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A network flow approach to a city emergency evacuation planning

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Two network flow methods are presented in this paper to optimize a city emergency evacuation plan. The problem is to assign each resident of the city to one of the places of refuge (PR) in preparation for major disasters. We model the city as an undirected graph, and by solving a shortest path problem on this graph, we obtain the shortest evacuation plan. The second model takes the capacity limit on each PR explicitly into account. The problem can then be transformed into a minimal cost flow problem on a slightly modified graph. We can evaluate the efficiency of the current city evacuation plan by comparing this against the optimal solutions of the above stated problems. Also, various pieces of information obtainable from these solutions can be utilized in evaluating the current evacuation policy. In addition, sensitivity analysis can be performed to answer various what-if *questions.*

1. Introduction

Under the Natural Disaster Prevention and Relief Law (Hoshino *et al.* 1992), every Japanese city has its own emergency evacuation plan in preparation for such major disasters as earthquakes, tidal waves, destructive fires etc. The plan includes places of refuge (PRs for short) and the allocation of all the residents to such PRs. Usually, schools or city parks are designated as PRs. In case of major disasters, the residents are advised to evacuate themselves to their respective PRs for emergency relief from the local and central governments. In Japan, a city is divided geographically into residential areas (RAs for short), and this serves as the basic element of the city addressing system as well as the lowest level of residents' community. In allocating residents to PRs, RAs are usually taken as a fundamental unit: thus people from the same RA are assigned to the same PR. Such an assignment of RAs to PRs, hereafter referred to as the city evacuation plan or CEP, has been drawn up by city officials (perhaps in an intuitive way). We wish to determine this assignment of each resident to PRs in more analytical manner.

Although literature abounds on mathematical approaches to evacuation problems (see, for example, Minieka 1973, Chalmet *et al.* 1982, Hamacher and Tufekci 1987), most of these studies are primarily concerned with the dynamic flow of evacuees in a building, and are not applicable to our problem. In this paper, we present two methods of optimizing the resident-to-PR assignment. To do this, a city is modelled as an undirected graph, with RAs and PRs as its nodes and the roads in between as its arcs. In the first model, we obtain the shortest evacuation plan (hereafter SEP for short) by solving a shortest path problem on an auxiliary graph obtained from the graph model of the city with minor modifications. Except for the optimal allocation of the residents to PRs, we obtain the following from this analysis.

(*a*) Distribution (and cumulative distribution) of population by the distance to evacuation (DTE for short), both for CEP and SEP.

(*b*) Distribution (and cumulative distribution) of population by the reduced distance to evacuation due to SEP.

(*c*) Number of evacuees allocated to each PR, both for CEP and SEP.

In calculating SEP we do not take the capacity of PRs into account, but this is important for more realistic modelling of the problem. This is probably considered in CEP, but it is, at most, implicit. We present the second model in which capacity limits on PRs are explictly introduced, and formulate the evacuation problem as a minimal cost network flow problem on another auxiliary graph. We term the resulting solution as the min-cost

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evacuation plan (MEP for short). Similar outputs can be obtained from this calculation as from the shortest path model. Comparing CEP against SEP and/or MEP and referring to the additional information obtained in the process of optimization, we can evaluate the current evacuation policy. Furthermore, sensitivity analysis can be performed on these models to answer various *what-if* questions.

2. Modelling of city evacuation planning

In formulating the problem, let us start by stating the case study of Yokosuka City, located 50 km south of Tokyo on the Miura peninsula. The city stretches approximately 10 km north to south and 8 km east to west. The population of 436 549 (as of 1990) resides in 305 distinct RAs (Yokosuka City 1991a). An RA is usually less than 1 km in diameter, with the average population of 1431 and the standard deviation 1294. There are 40 PRs throughout the city (Yokosuka City 1991b). Figure 1 gives a graph representation of the city. Here, dots (•) denote RAs, squares (■) PRs, and other nodes indicate supplementary points on road networks. In modelling the city evacuation system in the graph form of Fig. 1, we made the following simplifying assumptions.

Assumptions:

(1) *Residents of each RA live concentrated at its centre; therefore an RA is a point in the model.*

(2) *Residents evacuate themselves on foot at the uniform speed of* $4\ km\ h^{-1}$.

(3) *Each road is bi-directional, and takes the same time to move on both directions.*

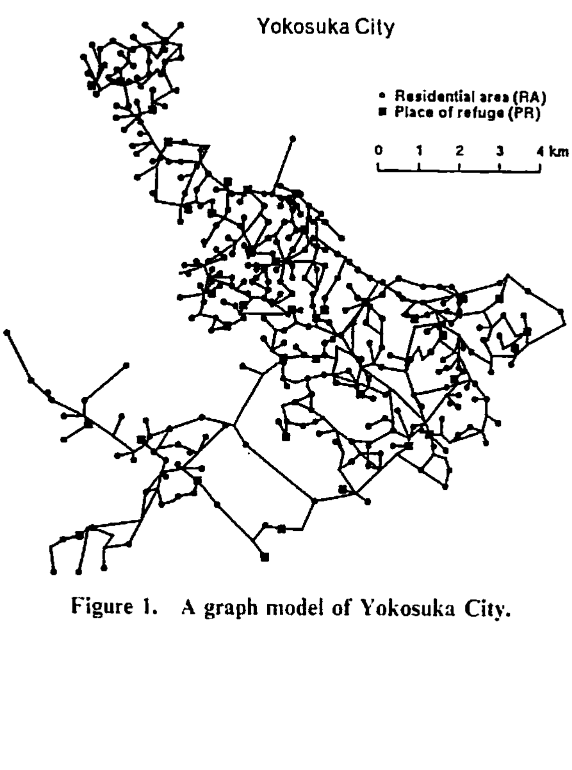

Figure 1. A graph model of Yokosuka City.

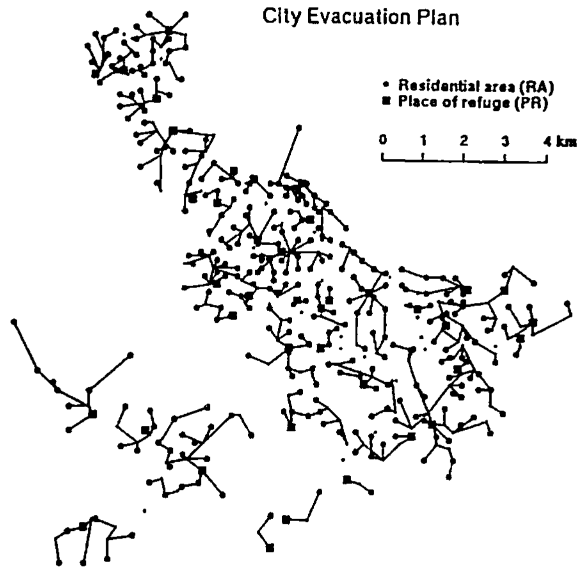

Figure 2. Yokosuka City evacuation plan (CEP).

(4) *No capacity constraint is imposed at any point on the road network.*

(5) *Each PR is unbounded in capacity; i.e. it admits infinitely many evacuees.*

Figure 2 shows the current CEP. Since every RA is uniquely assigned to a PR, the graph constitutes a forest (see, for example, Foulds 1991 for graph-theoretic terms) with each tree containing exactly one PR. Each tree in this figure represents the route of evacuation of every resident to their respective PRs in the city plan.

We now formulate the evacuation problem in mathematical terms. Let the graph of Fig. 1 be $G = (V, E)$, where V and E stand for the set of nodes and the set of arcs respectively, and V is decomposed as

$$V = R \cup D \cup N,$$

where

$R = \{r_1, r_2, \ldots, r_S\}$ is the set of RAs.

$D = \{d_1, d_2, \ldots, d_T\}$ is the set of PRs.

N is the set of other supplementary nodes of G.

Here, S and T are the numbers of RAs and PRs respectively. In the case of Yokosuka City these are $S = 305$ and $T = 40$, and the graph consists of $|V| = 503$ nodes and $|E| = 657$ arcs. The data required for the first model are:

(i) population P_i of the ith RA ($i = 1, 2, \ldots, S$);

(ii) distance d_{ij} (in km) of each arc $(i, j) \in E$.

3. Shortest evacuation planning

Given the data stated above and under the assumptions of the previous section, the optimal evacuation policy is to evacuate each resident to his/her nearest PR. This gives SEP, which minimizes the individual as well as total DTE. To calculate this, we modify the graph $G = (V, E)$ to obtain an auxiliary graph $G^+ = (V^+, E^+)$ in the following way: we introduce an additional node v^+ and the set of arcs from this node to every PR, and define

$$V^+ := V \cup \{v^+\},$$

$$E^+ := E \cup \{(v^+, d) | d \in D\}.$$

The distances of the newly added arcs are set to zero. Then, clearly, the shortest path from v^+ to each RA gives the optimal DTE and the nearest PR as well as the optimal route of evacuation for those in that RA. Thus, by applying the standard shortest path algorithm such as Dijkstra's method (Dijkstra 1959) with v^+ as the originating node, we obtain SEP.

Figure 3 is the result of this computation. As in CEP, this is a forest consisting of trees with exactly one PR in each. Figure 4 depicts the distribution of population by DTE both for CEP and SEP. The improvement of SEP (over CEP) in DTE may be better seen from the cumulative distribution of Fig. 5, where the percentage of the city population within an arbitrary distance to evacuation is given as a function of DTE. For example, while in CEP 55% of the city population is within 1 km of PRs, this percentage is more than 65% in SEP. Among the 305 RAs of the city, 81 save up to 1·2 km in DTE. Figure 6 gives the number of evacuees at each PR both for CEP and SEP. For the comparison of these two evacuation policies, readers are referred to Table 1, which also compares MEP to be stated in the following section. On average, DTE is reduced by 120 m more in SEP than in CEP, with the maximum being 1·5 km for an RA with 599 residents. It appears that the shortest path approach results in a more uniform evacuation plan than CEP in the sense that the standard deviation of DTE is smaller in SEP. This is also the case for the numbers of evacuees allocated to each PR.

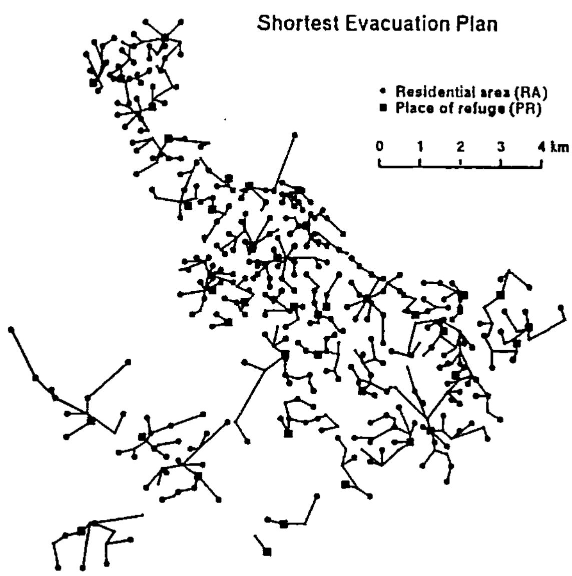

Figure 3. Shortest path evacuation plan (SEP).

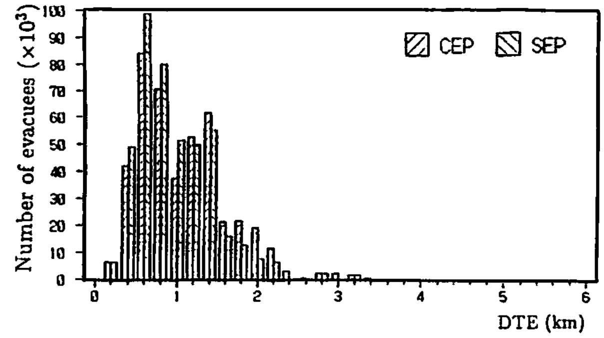

Figure 4. Distribution of population by distance to evacuation (DTE).

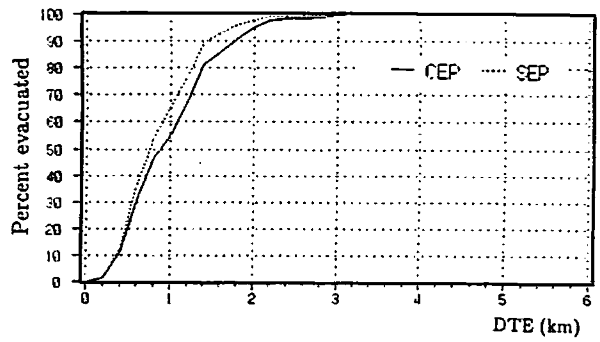

Figure 5. Cumulative distribution of population by DTE.

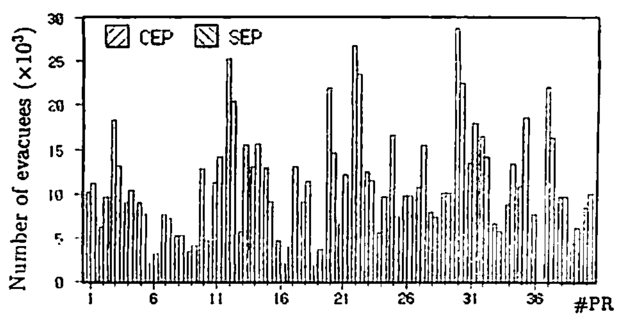

Figure 6. Number of evacuees allocated to each place of refuge (PR).

4. Evacuation with capacity constraints

In the SEP of the previous section, each PR is assumed to be unbounded in capacity (see assumption 5 of § 2). This is not the case in practice; PRs are usually of limited capacity. In this section, we present an evacuation model which explicitly takes capacity limits on PRs into account. Let the capacity of the jth PR be $C_j (j = 1, 2, \ldots, T)$. This specifies the upper limit of the number of evacuees acceptable in this PR.

We construct an auxiliary graph $G^* = (V^*, E^*)$ by modifying G as follows: we add two additional nodes, an imaginary source v_* and an imaginary sink v^*, and the set of arcs connecting v_* to RAs and v^* to PRs, and define

$$V^* := V \cup \{v_*, v^*\},$$

$$E^* := E \cup \{(v_*, r) | r \in R\} \cup \{(d, v^*) | d \in D\}.$$

Let the capacity $c(u, v)$ and the cost coefficient $k(u, v)$ be defined for each arc $(u, v) \in E^*$ as follows

$$c(u, v) := \begin{cases} P_i & \text{if } u = v_* \text{ and } v = r_i \\ C_j & \text{if } u = d_j \text{ and } v = v^* \\ \infty & \text{otherwise} \end{cases}$$

and

$$k(u, v) := \begin{cases} d_{ij} & \text{if } (u, v) = (i, j) \in E \\ 0 & \text{otherwise.} \end{cases}$$

We let the total supply of flow at the source v_* and the total demand at the sink v^* be identically $P := \sum_{i=1}^{S} P_i$; meaning that the whole city population must be evacuated. Let us consider a minimal cost network flow problem (Ford and Fulkerson 1962) on G^*. By $x(u, v)$ we denote the amount of flow on $(u, v) \in E^*$, i.e. the number of people who evacuate through this arc. We now consider the following.

Minimal cost network flow problem

$$\text{Minimize} \quad C := \sum_{(u, v) \in E} k(u, v) x(u, v)$$

$$\text{subject to} \quad \sum_{u \in V^*} x(u, v) = \sum_{w \in V^*} x(v, w) \quad \text{for all } v \in V$$

$$\sum_{d \in D} x(d, v^*) = P$$

$$0 \le x(u, v) \le c(u, v) \quad \text{for all } (u, v) \in E^*.$$

Owing to the above capacity constraints, we note that the amount of flow out of the ith RA is exactly P_i, and the flow into the jth PR is no more than C_j. The problem minimizes, under the capacity constraints, the total DTE travelled by the whole population in evacuation. However, for individual residents the allocated PR may not be their nearest. The solution to the above problem is MEP. In MEP, there may be more than one PR

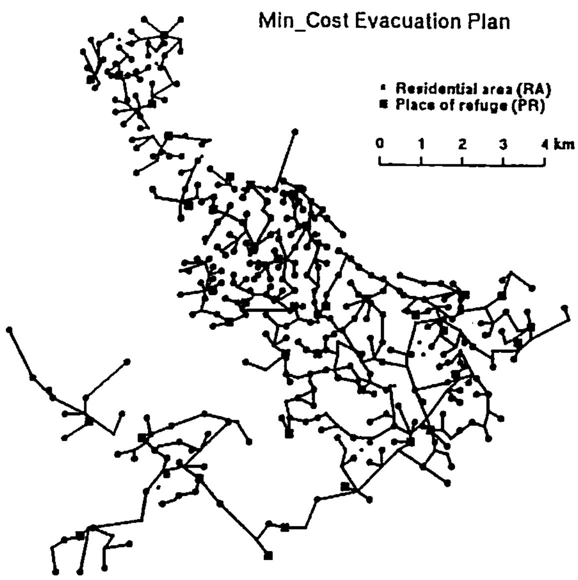

Figure 7. Min-cost evacuation plan (MEP).

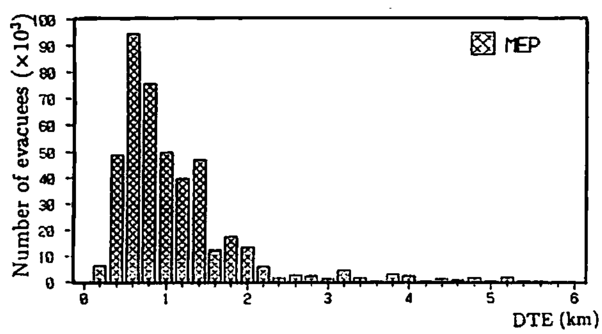

Figure 8. Distribution of population by DTE for MEP.

allocated to an RA, and therefore the result may not necessarily be a forest. Furthermore, due to the capacity limit of PRs, residents are often assigned to a more distant PR than in SEP or CEP.

In the case of Yokosuka City, let us assume the capacity of all PRs be fixed at 12 000. Figure 7 depicts an MEP, which is no longer a forest. Figure 8 shows the distribution of the population by DTE, and Fig. 9 gives its cumulative distribution together with those for CEP and SEP. Note that, due to the capacity limits on PRs, in MEP there exists a small percentage of people who are assigned to very distant PRs. For example, more than 7·3% must go more than 2 km in MEP, while this percentage is 4·9% in SEP and 2·4% in CEP. This results in the larger standard deviation of DTE in MEP than in CEP or SEP. On the other hand, city population is quite uniformly allocated to PRs in MEP, as we see from Fig. 10 which shows the number of evacuees at each PR.

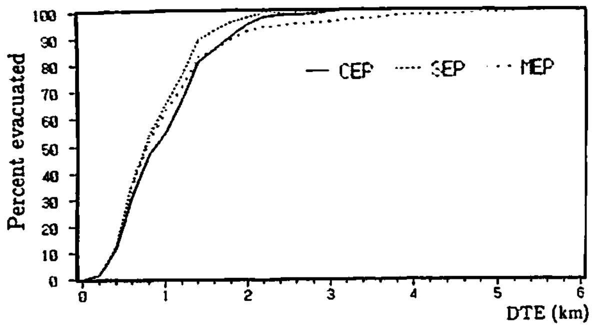

Figure 9. Cumulative distribution of population by DTE for three evacuation policies.

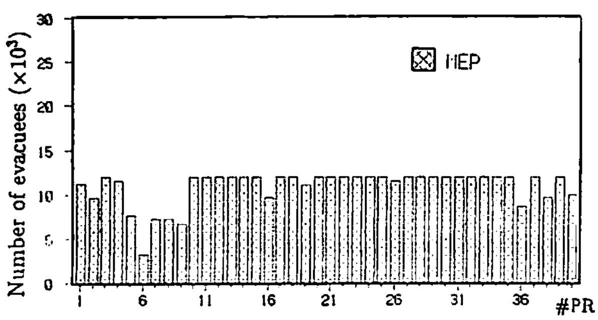

Figure 10. Number of evacuees at each PR in MEP.

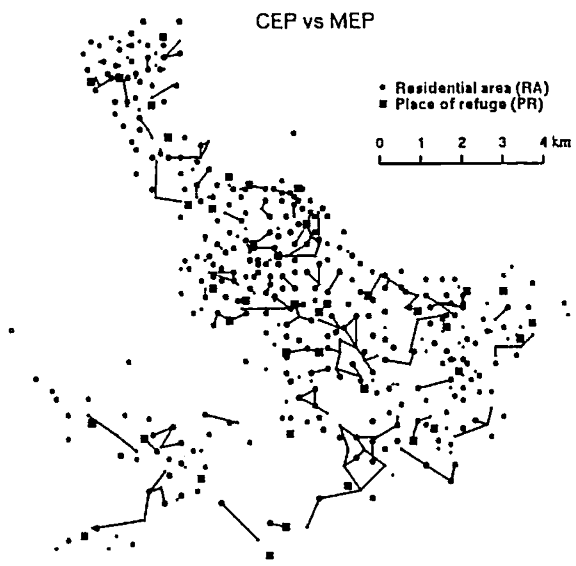

Figure 11. Difference of CEP and MEP.

Table 1 summarizes the comparison of the three evacuation policies. Finally, Fig. 11 shows the difference between CEP and MEP, indicating the arcs which are used in only one of these evacuation plans.

Table 1. Three evacuation policies compared

		CEP	OEP	MEP
DTE (km)	Mean	0·972	0·850	1·113
	S.D.	0·561	0·478	0·804
	Max	3·266	3·167	5·660
	Min	0·100	0·100	0·100
Evacuees per PR	Mean	10 917	10 917	10 917
	S.D.	6 551	5 427	1 988
	Max	28 769	23 519	12 000
	Min	1 828	0	3 253

5. Conclusions

In this paper, we have shown how network flow theory can be applied to city evacuation planning. The methods of this paper have been heavily dependent on the simplifying assumptions of § 2. Assumption 5 has been dropped in our second model. Assumption 4 would be similarly removed. Also, the road network can be made asymmetric to admit uni-directional traffic and slopes, which relaxes Assumption 3. Still, there exists a clear gap between the actual city geography and the graph representation of the problem in this paper. In particular, in the case study of Yokosuka City treated in this paper, the numbers of nodes and arcs are far less than those required for a reasonable description of the city. However, it is straightforward to improve the accuracy of the methods by constructing a more detailed model with increased numbers of RAs and connecting roads.

Once we have formulated the problem in the optimization form, we can perform various sensitivity analyses. This enables us to answer such *what-if* questions as the following.

(1) What is the effect of increasing (or decreasing) the capacity of each PR?
(2) What changes when a PR is added to or removed from the network?
(3) What happens if new roads are constructed?
(4) What will be the result of an increase (or decrease) of the city population?

This information, together with that obtained in the previous sections, is important in evaluating the existing evacuation policies, as well as in revising them.

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