Localized Minority Games and Emergence of Efficient Dynamic Order

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Abstract

We consider asymmetric coordination problems in which agents receive the gain if they choose the strategy as the majority does not. Such asymmetric coordination problems are well modeled as Minority Games. A basic assumptions of minority games is that agents play the game with all other agents by adapting to the global information, and we call them as Global Minority Games (GMG). In the simplest form of GMG agents are also modeled with their own rules, and they are updated and selected by natural selection. The idea of Local Minority Games (LMG) is proposed in which each agent play MG with their neighbor by reflecting limited ability (on the agent's part) to receive, decide, and act upon information they get in the course of interaction. We propose the rule of give-and-take that is significantly departed from the conventional assumption that agents update their behaviors in order to improve their payoff. With the rule of give-and-take, on the contrary, agents yield to others if they receive the payoff by changing the choice at the next time period. We show that GMG played by many agents with a selfish interest and LMG played with the rule of give-and-take are significantly different. With GMG, the main concern is whether interacting agents self-organize themselves into the efficient collective decision in time. However, with LMG, they have to self-organize an efficient behavior in both time and space.

Keyword: minority games, local interaction, give-and-take, self-organization, dynamic order

1 Introduction

Large-scale effects of locally interacting agents are called emergent properties of the system[Holland 95]. Emergent properties are often surprising because it can be hard to anticipate the full consequences of even simple forms of interaction. In this area of research, the most two frequently asked questions are how heterogeneous agents generate emergent coordination and how they manage and self-organize macroscopic orders from bottom up without any central authority. The answer will heavily depend on how they interact and adapt their behavior.

According to the bounded rationality [Simon 82], agents myopically evolve their behavior based on the rule, which is given as the function of the collective behavior and their idiosyncratic utilities. In the emergent system, we often find the micro-macro dynamics that relate the aggregate behavior with the underlying individual behaviors, and we also find that agents' rational behavior reflecting their micro-motives combined with the behavior of others produce unanticipated cyclic behavior.

The El Farol bar problem and its variants may provide a clean and simple example of asymmetric coordination problems [Arthur 94] [Fogel 99]. El Farol is the name of the bar, which is in downtown Santa Fe. Each of the staff in Santa Fe laboratory (after this, we call them "agents") will enjoy the night at El Farol very much if the bar is not crowded, however, each of them will suffer miserably if the bar is crowded. The only information available to agents is the number of visitors to the bar in previous nights.

Arthur used this very simple yet interesting problem to illustrate the effective uses of inductive reasoning of heterogeneous agents. Arthur examined the dynamic driving force behind this equilibrium. Agents make their choices by predicting ahead of time whether the attendance on the current night will exceed the capability and then take the appropriate course of action.

The above evolutionary process of collective action is guided by the self-interest seeking of heterogeneous agents. The mechanism has a strong similarity to the nature of a self-organizing and growing process. The growth starts from the set of the unstructured decision with heterogeneous motivations or interests. However, they are let to self-organize by establishing some stable collective decision as a whole.

The resulting dynamics can be quite complex. The stability of dynamic is determined by the distribution and combination of conformists and non-conformists. Conformists do what the majority does and non-conformist do the opposite. Conformists have the feature to accelerate their collective action to converge. On the other hand, non-conformist has the feature to oscillate it. By joining conformists and non-conformists, the property of lock-in of conformists and the property of oscillation of non-conformists are merged and they can produce a stable macro behavior.

Social interactions pose many coordination problems to individuals such as sharing and distributing limited resources in an efficient way. Many researches have investigated how agents' rational decisions combine with the decisions of the others to produce the macro behavior and some unanticipated results can be emerged. There is no presumption, however, collective decision of interacting rational agents leads to collectively satisfactory results. How well agents do for it in adapting to their social environment is not the same thing as how satisfactory a social environment they collectively create.

We consider asymmetric coordination problems in which agents receive the gain if they choose the strategy as the majority does not. Such asymmetric coordination problems are well modeled as Minority Games. Many researchers have investigated how do the heterogeneous and micro-world of individual behaviors generate and self-organize themselves to the global macroscopic orders of the whole. A basic assumptions of minority games is that agents play the game with all other agents by adapting to the global information, and we call them as Global Minority Games (GMG). In the simplest form of GMG agents are also modeled with their own rules, and they are updated and selected by natural selection. The idea of Local Minority Games (LMG) is also proposed in which each agent play MG with their neighbor by reflecting limited ability (on the agent's part) to receive, decide, and act upon information they get in the course of interaction [Moelbert 01].

We need to attempt to probe deeper understanding this issue by specifying how they adapt their decision. We aim at discovering fundamental local or micro mechanisms that are sufficient to generate the macroscopic structures and collective behaviors of interest by emphasizing learning strategies adapted by individuals. We propose the rule of give-and-take which is significantly departed from the conventional assumption that agents update their behaviors in order to improve their payoff. The collective decision is guided by the self-interest seeking by each agent, and it is commonly assumed that agents tend to choose the same action that yielded a higher payoff in the past, and to avoid actions with low payoff. With the rule of give-and-take, on the contrary, agents yield to others if they receive the payoff by changing the choice at the next time period. However, they choose randomly if they do not gain the payoff.

We show that GMG played by many agents with a selfish interest and LMG played with the rule of give-and-take are significantly different. With GMG, the main concern is whether interacting agents self-organize themselves into the efficient collective decision in time. However, with LMG, they have to self-organize an efficient behavior in both time and space. We investigate the self-organization process of locally interacting collective decision with the principle of give-and-take and show that efficient dynamic orders are emerged from the bottom up. They are let to self-organize into the whole collective decision by establishing efficient dynamic orders both in time and space depending on how many agents they locally interact.

2 Formalization of the Problem

2.1 El Farol Problem and Minority Game

The Arthur's El Farol model has been extended in the form as Minority Games, which show for the first time how equilibrium can be reached using inductive learning [Challet 97] [Challet 98].Minority Game is played by a collection of rational agents $G = \{A_i : 1 \leq i \leq N\}$. On each period of the stage of the game, each agent must choose privately and independently between two strategies $S = \{S_1, S_2\}$. The action of agent A_i at the time t is represented by $a_i(t) = 1$ if it chooses S_1 and $a_i(t) = -1$ if it chooses S_2. Given the actions of all agents, the payoff of agent A_i is given by:

$$\begin{array}{ll} \text{(i)} & u_i(t) = 1 \;\text{ if }\; a_i(t) = 1 \;\text{ and }\; p(t) \leq \theta \\ \text{(ii)} & u_i(t) = 1 \;\text{ if }\; a_i(t) = -1 \;\text{ and }\; p(t) > \theta \end{array} \tag{1}$$

where $p(t) = (\sum_i a_i(t) + N)/2N$ is a proportion of attendance and θ is the capacity rate. In El Farol problem, θ is 0.6 and in Minority Game, θ is 0.5. Each agent first receives aggregate information $p(t)$ of all agents' actions, and then decides whether to choose S_1 or S_2. Each agent is rewarded with a unitary payoff whenever the side it chose happens to be chosen by the minority of the agents, while agents on the majority side receive nothing.

Since $A(t) = \sum_i a_i(t)$ represents the difference between the number of agents who choose S_1 and the number of agents who choose S_2 at the time t, the payoff scheme in Eq.(1) can be summarized as:

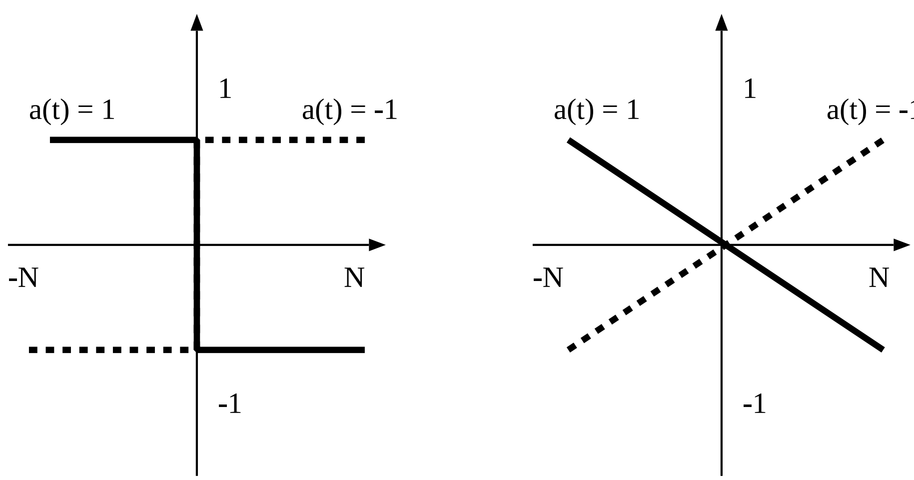
$$u_i(t) = -a_i(t)\text{sgn }(A(t)). \tag{2}$$

The payoff function in Eq.(2) becomes to be the step function as shown in Figure 1 (a).

We have another payoff scheme specified as follows:

$$u_i(t) = -a_i(t)A(t)/N. \tag{3}$$

The payoff function in Eq.(3), which is shown in Figure 1 (b) is linear with respect to the proportion of the attendance $p(t)$. Each agent also gets aggregate information $p(t)$ which represents all agents' actions, and then it decides whether to chooses S_1 or S_2. Each agent is rewarded with a payoff which is linearly decreasing function of the proportion of attendance $p(t)$.



(a) Payoff Scheme 1 (b) Payoff Scheme 2

Figure 1: The payoff schemes: (a) unitary award (Eq.(1)), (b) proportional award (Eq.(3))

Minority Game is characterized with many solutions. It is easy to see that this game has ${}_NC_{(N-1)/2}$ asymmetric Nash equilibrium in pure strategies in the case where exactly $(N-1)/2$ agents choose either one of the two sides. The game also presents a unique symmetric mixed strategy Nash equilibrium in which each agent selects the two sides with an equal probability. We analyze the structure of Minority Game to see what to expect. The social efficiency can be measured from the average payoff of one agent over a long-time period. Consider the extreme case where only one agent takes one side, and all the others take the other side at each time period. The lucky agent gets a reward, nothing for the others, and the average payoff per agent is $1/N$. Equally extreme situation is that when $(N-1)/2$ agents on one side, $(N+1)/2$ agents on the other side where the average payoff is about 0.5. From the society point of view, the latter situation is preferable.

Several methods have been suggested to lead an efficient outcome when agents learn from each other [Challet 97] [Weibull 96]. All agents have access to public information of $p(\tau)$, $\tau \leq t$. The past history available at the time t is represented by $\mu(t)$. Agents may behave differently because of their personal beliefs on the outcome of the next time period $p(t+1)$, which only depends on what agents do at the net time period $t+1$, and the past history $\mu(t)$ has no direct impact on it.

2.2 Minority Games as Asymmetric Games

The matching methodology also play an important role in the outcome of the game. Agents interact with all other agents, which is known as the uniform matching or random matching. Agents are not assumed to be knowledgeable enough to correctly anticipate all other agents' choices, however they can only access on the information about the aggregate behavior of the society with the random matching.

Agents are rewarded a unitary payoff whenever the side chosen happens to be chosen by the minority of the population. El Farol problem and Minority Games have a common feature that an agent's utility depend on the number of total participants. We now show the Minority Game an be represented as 2x2 games in which an agent play with the aggregate of society of the population N with payoff matrix in Table 1.

Table 1: The payoff matrix of Minority Game

	Opponent's behavior	
Own behavior	S_1 (Go)	S_2 (Stay)
S_1 (Go)	-1 -1	1 1
S_2 (Stay)	1 1	-1 -1

Let us suppose each agent plays with all other agents individually with the payoff matrix in Table 1. The payoffs of agent A_i from the play with all other agents with S_1 and S_2 are given:

$$\begin{aligned} \overline{U}_i(S_1) &= -n(t) + (N - n(t) - 1) = -A(t) - 1 \\ \overline{U}_i(S_2) &= n(t) - (N - n(t) - 1) = A(t) + 1, \end{aligned} \tag{4}$$

where $n(t)$ represents the number of agents who choose S_1. Dividing the above payoffs by N, we obtain the average payoff of each interaction with one agent as:

$$\begin{aligned} U_i(S_1) &= \overline{U}_i(S_1)/N \sim -A(t), \\ U_i(S_2) &= \overline{U}_i(S_2)/N \sim A(t). \end{aligned} \tag{5}$$

El Farol problem and Minority Game can be treated with following generic formulating: Let's suppose each agent play the two person game using the payoff matrix in Table

2 with aggregate of the society. The payoff when agent A_i chooses S_1 and the aggregate chooses S_2 is given by θ $(0 < \theta < 1)$, and the payoff when agent A_i chooses S_2 and the aggregate chooses S_1 is given by $1 - \theta$. El Farol problem can be modeled with $\theta = 0.6$, which is the capacity ratio of the bar, and the Minority Game is formulated with $\theta = 0.5$.

Table 2: The payoff matrix of generalized Minority Game

	Opponent's behavior	
Own behavior	S_1 (Go)	S_2 (Stay)
S_1 (Go)	0 0	2 (1 - θ) 2 θ
S_2 (Stay)	2 θ 2 (1 - θ)	0 0

Social efficiency of Minority Game also depends on the payoff scheme. We take the following two schemes:

Payoff Scheme 1 Let us suppose there exists some central authority, and it leads that a little bit larger number of agents than $N\theta$ choose S_1 if $\theta \geq 0.5$, and a little bit fewer agent than $N\theta$ choose S_1 if $\theta < 0.5$. In this case, the average payoff per agent is obtained as $\mathrm{Max}(\theta, 1-\theta)$. Similarly, if the central authority leads that a little bit fewer number if agents than $N\theta$ choose S_1 if $\theta \geq 0.5$, and a little bit larger than $N\theta$ choose S_1 if $\theta < 0.5$. In this case, the average payoff per agent is obtained as $\mathrm{Min}(\theta, 1-\theta)$. Then we have the following average payoffs as the best case and the worst case:

$$\begin{array}{ll} \mathrm{Max}_{0 \leq \theta \leq 1}(\theta, 1-\theta) & \text{[best case]} \\ \mathrm{Min}_{0 \leq \theta \leq 1}(\theta, 1-\theta) & \text{[worst case]} \end{array} \tag{6}$$

Therefore the average payoffs of the best case and the worst case become the same at $\theta = 0.5$.

Payoff Scheme 2 We now consider the payoff scheme 2 in Eq.3. The expected payoff of an agent who chooses S_1 is given $1 - \theta$, and that of an agent who chooses S_2 is θ, where θ denote the proportion of agents who chooses S_1. The average payoff of an agent therefore is given by $2\theta(1-\theta)$, which takes the maximum value 0.5 at $\theta = 0.5$.

3 Strategies for Adaptation

The decision theory is concerned with understanding and importing decision making of individuals. Individual decision-making may be defied, as intentional and reflective choice in response to perceive needs. This paper is about collective decision in which each autonomous agent adapts his rational decision to others' decisions. In examining collective decisions, we shall draw heavily on the individual decisions. Indeed, an organization or society does not make decisions, individual do. It might be argued that understanding how individual make decisions is sufficient to understand and improve collective decision. In this paper, we take a different view. Although individual decision is nested within important to understand, it is not sufficient to describe how a collection of agents arrives at specific decisions. How do the heterogeneous and micro-world of individual behaviors generate and self-organize themselves to the global macroscopic orders of the whole? We aim at discovering fundamental local adaptive rules that are sufficient to generate efficient macroscopic orders of interest. This type of self-organization is often referred as collective orders emerge from the bottom up.

In the simplest form our model agents are formulated with their own internal models and their rules of interaction, termed as local rules. These local rules can be specified with the assumption how they adapt their decisions to others.

3.1 The Best-Response Strategy

Game theory is typically based upon the assumption of a rational choice. Agents adopt actions that optimize their expected payoff given what they expect others to do. In this learning model, agents choose the best replies to the empirical frequencies distribution of the previous actions of the others.

3.2 The Mixed Nash Strategy

The minority game with the payoff matrix of Table 2 has three Nash equilibria, the pairs of the asymmetric pure strategies $(S_1,\ S_2)$, $(S_2,\ S_1)$, and one equilibrium of the mixed strategy. The symmetric Nash equilibrium is achieved with the mixed strategy is defined as $x = (\theta, 1-\theta)$ where θ is the probability to choose S_1.

3.3 The Give-and-Take Strategy

Several learning rules have been found to lead an efficient outcome when agents learn from each other. In this section, we propose the give-and-take strategy that departs from the conventional assumption such that agents update their behaviors in order to improve their measure functions such as payoffs. It is commonly assumed that agents tend to adopt actions that yield a higher payoff in the past, and to avoid actions that yield a low payoff. With the give-and-take strategy, on the contrary, agents are assumed that they yield to others if they receive the payoff by taking the opposite strategy at the next time period, and they choose randomly if they do not gain the payoff. We formalize the payoff scheme with give-and-take strategy as follows:

We denote the status of the collective choice by all agents with the following state variable $\omega(t)$ as follows:

$$\omega(t) = 0 \ \text{ if } \ A(t) < N\theta, \ \ \omega(t) = 1 \ \text{ if } \ A(t) \geq N\theta \tag{7}$$

Each agent receives common information on $\omega(t)$ which aggregate all agents' actions of the last time period, and then decides whether to choose S_1 or S_2 at the time period

$t+1$ by considering whether he is rewarded at the previous time t: The action $a_i(t+1)$ of agent A_i at the next time period $t+1$ is determined by the following rules:

$$\begin{array}{lll} \text{(i)} & (\omega(t)=0) \wedge (a_i(t)=1) & \Rightarrow a_i(t+1)=0 \\ \text{(ii)} & (\omega(t)=1) \wedge (a_i(t)=0) & \Rightarrow a_i(t+1)=1 \\ \text{(iii)} & (\omega(t)=1) \wedge (a_i(t)=1) & \Rightarrow a_i(t+1)=RND(x) \\ \text{(iv)} & (\omega(t)=0) \wedge (a_i(t)=0) & \Rightarrow a_i(t+1)=RND(y) \end{array} \tag{8}$$

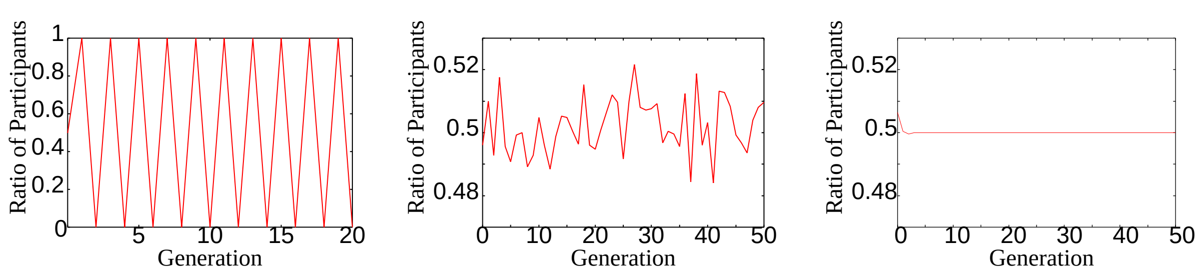
where $RND(x)$ represents the mixed strategy $x=(x,1-x)$ of choosing S_1 with the probability x and S_2 with $1-x$, and we set $x=0.5$.

4 Simulation Results

In this section, we demonstrate the efficacy of give-and-take strategy in Minority Game by comparison with two traditional strategies:

4.1 Global Minority Games

As the methodology of interaction, we consider the global interaction and the local interaction models. With global interaction, each agent interacts with all other agents. This model is also equivalent to the model in which each agent adapts to the aggregate information of the whole behavior. Each agent decides his optimal strategy based on the aggregated information of the behavior of all other agents. Each agent calculates his expected utility and plays his best response that maximizes his expected utility. An important assumption of global interaction is that they can receive knowledge of the current situation. With this assumption, agents gradually learn the strategy distribution of a population.

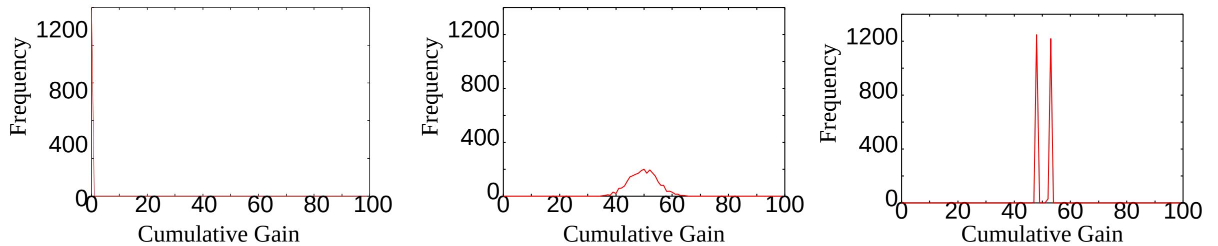


(a) The Best-Response Strategy

(b) The Mixed Nash Strategy

(c) The Give-and-Take Strategy

Figure 2: The ratio of participants: (a) best-response strategy, (b) mixed strategy, (c) give-and-take strategy



(a) The Best-Response Strategy (b) The Mixed Nash Strategy (c) The Give-and-Take Strategy

Figure 3: The distribution of agents' cumulative gain: (a) best-response strategy, (b) mixed strategy, (c) give-and-take strategy

4.2 Local Minority Games

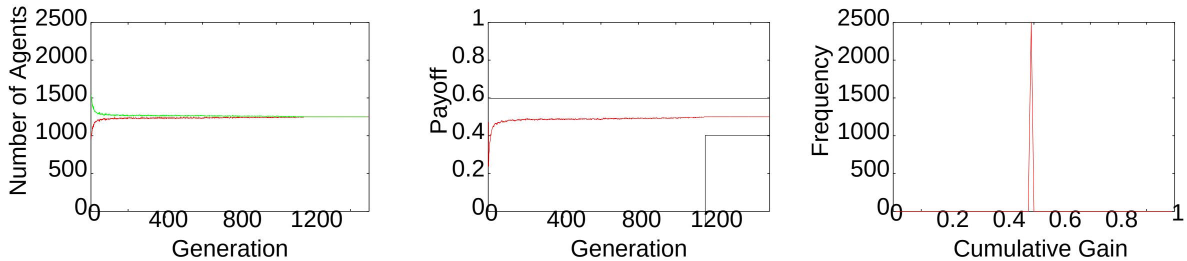
In many situations, agents are not assumed to be knowledgeable as to correctly guess or anticipate other agents' actions, or they are less sophisticated and that they do not know how to calculate best replies. The hypothesis of local interaction reflects limited ability of agents' parts to receive, decide, and act based upon information they receive in the course of interaction. We consider the lattice structure and each agent interacts with his neighbors.

5 Emergence of Efficient Dynamic Orders

In this section, we addressed the following question: how do locally interacting agents generate emergent properties of efficiency? These questions will depend crucially on how they interact and adapt their behavior.

We consider two models of localized minority games in which each agent play minority games with its 4 neighbors and 8 neighbors with the payoff matrix in Table 2. We arrange a population of agents in the area of 50×50 (2,500 agents in total) with no gap, and four corners and an edge of an area connect it with an opposite side. They generate meshed two patterns as shown in Figure 5 (left) when they interact with 4 neighbors. On the other hand, they generate stride patterns as shown in Figure 5 (right) when they interact with 8 neighbors.

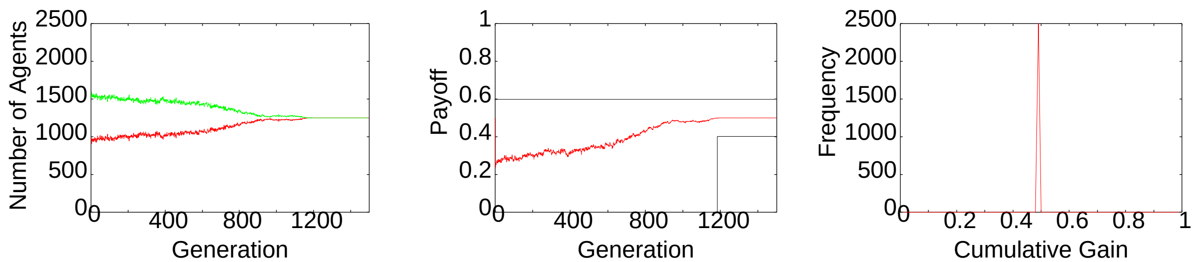
Emergent properties are often surprising because it can be hard to anticipate the full consequences of even simple forms of interaction. Agents myopically evolve their behaviors based on their give-and-take rules, which is given as the function of their idiosyncratic utilities and the action of their neighbors. We investigate how agents' behaviors reflecting their micro-motives combined with the behavior of others produce efficient dynamic orders of interests. We use the term emergent to denote stable macroscopic patterns arising from the idiosyncratic rules of agents. Form these simulation results, we can induce that knowing preferences, motives, or beliefs of agents can only provide a necessary but not a sufficient condition for the explanation of outcomes of the collective action. The interaction of many agents produces some kind of coherent, systematic behavior. The surprise consists precisely in the emergence of macrostructure from the bottom up, which



(a) The number of agents to choose S1 and S2 at each generation

(b) The maximum, average and minimum gain at each generation

(c) The gain distribution of the population



(a) The number of agents to choose S1 and S2 at each generation

(b) The maximum, average and minimum gain at each generation

(c) The gain distribution of the population

Figure 4: The performance of local minority games with give-and-take (Upper: 4 neighbors, Lower: 8 neighbors)

is from simple rules that outwardly appear quite remote from the collective phenomena they generate.

Natural evolution has ceased a multitude of systems in which the actions of simple and interacting components give rise to coordinated global information processing. Insect colonies, cellular assemblies, the retina, and the immune system have all been cited as examples of systems in which emergent computation occurs. This term refers to the appearance of global information-processing capabilities that are not explicitly represented in the system's elementary components or in their interconnection.

The local interacting agents with give-and-take guide the global adaptive process to realize the efficient collective action. The mechanism has a strong similarity to the nature of an emergence and growing process. The growth starts from the set of the unstructured decision. However, they are let to emergence by establish both efficient and equitable collective decision as a whole.

6 Conclusion

The interaction of heterogeneous agents produces some kind of coherent, systematic behavior. We investigated the macroscopic patterns arising from strategic interactions of heterogeneous agents who behave based on local rules. The hypotheses we employed in this paper reflect limited ability of agent to receive, decide, and act upon information they

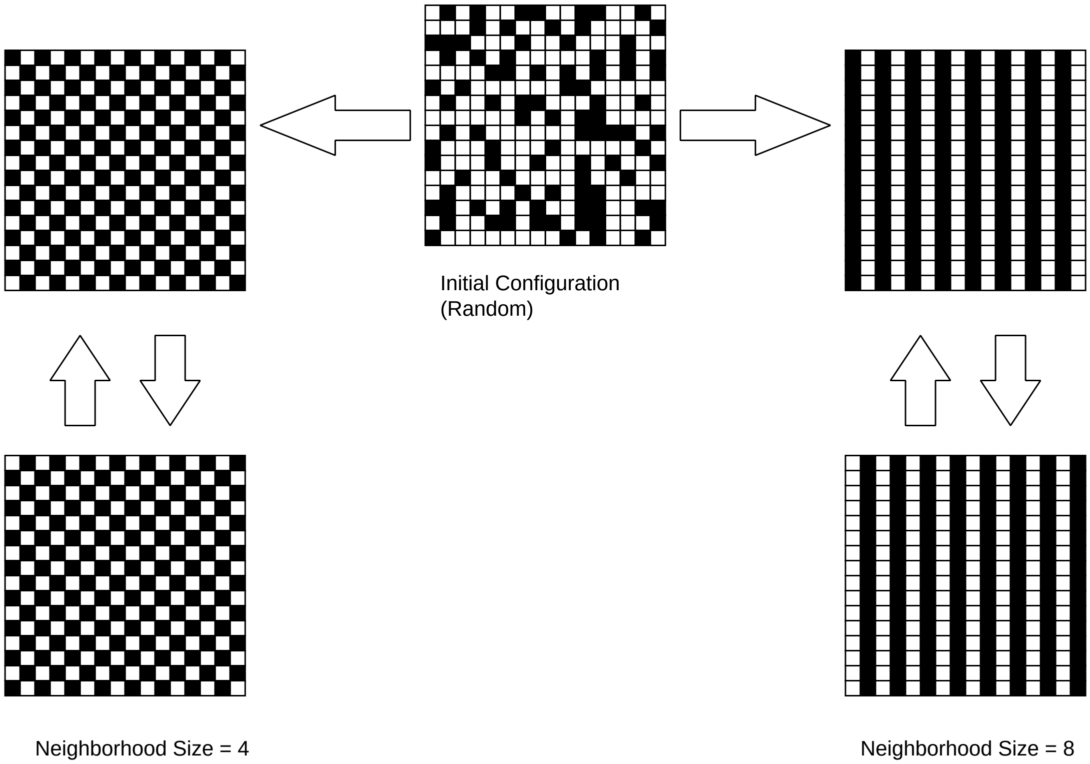


Figure 5: The spatio-temporal pattern produced by give-and-take strategy: (left) neighborhood size = 4, (right) neighborhood size = 8

get in the course of interactions. We considered specific strategic environments such as El Farol problem and Minority Game with several types of learning rule for agents. Among them, give-and-take learning which is introduced in this paper is superior to other learning rule. A rational approach is helpless in Minority Game by generating a large-scale social inefficiency. The emergent collective behavior of give-and-take learning is more efficient than that generated by the mixed Nash equilibrium strategies. We also show that give-and-take strategy has an ability to produce optimal spatio-temporal pattern when the games are played locally. These results implicate that we have a tools for examination how conventions evolve in a society that begins in an amorphous state where there is no established custom, and individuals rely on hearsay to determine what to do.

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