Diffusion and Cascade Dynamics on Growing Networks

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1 Introduction

Social networks largely determine how individuals behave and interact. The actions of individuals are influenced by friends and neighbors, and the relationships between individuals form the basis of social networks. A network consists of network nodes (i.e., people in social networks) connected by links. The specific pattern of connections defines the network topology. For many applications it is not necessary to exactly model the topology of a given real-world network, because the issue of interest often depends only on certain topological properties.

Recent studies revolve around the following two key aspects of research: the first is determining the values of important topological properties of a network that evolves over time and the second is evaluating the effect of these properties on network functions (Gross, 2008). One key aspect of network research is concerned with the dynamics of networks. Here, the topology of the network itself is regarded as a dynamic system. The other key aspect of network research focuses on the dynamics of the individual elements (nodes or links) of networks. Here, each node of the network represents a dynamic system. The individual systems are coupled according to the network topology.

One of the major focuses of this research is diffusion networks. However, little is known about the diffusion dynamics on growing networks. Other important dynamic processes include synchronization of individual systems and contact processes, such as opinion formation and epidemics. The abovementioned studies have made it clear that certain topological properties have a strong impact on the dynamics. For example, it was shown that vaccination of a fraction of the nodes cannot stop epidemics on a scale-free network.

Until recently, the two directions of network research were generally pursued independently. Even though strong interaction and cross-fertilization transpired, a given model would either describe the overall dynamics of a network or the dynamics within a network. Nevertheless, it is clear that in most real-world networks, the growth of the network is linked to its state. The topology of the network is regarded as a dynamic system, since it changes over time according to specific local rules. Investigations in this area have revealed that certain evolutionary rules give rise to peculiar network topologies with special properties.

We consider two types of diffusion, probabilistic diffusion and threshold-based diffusion or cascade dynamics on growing networks.

We begin by discussing several studies that illustrate the impact of the structure of social networks on macroscopic diffusion patterns. The discussions are divided into two classes. First, we discuss progressive diffusion, which means that once a node switches from one state to another, it remains in the same state in all subsequent time steps. For this type of diffusion, we focus on the correlations between social interaction patterns and the observable transmission rate at the individual level.

The largest eigenvalue of the adjacency matrix of a graph is the good measure of diffusion. One standard approach to designing desirable networks for maximizing (or minimizing) diffusion is to study the problem of adding edges to a graph so as to maximize its largest eigenvalue. This is a difficult combinatorial optimization, so we seek a heuristic for approximately solving the problem. We then propose a heuristic method for the problem. The heuristic is based on the independent growth of both nodes and edges, and therefore can be applied to very large graphs. A new algorithm for network with growth is proposed based on independent growth of both nodes and links of existing nodes that leads to the scale-free networks with the largest eigenvalue of the underlying graph.

The second class is threshold-based diffusion processes, where nodes over time can switch from one state to the other or vice versa, depending on the states of their neighbors. For threshold-based diffusion processes, we focus on decision making at the individual level. This class ties individuals to the analysis of the underlying games among a network of agents.

Which network structures favor the rapid spread of new ideas, behaviors, or technologies? This question has been studied extensively using epidemic models. Here we consider other scenarios where the individual behaviors are the result of a strategic choice among competing alternatives. In particular, we model cascade dynamics, which is also formulated as coordination games on networks. The cascade model is a type of diffusion in which innovation nodes can switch depending on the states of their neighbors. The switching behavior is described by the threshold rule. In this model, diffusion processes largely involve explicit individual choices. The results differ strongly from those provided by epidemic models. In an epidemic type of diffusion, networks having the largest eigenvalue of the adjacency matrix are optimal for fostering diffusion. In particular, it appears that innovation spreads much more widely on Barabási-Albert (BA) scale-free networks with a power index 3 than on other types of networks.

2 The Study of Diffusion and Cascade on Social Networks

Every day, billions of people worldwide make billions of decisions about many issues. People constantly interact with each other in different ways and for different purposes. The aggregation of these unmanaged individual decisions can lead to unpredictable outcomes, but, somehow these individual interactions usually exhibit some degree of coherence at the aggregate level. Therefore, aggregation may reveal structure and regularity. These emergent properties are the result of not only the behavior of individuals but also the interactions between them (Ball, 2004).

Diffusion is the process by which new products and practices are successfully introduced into society. The Bass model which formalizes the aggregate level of penetration of a new product, emphasizes two processes: external influence via advertising and mass media, and internal influence via word-of-mouth (Bass, 1969). The Bass model assumes all consumer populations are homogeneous, and such diffusion models are referred to as aggregate models. However, an individual decision rule can also be derived from the Bass model. The number of individuals who adopt a new product at a given time is a function of the number of individuals who have already adopted the product. If we calculate the expected number of adopters at a given time, the aggregate model displays a cumulative S curve of adopters (Rogers, 2005)

When we have to make a decision, we probably start by asking a few friends what they think. The decisions of friends probably have a major and possibly even determining impact on our own choices. Direct contagion effects may result from certain behaviors. This is particularly relevant when thinking of the spread of computer viruses and human diseases. The more infected friends that a given individual has, the more likely that the individual becomes infected.

The uniting feature of all these examples is that the networks of individuals can have a major impact on their lives. A person's behavior is most heavily influenced by the people with whom he or she is in contact with on a regular basis. Studying such situations requires an understanding of the basic social network structure at hand and its interplay of human decisions. The precise underlying social structure is important in terms of the reaction of the population to different products, diseases, and innovations. To see exactly the impact of the variations in social structure on the diffusion process, we concentrate on situations where the likelihood of individuals to adopt innovation increases with the number of friends they have who have already adopt the same innovation.

Diffusion has a social multiplier associated with it. That is, once a person influences others to adopt, it is more likely that their neighbors will adopt, and so on. In this situation, by slightly increasing the density of social networks, we see a dramatic change in the properties of the system as we leap from one setting where there is no diffusion to another where there is widespread diffusion.

Literature on the diffusion of innovation has examined the influence of the decisions of others on the decision making of an agent. Examples include studies on the diffusion of new technologies (Arthur, 1989) and the diffusion of conventions and social norms (Ellison, 1993; Morris, 2000). Spielman (2005) defined an innovation system as “a network of nodes (agents) along with the institutions, organizations, and policies that condition their behavior and performance with respect to generating, exchanging, and utilizing knowledge.” An innovation system reflects one aspect of value chain analysis by bringing actors together in the application of knowledge within a value chain.

Spielman’s definition highlights the need for a holistic view of the nature and structure of interactions among agents linked to one another within networks. The adoption of an innovation by one agent in a network can have positive or negative impacts on the behavior of other agents, although the impacts are often unintentional or unpredictable.

In most of the settings of diffusion models, coexistence is a typical outcome. An important aspect, however, that is arguably missing from the basic game-theoretic models of diffusion is a more detailed picture of what is happening at the coexistence boundary, where the basic form of the model posits adopting nodes linked to nodes that do not adopt. This aspect of diffusion models should be analyzed by formulating coordination games on a social network. Another important part of the research on innovation diffusion has focused on empirical studies showing the crucial elements of the structure of social interactions and on computational models that investigate the patterns of innovation diffusion through social networks (Rogers, 2005).

We describe the following technology adoption scenario proposed by Immorica (2007). Suppose we have two technologies, A and B, and agents must use the same technology to communicate. In a social network, G governs which agents talk to whom. Each edge (v, w) of G plays a coordination game with strategies A or B: if v and w each choose A, then they each receive a payoff of $1-\theta$ (since they can talk to each other using system A); if they each choose B, then each receive a payoff of θ. However, if v and w choose opposite technologies, then each receives a payoff of 0, a result that reflects the lack of interoperability.

The recent surge in network modeling of complex systems has set the stage for new studies of the fundamental and applied aspects of optimization in networked systems. Optimization, which is one of the broadest areas of research, has a very long history. Optimization comprises the evolutionary survival-of-the-fittest principle prominent in biology and engineering. Both natural and man-made systems and their ubiquitous attempts at optimization generate continuing appeal among researchers (Motter, 2007). The structural properties of networks can be derived from the dynamics by which the networks are created. The process by which new social ties are created is far from a random formation of connections. Formation of new links, for example, often happens by individuals introducing others to their friends. This process produces a power distribution of the number of connections.

In real societies, networks are created by individuals forming and maintaining social relationships, which in turn influence the dynamics of these processes. A challenging issue is to analyze the diffusion process with a dynamic network topology that is time varying, such as ad hoc networks and the information exchange of mobile users.

Diffusion is the process by which new products are invented and successfully introduced into a society (good diffusion), or infectious diseases spread (bad diffusion). Many studies shed light on how network topology interacts with the structure of social networked systems such as financial institutions to determine system-wide crises.

We consider two types of diffusion, probabilistic diffusion and threshold-based diffusion or cascade dynamics on growing networks.

3 A Growth Model of Social Networks

In societies, individuals are recognized and consequently they have identities. Human interactions are governed by formal and informal networks. Social networks formalize the description of the networks of the relations in a society. In social networks, individuals or social groups are represented as nodes, and the relations or flows between individuals or groups are portrayed as links. These links represent different types of relations between individuals, such as friendship, trust, collaboration, or the exchange of information or services. In a network of institutions, links represent the flow of money, products, or services, organizational relations, and so on.

Affiliation networks are composed of two types of nodes: institutions and individuals. Some links in the network connect nodes of one type to nodes of the other type (i.e., individuals and institutions). Nodes of the same type are connected together if both are connected to the same node of the other type. For example, one type of node may correspond to a supervisory board and another to an individual. Although the formalism of social networks is relatively old, the interest in networks, especially in social networks, has been increasing in recent years. Different types of networks occurring in nature and societies have been shown to share some universal properties.

The most promising models are those in which the structures of the social network determine the processes, which in turn shape the network structures. The aim of this paper is to investigate the network dynamics with growth properties. The topology of the network evolves according to specific rules.
The topology of the network itself is regarded as a dynamic system that changes in time according to specific, often local, rules. Investigations in this area have revealed that certain evolutionary rules give rise to peculiar network topologies with special properties. Gross (2007) proposed the concept of adaptive networks, which appear in many biological applications. He combined the topological evolution of a network with the dynamics in the network nodes. We review these recent developments and show that they can be viewed from a unique angle.

Some mechanisms of growing networks have been suggested, including preferential attachment-related rules in which external parameters have to be tuned. When dealing with biological networks, for example, the interplay between emergent properties derived from network growth and selection pressures has to be taken into account. Some models of growing networks are discussed below. Especially we propose a heuristic method for the problem. The heuristic is based on the independent growth of both nodes and edges, and therefore can be applied to very large graphs. A new algorithm for network with growth is proposed based on independent growth of both nodes and links of existing nodes that leads to the scale-free networks with the largest eigenvalue of the underlying graph.

(i) Random growth of links: a random network (RND)

We consider a graph *G(n, p)* that consists of n nodes (or vertices) joined by links (or edges) with some probability *p*. Specifically, each possible edge between two given nodes occurs with a probability *p*. The average number of links (the average degree) of a given node is $z=np$, and it can be easily shown that the probability p(k) that a vertex has a degree *k* follows a Poisson distribution. This network model displays a phase transition at a given critical average degree $z_c=1$. At this critical point, a giant component forms, and for $z > z_c$, a large fraction of the nodes are connected in a web, whereas for $z < z_c$, the graph is fragmented into small sub-graphs (Durrent, 2007).

(ii) Growth model with preferential attachment: a Barabási–Albert scale-free network (BA)

The Barabási–Albert (BA) model is an algorithm for generating random scale-free networks using a preferential attachment mechanism. At each time step, one new node is added to the initial network. A new node has m links connected to the nodes, which are selected in proportion to their degree k_i. The probability of a node for selection is set by $k_i / \sum k_i$. This method of adding a node to the network is called "preferential attachment". The degree distribution p(k) of the network shows the power law $p(k) \propto k^{-\alpha}$ and its power index of distribution is always a constant $\alpha = 3$, which is not related to the number of links m added by the new node added to the network at each time step. The change in the number m makes it possible to have a network with arbitrary average degree $z=2m$.

(iii) Independent growth of nodes and links: KN network (KN)

We propose a heuristic method for the problem. The heuristic is based on the independent growth of both nodes and edges, and therefore can be applied to very large graphs. A new algorithm for network with growth is proposed based on independent growth of both nodes and links of existing nodes that leads to the scale-free networks with the largest eigenvalue of the underlying graph.

The formation of the KN network consists of two steps at each time step. The first step adds a node to the network with preferential attachment. The second step adds m links to the network with preferential attachment. Therefore, at each time step, the network has one new node with a link and m new links. The degree distribution of the KN network also obeys power law $p(k) \propto k^{-\alpha}$ as in the scale-free network, but its power index $\alpha = 2 + 1/(2m+1)$ is not constant.

Figure 3.1 show the largest eigenvalues ($\lambda_{RND}, \lambda_{BA}, \lambda_{KN}$) of the adjacency matrices of the above three networks (RND, BA, KN) in terms of the average degrees (z). We can observe the following relations among these largest eigenvalues of these three networks.

$$\lambda_{RND} < \lambda_{BA} < \lambda_{KN} \tag{3.1}$$

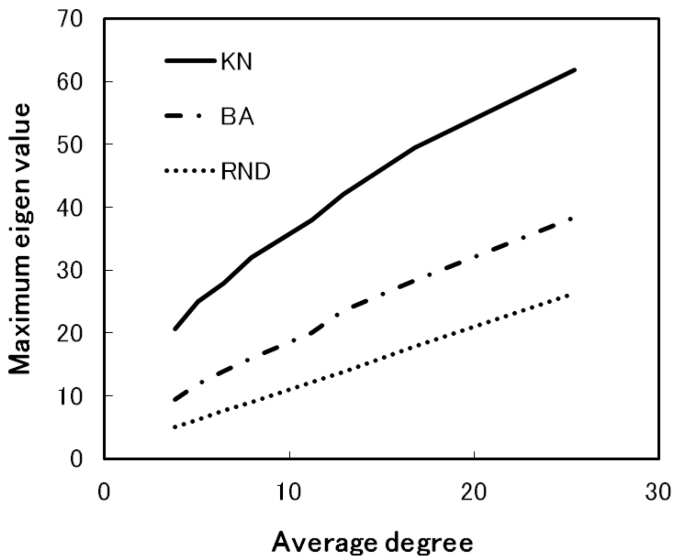

Figure 3.1 Comparison of the largest eigenvalues of the adjacency matrices of random network (RND), scale-free (BA), and KN network (KN). Each network has 500 nodes ($n=500$).

4 Probabilistic Diffusion

Important processes that take place within social networks, such as the spreading of opinions and innovations, are influenced by the topological properties of these networks. In turn, the opinions and practices of individuals can have a clear impact on network topology. From an applied point of view, it is desirable to compose an inventory of the types of microscopic dynamics investigated in social networks and their impact on emergent properties at the network level. Such an inventory could provide researchers with specific guidelines concerning the kinds of phenomena present in social systems where similar diffusion processes are at work.

The diffusion of innovations, new ideas, or rumors can be modeled after the spread of infectious diseases (Meyers, 2005), since epidemic models describe infection in terms of transmissibility and susceptibility. People are usually affected by word-of-mouth transmission, which is referred to as a social influence. What influences people to participate in the diffusion process? How do consumers choose which items to buy? These questions are constant in the minds of many scientists in different fields. Various models of human behavior span the extremes from simple and easily influenced behavior to interdependent and complex behavior, in which nothing is easily controlled or predicted.

Suppose that a number of consumers mutually transmit information about a newly launched product through a social network. Such diffusion on network is modeled by two representative types: epidemic and threshold-based diffusion. For instance, the diffusion of information, rumors, or gossip through social networks can be modeled after the spread of infectious diseases. Clearly, becoming infected is not an individual choice, and contagion in this case is not a strategic phenomenon. In the spread of a disease, some nodes (individuals) become infected initially through exogenous sources, and consequently some neighbors of these individuals are infected through contact. A node can possibly be immune; however, if a node is not immune, then it is sure to catch the disease if one of its neighbors is infected.

Therefore one basic diffusion model is the susceptible-infected-susceptible (SIS) model, in which nodes are initially susceptible to the disease and can become infected from infected neighbors. Once infected, a node continues to infect its neighbors until it is randomly removed from the system. The SIS model presumes that an infected node will eventually infect all of its susceptible neighbors. Infected nodes can either recover and stop transmitting the disease, or die and completely disappear from the network. In the SIS model, infected nodes can randomly recover; however, after recovery, the nodes are again susceptible to infection. The SIS model corresponds well with many real-world viral infections that cause individuals to transition back and forth between health and illness.

The diffusion process based on epidemic models describes the information spread in terms of transmissibility and forgetting at the individual level. Epidemic models are extended diffusion models that explicitly include decisions influenced by social situations and word-of-mouth processes. We view information propagation as a dynamic birth-death process with self-recovery. Specifically, an informed agent i propagates the information to another agent j in a single step with probability β, while at the same time an informed agent i may forget or lose interest with some probability δ. The ratio of the two factors, β/δ, defines the relative diffusion rate of the information.

We next formulate the epidemic type of diffusion on networks. Any graph G can be represented by its adjacency matrix, $A=(a_{ij})$, which is a real symmetric matrix, $a_{ij}=a_{ji}=1$, if vertices i and j are connected, or $a_{ij}=a_{ji}=0$, if these two vertices are not connected. The spectrum of the graph is the set of eigenvalues of the graph's adjacency matrix A.

We denote the probability that agent i is aware of particular information at time t as pi(t). The column vector $p(t)=(p_1(t),p_2(t),\ldots, p_N(t))$ represents the awareness probabilities of the agent population. The transition of the awareness probabilities is now described as

$$p(t+1) = (\beta A + (1-\delta)I)\,p(t) \tag{4.1}$$

where I is an $N\times N$ identity matrix. The long-term behavior of the above system is determined by the structure of the system matrix S= $\beta A + (1-\delta)I$. Wang et al (2003) also proved that the spectrum of system matrix S (the distribution of eigenvalues of S) is closely related to the spectrum of the adjacency matrix A. Therefore, we obtain

$$\lambda_i(S) = \beta\,\lambda_i(A) + (1-\delta), \quad i=1,2,\ldots,N, \tag{4.2}$$

where $\lambda_i(S)$ is the i-th largest eigenvalue of system matrix S. The largest eigenvalue $\lambda_i(S)$ is also called the principal eigenvalue of the system matrix. We denote the largest eigenvalue of the system matrix S as $\lambda_1(S)$.

$$\text{If } \lambda_1(S) < 1, \text{ then p(t) converges to the zero vector} \tag{4.3}$$

From (4.2) and (4.3), the diffusion process is a threshold phenomenon, and diffusion spreads throughout the entire network if

$$\beta/\delta > 1/\lambda_1(A). \tag{4.4}$$

Therefore, a high largest eigenvalue ($\lambda_1(A)$) leads to a high infection ratio. The rationale behind the high largest eigenvalue is that some clustered hub nodes are present with a high degree, which may foster the contagion and accelerate the spread of diffusion. The KN network has a very large principle eigenvalue; therefore, the information may spread widely, since this network has the largest principal eigenvalue.

On the other hand, homogeneous networks such as random regular network (RR) in which all nodes have the same number of links (degree) have the smallest eigenvalues (the same with the degree). Therefore RR network mostly prohibit probabilistic diffusion.

The largest eigenvalue of the adjacency matrix of a graph is the good measure of diffusion. One standard approach to designing desirable networks for maximizing (or minimizing) diffusion is to study the problem of adding edges to a graph so as to maximize its largest eigenvalue. This is a difficult combinatorial optimization, so we seek a heuristic for approximately solving the problem.

We perform numerical simulations to show the effects of the network topologies: a random network (RND), a scale-free network (BA) and a KN network. Each network has *n=500* nodes and the average degree is *z=4*. The diffusion rates on these different network topologies as shown in Figure 4.1. For the initial setting, we set all nodes to the same forgetting probability $\delta = 0.1$ and transmitting probability β (varied from 0.005 to 0.05). A node with a maximum degree is transmitted manually to start the diffusion simulation.

We observe the fraction of transmitted nodes at the steady state and plot the average fraction over 100 experiments. On the KN network, as β increases, the fraction of informed nodes begins to increase more easily than that on other networks, but the diffusion starts at a lower diffusion probability (β). It is well-known that the BA network also greatly promotes diffusion, and so it is the most diffusive among them.

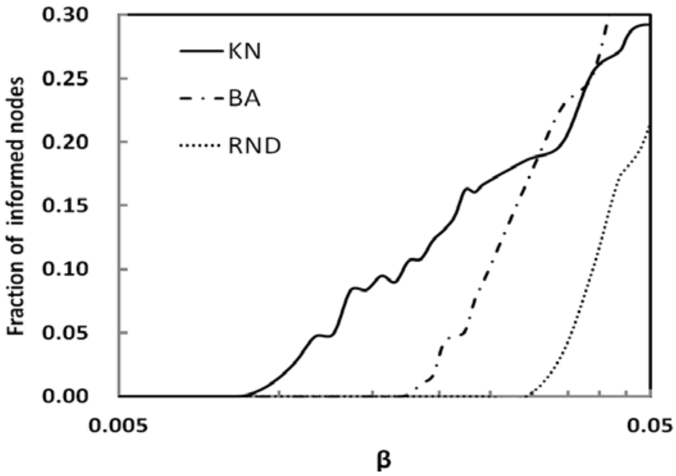

Figure 4.1 Diffusion simulation on RND, BA, and KN networks: We plot the relationship between the fraction of informed nodes and the transmitting probability β when each node has the same forgetting probability: $\delta = 0.1$.

5 Threshold-based Diffusion

Important processes that take place within social networks, such as the spreading of opinions and innovations, are influenced by the topological properties of these networks. In turn, the opinions and practices of individuals can have a clear impact on network topology. From an applied point of view, it is desirable to compose an inventory of the types of microscopic dynamics investigated in social networks and their impact on emergent properties at the network level. Such an inventory could provide researchers with specific guidelines concerning the kinds of phenomena present in social systems where similar diffusion processes are at work.

A fundamental problem in the social sciences is to understand the ways in which new ideas, behaviors, and practices diffuse throughout human populations. Another active line of research in economics and mathematical sociology is concerned with modeling the types of diffusion processes as binary decisions. Some examples include the adoption of new technologies, the emergence of new social norms or organizational conventions, and the spread of languages (Immorica, 2007, Spielman, 2005).

For simple diffusion, such as the spread of disease or rumors, in which a single active node efficiently triggers the activation of its neighbors, hub nodes with many connections or random links connecting distant nodes allow a large-scale diffusion, as discussed in the previous section. However, not all propagation is the simple activation of nodes after exposure to multiple active neighbors. This type of diffusion is modeled as threshold-based diffusion.

To illustrate how the threshold-based approach represents a potentially essential change in modeling, we consider an individual's decision whether to buy a new product, where the benefits to that individual from using that product increase with the number of friends who also use the product. People differ in a variety of ways that affect their decisions of whether to purchase a product.

From the social network perspective, these people might differ in terms of the number of friends with whom they interact, thus making the product more valuable to certain individuals. Therefore, each individual has some threshold that determines the proportion of neighbors required to activate the purchase. The most readily apparent result of this added social network factor is that it affects the threshold or tipping point. If the first adopters of a product reach the threshold, the product may spread significantly.

Many social interactions exist with positive externalities, in which the underlying game-theoretic model is formulated as a coordination game with multiple equilibria. We begin by discussing one of the most basic game-theoretic diffusion models proposed by Morris (2000). Morris provided a set of elegant game-theoretic characterizations for qualitatively different types of equilibria in the underlying network topology and the quality of *A* relative to *B* (i.e., the relative sizes of $1-\theta$ and θ). A cascading sequence of nodes may switch to *A* (x_i=1) such that network-wide equilibrium is reached in the limit, with all nodes adopting *A*. Or, it may involve coexistence with nodes partitioned into a set adopting A and a set adopting *B* (x_i=0).

This interdependence can be described as a threshold-based cascade model (Watts, 2007). We consider a population of N agents. Each faces a binary problem between two choices: *A* or *B*. For any agent, the payoff for choice *A* or *B* depends on whether other connected agents choose *A* or *B.* The payoff for each agent is given as an explicit function of the actions of all other agents. The adoption rule of an agent is

$$\begin{aligned} &If \quad p > \theta \quad then \quad choose \quad A \\ &\qquad\qquad otherwise \quad choose \quad B \end{aligned} \tag{5.1}$$

where p represents the ratio of neighbors who choose *A*.

Let us suppose that all agents (nodes) initially adopt *B*. Thereafter, a small number of agents begin adopting *A* instead of *B*. If we apply best-response updates to nodes in the network, then the nodes will in effect repeatedly apply the following simple rule: switch to *A* if the proportion of neighbors who have already adopted *A* is larger than the threshold θ.

López-Pintado (2006) and Watts (2002, 2007) studied the problem of spreading new technology *A* starting from the initial condition where all agents use old technology B. Assuming myopic-best response dynamics, López-Pintado and Watts showed that a threshold of the payoff parameter θ exists such that below the threshold, contagion of the new technology occurs by initiating the new technology by one agent, and eventually all the agents adopt the new technology.

López-Pintado investigates a cascade condition by using mean-field analysis (López, 2007). We describe the degree distribution, the network connectivity, by P_k . We also define an average degree where $\langle k \rangle$

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle = z \tag{5.2}$$

A node which can be activated by just one contact being active is defined to be vulnerable node. A node with having degree k and the threshold value θ is called a vulnerable node if these two values satisfy:

$$k < 1/\theta \tag{5.3}$$

A cascade progresses through the network when these vulnerable nodes are connected to each other. Therefore, we consider the condition that these nodes are connected as a cascade. The probability β_k that linked degree k node is vulnerable is as follows:

$$\beta_k = \int_{\theta=0}^{1/k} f(\theta) d\theta \tag{5.4}$$

where f(θ) is the distribution of node threshold θ.

Then the cascade condition is derived as follows:

$$\sum_{k=1}^{\infty} (k-1)\beta_k kP_k / z \geq 1 \tag{5.5}$$

Then the cascade condition depends on the distribution of the network connectivity, P_k , the probability β_k that linked degree k node is vulnerable and the average degree z From this cascade condition, we can imply that the higher the average connectivity of the network is the lower the threshold.

We now obtain and compare the minimum levels of threshold of a KN network (KN), a random network (RND) and a scale-free network (BA). These minimum thresholds on these three networks are represented as θ^*_{KN} , θ^*_{RND} and θ^*_{BA} respectively and a scale-free network with BA model (BA) has the highest threshold, and a KN network with the maximum largest eigenvalue has the lowest:

$$\theta^*_{KN} < \theta^*_{RND} < \theta^*_{BA} \tag{5.6}$$

Therefore a class of scale-free network (BA), which is extremely popular in epidemiology does also support this threshold-based contagion. However cascade is mostly unsuccessful accomplished by a higher variance network such as a KN network.

With a close look at cascade conditions, we can derive cascade areas in which a cascade can spread through an entire network in different network topologies. In Figure 5.1 we show the cascade areas derived theoretically in these various networks. As can be seen in Figure 5.1, cascade is characterized as a critical phenomenon which depends on both agent threshold θ and the average degree z of the network topology, and we can characterize cascade as follows:

(i) A critical threshold θ^*_c : There exists a critical threshold θ^*_c such that cascade occurs such that a large proportion of agents choose A with an infinitesimally small fraction of agent choosing A if $\theta < \theta^*_c$.

(ii) A critical average degree z_c^*: There exists a critical average z_c^* such that cascade occurs such that a large proportion of agents choose A with an infinitesimally small fraction of agent choosing A if $1 \le z < z_c^*$.

In Figure 5.1 we show that

(iii) For critical threshold θ_c^*: we have the following inequalities among critical thresholds of various networks, and Exponential network has the highest critical threshold, the KN and homogeneous regular network has the lowest critical threshold.

$$\theta_{KN}^* < \theta_{RND}^* < \theta_{BA}^* \tag{5.7}$$

(iv) For critical average degree z_c^*: we have the following inequalities among critical thresholds of various networks, and Exponential network has the highest critical threshold, the KN and homogeneous regular network has the lowest critical threshold.

$$1 \le z_{KN}^* < z_{RND}^* < z_{BA}^* \tag{5.8}$$

Therefore the scale-free BA network has the largest cascade area and the KN has the smallest cascade area

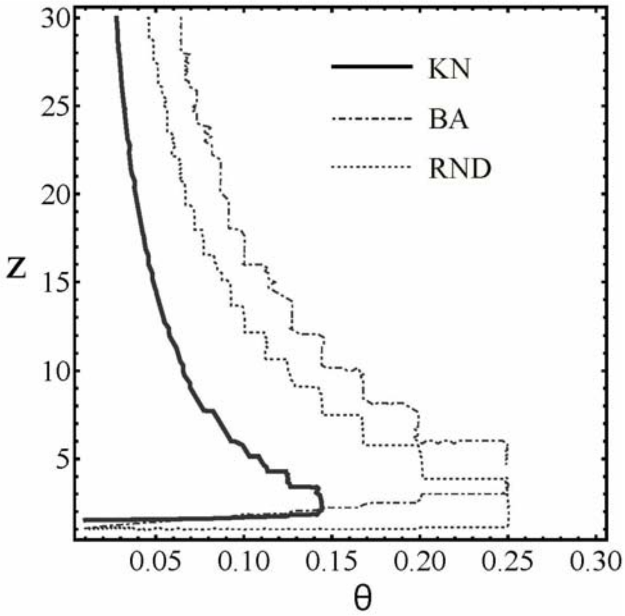

Figure 5.1 Cascade areas as the function of the threshold θ and the average degree z (theoretical results). Each line is derived mathematically using the equation and each point is the result of the numerical simulation. The x coordinates denote the threshold values of each agent (θ) and the y coordinates denote the average degree (z). A scale-free network (BA) is indicated by the short dashed line and circles, the random network (RND) by the solid line and triangles, and the KN network by the long dashed line and × characters.

Cascade phenomena depend on the dependence of the vulnerability of interconnected systems to global cascades on the network of interpersonal influences governing the information that individuals have about the world and therefore their decisions. We considered networks with growing properties and the same numbers of vertices and links specified by the degree distribution. From the results in Figure 5.1 and Figure 5.2, the scale-free network (BA) with power index α=3 has the largest cascade areas and fosters the highest cascade-based diffusion. Therefore we obtain the results in some important respects. (*i*) Unlike probabilistic diffusion models, where contagion events between pairs of individuals are stochastic and independently, the KN network with the largest eigenvalues have small cascade areas, and (*ii*) the scale-free network (BA) with power index α=3 has the larger area of cascade area among three different networks.

The rationale behind this result is the existence of a giant component of vulnerable nodes in the BA scale-free network. In contrast, in the KN network with a few well-connected hub nodes have the smallest cascade area, and cascade-type diffusion is the most difficult.

Global cascades in social and economic systems, as well as cascading failures in engineered networks, display two striking qualitative features: they occur rarely, and, by definition, are large when they do occur. Based on the theoretical and simulation results, power-law distributions of cascades satisfy the claim of infrequent, large events. Unfortunately, a lack of empirical data detailing cascade size distributions prevents us from determining which distribution best describes these systems.

Here we analyzed both epidemic and threshold-based cascades. When interpersonal influences are sufficiently sparse, and diffusion among the nodes occurs independently (independent cascade), a larger cascade is expected on the network with the largest eigenvalue. In this case, the hub nodes that are most highly connected with each other easily trigger diffusion. In contrast, a cascade is limited on the homogeneous network, where each node has the same degree. When each node has the same number of links (degree), cascades exhibit the power-law distribution at the corresponding critical point, In this case, the BA network with the power index α=3 is much more likely to foster cascades.

The KN network displays two faces, characterized as a promotion-yet-inhibition, which promotes epidemic-type propagation but inhibits global cascades based on individual strategic choices. Homogeneous networks such as regular or random regular networks inhibit epidemic-type propagation. Homogeneous networks also inhibit cascade, threshold-type propagation, based on individual strategic choices in a dense network where the degree of each node is high, however they promote threshold-type propagation in a sparse network where the degree of each node is low.

6 Conclusions

In this paper, we provided an overview of research that examines the impacts of social network structure on various types of diffusion processes. Some diffusion processes are progressive in the sense that once a node switches from one state A to another state B, it remains in the B state in all subsequent time steps. It was shown that KN networks with a higher largest eigenvalue foster probabilistic diffusion. The other type of diffusion is non-progressive and threshold-based diffusion, in which, as time progresses, nodes can switch from state A to B or from B to A, depending on the states of their neighbors. In a threshold-based model, such as innovation diffusion, the diffusion process involves individual decisions. In this case, the BA scale-free network with a few hub nodes fosters the most diffusion.

The diffusion process enhances innovation via the feedback of information about the innovation’s utility across different users. This feedback can then be used to improve the innovation. Most human social activities are substantially free of centralized management, and, although people may care about the end result of the aggregate, the decisions and behaviors of individuals are typically motivated by self interests. To make the connection between microscopic behavior and macroscopic patterns of interest, we need to look at the system of interactions among agents. This type of dynamics can be described as the interplay between the dynamics of each agent (node) and the network topology.

In future work, we intend to focus on both the microscopic and macroscopic dynamics, and especially the bi-directional interactions between them. Social network topologies determine a basic and important form of social interactions among agents, and the microscopic behaviors of agents largely determine the diffusion patterns observed at the macroscopic level.

The network topology is crucial for determining the diffusion and cascade dynamics. The network topology also has a direct impact on other important network performances, including robustness and resilience. The increasing scale, complexity, and interdependencies between policies and configurations of emerging networks call for a shift from the current human-centric network management toward more autonomic and adaptive mechanisms. However, we still lack the proper methodologies to control the network topology. Another challenging issue is to explore an autonomic topology control approach that builds on the concepts of self-organization to evolve and adapt the network topology to satisfy dynamically changing requirements. Adaptation may be performed in a decentralized manner, in which local nodes maintain neighborhood information and apply local neighbor selection policies to shift to the desirable network topology. It is hoped that the combined framework of analyzing diffusion and cascades will stimulate theoretical and empirical efforts to explore network management methodologies.

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