Evolutionary Optimized Consensus and Synchronization Networks

Toshihiko Yamamoto, Hiroshi Sato, Akira Namatame*
Department of Computer Science
National Defense Academy
1-10-20 Hashirimizu, Yokosuka, Kanagawa, 239-8686 Japan
E-mail: g47038,hsato,nama@nda.ac.jp
*Corresponding author

Abstract: There is consensus problem as an important characteristic for coordinated control problem in collective behavior, the interaction between agents and factors. Consensus problem is closely related to the complex networks. Recently, many studies are being considered in the complex network structure, the question what network is the most suitable to the property of the purpose has not been answered yet in many areas. In the previous study, network model has been created under the regular rules, and been investigated their characteristics. But in this study, network is evolved to suit the characteristics of the objection by evolutionary algorithm and we create optimized network. As a function of the adaptive optimization, we consider the objection that combinate consensus, synchronization index and the density of the link, and create the optimized network which is suitable to the property of the objective function by evolutionary algorithms. Optimal networks that we design have better synchronization and consensus property in terms of the convergence speed and network eigenvalues. We show that the convergence speed is faster in evolutionary optimized networks than previous networks which are known as better synchronization networks. As a result, we generate optimal consensus and synchronous network.

Keywords: Optimal network, Genetic Algorithm, Synchronization, Consensus, Complex networks

Reference to this paper should be made as follows: T. Yamamoto, H. Sato and A. Namatame. (xxxx) 'Evolutionary Optimized Consensus and Synchronization Networks', *Int. J. Bio-Inspired Computation,*, Vol. x, No. x, pp.xxx–xxx.

Biographical notes: Toshihiko Yamamoto is a Graduate student of Computer Science at National Defense Academy of Japan. He holds the degrees of Engineering in Computer Science at National Defense Academy of Japan. His research interests include Complex Networks, Evolutionary Computation and Multi-agents.

Hiroshi Sato is Assistant Professor of Dept. of Computer Science, National Defense Academy of Japan. He holds degrees of Physics from Keio University, and Master and Doctor of Engineering from Tokyo Institute of Technology in Japan. His research interests include Agent-based Simulation, Evolutionary Computation, and Artificial Intelligence.

Akira Namatame is a Professor of Dept. of Computer Science, National Defense Academy of Japan. He holds the degrees of Engineering in Applied Physics from National Defense Academy, Master of Science in Operations Research and Ph.D, in Engineering-Economic System from Stanford University. His research interests include Multi-agents, Game Theory, Evolution and Learning, Complex Networks, Economic Sciences with Interaction Agents and A Science of Collectives.

1 Introduction

Synchronization and collective behavior phenomena such as herd behavior and cries in nature Neda et al. (2000)Strogatz (2000)Buck and Buck (1976) and human society consensus of opinion, the problem of coordinated control of agents robots Olfati-Saber and Murray (2004)Olfati-Saber et al. (2007)Ren and Beard (2008) are being treated as important issues by the interaction between elements of emergent phenomena. And the problem can be viewed as a consensus problem. It is closely related to the properties of complex networks. Recently, many studies are being considered in the complex network structure, the question what network is the most suitable to the property of the purpose has not been answered yet in many areas. Many essential features displayed by complex systems emerge from their underlying network structure Strogatz (2001)Kauffman

(1993). Different mechanisms have been suggested to explain the emergence of the striking features displayed by complex networks. When dealing with biological networks, the interplay between emergent properties derived from network growth and selection pressures has to be taken into account. As an example, metabolic networks seem to result from an evolutionary history in which both preferential attachment and optimization are present. This view corresponds to Kauffman's suggestion that evolution would operate by taking advantage of some robust, generic mechanisms of structure formation Kauffman (1993). Synchronous behavior is also affected by the network structure. The range of stability of a synchronized state is a measure of the system ability to yield a coherent response and to distribute information efficiently among its elements, while a loss of stability fosters pattern formation Barahona (2002).

In the previous study, network model has been created under the regular rules, and been investigated their characteristics. As prominent examples, random network Erdős and Rényi (1959), small-world network Watts and Strogatz (1998), scale free network Barabási and Albert (1999) are raised. And there are many other network models based on local making rule. But in this study, network is evolved to suit the characteristics of the objection by evolutionary algorithm and we create optimized network. As a function of the adaptive optimization, we consider the objection that combinate consensus, synchronization index and the density of the link, and create the optimized network which is suitable to the property of the objective function by evolutionary algorithms. Optimal networks that we design have better synchronization and consensus property in terms of the convergence speed and network eigenvalues. We show that the convergence speed is faster in evolutionary optimized networks than previous networks which are known as better synchronization networks. As a result, we generate optimal consensus and synchronous network.

2 Consensus Problems and Synchronization

Consensus problems have a long history in computer science and control theory Olfati-Saber et al. (2007). In networks of agents, consensus means to reach an agreement regarding a certain quantity of interest that depends on the state of all agents. A consensus algorithm is an interaction rule that specifies the information exchange between an agent and all of its neighbors on the network. The theoretical framework for solving consensus problems for networked systems was introduced by Olfati-Saber and colleagues Olfati-Saber and Murray (2004).

The analysis of consensus problems relies heavily on matrix theory and spectral graph theory. The interaction topology of a network of agents is represented using a directed graph G with the set of nodes and edges. We denote neighbors of agent i with n_i.

Consider a network of decision-making agents with each internal dynamics

$$\dot{x}_i = u_i \tag{1}$$

We are interested in reaching a consensus via local communication with their neighbors on a graph $G = (V, E)$. $x_i \in R$ is the state of an agent i and $u_i \in R$ is the input to that agent. Let $A = [a_{ij}]$ be the adjacency matrix of graph G. The set of neighbors of agent i is N_i and defined by

$$N_i \triangleq \left\{ j \in V : a_{ij} \neq 0 \right\} \tag{2}$$

Agent i communicates with agent j if j is a neighbor of i. The set of all nodes and their neighbors defines the edge set of the graph as $E = (i, j) \in V \times V$.

A dynamic graph $G(t) = (V, E(t))$ is a graph in which the set of edges $E(t)$ and the adjacency matrix $A(t)$ may be time-varying n general. Clearly, the set of neighbors $N_i(t)$ of every agent in a dynamic graph is a time-varying set as well. Dynamic graphs are useful for describing the network topologies such are mobile sensor networks and flocks Olfati-Saber (2006).

It is shown that the linear system

$$\dot{x}_i = \sum_{j \in N_i} \alpha_{ij} \left(x_j(t) - x_i(t) \right) \tag{3}$$

where a_{ij} is the weight of agent i on agent j. Here, reaching a consensus means asymptotically converging to the same internal state by way of an agreement characterized by the following equation:

$$x_1 = x_2 = \cdots = x_n = \alpha \tag{4}$$

Assuming that the underlying graph G is undirected ($a_{ij} = a_{ji}$ for all i, j), the collective dynamics converge to the average of the initial states of all agents:

$$\alpha = \frac{1}{n} \sum_{i=1}^{n} x_i(0) \tag{5}$$

The dynamics of system in Eq. 3 can be expressed as

$$\dot{\mathbf{x}} = -\mathbf{L}\mathbf{x}(t) \tag{6}$$

$\mathbf{L}$ is the graph Laplacian of the network G; the graph Laplacian is defined as Merris (1998)

$$\mathbf{L} = D - A \tag{7}$$

where $D = \mathrm{diag}(d_1, d_2, \cdots, d_n)$ is the diagonal matrix with elements $d_i = \sum_{j \neq i} a_{ij}$ and A is the binary adjacency matrix ($n \times n$ matrix) with elements a_{ij} for all i, j where is 1 if agent i and agent j is connected or 0 if they are disconnected.

Notice that because our networks are undirected, L is a symmetric matrix with all real entries, and therefore a Hermitian matrix. In our case this is always met with equality, since the diagonal entry of each row in L is the degree of node i, and each link connected to i results in -1 in the same row. So the sum of all off diagonals in a row is a_{ii}. Therefore L is a positive semi-definite matrix. Since L is semi-definite (and therefore also Hermitian), we will adopt the ordering convention

$$0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n \tag{8}$$

We can also ask how synchronizability depend upon network topology does in the same framework. An answer to this question was given in a seminal work by Barahona and Pecora who established the following criterion to determine the stability of fully synchronized states on networks Barahona (2002)Donetti et al. (2005). Consider a general dynamical process

$$\dot{x}_i = F(x_i) - \sigma \sum_j L_{ij} H(X_i) \tag{9}$$

where x_i with $i \in 1, 2, \ldots, n$ are dynamical variables, F and H are an evolution and a coupling function respectively, and σ is a constant. A standard linear stability analysis can be performed by i) expanding around a fully synchronized state

$$x_1 = x_2 = \cdots = x_n = x^s \tag{10}$$

with x^s solution of $\dot{x}^s = F(x^s)$, ii) diagonalizing L to find its n eigenvalues, and iii) writing equations for the normal modes y_i of perturbations

$$\dot{y}_i = \left[F'(x^S) - \sigma\lambda_i H'(x^s)\right] y_i \tag{11}$$

all of them with the same form but different effective couplings $\alpha = \sigma\lambda_i$. Barahona and Pecora noticed that the maximum Lyapunov exponent for Eq. 11 is, in general, negative only within a bounded interval $[\alpha_A, \alpha_B]$, and that it is a decreasing (increasing) function below (above). Requiring all effective couplings to lie within such an interval,

$$\alpha_A < \sigma\lambda_2 \leq \ldots \leq \sigma\lambda_n < \alpha_B \tag{12}$$

one concludes that a synchronized state is linearly stable if and only if $\frac{\lambda_n}{\lambda_2} < \frac{\alpha_B}{\alpha_A}$ for the corresponding network. It is remarkable that the left hand side depends only on the network topology while the right hand side depends exclusively on the dynamics (through F and G, and x_s).

Therefore, the interval in which the synchronized state is stable is larger for a smaller ratio of the two eigenvalues λ_n/λ_2, and a network has a more robust synchronized state if the ratio

$$Q = \frac{\lambda_n}{\lambda_2} \tag{13}$$

which is also known as algebraic connectivity, is as small as possible Olfati-Saber et al. (2007)Lovász and Sós (1981).

Also, as the range of variability of λ_n is limited (it is related to the maximum connectivity) minimizing Q gives very similar results to maximizing the denominator λ_2 in most cases. Especially, λ_n expresses robustness to delays i.e. if λ_n is small, the network has good consensus. Indeed, as argued in Xiao and Boyd (2004), in cases where the maximum Lyapunov exponent is negative in an un-bounded from above interval, the best synchronizability is obtained by maximizing the algebraic connectivity.

3 Definition of Objective Functions

In this simulation, we define that the objective function include both eigenvalue ratio and link cost. Thus, we will optimize networks through eigenvalue ratio and link cost on same time.

3.1 Eigenvalue ratio

We want to optimize a network which have minimum eigenvalue ratio, so Eq. 13 is set to object function. We can optimize a network through selection of minimize network. So, we optimize a network both maximization of λ_2 and minimization λ_n that is, we obtain a network with the minimum the λ_n/λ_2 algebraic connectivity.

3.2 Average degree

Many essential features of links are displayed by complex systems: for example, memory, stability and homeostasis emerge from the underlying network structure Strogatz (2001). Different networks exhibit different features at different levels, but most complex networks are extremely sparse and exhibit the so-called small-world phenomenon Watts and Strogatz (1998).

We can predict that the network of minimum eigenvalue ratio is complete network (Thus $\lambda_n/\lambda_2 \geq 1$). Therefore, we add eigenvalue ratio to Average degree. And, average degree is defined

$$\langle k \rangle = \frac{2}{n} \sum_{k=1}^{n} a_{ij} \tag{14}$$

3.3 Weighted object function

In this simulation, the evaluation function of our optimization algorithm is optimization of both Eq. 13 and Eq. 14 at the same time.

In this simulation, we will optimize networks through importance of link constraint.

There is a large gap between eigenvalue ratio λ_n/λ_2 and average degree $\langle k \rangle$. And, in this case, we use any constant for balance. Because, optimized networks are much alike in characteristic i.e. there is not important.

Therefore, the object function optimized for consensus and synchronization is defined as

$$E(\omega) = \omega \frac{1}{\lambda_2} + (1-\omega)\langle k \rangle \tag{15}$$

$$E(\omega) = \omega \left(\frac{\lambda_n}{\lambda_2}\right) + (1-\omega)\langle k \rangle \tag{16}$$

where ω $(0 \leq \omega \leq 1)$ is a parameter controlling objects $1/\lambda_2$, λ_n/λ_2 and $\langle k \rangle$. We know that $\omega = 0$ is the minimization problem for just average degree $\langle k \rangle$ and $\omega = 1$ is the minimization problem for just eigenvalue $1/\lambda_2$, λ_n/λ_2.

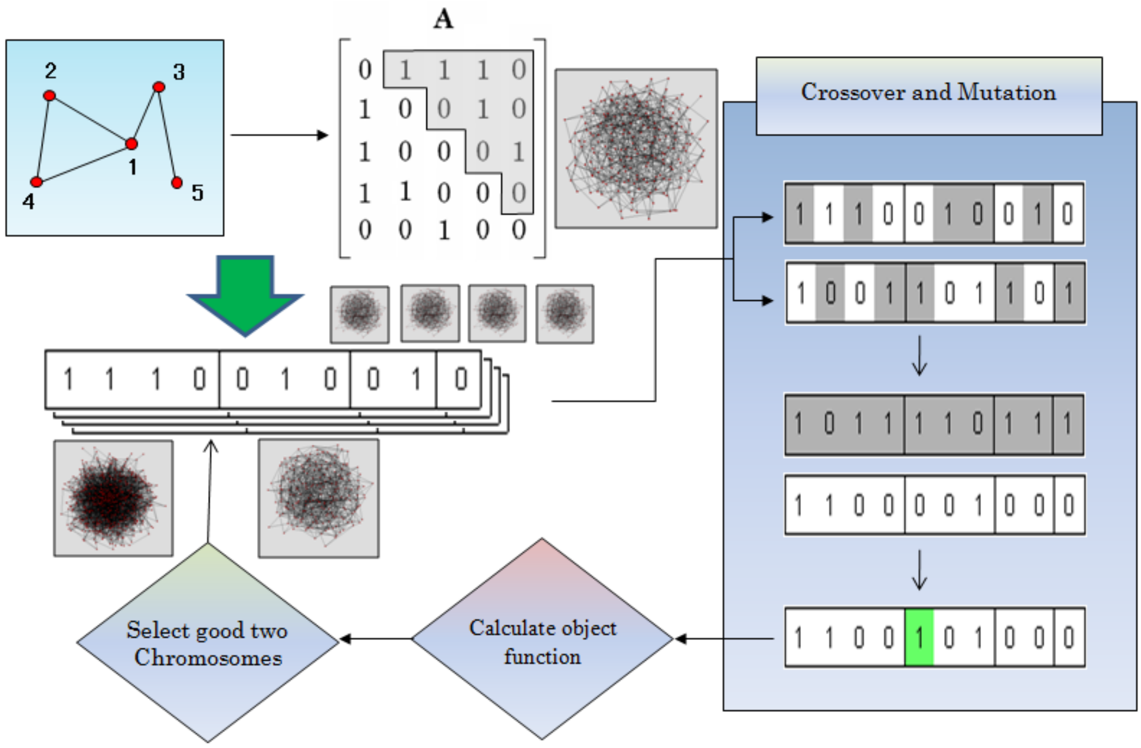

Figure 1 The basic scheme of the genetic algorithm for optimizing networks. Starting from a given adjacency matrix A, the algorithm performs a change in a small number of bits. The object function $E(\omega)$ is then evaluated and the new matrix is accepted provided that a object value is achieved.

4 The Evolutionary Optimization Method

Initially we prepare 50 undirected graphs, each graph has a fixed number of nodes n and links defined by the binary adjacency matrix $A = [a_{ij}]$, $1 \leq i,\ j \leq n$. The adjacency matrix A is $n \times n$ matrix because of award for all nodes and a symmetry matrix because of an undirected graph.

4.1 Initial networks

Initially 50 networks were generated by a specified probability about links. One of the initially generated networks is shown in Figure. 2. The average link number per node is 7, and the degree distribution obeys a Poisson distribution. In other words, an initially designed network is a random network. We generate ten random networks that resemble this and we use the genetic algorithm to obtain better networks in terms of improving the fitness function in (15),(16).

4.2 Genetic algorithm

In this study, the system uses the genetic algorithm to generate an optimized network structure. Especially we use the MGG model for the change of generations Sato et al. (1997). Especially we select two best networks and their adjacency matrices are crossover as shown in Figure. 1 in order to generate two better networks.

We use crossover rate at 0.7, and mutation rate is set at $2/{}_nC_2$, i.e. reverses of two links per one generation. We create 50 different networks as individuals at the beginning.

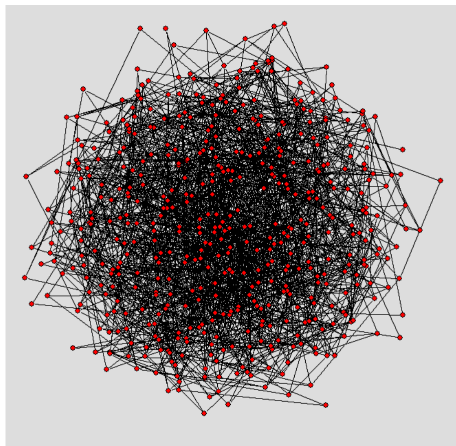

Figure 2 The initial network

And, we stop until the object function has almost the same value match on ${}_nC_2$ generations.

The model uses an encoded network by the binary adjacency matrix for the mutation and crossover. Next, the most suitable matrices among the parents and children matrices are chosen, and the others are eliminated.

We used the multi-point crossover. After crossover, each element in the matrix switches to a reverse state with a specific probability. In this paper, the network is an undirected graph, and so, if one element is reversed, the symmetry element is reversed at the same time.

There is the possibility that an isolated network appears after crossover and mutation. In this paper, when an isolated node appears in a new network that the node has 0 distances to another node, we dump the network. Therefore, we can use non-isolated matrices. After long generations have passed, we can obtain an optimal network which minimizes the fitness function defined in (15),(16).

5 Optimized networks generated

5.1 Previous conventional network models

On the network consensus and synchronization problem, That Laplacian matrix 2nd smallest eigenvalue is high means the algebraic connectivity of the network is high, and the network have the characteristics of fast convergence to the solution of the consensus and synchronization problem. Previous conventional network models which are valid for consensus and synchronization problem are small-world networks Watts and Strogatz (1998). In previous studies, small-world networks' the convergence speed is faster and more effective than regular network Hovareshti and Baras (2006)Olfati-Saber (2005)Olfati-Saber et al. (2007).

Regular network models have the same degree connected to neighborhood nodes, such network model of λ_2 is low, so convergence speed is slow. But small-world networks are randomly chosen nodes and rewired by link probability $0 \leq p \leq 1$, so λ_2 have higher value. In the case of probability $p = 0$, the network is regular network, and when probability is $p = 1$ the network is random network Erdős and Rényi (1959). Random network and small-world networks whose p is close to $p = 1$ have high algebraic connectivity property, therefore the networks show fast convergence and good consensus and synchronization property.

Furthermore, random regular(or Ramanujan graph) network is the network model which have higher algebraic connectivity rather than random network and small-world networks Kar et al. (2008)Pikovsky et al. (2003)Bollobas (2001). Random regular network have the same degree as regular network. Regular network have the same degree connected to neighborhood nodes, but random regular network have links that stretched to other nodes randomly. Random regular network have high homogeneous property and larger algebraic connectivity because of the same degree and random links that stretched to other nodes. Therefore, Random network and small-world networks whose p is close to $p = 1$ have high algebraic connectivity property, but random regular network have larger algebraic connectivity than random network and small-world networks.

These previous network models(regular network, random network, small-world network and random regular network)(Figure.3) are the comparing models with our optimized networks. Figure.4 shows all Laplacian eigenvalue transition of the previous network models(regular network, random network, small-world networks and random regular network), and 2nd Laplacian eigenvalue of the previous network models are shown in Figure.5 to compare algebraic connectivity. In figure, RG,SW,ER,RR represents regular network, small-world networks, random network, random regular network respectively. Number of nodes is 500, so the number of all eigenvalues are 500.

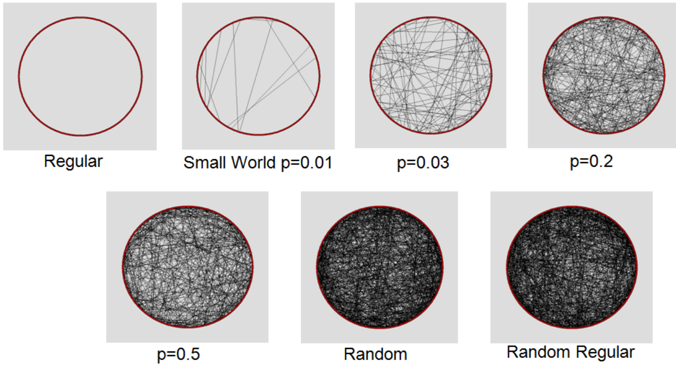

Figure 3 Previous network models we compare(regular network, random network, small-world network and random regular network)

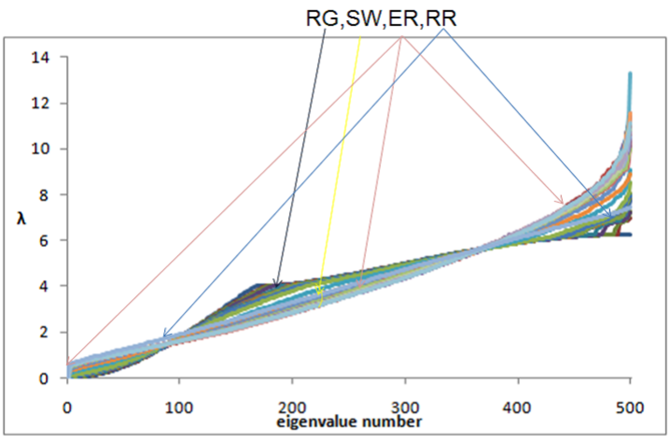

Figure 4 All Laplacian eigenvalue transition of the previous network models

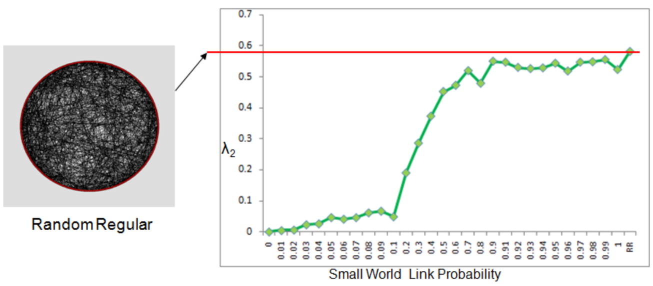

Figure 5 2nd smallest Laplacian eigenvalues of the previous network models

From Figure.5 regular network is low algebraic connectivity, and small-world networks' algebraic connectivity are becoming higer as p is increasing. Random network and small-world networks whose p is close to $p = 1$ have high algebraic connectivity property of all network based on link probablity p, and random regular network have larger algebraic connectivity than those networks.

5.2 Optimized Networks Comparison with Previous Networks

The structure of the optimal networks generated evolutionary algorithm is shown in Figure.6,7 by degree distribution. In order to examine how optimal networks are suitable and effective on the consensus and synchronization problem, We compare optimal networks with the previous network models. And the average degree is almost equivalent to assure comparing, the average degree is 4. Optimized networks are homogeneous networks having almost the same degree and node-to-node distance. However the optimized network is not random regular networks in which all nodes have the same degrees nor random networks whose degree distribution is poisson degree distribution.

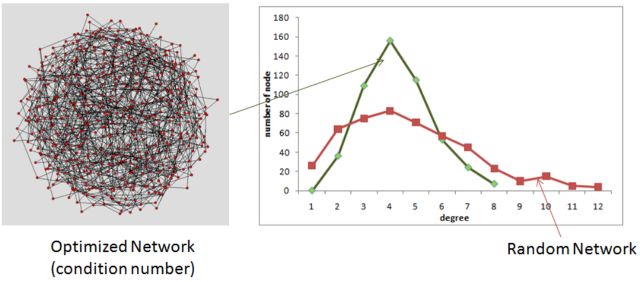

Figure 6 Optimized networks(condition number) and degree distribution

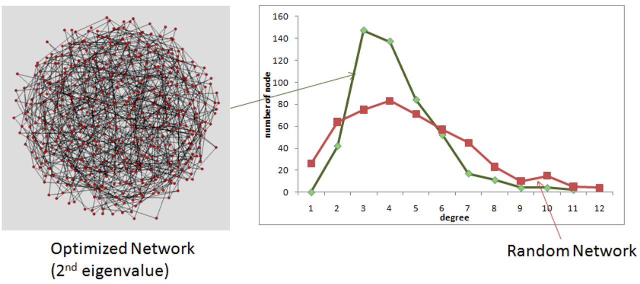

Figure 7 Optimized networks(2nd eigenvalue) and degree distribution

In the previous network models, we can understand that random regular network is the highest algebraic connectivity. Figure.8 shows that optimized networks comparing with the previous network model by 2 smallest eigenvalue. Optimzed networks by λ_2 or by condition number have property whose algebraic connectivity is superior than the previous network models. From Figure.8 we can see that the 2nd smallest eigenvalue is much greater in evolutionary optimized networks than random regular(Ramanujan graph) networks which was the highest algebraic connectivity networks in the previous network models(regular network, random network, small-world network and random regular network).

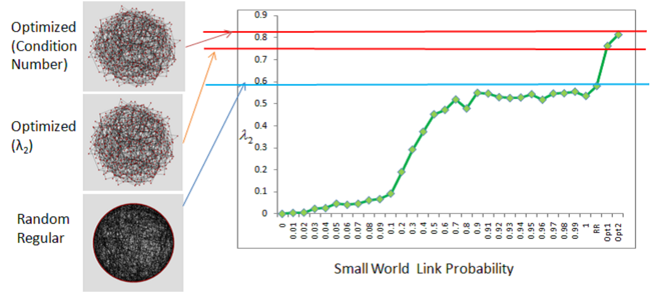

Figure 8 2nd smallest Laplacian eigenvalues of the optimized networks and previous network models

To be considered stable conditions for consensus and synchronization robustness of information and communication by time delay and fast convergence speed, Smaller the Laplacian matrix eigenvalue ratio of 2nd smallest eigenvalue and maximum eigenvalue Q(condition number) is, the better network structure has properties for consensus and synchronization. Figure.9 shows that optimized networks comparing with the previous network model by maximum eigenvalues. And Figure.10 shows that optimized networks comparing with the previous network model by Q(condition number).

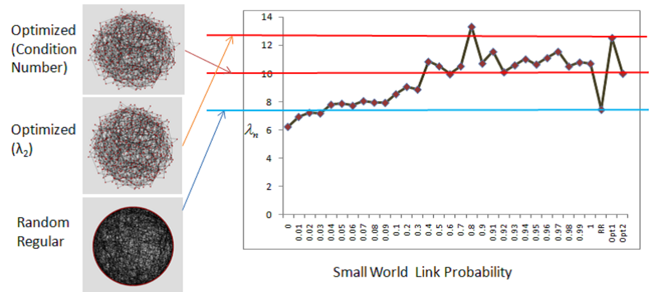

Figure 9 Maximum Laplacian eigenvalues of the optimized networks and previous network models

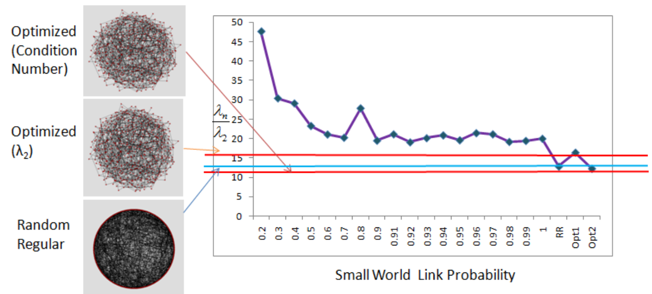

Figure 10 Q(condition number) of the optimized networks and previous network models

As the probability rises from small-world links, the maximum eigenvalue go up high in Figure.9, which shows the characteristics of random regular network lower maximum eigenvalue. Because random regular network has similar characteristics of regular network which has same degree for all nodes. As we see in Figure.5, random regular network have larger algebraic connectivity

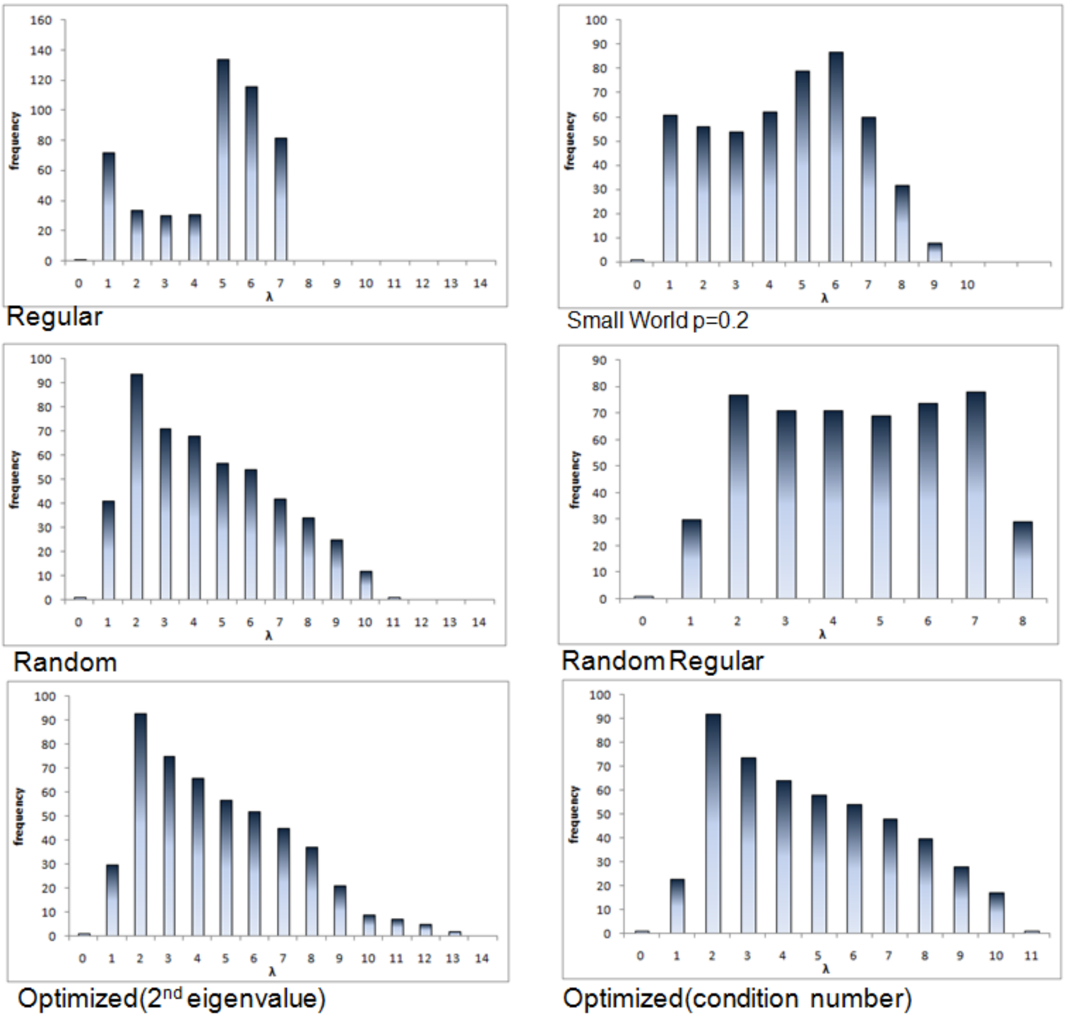

Figure 11 Network eigenvalue histogram

than random network and small-world networks. Random regular network have high homogeneous property and larger algebraic connectivity because of the same degree and random links that stretched to other nodes. But, optimized networks we propose have comparatively better maximum eigenvalue property than small-world networks whose p is close to $p = 1$, and furthermore, optimized networks have far better algebraic connectivity and Q(condition number) values than random regular networks which are currently known as the best networks in the literatures and previous network models. That is why we can say that our optimized networks are better networks for consensus and synchronization in comparison to all the network models which are known.

We can understand these networks properties from all Laplacian eigenvalues transition. These all eigenvalues transition of the optimized networks, regular network, random network, small-world networks($p = 0.2$ as representation of algebraic connectivity starting going up high) and random regular network are shown in Figure.12.

The 2nd smallest Laplacian eigenvalue of optimized networks is highest, and random regular network, random network, small-world networks($p = 0.2$ as representation)

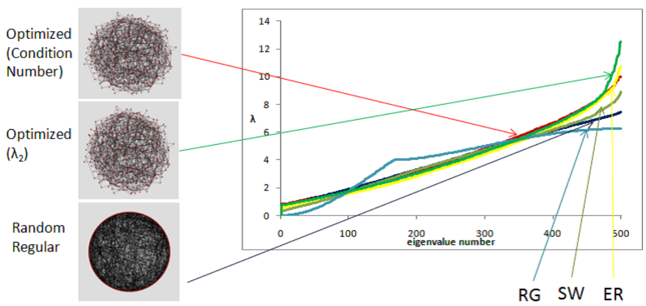

Figure 12 All Laplacian eigenvalues transition of the representative networks

follow, regular network is the lowest. Eigenvalues of the Laplacian matrix of regular network is fluctuating eigenvalue transition in the middle rising, the maximum and minimum eigenvalue are relatively low. Eigenvalues of the Laplacian matrix of random network increase exponentially, small-world networks have intermediate characteristics between regular network and random network. Eigenvalues of the Laplacian matrix of random regular network increases linearly because of homogeneous property. And optimized network(optimized by λ_2) has the higher algebraic connectivity than previous network models, but maximum eigenvalue property is higher than optimized network(optimized by condition number). So optimized network(optimized by λ_2) dose not have better property for consensus and synchronization than optimized network(optimized by condition number). And optimized network(optimized by condition number) has better network property for consensus and synchronization which has the highest algebraic connectivity and comparatively lower maximum eigenvalue property than small-world networks whose p is close to $p = 1$. Because optimized network(optimized by condition number) is considered by both maximum eigenvalue and 2nd smallest eigenvalue. As a result, optimized networks have the lowest Q(condition number), and we can say optimized networks have the best property for consensus and synchronization.

These networks eigenvalues characteristics appears in the histogram of the eigenvalues, we can explore all these property from Figure.11 which shows the eigenvalues frequency histogram of the each networks.

Regular network has low algebraic connectivity and comparatively less overall eigenvalues, so eigenvalues increase and sudden change in the trends that have become characteristic of low frequency and unevenness of the like valley eigenvalues histogram. small-world networks have intermediate characteristics between regular network and random network, and eigenvalue increase more than regular network about the point of the low frequency and unevenness of the like valley eigenvalues histogram.

Random network shows high frequency of low eigenvalues, the eigenvalue histogram has fewer frequency in higher eigenvalues. Curve eigenvalues frequency transition tend to be relatively high algebraic connectivity. Random regular network is also little difference in the histogram of eigenvalues, which appeared homogeneous characteristics. The linear eigenvalues frequency trends include high homogeneity, so random regular network are currently known as the best consensus and synchronous networks in the literatures and previous network models.

Optimized network(optimized by λ_2) shows the curve change of eigenvalues frequency, eigenvalues histogram tend to become the width of the eigenvalue spreading. So, the largest Laplacian eigenvalue is larger and condition number is larger than optimized network(optimized by condition number).

And optimized network(optimized by condition number) shows the well-regulated curve change of eigenvalues frequency, and eigenvalues histogram tend to become compact about the width of the histogram, therefore, optimized network(optimized by condition number) has relatively low maximum Laplacian eigenvalue and very high 2nd smallest Laplacian eigenvalue, and has better network property for consensus and synchronization which has the highest algebraic connectivity and the lowest condition number property. We can conclude that optimized networks have the best property for consensus and synchronization whose convergence speed is much faster than previous network models.

Furthermore, we show the property of the optimized networks(condition number) by different $\omega(0 \le \omega \le 1)$. Figure.13 shows the 2nd smallest Laplacian eigenvalues of the optimized networks(condition number) by different ω.

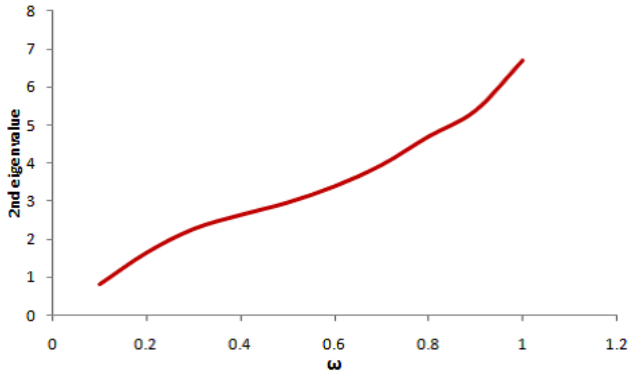

Figure 13 2nd smallest Laplacian eigenvalues of the optimized networks(optimized by condition number) by different ω

The higher weight ω is, the higher the algebraic connectivity is. Because if the ω is higher, link density objection become less constrained, so the number of links is increasing. Algebraic connectivity have the property that it is based in proportion to the number of links. Therefore, by the weighting of each ω, the networks which are optimized have the characteristics of the different average degree.

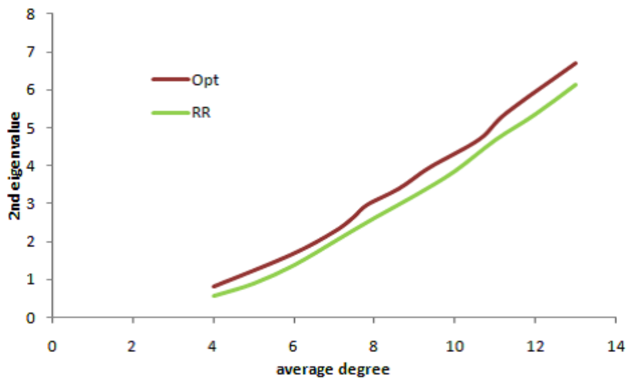

Figure 14 Average degree versus 2nd smallest Laplacian eigenvalues of the optimized networks(optimized by condition number) by different $\omega(0 \le \omega \le 1)$ and random regular networks

From Figure.14, we can understand the property that average degree versus 2nd smallest Laplacian eigenvalues of the optimized networks(optimized by condition number) by different $\omega(0 \le \omega \le 1)$ and random regular networks. Both optimized networks(optimized by condition number) and random regular networks have the algebraic connectivity which is based in proportional to the average degree. We can see that optimized networks have greater algebraic connectivity value than random regular networks in different average degree case from Figure.14. Random regular network is the best network as previous network

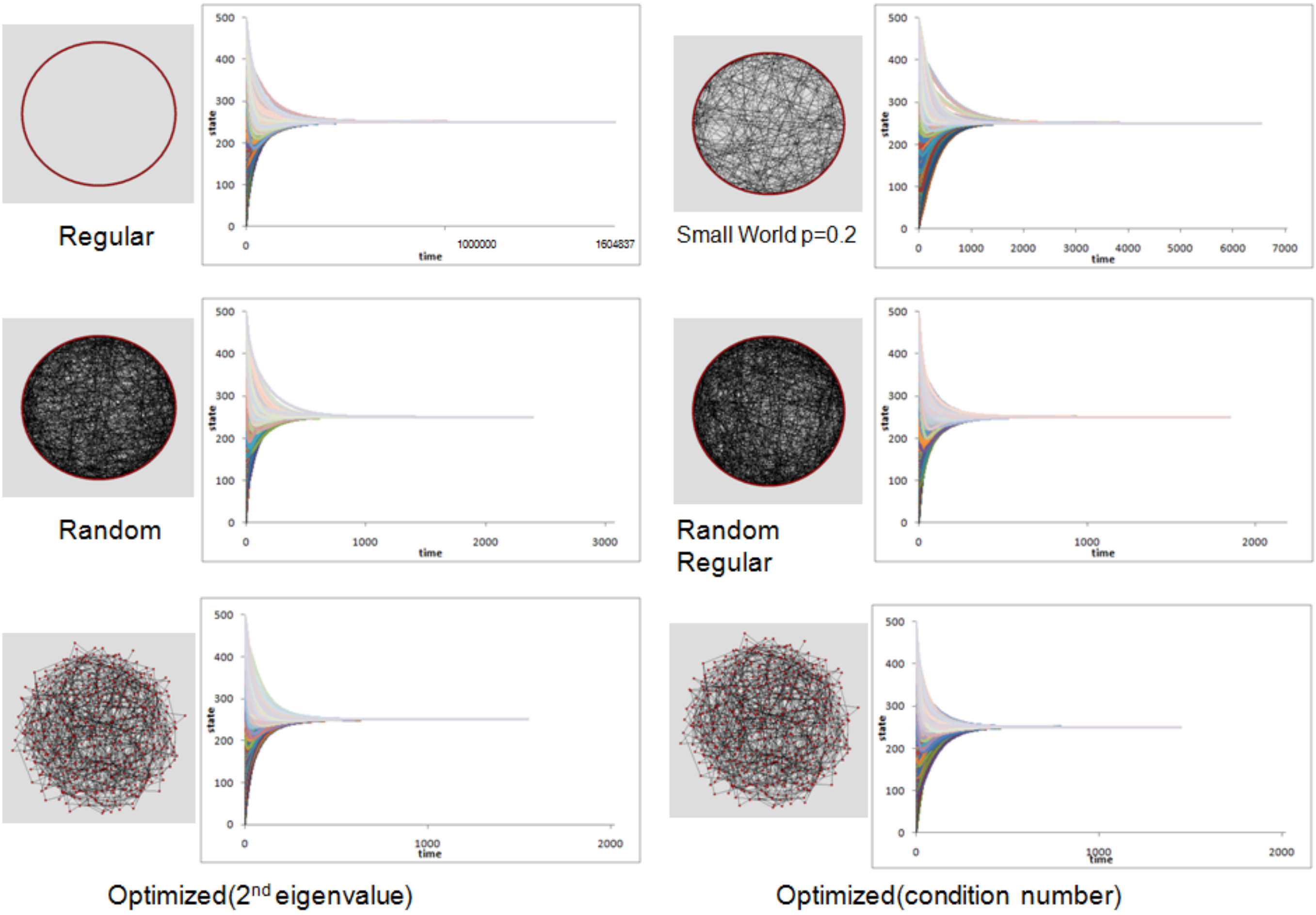

Figure 15 Network convergence process

models for consensus and synchronization. Because random regular network has the highest algebraic connectivity in all previous network models. Therefore, optimized networks(optimized by condition number) have better network property for consensus and synchronization in different average degree case than random regular networks which have the highest algebraic connectivity in all previous network models. And we can say that optimized networks generated by different ω have the best property for consensus and synchronization in different average degree case.

5.3 *Comparison of consensus and synchronization convergence speed*

So far, by comparing the optimal networks generated by evolutionary algorithm with the previous network models, we can understand that optimized networks have the best property for consensus and synchronization whose algebraic connectivity and Q(condition number) are better than previous network models.

To examine how fast the process of achieving consensus of the networks are, we compare convergence speed property of the networks which we see so far about the algebraic connectivity and Q(condition number) on the consensus problems.

As the following equation Eq.17, the initial various values of each nodes are given at first.

$$x_i(0) = i(i = 1, 2, \cdots, n) \tag{17}$$

And each n agents(nodes) for the system, the state of each agents x_i, $(1 \leq i < n)$ shown in chapter.2 converge to a constant value, Comparing the time required for achieving consensus. That is, until asymptotically converging to the same internal state by way of an agreement characterized by the following equation:

$$x_1 = x_2 = \cdots = x_n \tag{18}$$

This means the collective dynamics converge to the average of the initial states of all agents, the results of the convergence process of the each networks are shown in Figure.15.

Regular network, the algebraic connectivity is so low that the convergence time is very much time step. It takes a lot of time. Small-world network (represented $p = 0.2$) is faster convergence time compared to the regular network, which is 6532 convergence time step.

For $p = 1$, the random network, the time step of convergence is 2393 time step, random network is one of the fastest networks in the models based on the link probability p.

Random regular network has higher algebraic connectivity than random networks, so convergence becomes faster, the time is 1848 step. Optimized network(optimized by λ_2) convergence time is 1542 time step, which is faster than random regular network which is fastest in the previous network models. We can see that the network optimized by only λ_2 has better consensus and synchronization property than random regular network. Optimized network(optimized by condition number) convergence time is 1443 time step, which is faster than the optimized network(optimized by λ_2). The convergence time of optimized network(optimized by condition number) is fastest of all the networks we compare. So far, we can see that the algebraic connectivity is greater in evolutionary optimized networks than random regular(Ramanujan graph) networks which are the highest algebraic connectivity networks in the previous network models(regular network, random network, small-world network and random regular network). And optimized networks have far better algebraic connectivity and Q(condition number) values than random regular networks. These property we examine also corresponds to the actual convergence time, the convergence time of optimized network(optimized by condition number) is fastest of all the network models. This optimized networks are suitable to call ”optimal”, which has best convergence property for consensus and synchronization.

Optimal networks we obtained in this study, show better synchronization and consensus property to see the convergence speed and network eigenvalues. In the previous study, network model has been created under the regular rules, and been investigated their characteristics. But in this study, network is evolved to suit the characteristics of the objection by evolutionary algorithm and we create optimized network. Consensus and synchronization problem is closely related to the variety of issues such as collective behavior in nature, the interaction between agents and factor, as a matter of the robot control and many agents coordination, understanding the property of nature synchronization and building efficient wireless sensor networks, etc. Consensus and synchronization problem is associated with various issues, therefore, the optimal networks which we proposed indicate its significance and effectiveness as one of the best network structures.

6 Conclusion and Remarks

In this study, we proposed the optimal networks based on evolutionary algorithm which consider link density and Laplacian matrix eigenvalue. We show that the 2nd smallest eigenvalue and Q(condition number) are greater in evolutionary optimized networks than random regular(Ramanujan graph) networks which are the highest algebraic connectivity networks in the previous network models(regular network, random network, small-world network and random regular network). And the convergence speed is faster in evolutionary optimized networks than random regular(Ramanujan graph) networks which are currently known as the best consensus and synchronization networks in the previous network models. We can conclude optimal networks are the most suitable to the property of consensus and synchronization. Consensus and synchronization problem is closely related to the variety of issues, so the optimal networks which we proposed have significance to design and consider about the network system and society.

References

Barabási, A. L. Albert, R. (1999), ‘Emergence of scaling in random networks’, *Science* **286**(5439), 509–512.

Barahona, M. (2002), ‘Synchronization in small-world systems’, *Physical Review Letters* **89**(5), 054101.

Bollobas, B. (2001), *Random graphs*, Cambridge Univ Pr.

Buck, J. Buck, E. (1976), ‘Synchronous fireflies’, *Scientific American* **234**, 74–85.

Donetti, L., Hurtado, P. I. Munoz, M. A. (2005), ‘Entangled networks, synchronization and optimal network topology’.

Erdős, P. Rényi, A. (1959), ‘On random graphs. I’, *Publ. Math. Debrecen* **6**, 290–297.

Hovareshti, P. Baras, J. (2006), Consensus problems on small world graphs: A structural study, Technical report, Institute for Systems Research.

Kar, S., Aldosari, S. Moura, J. (2008), ‘Topology for distributed inference on graphs’, *IEEE Transactions on] Signal Processing, IEEE Transactions on [see also Acoustics, Speech, and Signal Processing* **56**(6), 2609–2613.

Kauffman, S. A. (1993), *The Origins of Order: Self-Organization and Selection in Evolution*, Oxford University Press.

Lovász, L. Sós, V. T. (1981), *Algebraic Methods in Graph Theory*, Vol. 1, Amsterdam ; New York : North-Holland.

Merris, R. (1998), ‘Laplacian graph eigenvectors’, *Linear Algebra and its Applications* **278**(1-3), 221–236.

Neda, Z., Ravasz, E., Brechet, Y., Vicsek, T. Barabasi, A. L. (2000), ‘The sound of many hands clapping’, *Nature* **403**, 849–850.

Olfati-Saber, R. (2005), Ultrafast consensus in small-world networks, *in* ‘Proceedings Proc. of American Control Conference’, pp. 2371–2378.

Olfati-Saber, R. (2006), ‘Flocking for multi-agent dynamic systems: Algorithms and theory’, *IEEE Transactions on Automatic Control* **51**, 401–420.

Olfati-Saber, R., Fax, J. A. Murray, R. M. (2007), Consensus and cooperation in networked multi-agent systems, *in* ‘Proceedings of the IEEE’, Vol. 95, pp. 215–233.

Olfati-Saber, R. Murray, R. M. (2004), ‘Consensus problems in networks of agents with switching topology and time-delays’, *Automatic Control, IEEE Transactions on* **49**(9), 1520–1533.

Pikovsky, A., Rosenblum, M. Kurths, J. (2003), *Synchronization : A Universal Concept in Nonlinear Sciences (Cambridge Nonlinear Science Series)*, Cambridge University Press.

Ren, W. Beard, R. W. (2008), *Distributed Consensus in Multi-vehicle Cooperative Control*, Communications and Control Engineering, Springer-Verlag.

Sato, H., Isao, O. Shigenobu, K. (1997), ‘A new generation alternation model of genetic algorithms and its assessment.’, *Journal of Japanese Society for Artificial Intelligence* **12**(5), 734–744.

Strogatz, S. H. (2000), ‘From kuramoto to crawford: exploring the onset of synchronization in populations of coupled oscillators’, *Phys. D* **143**(1-4), 1–20.

Strogatz, S. H. (2001), ‘Exploring complex networks.’, *Nature* **410**(6825), 268–276.

Watts, D. J. Strogatz, S. H. (1998), ‘Collective dynamics of ’small-world’ networks.’, *Nature* **393**(6684), 440–442.

Xiao, L. Boyd, S. (2004), ‘Fast linear iterations for distributed averaging’, *Systems & Control Letters* **53**(1), 65–78.