The Design of Desired Collectives with Agent-based Simulation

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Abstract

In this paper, I consider the issue of efficient and equitable utilization of limited resources by a collection of interacting agents. There is no presumption that a collective decision of interacting agents leads to a collectively satisfactory result without any central authority. How well agents do for it in adapting to their environment is not the same thing as how satisfactory an environment they collectively create. Agents normally react to others' decisions, and the resulting volatile collective decision is often far from being efficient. By means of agent-based simulation, I explore a new approach for designing desirable collectives. There are two closely related issues concerning collectives, the forward problem and the inverse problem. The forward problem is to investigate what a collective of interacting agents determines its complex emergent behavior. The inverse problem is to investigate how each agent learns to behave for a desirable collective. I discuss how agent-based simulation contributes to solve these two issues.

Keywords: forward problem, inverse problem, congestion, collective learning, coupling, efficiency, equity

1. Introduction

Today we have many challenges for designing large-scale and complex systems consisting of multiple physically and geographically distributed processors. Such systems offer the promise of speed, reliability, extensibility and the potential for increased tolerance to uncertain date and knowledge. The conventional approach requires a unifying control or coordination mechanism in order to extend the partial views and the incomplete knowledge of components and to guide a global solution. By true meaning of distribution that both control and knowledge are physically and often geographically distributed, we mean that there is neither global control nor global knowledge storage. There are two methodologies that need to be developed to address this view of complex systems. First, distributed coordination mechanisms that enable systems to organize themselves and accomplish critical tasks with available resources. Second, self-organizing techniques that enable systems to improve with experience themselves.

The design of efficient collectives from bottom up becomes is an important issue in many areas [3][8][12]. Collective means any pair of a complex system of autonomous components, together with a performance criterion by which we rank the behavior of the overall system. The performance of the collective system which consists of many interacting agents should be described on two different levels: the microscopic level, where the decisions of the individual agents occur and the macroscopic level where collective behavior can be observed. To understand the role of a link between these two levels remains one of the challenges of complex system theory [14][15]. In examining collective, we shall draw heavily on the individual decisions. It might be argued that understanding how individuals make decisions is sufficient to understand the collective action. In this presentation, I will take a different view. Although individual decision is nested within important to understand, it is not sufficient to describe how a collection of agents arrives at specific decisions. These situations, in which an agent decision depends on the decisions of others are the ones that usually do not permit any simple summation or extrapolation to the aggregates. To make that connection we usually have to look at the system of interactions among agents [15].

Wolpert and Tumer propose that the fundamental issue is to focus on improving our formal understanding of two closely related issues concerning collective [22][23][24]:

(1) The forward problem of how the fine-grained structure of the system underlying a collective determines its complex emergent behavior and therefore its performance.

(2) The inverse problem of how to design the structure of the system underlying a collective to induce optimal performance.

Agent-based simulation involves the study of many independent agents and their interactions. It is regarded as a new way of doing science through experiments [2]. Like deduction, it starts with a set of explicit assumptions. But unlike deduction, it does not prove theorems. Instead, an agent-based model generates simulated data that can be analyzed inductively. Unlike typical induction, however, the simulated data come from a rigorously specified set of rules rather than direct measurement of the real world. When the agents use adaptive rather than optimizing strategies, deducing the consequences is often impossible, and then simulation becomes necessary. We specify how the agents interact, and then observe properties that occur at the macro level. The connection between micro-motivation and macro-outcomes will be developed through agent-based simulation, in which a population of agents is instantiated to interact according to fixed or evolving rules of behavior. Collective phenomena to be studied include the dynamics of markets, the emergence of social norms and conventions, and daily-phenomena such as traffic jams.

In examining collective decisions, we shall draw heavily on the individual adaptive decisions. Within the scope of our model, we treat models in which agents make deliberate decisions by applying rational procedures, which also guide their reasoning. An interesting problem is then under what circumstances will a collection of agents realizes some particular stable situations, and whether they satisfy the conditions of efficiency? An agent wishes to maximize her own utility and a system designer wish to implement a decentralized algorithm for maximizing the whole utility of a group. While all agents understand the outcome is inefficient, acting independently is powerless to manage this collective about what to do and also how to decide. The question of whether interacting agents self-organize efficient macroscopic orders from bottom up depends on how they interact each other. We attempt to probe deeper understanding this issue by specifying how they interact each other.

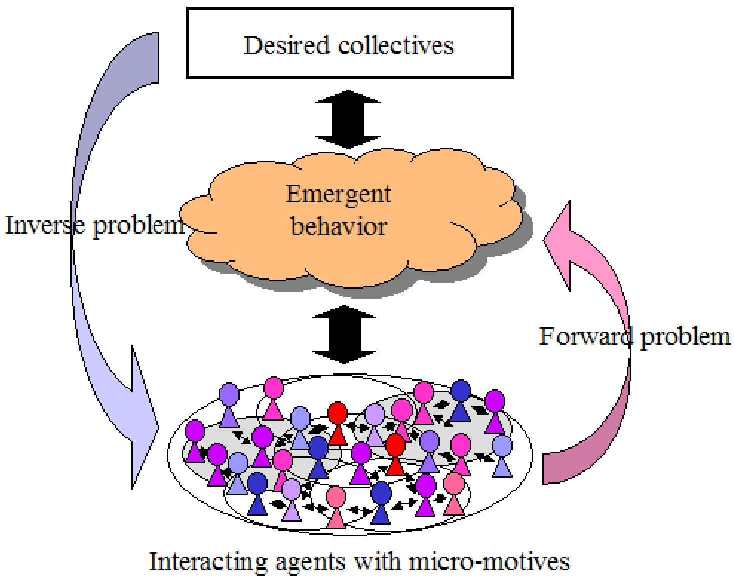


Figure 1 Solving forward and inverse problems with agent-based simulation

2. Collectives with Logic of Minority

We can classify interdependency among agents into two types. The typical interaction is the situation where the increased effort by some agents leads the remaining agents to follow suit, which gives multiplier effects [8]. We characterize this type of situation as agents behave based on the logic of majority since they receive higher payoff if they select the same strategy as the majority does.

On the other hand, there are many situations where agents receive payoff if they select the distinct strategy as the majority does. Let consider a competitive routing problem of networks, in which the paths from sources to destination have to be established by independent agents. For example, in the context of traffic networks, agents have to determine their routes independently. In telecommunication networks, for instance, they have to decide on what fraction of their traffic to send on each link of the network. We distinguish this type of interaction where agents behave based on the logic of minority. In this paper, we investigate collectives with logic of minority, and we have to utilize different methodology. Many agents who behave based on the logic of minority generate complex behavior of interest.

(1)Congestion Problems and Network Equilibrium Models

Network equilibrium models are commonly used for the prediction of traffic patterns in transportation networks that are subject to congestion [4][5]. The idea of traffic equilibrium originated as early as 1924, when F.M. Knight gave a simple and intuitive description of a postulate of route choice under congested conditions [17]. Suppose that between two points there are two highways, one of which is broad enough to accommodate without crowding all the traffic which may care to use it, but is poorly graded and surfaced; while the other is a much better road, but narrow and quite limited in capacity. If a large number of trucks operate between the two termini and are free to choose either of the two routes, they will tend to distribute themselves between the roads in such proportions that the cost per unit of transportation will be the same for every truck on both routes. As more trucks use the narrower and better road, congestion develops, until at a certain point it becomes equally profitable to use the broader but poorer highway."

In 1952 Wardrop stated two principles that formalize this notion of equilibrium and introduced the alternative behavior postulate of the minimization of the total travel costs [22]. Wardrop's first principle of route choice, which is identical to the notion postulated by Knight, became accepted as a sound and simple behavioral principle to describe the spreading of trips over alternate routes due to congested conditions. Wardrop's first principle: "The journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route." Each user non-cooperatively seeks to minimize his cost of transportation. The traffic flows that satisfy this principle are usually referred to as "user equilibrium" (UE) flows, since each user chooses the route that is the best. Specifically, a user-optimized equilibrium is reached when no user may lower his transportation cost through unilateral action. On the other hand, the system optimal is characterized by Wardrop's second principle (SO): "At equilibrium the average journey time is minimum." This implies that each user behaves cooperatively in choosing own route to ensure the most efficient use of the whole system. Traffic flows satisfying Wardrop's second principle is generally known as the system optimal (SO), and which requires that users cooperate fully or that a central authority controls the transportation system.

We formulate network equilibrium models as the collectives with the logic of minority [17] as follows; Let suppose that there are two alternative route A and B to commute for a collection of agents. Each agent has the following two strategies:

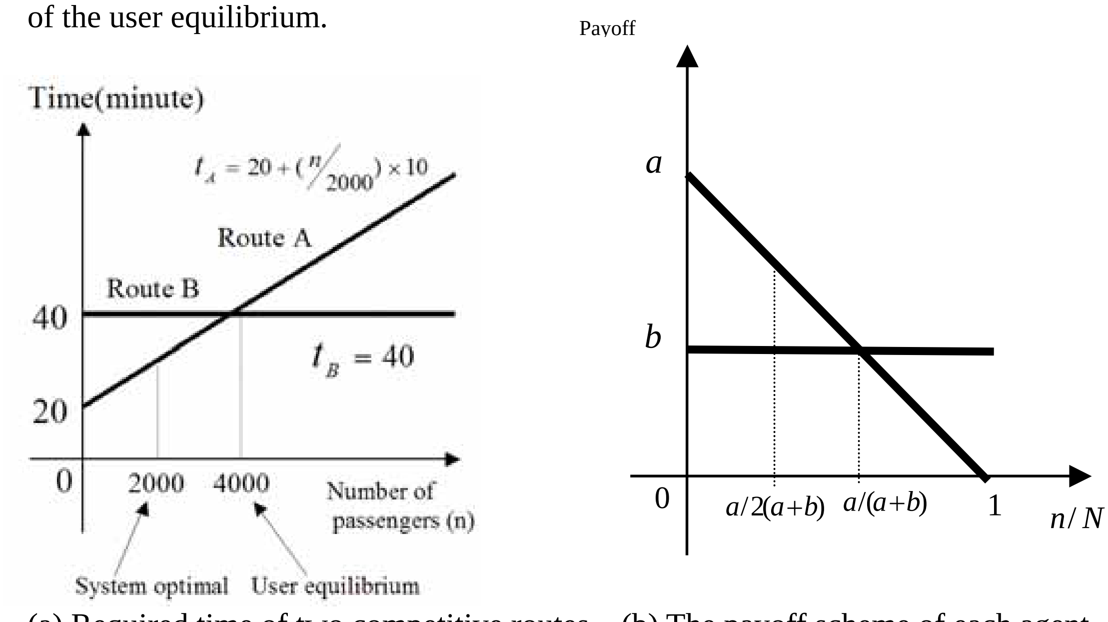
S_1(Route A): Uses a train, S_2(Route B): Uses a private car (2.1)

The required time if an agent chooses the public transportation, a train (Route A) is 40 minutes, which is constant without regarding the number of agents on the Route A. On the other hand, the required time when an agent chooses a car and takes the Route B is the increasing function of the number of agents on the same route as shown in Figure 2(a). User equilibrium is realized at the intersection in Figure 2(a). However, system optimal achieved at n=2,000, which is the half of the user equilibrium.

If the utility of an agent is defined as the benefit minus time (cost), we can reformulate the above network equilibrium model in which each agent receives the utility when she chooses distinct strategy from the majority does. We define the utility function of each agent as follows:

$$U(S_1) = a(1 - n/N), \qquad U(S_2) = b \qquad (2.2)$$

The payoff of each agent under the strategy S_1 is defined as a linearly decreasing function of the ratio *n/N* of agents to choose the same strategy S_1 (Route A) as shown in Figure 2. On the other hand, the payoff when she chooses the opposite strategy S_2 (Route B) is constant without regarding the choices of the others. The user equilibrium is reached at the intersection in Figure 2(b), which is achieved at n/N=a/(a+b), and the system optimal is achieved at n/N=a/2(a+b), which is also the half of the user equilibrium.



(a) Required time of two competitive routes (b) The payoff scheme of each agent

Figure 2 A Route Selection Problem

The market entry game is a stylized representation of a common problem based on the logic of minority: a number of agents have to choose independently whether or not to undertake some activity, such as enter a market, go to a bar, or drive on a road, the utility from which is decreasing in the number of the participants [9][20]. The main purpose of market entry games is to understand if and how competitive markets coordinate decentralized allocation decisions. An iterated market entry games are intended to simulate a situation where a newly emergent market opportunity may be fruitfully exploited by no more than a fixed and commonly known number of firms. The choice of market entry games for studying coordination is quite natural. When there are too many potential entrants wishing to exploit a new market opportunity, a problem arises regarding how entry may be coordinated. Without coordination, too many firms may decide to enter the market and consequently sustain losses. Conversely, fully aware of the consequences of excessive entry, firms may be reluctant to enter and exploit the market in the first place.

Market entry games typically admit a large number of Nash equilibria. Given this multiplicity of equilibrium outcomes, an obvious question is which type of equilibrium are agents likely coordinate upon? They showed that there is no support for convergence to equilibrium play on either the aggregate or individual level or for any trend across rounds of play to maximize total group payoff by lowering the frequency of entry. The coordination failure is attributed to certain features of the payoff function that induce strong competition in the attempt to penetrate the market. Rapoport and his colleagues observed that under the linear payoff condition experimental data were inconsistent with the mixed strategy equilibrium and that inexperienced subjects failed to converge to a pure-strategy equilibrium [19]. They find no support for the symmetric mixed strategy equilibrium solution on the individual level and on the aggregate level. They also report that the subjects recall the last outcome and adjust their behavior accordingly, entering more frequent after a successful entry and staying out more often after a previous successful non-entry. They also suggest that coordination success or failure may depend on certain features of the payoff function and the stability of the associated equilibrium solutions. As we will discuss in Section 7, the market-entry games is also modeled with the payoff scheme of (2.2).

(2) The EL Farol bar problem and minority games

The EL Farol bar problem and its variants minority games have regained interests of many researchers [1][6][17]. Let consider the following specific situation: each agent faces the binary choice of the two strategies;

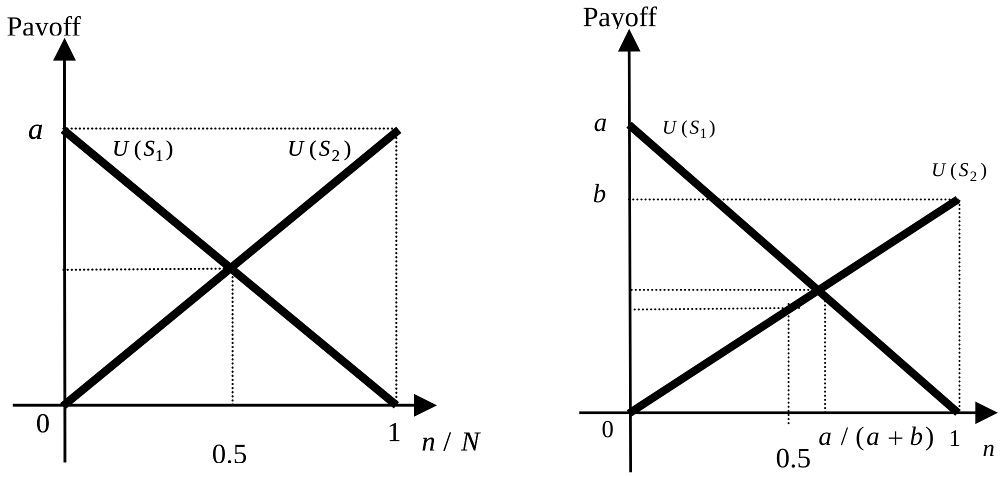
S_1: Goes to the bar, S_2: Stays at home (2.3)

The utility of each agent is determined what the majority does, and each agent gains utility only if she chooses the opposite strategy of the majority does. We define the utility function of each agent as follows:

$$U(S_1) = a(1 - n/N), \qquad U(S_2) = b(n/N) \tag{2.4}$$

The payoff of each agent under the strategy S_1 is given as a linearly decreasing function of the ratio of agents to choose the same the strategy S_1(*n/N*) ass shown in Figure 3. On the other hand, the payoff when she chooses the opposite strategy S_2 is given as a linearly increasing function of the same ratio. Consider the extreme case where only one agent take one action S_1, and all the other agents take the other strategy S_2. The lucky agent gets a reward *a*, nothing for the others.

With the payoff scheme defined in Figure 2(a) with *a*=*b*, an agent receives the same payoff under S_1 and S_2 when she belongs to the minority side. On the other hand, with the payoff scheme in Figure 2(b) with $a \neq b$, each agent is more likely to prefer to choose S_1. Each user non-cooperatively seeks to maximize her utility, and "Nash equilibrium" (the same as user equilibrium of the network equilibrium model) is achieved when each user chooses the route that is the best. Specifically, a user-optimized Nash equilibrium is reached when no agent may improve her utility through unilateral action. Such Nash equilibrium is achieved at the intersection in Figure 4, and it is given as *n/N=a/(a+b).* The collective efficiency (system optimal) can be measured from the average payoff of all agents, and which is achieved at *n/N=0.5.* Therefore under the payoff scheme of Figure 2(a), collectives at user-optimized Nash equilibrium and at the system efficiency become the same. On the other hand, under the payoff scheme in Figure 2(b), user-optimized Nash equilibrium and at the system efficiency are different..



(a) The payoff scheme with $a = b$ (b) The payoff scheme with $a \neq b$

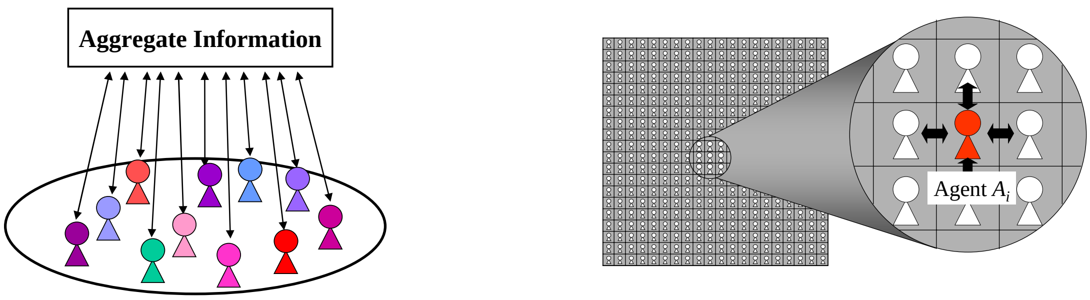
Figure 3: The utility of an agent as the function of the ratio of agents to choose S_1(*n/N)*

4. Reduction of Global Payoff Functions to Local Payoff Functions

With the view of reductionism, every phenomenon we observe can be reduced to a collection of components whose movement is governed by the deterministic laws of nature. In such reductionism, there seems to be no place for novelty or creativity. Twentieth century science has slowly come to the conclusion that such a philosophy will never allow us to explain or model the complex world that surrounds us.

It is also important to consider with whom an agent interacts and how each agent decides his action depending on others' actions. Agents may adapt based on the aggregate information representing the current status of the whole system (global adaptation) as shown in Figure 4(a). In this case, each agent chooses an optimal decision based on aggregate information about how all other agents behaved in the past. An agent calculates her reward and plays her best response strategy. An important assumption of global adaptation is that they receive knowledge of the aggregate.

The dispersion problem with N-persons formulated with the payoff function in (3.1) or (3.2) can be decomposed into the pair-wise interaction problem between individuals with the payoff matrix given in Table 1(a) or (b).



(a) Interaction between agents and collective (b) Local interactions of agents

Figure 4 Reduction of global interaction into local interaction

Table 1 Decomposition of dispersion problems into the payoff matrix

(a)Case1: Desired collective: *n/N*=0.5

Own choice \ The choice of the other	S_1	S_2
S_1	0 0	$1-\theta$ θ
S_2	θ $1-\theta$	0 0

(b) Case2: Desired collective: *n/N*=1-θ

Own choice \ The choice of the other	S_1	S_2
S_1	0 0	$1-\theta$ 1
S_2	1 $1-\theta$	$1-\theta$ $1-\theta$

$(\theta = a/(a+b))$

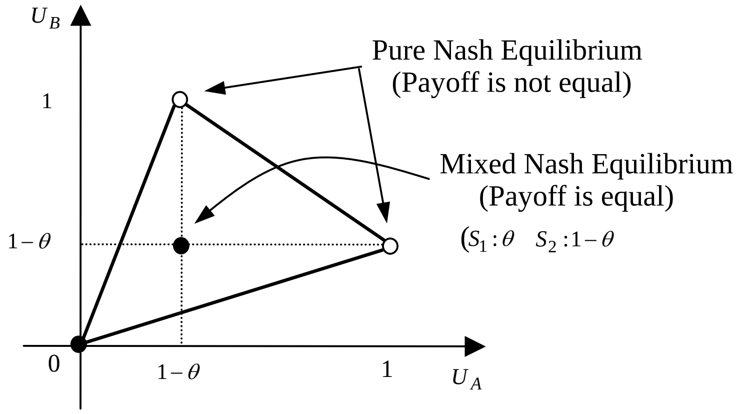


Figure 5 The utilities at Nash equilibria of the payoff matrix Table 1(b)

The 2x2 games with the payoff matrices in Table 1(a) and (b) have two pure Nash equilibria of (S_1, S_2) and (S_1, S_2), and one mixed Nash equilibrium as shown in Figure 5. If both agents choose S_1 with probability θ and S_2 with the probability 1-θ, they realize a mixed equilibrium. At the two pure equilibria (S_1, S_2) and (S_1, S_2) of the payoff matrix Table 1(b), for instance, they receive different payoffs 1 and 1-θ respectively. However, in an equilibrium situation involving mixed strategies both agents receive the same payoff $1-\theta$, and this mixed equilibrium situation seems fair than the first two asymmetric equilibrium situations. However, payoff for each agent is less than in the case of the first two cases. Since the sum of the payoffs at one of the pure equilibria is 2+θ, and that is at the fair mixed equilibrium is 2(1-θ). We thus observe that the criteria for stability (Nash equilibrium) and fairness contradicts with the criteria of stability (Nash equilibrium) and efficiency

5. Coupling Rules of Agents and Co-evolution of Coupling Rules

An important aspect of collectives is the learning strategy adapted by individuals. We make a distinction between evolutionary systems and adaptive systems. The adaptation may be at the individual level through learning, or it may be at the population level through differential survival and reproduction of the more successful individuals. Either way, the consequences of adaptive processes are often very hard to deduce when there are many interacting agents following rules that have nonlinear effects.

Several learning rules have been found to lead an efficient outcome when agents learn from each other. Among the adaptive mechanisms that have been discussed in the learning literature are the following [10].

(1) Reinforcement learning

Agents tend to adopt actions that yielded a higher payoff in the past, and to avoid actions that yielded a low payoff. Payoff describes choice behavior, but it is one's own past payoffs that matter, not the payoffs of the others. The basic premise is that the probability of taking an action in the present increases with the payoff that resulted from taking that action in the past.

(2) Best response learning

Agents adopt actions that optimize their expected payoff given what they expect others to do. In this learning model, agents choose the best replies to the empirical frequencies distribution of the previous actions of the others.

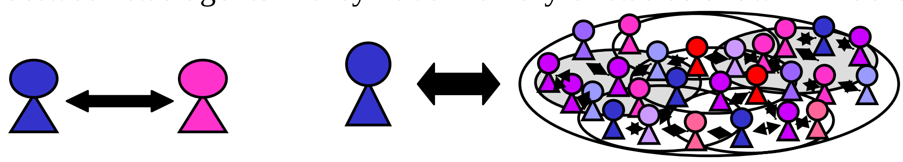
(3) Evolutionary learning

Agents who use high-off payoff strategies are at a productive advantage compared to agents who use low-payoff strategies, hence the latter decrease in frequency in the population over time (natural selection). In the standard model of this situation agents are viewed as being genetically coded with a strategy and selection pressure favors agents that are fitter, i.e., whose strategy yields a higher payoff against the population.

Many adaptive models have been proposed such as best-response dynamics or payoff improving learning, and soon. However, it is notoriously difficult to formulate adaptive dynamics that guarantee convergence to Nash equilibrium. We call a dynamical system uncoupled if the adaptive dynamic for each agent does not depend on the payoff functions of the other agents. Hart showed that there are no uncoupled dynamics that are guaranteed to converge to Nash equilibrium [10]. Therefore coupling among agents, that is, the adjustment of an agent's strategy depend on the payoff functions of the other agents is a basic condition for adaptive dynamics.

We take a different approach by focusing co-evolution. Co-evolutionary dynamics differ, in this sense, from the common use of the genetic algorithm, in which a fixed goal is used in the fitness function and where there is no interaction between individuals. In the genetic algorithm, the focus is on the final result what is the best or a good solution. In models of co-evolutionary systems, one is usually interested in the transient phenomenon of evolution, which in the case of open-ended evolution never reaches an attractor.

Each strategy in the repeated game is represented as a binary string so that the genetic operators can be applied, and we represent each strategy as S_1=0 and S_2=1. We use a memory of an agent, which means that the outcomes of the previous moves are used to make the current choice. There are 16 possible combinations of strategies between two agents if they have memory of two as shown in Table 2.



(a) Coupling between agents (b) Coupling between agent and collective

Figure 6 The ways of coupling

Table 2. A coupling rule between agents
(own: own action, opp: opponent's action, S_1=0, S_2=1, #: 0 or 1)

pattern No.	past action				next action
	t-2		t-1		
	own	opp	own	opp	
#1	0	0	0	1	#
#2	0	0	1	0	#
#3	0	0	1	1	#
#4	0	1	0	0	#
#5	0	1	0	1	#
#6	0	1	1	0	#
#7	0	1	1	1	#
#8	1	0	0	0	#
#9	1	0	0	1	#
#10	1	0	1	0	#
#11	1	0	1	1	#
#12	1	1	0	0	#
#13	1	1	0	1	#
#14	1	1	1	0	#
#15	1	1	1	1	#
#16	0	0	0	0	#

(# represents 0 or 1)

We can fully describe a deterministic strategy by recording what the strategy will do in each of the 16 different situations that can arise in the iterated game.

In order to accomplish a coupling rule in Table 2, we treat each strategy as deterministic bit strings. Since no memory exists at the start, an extra 4 bits are needed to specify a hypothetical history as shown in Figure 7. At each generation, agents repeatedly play the game for T iterations. Agent i, $i \in$ [1…N] uses a binary string i to choose his strategy at iteration t ,$t \in$[1…T]. Each position of a binary string in Figure 7 as follows: The first position, p_1 encodes the action that agent takes at iteration $t = 1$. A position p_j, $j \in$[2,3] encodes the memories that agent i takes at iteration $t - 1$ and his opponent. A position p_j , j$\in$[4…7] , encodes the action that agent i takes at iteration $t > 1$, corresponding to the position p_j, $j \in$[2,3]. An agent i compares the position p_j, $j \in$[2,3], and decision table as shown in Table.4, and then, an agent i decide the next action. Here is an example of binary string given the agent's action taken in the previous iteration.

We use evolutionary learning where agents learn from the most successful neighbor. Each agent adapts the most successful strategy as guides for their own decision (individual learning). Hence their success depends in large part on how well they learn from their neighbors. If the neighbor is doing well, its strategy can be imitated by others. A part of the list is replaced with that of the most successful neighbor.

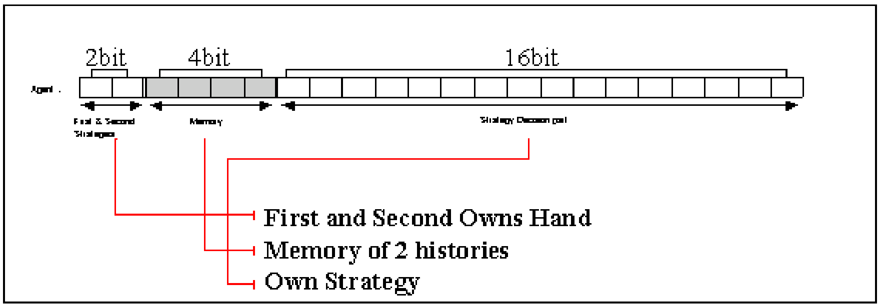
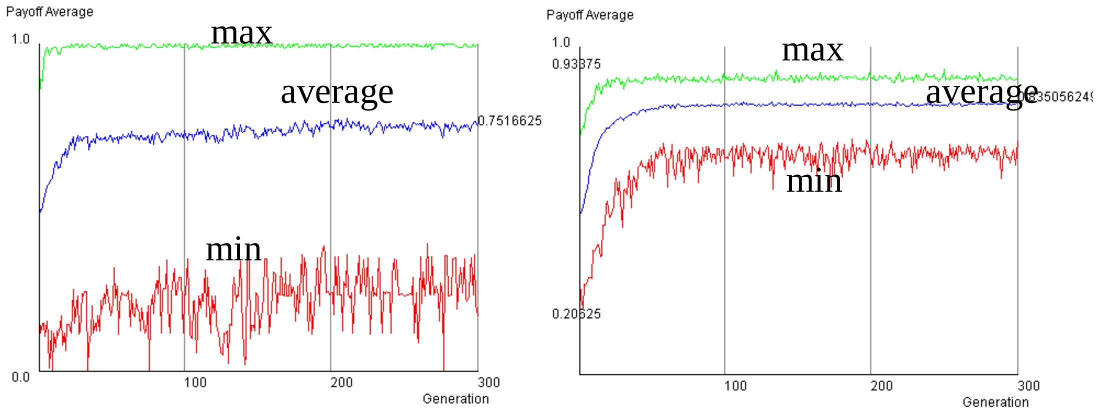


Figure 7　　Representation of the coupling rule in Table2

We showed the simulation result in Figure 8(a). The average payoff per an agent was gradually increased to 0.75. We also show the highest payoff and lowest payoff, and there are lucky agents got the maximum payoff of 1 since they were and the unlucky agents got nothing since they were always belong to the majority side. We also consider the implementation error of the strategy. Agents choose their strategy specified by the coupling rule. We showed the simulation results with the error rate 5% in Figure 8(b). Consequently, we can conclude that co-evolution learning leads to a more efficient situation when agents have small chances of making mistakes. As shown Figure 8(b), the highest payoff and the lowest payoff become to be close by implying that each agent acquires the almost the same payoff in the long-run.



(a)error rate: 0%

(b)error rate : 5%

Figure 8　The average payoff under evolutionary learning

The advantage of agent-based simulation is that we can obtain microscopic rules which induce emergent behavior at the macro level. Specifically we can investigate what agents have learned by evolving their coupling rules. At beginning, each agent has a different coupling rule specified by the 16 bits information. In Figure 9, we showed the acquired coupling rules by 400 agents under which collective behavior was obtained as shown in Figure 8(b). All rules of agents were aggregated into 15 types. The numbers of the blanket represent the number of agents who acquired the same type. Those 15 types of rules have some commonality in bit lists from Table 3, and we can conclude the following result. Although we specify a coupling rules between agents so that they make choices based on the choices made at the previous two periods each agent has learnt to make choice based on the only previous period. And if an agent chooses $S_1(0)$ and her opponent chooses $S_2(1)$ at the previous time, then she chooses $S_2(1)$. If she chooses $S_2(1)$ and her opponent chooses $S_1(0)$ at the previous period, then she chooses $S_1(0)$. These acquired rules are interpreted as follows: if they gain then they change their strategies, which we can call the give-and-take rule.

Type01: 0 1 0 1 1 1 0 0 0 1 0 1 1 1 0 0 (10)
Type02: 0 1 0 1 1 1 0 0 0 1 0 1 0 1 0 0 (31)
Type03: 1 1 0 0 1 1 0 0 0 1 0 1 0 1 0 0 (72)
Type04: 0 1 0 0 1 1 0 0 0 1 0 1 0 1 0 0 (61)
Type05: 0 1 0 0 1 1 0 0 0 1 0 1 1 1 0 0 (10)
Type06: 1 1 0 0 1 1 0 0 0 1 0 1 1 1 0 0 (13)
Type07: 0 1 0 0 1 0 0 0 0 1 0 1 0 1 0 0 (40)
Type08: 1 1 0 1 1 1 0 0 0 1 0 1 0 1 0 0 (37)
Type09: 1 1 0 0 1 0 0 0 0 1 0 1 0 1 0 0 (43)
Type10: 0 1 0 1 1 0 0 0 0 1 0 1 0 1 0 0 (31)
Type11: 1 1 0 1 1 0 0 0 0 1 0 1 0 1 0 0 (35)
Type12: 0 1 0 0 1 0 0 0 0 1 0 1 1 1 0 0 (2)
Type13: 1 1 0 1 1 1 0 0 0 1 0 1 1 1 0 0 (12)
Type14: 0 1 0 1 1 0 0 0 0 1 0 1 1 1 0 0 (1)
Type15: 1 1 0 0 1 1 0 0 0 1 1 1 0 1 0 0 (2)

Figure 9 The meta-rules which are acquired by 400 agents (Then number of the blanket represent the number of agents who acquired the same meta-rules)

Table 3 Learnt coupling rules which are characterized as give-and-take

	past actions				
bit	t-2		t-1		next
position	own	opp	own	opp	action
#2	0	0	0	1	1
#3	0	0	1	0	0
#5	0	1	0	0	1
#6	0	1	0	1	0 or 1
#7	0	1	1	0	0
#8	0	1	1	1	0
#9	1	0	0	0	0
#10	1	0	0	1	1
#11	1	0	1	0	0
#12	1	0	1	1	1
#14	1	1	0	1	1
#15	1	0	1	0	0
#16	1	1	1	1	0

From this simulation result we can conclude that some mistakes in co-evolutionary learning of coupling rules help agents to behave based on the principle of give-and-take in the long run, which lead a collective behavior to be both efficient and equitable. For self-organizing systems, fitter usually means better or with more potential for growth. However, the dynamics implied by a fitness landscape does not in general lead to the overall fittest state, and the system has no choice but to follow the path of steepest descent. This path will in general end in a *local* minimum of the potential, not in the *global* minimum. Apart from changing the fitness function, the only way to get the system out of a local minimum is to add a degree of indeterminism by adding some noise to the co-evolutionary dynamics, that is, to give the system the possibility to make transitions to states other than the locally most fittest one. This can be seen as the injection of noise or random perturbations into the system, which makes it deviate from its preferred trajectory. Physically, this is usually the effect of outside perturbations. Such perturbations can push the system upwards, towards a higher potential. This may be sufficient to let the system escape from a local minimum.

6. Synthesis of Agents Behaving with Coupling Rules

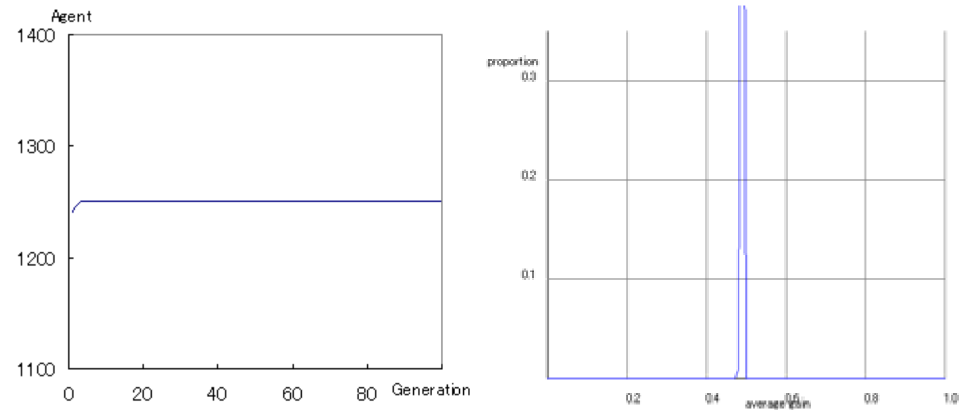
The give-and-take rule departs from the conventional assumption such that agents update their behaviors in order to improve their payoffs. It is commonly assumed that agents tend to adopt actions that yield a higher payoff in the past, and to avoid actions that yield a low payoff. With the give-and-take, on the contrary, agents are assumed that they yield to others if they receive the payoff by taking the opposite strategy at the next time period, and they choose randomly if they do not gain the payoff.

We formalize the payoff scheme with give-and-take strategy. We denote the status of the collective choice by the state variable $\omega(t)$ as follows:

(1) $\omega(t) = 0$ if $A(t) < N\theta$ (The bar is less crowded) (6.1)

(2) $\omega(t) = 1$ if $A(t) \geq N\theta$ (The bar is over crowded) (6.2)

In the case when the attendance is below than the capacity, the number of agents to stay at home is greater than the capacity. We also consider a population of agents with N = 2,500 with the capacity rate θ = 0.5. Figure 10 showed the simulation result when all agents adapt the give-and take learning rules in (6.2). Figure 10(a) shows the number of agents having chosen S_1 and S_2 over time, and it is shown that the average number of agents who choose S_1 (Go to the bar) converges to the capacity, indicating that collective behavior satisfies the constraint. Figure 10(b) shows the proportion of agents with the same average payoff. The majority of agents receive the average payoff 0.5. This result indicates emerged collective behavior is not only efficient, but to be fair under give-and-take strategy.

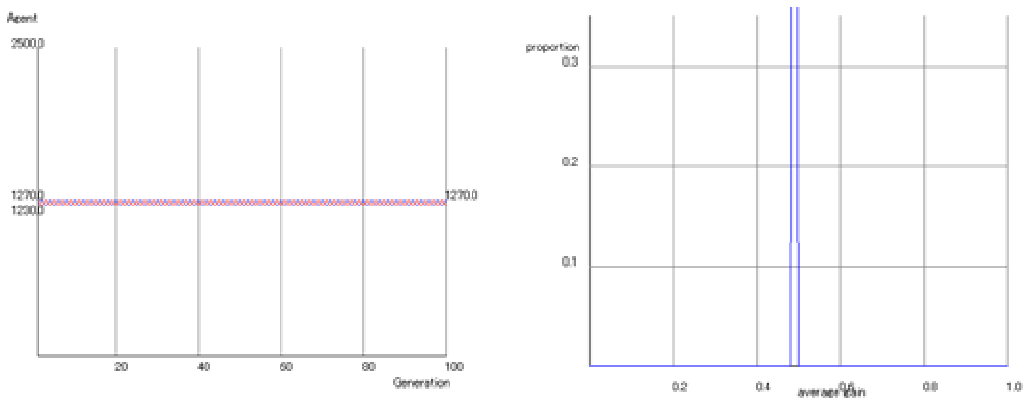


(a) (b)

(a)The dynamic changing of numbers of agents of having chosen S_1 and S_2,

(b)The proportion of agents with the same payoff

Figure 10 The simulation result with the rule of give-and-take (θ = 0.5)



(a) The dynamic changing of numbers of agents of having chosen S_1 and S_2,

(b) The proportion of agents with the same payoff

Figure 11 The simulation result with the rule of give-and-take (θ = 0.6)

As shown in Section 3, the average payoff per agent is given by $2\theta(1-\theta)$, which takes the minimum value at the capacity rate θ = 0.5. We also evaluate the performance of give-and take learning with the capacity rate θ = 0.6, the capacity rate of the El Farol problem. In Figure 11, we showed the simulation result. Figure 11(a) shows the number of agents to go having chosen S_1 and S_2 over time. It is shown that the proportion of agents by choosing S_1 converges to 0.5 which is less than the capacity $\theta = 0.6$. In Figure 11(b), we showed the proportion of agents with the same average payoff, and the majority of agents received the payoff of 0.5.This result indicates that even if the capacity rate θ is increased from 0.5, a collection of agents split into two groups, a group of going to the bar and a group of staying at home in asymmetric situations.

7. Exogenously Design of Individual Payoff Functions

Many researchers have pointed out that equilibrium theory does not resolve the question of how a collection of agents behave in a particular interdependent decision situation. They argued that “It is hard to see what can advance the discussion short of assembling a collection of people, putting them in the situation of interest, and observing what they do” [19]. Faced with this challenge, experimenters have devised a large variety of games with multiple pure-strategy equilibria coordination games investigated to study coordination with no pre-play communication in the controlled environment of the laboratory.

Included in these games are market entry games whose purpose is to understand if and how competitive markets coordinate decentralized allocation decisions. An iterated market entry games are intended to simulate a situation where a newly emergent market opportunity may be fruitfully exploited by no more than a fixed and commonly known number of firms. The choice of market entry games for studying coordination problems is quite natural. When there are too many potential entrants wishing to exploit a new market opportunity, a problem arises regarding how entry may be coordinated. Without coordination, too many firms may decide to enter the market and consequently sustain losses. Conversely, fully aware of the consequences of excessive entry, firms may be reluctant to enter and exploit the market in the first place.

Rapoport investigated coordination behavior in yet another class of market entry games which include only a single market with a commonly known capacity [21]. This game is played by collection N of n agents. On each period, an integer c ($1 < c < n$), interpreted as the “capacity of the market”, is publicly announced. Then each agent i ($i \in N$) must decide privately whether to enter the market ($x = 1$) or stay out of it ($x = 0$). The payoff for each stage game is determined from

$$U_i(\mathbf{x}) = \begin{cases} v & \text{if} \quad x = 0 \\ v + r(c - m) & \text{if} \quad x = 1 \end{cases} \tag{7.1}$$

where $U_i(\mathbf{x})$ is agent i’s payoff , and $\mathbf{x} = (x_1, x_2, \dots, x_n)$ is the vector of individual decisions, m is the number of entrants ($0 < m < n$), and v, r, and c are positive constants. The constant c represents the capacity of the market (or road or bar). This market entry game has complete information, binary actions, and an incentive to enter the market which decreases linearly in the number of entrants. It allows each agent the option of staying out of the market and receiving a positive or negative payoff v, which does not depend on the decisions of the of the other n - 1 agents.

Market entry games typically admit a large number of Nash equilibria. Pure equilibria involve coordination on asymmetric outcomes where some agents enter and some stay out. The only symmetric outcome is mixed, requiring randomization over the entry decision. There also exist asymmetric mixed equilibria, where some play pure while others randomize. Given this multiplicity of equilibrium outcomes, an obvious question is which type of equilibrium are agents likely coordinate upon? It is known that there is no support for convergence to equilibrium play on either the aggregate or individual level or for any trend across rounds of play to maximize total group payoff by lowering the frequency of entry.

The coordination failure is attributed to certain features of the payoff function that induce strong competition in the attempt to penetrate the market.

Another observation conclusion concerns a shift in emphasis in the research on coordination in iterated market entry games. Clearly, there is no compelling reason to believe, *a priori*, that agents sharing no common history on which to condition their choices will achieve perfect coordination in these games. Nor is there any particular reason to believe that agents can randomize their choices across iterations of the same stage game, producing sequences of responses that pass statistical tests for independent and identically distributed events. They also report that the subjects recall the last outcome and adjust their behavior accordingly, entering more frequent after a successful entry and staying out more often after a previous successful non-entry. They also suggest that coordination success or failure may depend on certain features of the payoff function and the stability of the associated equilibrium solutions.

Previous experimental studied have yielded that Nash equilibrim fails as a predictor of human behavior. Duffy and Hopkins studied the long-run predictions of learning models in the context of the market entry games [8]. They investigated the hypothesis that individual should learn equilibrium behavior. Agents are modeled to learn with reinforcement learning, and the probability of entering or staying out is reinforced with performance of the previous play. They showed a collection of agents converge to asymmetric pure equilibrium that involves what they call “sorting” where some agents always enter and the remaining agents always stay out. Similar observation has been also reported by Bell[6]. She studied the long-run predictions of learning models in the context of the minority games. Agents are modeled to learn with reinforcement learning.

She showed a collection of agents converge to asymmetric pure equilibrium that involves what they call “sorting” where some agents always enter and the remaining agents always stay out.

The utility of each agent is determined what the majority does, and each agent gains utility only if she chooses the opposite strategy of the majority does. We define the utility function of each agent as follows:

$$U(S_1) = 1 - n/N, \qquad U(S_2) = 1 - \theta \tag{7.2}$$

The payoffs of an each agent under both strategies are shown in Figure 12(a), which has the similar structure with that of Figure 2(a). With this payoff scheme, collectives with Nash equilibrium and Pareto-optimal are different, and it becomes difficult to derives desired collective.

Let suppose each agent can receives an extra payoff from the outside, and then the utility function of each agent is given as follows:

$$U(S_1) = 1 - n/N + \theta(n/N), \qquad U(S_1) = 1 - \theta + \theta(n/N) \tag{7.3}$$

The payoffs of an each agent under both strategies are shown in Figure 12(b), which has the similar structure with that of Figure 2(b). With this payoff scheme, collectives with Nash equilibrium and Pareto-optimal are the same, and interacting agents evolve their behavior to reach to a desired collective.

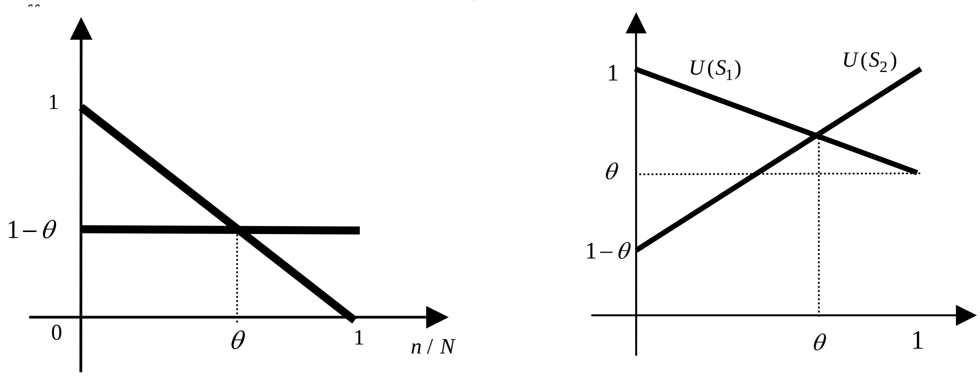


Figure 12　Modification of the payoff functions through subsidy

8. Summary: Evolutionary Design of Complex Systems with Agent-based Simulation

Many fundamental changes to a society result from collective behaviors of groups of interacting agents. How do heterogeneous micro-world of individuals generate the global macroscopic orders and regularities of society? Much of hidden knowledge is underlying accumulated interactions. There is no presumption that a collection of interacting agents leads to collectively satisfactory results without any central authority. The system performance of interacting agents crucially depends on the type of interactions among agents as well as how they adapt to others. There are two closely related issues concerning collective, (1) the forward problem of how the fine-grained structure of the system underlying a collective determines its complex emergent behavior and therefore its performance, and (2) the inverse problem of how to design the structure of the system underlying a collective to induce optimal performance. We discuss how agent-based simulation contributes to solve these two issues.

In this paper we addressed the issue of collective decisions by agents in which they have to realize both efficient and equitable utilization of limited resources. Agents normally react to aggregate of others' decisions, and the resulting volatile collective decision is often far from being efficient. By means of experiments, we showed that the overall performance depends crucially on the types of interacting decisions as well as the heterogeneity of agents in term of their preferences. We considered two different types of interaction formulated as the coordination problem and the dispersion problem. We showed that the most crucial factor that considerably improves the performance of the system of interacting agents is the endogenous selection of the partners and reinforcement of preferences at individual levels.

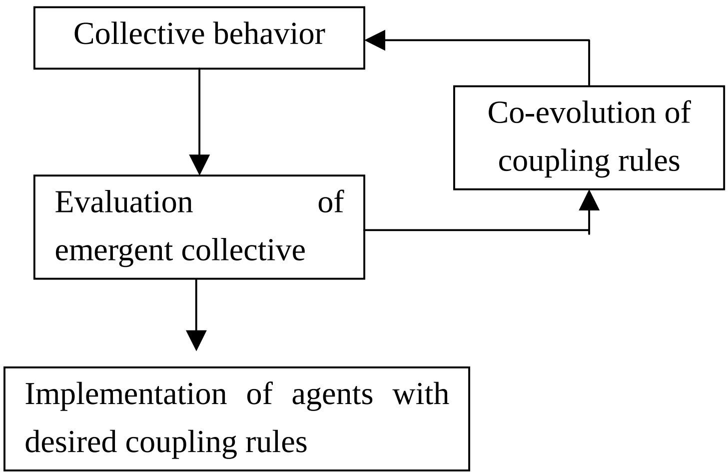


Figure 13 An evolutionary design with agent-based simulation

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