 chooser.

(3) Type 3: conformist

If agent i is conformist ($\beta_i > 0$) and her payoff parameters satisfies $|\alpha_i| < \beta_i$, her rational decision depends on the proportion of agents who make the same decision. Since he prefers what the majority does, this type of an agent is regarded as a conformist.

(4) Type 4: nonconformist

If agent i is nonconformist ($\beta_i < 0$) and her payoff parameters satisfies $|\alpha_i| < -\beta_i$, her rational decision depends on the proportion of agents who make the opposite decision. Since she prefers what the majority does not, this type of an agent is regarded as a nonconformist.

In Figure 2, the x -axis represents the parameter value of α_i , each agent's preference level over two choices (purposive behavior), and the y -axis represents the parameter value of β_i , the level of consistency with the others' decisions

1 **Figure 2.** *Classification of agent types from their idiosyncratic payoffs (α_i, β_i)*

19 (contingent behavior). An infinite number of possible decision rules of hetero-
 20 geneous agents can be classified into the above four types.

5. Heterogeneity in preferences

25 In human societies, an essential element is that individuals differ from each
 26 other. The diversity comes into play in many instances of collective behavior. In
 27 this section, we depart from the assumption of homogeneity with respect to the
 28 payoff parameters. In particular, we consider continuum games with infinite
 29 number of heterogeneous agents with respect to their preferences.

30 We assume that a conformist has the payoff matrix in Table 4, and a non-
 31 conformist in Table 5. We should remark that the payoff matrices in Tables 1
 32 and 2 are strategically equivalent with those in Tables 4 and 5. The heterogeneity
 33 among agents can be described by their payoff parameter θ ensuring an enor-
 34 mous simplification in the present analysis. In our model, agents have idiosyn-
 35 cratic payoff parameter θ . The diversity of a collection of heterogeneous agents is
 36 characterized by the distribution function of the payoff parameter θ . We con-
 37 sider a collection of N agents, and denote the number of agents with the same
 38 parameter value θ by $n(\theta)$. We define the density of θ by $f(\theta)$, which is obtained
 39 by dividing $n(\theta)$ by the total number of agent N , i.e.

$$f(\theta) = n(\theta)/N \quad (5)$$

41 As specific examples, we consider three density functions in Figure 3. In Fig-
 42 ure 3(case 1), all agents have the same payoff parameter value $\theta = 0.5$, and in
 43 Figure 3(case 2), a half of the population have the payoff parameter value $\theta = 0$,
 and the rest of them have $\theta = 1$.

Table 4. The payoff matrix of a conformist

$$\theta(1 = +\alpha/\beta)/2(0 \leq \theta \leq 1)$$

Choice of agent i	Choice of other agents	
	S_1	S_2
S_1	$1-\theta$	0
S_2	0	θ

Table 5. The payoff matrix of a nonconformist

$$\theta(1 = -\alpha/\beta)/2(0 \leq \theta \leq 1)$$

Choice of agent i	Choice of other agents	
	S_1	S_2
S_1	0	θ
S_2	$1-\theta$	0

In examining collective effects, we shall draw heavily on the individual adaptive decisions. Within the scope of our model, we treat the case in which agents make deliberate decisions by applying rational procedures, which also guide their reasoning. In order to describe the adaptation process at the individual level, we may have two fundamental models, global interaction and local interaction. It is important to consider with whom an agent interacts and how she decides her action depending on others' actions. Agents may adapt based on the aggregate information representing the current status of the whole system (global information). In this case, each agent chooses an optimal decision based on aggregate information about how all other agents behaved in the past. In many situations, agents are not assumed to be able to correctly guess or anticipate other agents' actions, or they are not able to know how to calculate best replies. With local adaptation, each agent is modeled to adapt to local information (Kirman, 1997; Tennenholtz, 2002).

As a specific model, we consider the lattice structure as shown in Figure 4. We arrange a collection of heterogeneous agents in the torus of 50×50 (2500) agents where the four corners and an edge of an area connect it with an opposite side. The consequence of their actions also gives an effect on agents with whom they are not directly linked. The hypothesis of local adaptation also reflects limited ability of agents to receive, decide, and act based upon information they receive in the course of interaction.

1 **Figure 3. Some density functions of the payoff parameter: $f(\theta)$ Case 1: Distri-**
 3 **bution with one-peak; Case 2: Distribution with the two-peaks; Case 3: Normal**
 5 **distribution; Case 4: Uniform distribution; and Case 5: Polarized distribution**

(Case 1)

(Case 2)

(Case 3)

(Case 4)

(Case 5)

Global adaptation rule of a conformist

37 Agents adopt actions that optimize their expected payoff given what they
 39 expect the others to do. In this model, agents choose the best replies to the
 empirical frequencies distribution of the previous actions of the others. The main
 point is that an agent's decision depends on what he knows about the others. We
 41 obtain the adaptive rule of each agent as her best response. We denote the
 proportion of agents having chosen S_1 at time t by $p(t)$. The optimal adaptive
 43 rule of an agent is obtained as the function of the aggregate information on
 collective $p(t)$ and her idiosyncratic payoff parameter θ as follows:

Figure 4. Adaptation to (a) global information and (b) local information

- (i) If $p(t) \geq \theta$, choose S_1
- (ii) If $p(t) < \theta$, choose S_2 . (6)

Local adaptation rule of a conformist

- (i) If $p_i(t) \geq \theta$, choose S_1
- (ii) If $p_i(t) < \theta$, choose S_2 . (7)

We denote the proportion of the neighbors of an agent having chosen S_1 at time t by $p_i(t)$. The optimal adaptive rule with local adaptation is obtained as follows:

We evaluate the collective result of interacting heterogeneous agents with two criteria, efficiency and equity. Efficiency is evaluated by the average utility, which also stands for the measure of the desirability at the macro level. Equity is evaluated the utility distribution, which stands for the measure of the desirability at the micro level. The Gini ratio is often used to measure the dispersion of the utility distribution of a society. The pairs of the average payoff U and equity E of all heterogeneous populations of conformists both under global adaptation and local adaptation are shown in Figure 5. The effect of local adaptation compared with global adaptation has been discussed by many researchers (Kirman, 1997; Tennenholtz, 2002). With these simulation results, both efficiency and equity are low when heterogeneous agents adapt locally. The merit of local adaptation is enhanced for the collective of conformists.

Global adaptation rule of a nonconformist

The global adaptive rules of a nonconformist is obtained as follows:

- (i) If $p(t) \leq \theta$, choose S_1

1 **Figure 5. Efficiency U and equity E for conformists (the initial proportion is $p(0) = 0.5$) (a) Global & (b) Local adaptation**

17 (ii) If $p(t) > \theta$, choose S_2 (8)

21 *Local adaptation rule of a nonconformist*

23 The local adaptive rule of a nonconformist is obtained as follows:

25 (i) If $p_i(t) \leq \theta$, choose S_1

27 (ii) If $p_i(t) > \theta$, choose S_2 . (9)

29 We evaluate a collective of nonconformists with two criteria, efficiency and
 31 equity. The pairs of the average payoff U and equity E of all heterogeneous
 33 populations of nonconformists both under global adaptation and local adap-
 35 tation are shown in Figure 6. As shown in Figure 6, both efficiency and equity
 are low when conformists adapt locally. On the other hand, the merit of local
 adaptation is enhanced for the collective of nonconformists.

37 6. Selective interaction of heterogeneous agents

We formalize our idea by modeling a population of heterogeneous agents in
 39 which agents are repeatedly matched within a long-time period to play coordi-
 41 nation games, see Table 4. There are many parameters to be considered such
 43 as payoff matrix, population structure, population configuration, the number of
 agents, and so on. Among these parameters, we examine the effects of hetero-
 geneity in payoffs θ and the configuration of locating agents. The interaction
 methodology also plays an important role in the outcome. We consider two

Figure 6. Efficiency U and equity E for nonconformists (the initial proportion is $p(0) = 0.5$). (a) Global adaptation and (b) Local adaptation

fundamental models, random (or global) interaction and local interaction. Agents myopically evolve their actions based on their own rules obtained as a function of their idiosyncratic utility and of the actions of their neighbors.

Assignment of heterogeneous agents also becomes important. We investigate the relation between the collective behavior and the assignment of heterogeneous agents. Our interest is to investigate whether that systems, in which many locally connected processors with no central control, can produce efficient collective performance. We evaluate the collective behavior from the criteria of both efficiency and equity.

The crucial concept for describing heterogeneity of agents is their preference characterized by the value of payoff parameter θ . Heterogeneity of preferences makes it possible to have a different type of interaction, selective interaction. This is possible because each agent has a different payoff parameter. We classify heterogeneous agents into the following two types:

Type 1: The payoff parameter value θ is less than 0.5 (such an agent prefers S_1 to S_2).

Type 2: The payoff parameter value θ is greater than 0.5 (such an agent prefers S_2 to S_1).

We also classify interaction types into the following three types as shown in Figure 7:

- (1) Random assignment: each agent has a chance to interact with neighbors of any type.
- (2) Sorted assignment: each agent interacts with neighbors of the same type.
- (3) Mixed assignment: each agent interacts with neighbors of the opposite type.

1 **Figure 7. Assignment of heterogeneous agents (a) Random, (b) sorting, and (c) Mixed**

11

13 We describe the selective interaction process of the agents. An agent of type 1 ($\theta \leq 0.5$) chooses S_1 , since such an agent prefers S_1 to S_2 . On the other hand, an
 15 agent of type 2 ($\theta > 0.5$) chooses S_2 , since such an agent prefers S_2 to S_2 . In Figure 8, we represent the selection process of an agent. Each agent interacts
 17 with neighbors by choosing her preferred strategy. If she receives the average payoff per one neighbor more than 0.5, she remains in the same location, oth-
 19 erwise she moves to another location and interacts with different neighbors.

At the beginning, we set two types and heterogeneous agents randomly in the
 21 lattice of 50×50 agents. Conformists and nonconformists of type 1 and type 2 are randomly assigned as shown in Figures 9(a) and 10(a). Each agent interacts
 23 with current 8 neighbors. If an agent receives an average payoff per one neighbor less than 0.5, then he is defined as “unhappy agent”. We choose randomly any
 25 two unhappy agents on the lattice, and we replace their locations so that all unhappy agents can change their current neighbors. After a few hundreds of
 27 rounds, a collection of heterogeneous conformists could self-organize both their locations and their actions so that they can realize homogeneous interaction as
 29 shown in Figures 9(b) (conformists) and 10(b) (nonconformists).

We evaluate collectives of both conformists and nonconformists with two
 31 criteria, efficiency and equity. The pairs of the average payoff U and equity E under selective interaction are shown in Figure 11. Compared with the results
 33 shown in Figure 5, both efficiency and equity are improved when agents interact with the fixed neighbors. Therefore, a collective of heterogeneous agents can
 35 achieve the highest efficiency and equity under selective interaction.

37 7. Selective interaction with reinforcing preferences

39 Game theory is typically based upon the assumption of a rational choice (Fudenberg and Levine, 1998). The real advantage of the rational-choice as-
 41 sumption is that it often allows deduction. The main alternative to the assumption of rational choice is the adaptive approach with reinforcement learning.
 43 With reinforcement learning, agents tend to adopt actions that yielded a high payoff, and to avoid actions that yielded a low payoff. Agents will try any of the

Figure 8. The process of selective interaction **Figure 9. Locations of conformists after selective interactions (a) Random assignment and (b) structured assignment**

binary choices, and repeat the strategy that led to high payoffs in the past experiences. Propensity of trying a strategy is increased according to the associated payoff. Payoff describes choice behavior, but it is one's own past payoffs that matter, not the payoffs of the others. The basic premise is that the probability of taking an action in the present increases with the payoff that resulted from taking that action in the past.

In this section, we consider a collection of homogeneous agents who initially have the same payoff parameter with $\theta = 0.5$ in Table 4, and the threshold density is shown in Case 1 in Figure 3. Each agent repeats $T(=10)$ interactions with neighbors. If agents receive an average payoff per one neighbor more than

1 **Figure 10.** *Locations of nonconformist agents after selective interaction (a)*
3 *Random assignment and (b) structured assignment*

17 **Figure 11.** *Efficiency U and equity E under structured assignment (a) onformist*
19 *under sorted assignment and (b) nonconformist under mixed assignment.*

31 **Table 6.** *Reinforcement of learning the payoff-matrix (a) The payoff matrix of a*
33 *conformist and (b) The payoff matrix of a nonconformist*

Choice of agents		Choice of the other agents	Choice of agents		Choice of the other agents
	S_1	S_2		S_1	S_2
S_1	$1-\theta+\Delta\theta$	0	S_1	0	$\theta-\Delta\theta$
S_2	0	$\theta-\Delta\theta$	S_2	$1-\theta+\Delta\theta$	0

39 0.5 by choosing S_1 , they increase $1-\theta$ by $\Delta\theta = 0.01$, hence they decrease θ by
41 $\Delta\theta = 0.01$ as shown in Table 6. Similarly, agents who receive an average payoff
43 more than 0.5 by choosing S_2 , they increase θ by $\Delta\theta = 0.01$. In Figure 12, we
represent the processes of collective reinforcement learning.

Figure 12. The reinforcement process of an agent's preference. Count: the number of successful gain

(1) A collective of conformists

We show the transition of the density function of a collective of conformists in Figure 13. In Figure 13(a), all conformists have the same value $\theta = 0.5$ at the beginning, and the initial proportion of agents who choose S_1 is set to $p(0) = 0.25$. A collection of identical conformists gradually self-reinforce their payoff value (preference), and after a repetition of $T = 600$ rounds, all agents have the same value of $\theta = 1$. In Figure 13(c), the initial proportion $p(0)$ is set to $p(0) = 0.5$. Identical conformists with the same payoff parameter $\theta = 0.5$ self-reinforce their payoff parameters, and after a repetition of $T = 600$ rounds, all agents have the same parameter $\theta = 0$. In Figure 13(b), the initial proportion is set to $p(0) = 0.5$, and after learning process of $T = 600$ periods, a half of agents have the same payoff parameter $\theta = 0$, and the rest of agents have $\theta = 1$. In all three cases, a collection of identical agents at the beginning collectively reinforce their preferences and evolve into heterogeneous agents, so that the most efficient and equitable collective action can be self-organized. However, the result of collective learning heavily depends on the initial ratio of choosing either strategy.

(2) A collective of nonconformists

We show the transition of the density function of a collective of nonconformists in Figure 14. Figure 14(a) gives the results when all conformists have the same value $\theta = 0.5$ at the beginning, and the initial proportion of agents to choose S_1 is set to $p(0) = 0.5$. A collection of identical conformists gradually self-

1 **Figure 13.** *Distribution of θ under reinforcement learning (conformists) (a)*
 $p(0) = 0.25$, (b) $p(0) = 0.5$, and (c) $p(0) = 0.75$

29 **Figure 14.** *Distribution of θ under reinforcement learning (nonconformists) (a)*
 $p(0) = 0.5$ (b) $(P0) = 0.3$ or $P(0) = 0.7$

43 reinforce their payoff value (preference), and after the repetition of $T = 600$
 periods, they split into two groups: agents with the payoff parameter $\theta = 0$, and

the agents with $\theta = 1$. Figure 14(b) shows the results when the initial proportion of agents to choose S_1 at beginning is set to either $p(0) = 0.25$ or $p(0) = 0.75$. They remain as homogeneous agents with the same payoff parameter $\theta = 0.5$ and they fail to reinforce their preferences. The success of reinforcing nonconformists' preferences to induce desired collective behavior heavily depends on the initial condition.

(3) A mixed collective of conformists and nonconformists

We have observed that the success of collective reinforcement learning of conformists and nonconformists heavily depends on the initial condition to choose either strategy. We now consider a mixed case in which conformists who behave with the majority rule and nonconformists who behave with the minority rule coexist in the same population. We assume that the ratios of conformists and nonconformists are the same.

Figure 15 shows the results when the initial proportion of agents to choose S_1 is set to $p(0) = 0.25$. A collection of identical agents who have the same payoff parameter $\theta = 0.5$ initially, gradually self-reinforce their preferences, and after the repetition of $T = 1000$ periods, 75% of the agents reinforced the payoff parameter up to $\theta = 1$, and the rest of 25% of agents reinforced to $\theta = 0$. The ratio of agents who choose S_1 is 75%, and all conformists (50% of the total population) choose S_1 and 50% of nonconformists (25% of the total population) choose S_1 . The ratio of agents who choose S_2 is 25%, and they are all nonconformists.

Figure 16 shows the results when the initial proportion of agents to choose S_1 is set to $p(0) = 0.75$. In this case, a collection of identical agents gradually self-reinforce their payoff parameters, and after the repetition of $T = 1000$ periods, 25% of agents have $\theta = 1$, and the rest 75% of agents have $\theta = 0$. The ratio of agents to choose S_1 is 25%. In this case, all conformists (50% of the total

Figure 15. Distribution of θ under reinforcement learning (conformists and nonconformists with initial ratio $p(0) = 0.75$)

1 **Figure 16.** *Distribution of θ under reinforcement learning (conformists and non-*
 3 *conformists with initial ratio $p(0) = 0.5$)*

15 population) choose S_2 and 50% of nonconformists (25% of the total population)
 choose S_2 . The ratio of agents to choose S_1 is 25%, and they are all nonconformists.

17 Figure 17 shows the results when the initial proportion is set as $p(0) = 0.5$. In
 this case, a half of agents reinforce to have $\theta = 1$, and the rest of agents have
 19 $\theta = 0$. The ratio of agents who choose S_1 is 50%, and a half of conformists (25%
 of the total population) choose S_1 and 50% of nonconformists (25% of the total
 21 population) also chose S_1 . The same thing is true for agents who chose S_2 .

In all three cases, a collection of identical agents collectively reinforce their
 23 preferences and evolve into heterogeneous agents, so that the most efficient and
 equitable collective action can be self-organized. The success of collective re-
 25 inforcement learning therefore depends on the coexistence of conformists and
 nonconformists who behave with the opposite rules.

27

8. Conclusion

29

There is no presumption that a collective of interacting agents leads to a sat-
 31 isfactory performance. Agents normally react to others' decisions, and the re-
 sulting volatile collective action is often far from being efficient. In this paper, we
 33 investigated how the preferences of agents can be reinforced so that their self-
 interested behavior collectively gives rise to a desired outcome. There are many
 35 parameters to be considered such as payoff matrix, population structure, pop-
 ulation configuration, number of agents, and so on. Among these parameters,
 37 we examine the heterogeneity of payoffs and the configuration of locating het-
 erogeneous agents. Agents myopically evolve their actions based on their own
 39 rules obtained as a function of their idiosyncratic utility and of the actions of
 their neighbors. Assignment of heterogeneous agents also becomes to be im-
 41 portant. We examined the interaction between partner choice and individual
 preferences. Agents choose their partners and also decide on a mode of behavior
 43 in interactions with these partners. We also investigated how interacting homo-
 geneous agents evolves into heterogeneous agents with diverse preferences by

Figure 17. Distribution of θ under reinforcement learning (conformists and non-conformists with initial ratio $p(0) = 0.25$)

reinforcing their preferences so that they can realize the most efficient and equitable collective action. We showed that the most crucial factor that considerably improves the performance of the system of interacting agents are the endogenous selection of the partners and reinforcement of preferences at individual level.

Uncited References

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