Agent-based Modeling for the Study of Diffusion Dynamics

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Abstract

Diffusion is the process in which the successful introduction of new products and practices into society along with invention. Many studies of the diffusion of individual innovations exist, and exhibit some commonalities such as the famous S-shaped diffusion curve. New ideas, products, and innovation often take time to diffuse, a fact that is often attributed to some form of heterogeneity among people. Then a basic puzzle posed by innovation diffusion is why there is often a long lag between an innovation's first appearance and the time when a substantial number of people have adopted it. There is an extensive theoretical and empirical literature on this phenomenon and the mechanisms that might give rise to it. The diffusion process enhances an innovation via the feedback of information about its utility across different users that can be used to improve it. This aspect is similar to the micro-macro loop which is essential part of emergent dynamics.

In this paper we discuss how micro-macro loop formed via social network impacts on the qualitative aspects of diffusion process. The diffusion process enhances an innovation via the feedback of information about its utility across different users that can be used to improve it. This aspect of the diffusion process is characterized as the micro-macro loop. Understanding the nature of the relationship between different levels at which macroscopic phenomena can be observed has been made possible due to the tools and insights generated in the agent research. The direction of the research to come is to understand how the agent based modeling is essential for the study of emergence in the diffusion process.

Keywords: contagion, diffusion, innovations, interaction, social influence, consensus, synchronization

1 Introduction

For decades, social scientists, economists and physicists have been interested in the fundamental and common question of how infectious diseases, new technologies practices, or new trends spread through the society. When a new technology appears into a society, its members have the chance to become aware of the innovation and to relate themselves to it.

The main study on diffusion modeling is based on the Bass model (Bass, 1969). The Bass diffusion model describes the process how new products get adopted as an interaction between users and potential users. When the innovation is a good whose consumption is individual, single consumers can decide whether to adopt it or not. The Bass model also formalizes the aggregate level of penetration of a new product emphasizing two processes: external influence via advertising and mass media, and internal influence via word-of-mouth.

The Bass model assumes all consumers to be homogeneous, and such diffusion models are referred to as aggregate models. If we calculate the expected number of adopters at a

given time, the aggregate model displays a cumulative S curve of adopters. However, it has been recognized widely that the familiar Bass aggregate model does not formalize the emergent aspect of the complex diffusion process. For instance, new ideas, products, and innovation often take time to diffuse. This stylized fact is often attributed to some form of heterogeneity among people. The market is currently huge and is more complex than ever before. The amount of information has considerably increased and a consumer has a large amount of information to consider. It is not hard to imagine that a consumer spends much time thinking and hesitating before making a decision. Thus, a change in personal decision requires more time than before.

Rosenberg made the following observation about the diffusion of innovations: in the history of diffusion of many innovations, one cannot help being struck by two characteristics of the diffusion process: its apparent overall slowness on the one hand, and the wide variations in the rates of acceptance of different inventions, on the other. (Rosenberg, 1972). Empirical measurement and study since then has confirmed his view. Why is diffusion sometimes slow? Why is it faster in some regions than others and for some innovations than for others? What factors explain the wide variation in the rate at which it occurs? Hall provides a historical and comparative perspective on diffusion that looks at the broad determinants, economic, social, and institutional (Hall, 2003). Nowadays, the market occasionally accepts innovations very slowly, despite superior technological advances. Chakravorti also explains many examples of the slow pace of fast change (Chakravorti, 2003).

Consumers may realize different benefits and costs from the innovation, have different beliefs about its benefits and costs, hear about it at different times, or delay in acting on their information. Young (2007) analyzes the effect of incorporating heterogeneity into three broad classes of models, contagion, social influence, and social learning. In addition, when a consumer has many neighbours, these represent many information providers and the consumer may have problems in handling their information effectively. Explanation of this relationship calls for examining the types of social interactions that link various types of heterogeneous individuals in a society.

We can also derive an individual decision rule from the Bass model: the number of individuals who adopt at a given time is a function of the number of individuals who have already adopted. The probability that an individual adopts increases as a function of the number of its adopted neighbors. Naturally, if a person has more friends already using a certain service or product, she will adopt with a higher probability. However, it is crucial and difficult to estimate the neighbors' effects on an individual decision.

Our societies consist of individuals and the social systems which largely determine how they behave and interact. Actions of individuals are influenced by their friends, acquaintances and neighbors, and their relationships determine the topologies of social networks. One of the cardinal rules of human behavior is "birds of a feather flock together". Friends of friends become friends and this property foster to emerge dense clusters of connections throughout the social networks.

Spielman defines an innovation system as "a network of agents, along with the institutions, organizations, and policies that condition their behavior and performance with respect to generating, exchanging, and utilizing knowledge" (Spielman, 2005). This definition

highlights the need for a holistic view of the nature and structure of interactions among agents linked to one another within social networks. The adoption of an innovation by one agent can have positive or negative impacts on the behavior of other agents through social networks, which often brings unintentional or unpredictable outcome.

However, little is known, however about the dynamic processes on networks, and how these processes depend on network properties. In social systems that involve large numbers of interacting agents, emergent global stylized facts that arise from local interactions are a critical concept. The emergence of the Internet, for instance, marked the appearance of totally new forms of social and economic exchange (Sole, 2003). As a technological innovation, the Internet provided a new stage for communication and information processing within societies, leading to the creation of new previously non-existent system structures.

We investigate two basic factors, individual decision making process and the agent networks, and discus how these two factors are related and impacts on macroscopic diffusion patterns. The discussions are divided into three classes. First, we discuss the progressive diffusion processes. Many diffusion processes are progressive in the sense that once a node switch from one state to another state, it remains with the same state in all subsequent time steps. This type of diffusion process is progressive. In this class of the diffusion process, we concentrate on the correlations between social interaction patterns and observable transmission rate at the individual level.

2. Research on Networked Agents and Diffusion Dynamics

Every day, billions of people worldwide make billions of decisions about many things. The aggregation of these unmanaged individual decisions often leads to unpredictable outcomes. People constantly interact with each other in different ways and for different purposes. Somehow, these individual interactions exhibit some degree of coherence at the aggregate level, and therefore aggregation may reveal structure and regularity. The individuals involved may have a very limited view of the whole system, but their activities are coordinated to a large degree and produce desirable outcomes at the aggregate level, often exhibiting the features of emergent properties—properties of the system that individual components do not have. These emergent properties are the result of not only the behavior of individuals but the interactions between them (Ball, 2004).

Spielman (2005) defines an innovation system as “a network of agents, along with the institutions, organizations, and policies that condition their behaviour and performance with respect to generating, exchanging, and utilizing knowledge”. An innovation system reflects one aspect of value chain analysis by bringing actors together in the application of knowledge within a value chain. This definition highlights the need for a holistic approach to the nature and structure of interaction between agents linked together within networks. The adoption of an innovation by one agent in a networked system can have positive or negative impacts on other agents that are often unpredictable or unintended.

The growing literature on social interactions has examined how decision-making by an agent is influenced by the decisions of others in the context of a multitude of social phenomena. Examples include studies on the diffusion of new technologies (Arthur, 1989),

herding behavior in stock markets (Banerjee, 1992) and the diffusion of conventions and social norms (Ellison 1993; Morris, 2000), While the research on the diffusion recognizes the importance of complex interactions among agents, it rarely explains or measures the unintended impacts, both positive and negative, on agents within a network. More to the point, current methodologies provide insufficient explication of the network externalities concept. This suggests the need for alternative tools to provide a more enhanced understanding of these interactions.

Recently, the study of intelligence emerged from interactions among many agents has been popular. In this study it is recognized that a network structure of the agents plays an important role. The current state-of-the art in agent-based modeling tends to be a mass of agents that have a series of states that they can express as a result of the network structure in which they are embedded. Agent interactions of all kinds are usually structured with complex networks. Research on complex networks focuses on scale-freeness of various kind of networks.

Computational modeling of dynamic agent interactions on richly structured networks is important for understanding the sometimes counter-intuitive dynamics of such loosely coupled systems of interactions. Yet our tools to model, understand, and predict dynamic agent interactions and their behavior on complex networks have lagged far behind. Even recent progress in network modeling has not yet offered us any capability to model dynamic processes among agents who interact at all scales on such as small-world and scale-free networks. Generally the high-dimensional, non-linear nature of the resulting network-centric multi-agent systems makes them difficult or impossible to analyze using traditional methods. Agents follow local rules under complex network constraints. The idea of combining multi-agent systems and complex networks is also particularly rich and fresh to foster the research on the study of very large-scale multi-agent systems.

We intend to turn this into an engineering methodology to design complex agent networks. Multi-agent network dynamics involves the study of many agents, constituent components generally active ones with a simple structures and whose behavior is assumed to follow local rules, and their interactions on complex network. A basic methodology is to specify how the agents interact, and then observe emergent intelligence that occur at the collective level in order to discover basic principles and key mechanisms for understanding and shaping the resulting intelligent behavior on network dynamics.

Agent networks have been used to create systems for supporting real communities that interchange information. For examples multi-agent systems are designed that exploit social networks on the WWW. Networks are represented as graphs, where nodes represent agents and the edges represent the relationship between them. Links can be directed or undirected as well as weighted or not.

Conformity could be overwhelming almost everyone in a community winds up making the same choice without any enforcement. One of the surprising results of the cascade models is that massive-conformity can be very fragile. Even though a million people may have chosen one action, seemingly little information can induce the next million people to choose the opposite action. Fragility is an integral component of the information cascade model.

Despite the different concepts that these mechanisms support, it is possible to agree on

the fact that all of them have an effect on the system both at the microscopic and the macroscopic level. At microscopic level, they help individual agents to reduce complexity and uncertainty. At macroscopic level, they facilitate coordination and cooperation among agents and, so, they have an impact on the success of the multi-agent system as a whole.

3. A Binary Choice Model

Many works have been focused on the diffusion of innovations or ideas. These works contend that the choices made by agents are quasi-rational: they reflect both an attempt to assess the imperfect data surrounding such innovations as well as a reliance on social cues, i.e., what other people have done.

An agent's behavior is called "purposive behavior", if it is based on the notion of having preference, pursuing a goal, or maximizing his interest. The goal or purpose of an agent often relates directly to other agents, or their behavior is constrained by other agents who are also pursuing their goal or interest. We call this type of behavior "contingent behavior", which also depends on what others are doing. In these situations, they usually do not permit simple summation or extrapolation to the aggregate. To make the connection we usually have to look at the interaction among agents, and between agents and the collectivity.

In this section we present a model that considers binary choice problems as those for which (1) agents have their prior preferences over two alternatives, and (2) agents rely heavily on the prior choices by other agents in similar decision environments. By combining two factors, an agent's decision making is modeled as a stochastic process. Then we show that the collective dynamics leads to outcomes that appear to be deterministic in spite of being governed by a stochastic process at the individual level.

In other words, when the objective evidence for the adoption of a new technology is weak, any sample path of this process quickly settles down to a fraction of agents that is not predetermined by the initial conditions: ex ante, every outcome is just as unlikely as every other and a large-scale cascade occurs. In the case when the objective evidence is strong, the process settles down to a value that is determined by the quality of the evidence. In particular, the more the social cues favor innovation A over B, the more likely it is that an agent will select A. Yet the reasoning that underlies such choices is adaptively rational rather than fully rational: it can be executed by feasible psychological mechanisms and it is consistent with the hardnosed experimental studies of imitation mentioned above.

3.1 A Rational Decision Model without Social Influence: A Logit Model

The Logit model is based on the stochastic utility theory in individual decision-making. In the stochastic utility theory, an agent is assumed to behave rationally by selecting the option that brings a high utility. But, the individual utility contains some random element. This uncertain factor is treated as a random variable in the stochastic utility theory.

Let the utilities associated with the choices of A and B are given by adding some random terms as follows:

$V_i = U_i + \varepsilon_i$: the utility of choosing i, i=A,B

where U_i, i=A,B, are deterministic utilities of the choices of A and B and ε_i, i=A,B, are

random variables. The probability of choosing A is then given

$$\mu = \Pr(V_A > V_B) = \Pr(U_A + \varepsilon_A > U_B + \varepsilon_B)$$
$$= \Pr(U_A - U_B > \varepsilon_B - \varepsilon_A) \tag{3.1}$$

Denoting the joint probability density function of the random variables by $f(\varepsilon_1, \varepsilon_2)$, we can derive

$$p_i = \int_{\varepsilon_1=-\infty}^{+\infty} \int_{\varepsilon_2=-\infty}^{V_A - V_B + \varepsilon_1} f(\varepsilon_1, \varepsilon_2) d\varepsilon_2 d\varepsilon_1 \tag{3.2}$$

Assuming that random variables are independent and follow the gumbel density function

$$F(x)=exp\{-exp(-x/\lambda)\} \tag{3.3}$$

we can obtain the next expression by the substitution integration (Levy, 2000).

$$\mu = \frac{1}{1 + \exp\{-(U_A - U_B)/\lambda\}} \tag{3.4}$$

The choice probability of A in (3.4) is described in Figure 1 as the function of the difference of the utility between the choices of A and B. The parameter λ in (3.4) represents some kind temperature and the probability function of (3.4) becomes a step function with a low value of λ. However the binary choice becomes a random choice with a high value of λ.

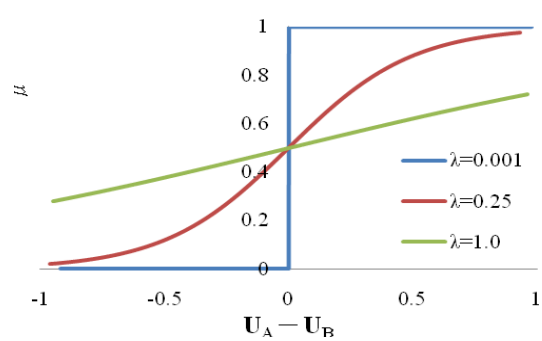


Figure 1: Individual choice probability in (4.4) in terms of λ

3.2 An Individual Decision Model with Social Influence

We present a model of sequential decision-makings by heterogeneous agents. Each agent faces in sequence a binary decision and she makes her decision by reflecting her preference over two decision alternatives as well as social influence. This type of a sequential decision model is important for the analysis innovation diffusion or management fads in uncertain environments in which many consumers (agents) lack clear objective evidence about the merits between two alternatives. The choices made by heterogeneous agents reflect both their own evaluations and the social influence received from other agents. We show the

collective dynamics lead to outcomes that appear to be deterministic in spite of being governed by a stochastic decision models at the individual levels. We investigate the level of cascade size observed in collective dynamics and find the relationship between agents' preferences and the social influence level on the cascade size. When the social pressure is strong, the evolution of the collective decision process settles down to a fraction of agents is not predetermined by objective evidence about the merits of two alternatives, and cascade occurs at a large-scale. When the social pressure is relatively strong, the proportion of adopters is determined by the agents' preferences.

An externality occurs when individuals care about others' choices and each individual's choice affects others' choices. For instance, when deciding which movies to visit, which new technologies to adopt or which job candidates to be decided, we often have little information with which to evaluate the alternatives. Therefore we rely on the recommendation of friends or simply pick the one to which most people are enjoying. Even when we have access to plentiful information, we often lack the ability to make sense of it and we rely on the advice of trusted friends or colleagues.

We assume that there are two factors to individual decisions. The first is based on individual judgment (preference) and the second on social influence. Regarding the first factor, we assume that an agent isolated from social influence would choose the objectively the option A with probability μ given in (4.4). Thus p reflects the quality of the evidence about the relative merits of A versus B of each agent. We assume heterogeneity among agents and they take some random value in [0, 1] in general. If an agent's preference over A is strong enough, then μ will be close to 1, and if the two alternatives are nearly interchangeable then p will be close to 0.5

We assume that the impact of social influence is linearly increasing in the proportion of the agents who have adopted. Thus, if A_t denotes the number of agents who have chosen A by period t and B_t denotes the number who have chosen B, then the social pressure at time t+1 to an agent who chooses is A is simply the ratio of $F_t = A_t/(A_t + B_t)$.

We assume that an agent's choice is simply a weighted average of the two factors, individual preference and social influence. An agent's decision for a particular alternative depend on how other agents decide to do so, partly because of social influence, partly because one does want to choose the majority side. Then the probability to choose A is in period $t+1$ is

$$p[\text{agent in period } t+1 \text{ chooses } A] = p_{t+1} = (1-\alpha)\,\mu + \alpha F_t. \qquad (3.4)$$

where μ is the choice probability of an agent without social influence as given in (3.1), and α $(0 < \alpha < 1)$ is the strength of social influence.

4. Diffusion Dynamics as Sequential Decisions

In this section we investigate properties of the collective decision process when a large number of agents (N=1,000) sequentially make decisions with the decision rule described in (3.4). We consider how a new alternative technology A diffuses through a population of

agents while all agents enjoy an old technology B. In every period one agent makes up his mind about whether to adopt A. In this sense the formulation is like most contagion models. The diffusion continues until everyone in the population has selected A. The numbers of A-choice at t will be denoted by A_t and the ratio of A-choosers $F_t=A_t/N$ is defined as the penetration ratio. The question is how follow-on agents decide to adopt A. At each time period, one agent is chosen from the population as a target agent, and this target agent has not chosen A before, he makes his choice of A with probability in (4.4) by considering both his preference and social influence. If the target agent has already chosen A, we select another target agent from the population.

First let us examine how it behaves over time by considering first the two extreme cases: α=0 and α =1.

(1) α = 0 (independent decisions without social influence)

In this case, the sequential decision process becomes simply an accumulation of independent decisions. The simulation results with 1,000 agents are shown in Figure 2. For any value λ, eventually all agents become to adopt a new technology.

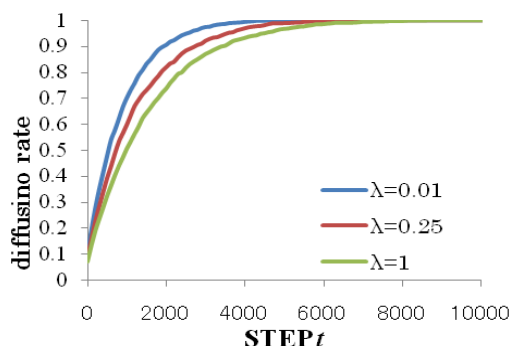


Figure 2: Simulations of the complex process after 1000 rounds. In all cases (λ = 0.01, 0.25 and 1) we choose μ = 1, and all agents are interested in a new technology. In each figure three sample paths are shown. As can be seen, the convergence rate for small λ is much slower than the rate for large λ.

(2) α = 1 (pure social influence)

In this case, the sequential diffusion process looks like the well-known the Polya's urn process. That is the expected proportion exactly equals the current proportion with pure social influence (α = 1). Some sample paths are shown in Figure 2. It is easy to show that this martingale property holds in general: on average the pure social influence process stays exactly where it currently is. The pure social influence process is strongly path dependent and in expectation it tends to stay wherever it is.

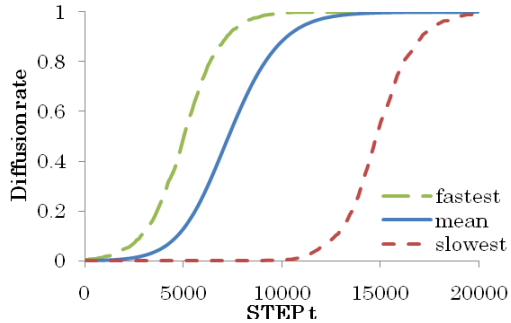


Figure 3: A simulation of the diffusion process for 1,000 rounds. In this figure three sample paths are shown: fastest case, slowest case and the average. As can be seen, the diffusion process becomes to be slow and unpredictable under pure social influence (α=1).

If we compare the two processes, the pure social process is less predictable as measured by the penetration rate of *Ft*. In one case, the diffusion speed becomes relatively high, but in the other case it becomes to be extremely slow. Thus every feasible outcome is equally likely at every time: given a fair start, the process with pure social pressure is completely blind, and the diffusion process becomes completely unpredictable. When the diffusion is purely a matter of social construction, every feasible outcome is equally likely; this is intuitively very random.

(3) $0 < \alpha < 1$ (mixed decisions with a partial social influence):
In this case, the sequential diffusion process becomes a mixed between two extreme cases with α=0 and α=1. If we calculate the expected number of adopters at a given time, the aggregate model displays a cumulative S curve of adopters.

When the innovation is a good whose consumption is individual, single consumers can decide whether to adopt it or not. The Bass model formalizes the aggregate level of penetration of a new product emphasizing two processes: external influence via advertising and mass media, and internal influence via word-of-mouth. The Bass model assumes all consumers to be homogeneous, and such diffusion models are referred to as aggregate models.

We can derive an individual decision rule from the Bass model: the number of individuals who adopt at a given time is a function of the number of individuals who have already adopted. The probability that an individual adopts increases as a function of the number of its adopted neighbors. Naturally, if a person has more friends already using a certain service or product, she will adopt with a higher probability.

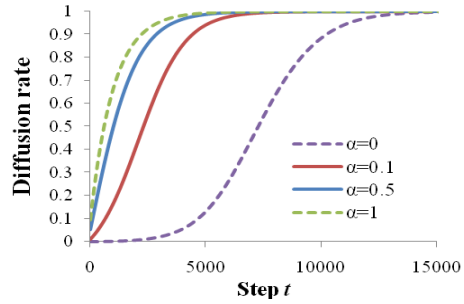


Figure 4: Simulations of the complex process after 1000 rounds. In both cases (α = 0.1 and α =0.5) we choose μ = 1. As can be seen, the diffusion process becomes for large α, i.e., each agent puts high weight on social trend.

5. Diffusion Dynamics on Social Networks

Important processes that take place within social networks, such as the spreading of opinions and innovations, are influenced by the topological properties of those networks. In turn, the opinions and practices of individuals can have a clear impact on network topology, for instance, when conflicting opinions lead to the breakup of social interaction. From an applied point of view, it is desirable to compose an inventory of the types of microscopic dynamics that have been investigated in social networks and their impact on emergent properties at the network level. Such an inventory could provide researchers with specific guidelines concerning the kinds of phenomena present in social systems where similar diffusion processes are at work.

A large number of interactions is not enough to guarantee emergent patterns in a system; in some cases, a certain critical mass must be reached before emergent patterns can be generated—a system must reach a combined threshold of diversity, organization, and connectivity before emergent properties appear. Topological network properties should be derived from the dynamics by which the networks are created. Formation of new links, for example, may occur when individuals introduce their friends to new people, creating new connections between existing nodes. In reality, social networks are formed by social processes in which individuals create and maintain social relationships; in turn, these social processes influence the dynamics of social networks. This process results in the self-organization of social systems, in which social relations depend on and in turn influence the relationships among agents. The direction of this research to come is to understand how the structure of social networks determines the diffusion dynamics occurring in social networks.

One of the major focuses of this research is the on networks. Here, each node of the network represents a dynamical system. Individual systems are coupled according to the network topology. Thus, the topology of the network remains static while the states of the nodes change dynamically. Important processes studied within this framework include synchronization of the individual dynamical systems and contact processes, such as opinion formation and epidemic spreading. Studies like these have clarified that certain topological

properties have strong impacts on the dynamics of and on networks.

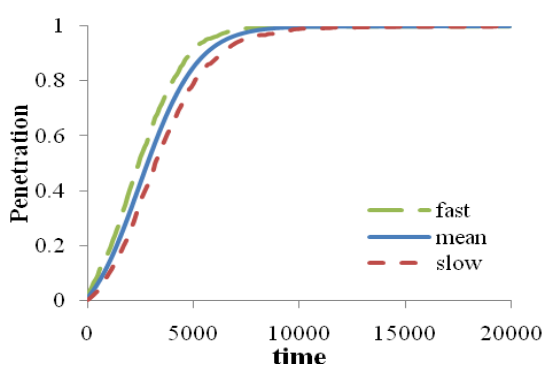
In this section we investigate properties of the collective decision process when a large number of agents (N=10,000) sequentially make decisions with the decision rule described in (3.4). We consider how a new alternative technology A diffuses through a population of agents while all agents enjoy an old technology B. In every period one agent makes up his mind about whether to adopt A. In this sense the formulation is like most contagion models. The diffusion continues until everyone in the population has selected A. The numbers of A-choice at t will be denoted by A_t and the ratio of A-choosers $F_t=A_t/N$ is defined as the penetration ratio. The question is how follow-on agents decide to adopt A. At each time period, one agent is chosen from the population as a target agent, and this target agent has not chosen A before, he makes his choice of A with probability in (4.4) by considering both his preference and social influence. If the target agent has already chosen A, we select another target agent from the population.

$$p[\text{agent in period } t+1 \text{ chooses } A] = (1-\alpha)\,\mu + \alpha Nb \qquad (5.1)$$

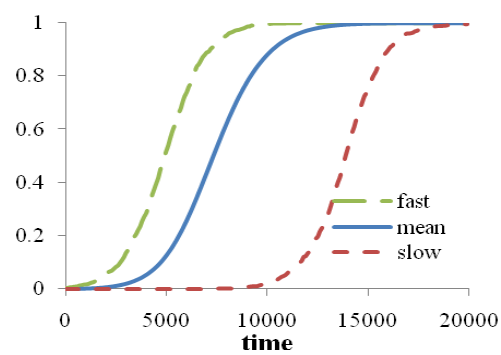
where Nb is the ratio of A-choosers of the neighbors who are connected to the target agent to make choice at time t+1. In this simulation, we only consider the case of pure social influence (α=1).

As examples we consider three cases (a) regular networks where all agents are connected to 10 neighbors, (b) complete network where all agents connected to all other agents, and (c) scale-free networks, where there some agents who connect to many agents but the rest s are connected to a few agent.

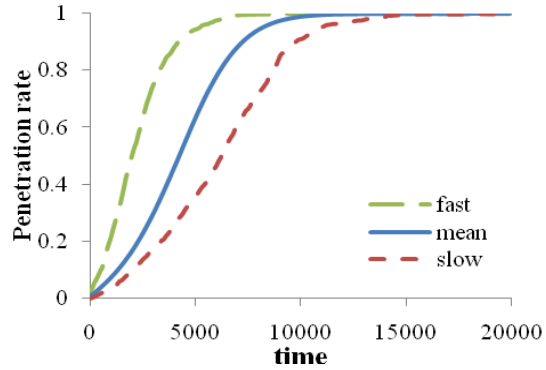
The diffusion dynamic process in term of the penetration rates are shown in Figure 5. Comparing these results, the fastest diffusion is possible in the case when agents are locally connected. On the other hand, when agents are fully connected, the diffusion process is the slowest and the process is also unstable: in one case it is relatively fast, but in another case it is extremely slow. In the case of scale-free network, the diffusion pattern takes the middle of the two extreme cases.



(a) regular network



(b) complete network



(c) scale-free network

Figure 5: Simulations of the complex process after 1000 rounds. Under the different agent network topologies,: (a) regular network, (b) complete network, and (c) scale-free network. In all cases, we set α =1 and μ =1 in (5.1). In each figure three sample paths are shown: fastest case, slowest case and the average. As can be seen, the diffusion process becomes to be slow and unpredictable if all agents are connected each other (complex network).

6. Conclusion

Psychologists have argued that when faced with choice problems, people often use a reasonable heuristic method such as following social trend. We presented a model of diffusion that combines this heuristic with agents' efforts to make rationally grounded decisions based on their preferences. We established stochastic processes that exhibit clear properties of individual decisions combined with the collective dynamics that lead to outcomes that appear to be deterministic.

When agents make decisions under weak social influence, the diffusion process settles down to a value that is determined. In this case the proportion of adopters settle into by the distribution of the agents' preferences and pure herding does not occur.

When agents make choices under strong social influence by referring to social trend, the diffusion process settles down to a value that is not predetermined by the distribution of the agents' preferences. Even if when the objective evidence for the adoption of a new technology is strong, any sample path of this process quickly settles down to a fraction of adopters that is not predetermined by the initial conditions: ex ante, every outcome is just as likely as every other.

For decades, social scientists, economists and physicists have been interested in the fundamental and common question of how infectious diseases, new technologies practices, or new trends spread through the society. The main study on diffusion modeling is based on the Bass model. The Bass diffusion model describes the process how new products get adopted as an interaction between users and potential users. The Bass model assumes all consumers to be homogeneous, and such diffusion models are referred to as aggregate models. If we calculate the expected number of adopters at a given time, the aggregate model displays a cumulative S curve of adopters.

We can also derive an individual decision rule from the aggregated diffusion patterns. The number of individuals who adopt at a given time is a function of the number of individuals who have already adopted. The probability that an individual adopts increases as a function of the number of its adopted neighbors. Naturally, if a person has more friends already using a certain service or product, she will adopt with a higher probability.

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