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A New Approach to the Problem of Ship Wave

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Abstract

The author deals with the problem of the waveless ship.

This is a historically old but always interesting problem in theory and application.

Introducing an auxiliary function from which we have the velocity potential or singularity distribution by the operation adjoint to the one to be satisfied by the potential at the free surface, he gives the waveless condition for the singularity distributions of various types in two and three dimensional case of steady motion, and also some characters and examples.

The adjoint operator to the one for the free surface condition plays naturally an important role in solving wave problems of all types, and we may hope a new approach to them.

Introduction

The principal theme of this paper is a problem of the waveless distribution, that is, a singularity distribution which leaves no regular wave system in the rear.

This problem was studied at an earlier stage of ship wave researches by Lord Kelvin and Sir Havelock in two dimensional problem, but after them no one seems to have been interested with.

This is, however, clearly an important problem especially for the theoretical point of view, eventhough the waveless ship would not be obtained in practice.

The author seeks a condition under which a singularity distribution is waveless, introducing an auxiliary function from which we have the velocity potential by the operation adjoint to the one for the free surface condition of the petential.

The introduction of this auxiliary function is a very natural way to solve the problem as the differential equation, but we have not found it explicity in the literature.

In this paper, we deal only with the wave problem when a ship moves along a straight course with constant speed in calm sea, while the method proposed here are applicable to other problems, say, in unsteady motion.

1. Two dimensional problem, in general

Consider the two dimensional fluid motion due to a fixed cylinder with some section placed on the surface of a uniform stream with great depth.

Take the origin in the cylinder and on the undisturbed surface of the stream, with Ox horizontal and Oy vertically upwards and suppose the stream to be of the unit velocity in the negative direction of Ox.

We write the complex velocity potential of the motion as follows,

$$f(z)=\varphi(x, y)+i\psi(x, y), \tag{1.1}$$

where $z=x+iy$, φ is the velocity potential and ψ is the stream function.

The solution must satisfy two conditions at the free surface.

Firstly, the pressure must be constant, namely,

$$p/\rho=\text{const.}-gy-q^2/2+\mu\varphi=\text{const.} \tag{1.2}$$

where ρ is the fluid density, g is the gravity constant, q is the velocity and μ is an artificial frictional coefficient to avoid a mathematical indeterminancy of the solution and to tend to zero after the calculation.

Neglecting the second order quantity, this is approximately

$$p/\rho=\text{const.}-gy-\partial\varphi/\partial x+\mu\varphi, \tag{1.3}$$

The second condition is a kinematical one, this is, neglecting higher order terms,

$$d\eta(x)/dx=\partial\varphi(x, 0)/\partial y, \qquad \text{or} \qquad \eta(x)=-\psi(x, 0), \tag{1.4}$$

where η is the surface elevation.

Taking an appropriate constant and putting (1.4) into (1.3), the free surface condition is to be

$$Re.[(d/dz+ig-\mu)f(z)]=0, \text{ for } y=0, \tag{1.5}$$

where $Re.$ means the real part to be taken.

The function regular in the fluid and satisfying the above condition is given in the next form,

$$f(z)=\frac{1}{2\pi i}\int_C \frac{f(\zeta)}{z-\zeta}\,d\zeta+\lim_{\mu\to+0}\int_0^\infty \frac{\kappa+g-\mu i}{\kappa-g-\mu i}\,\overline{F(\kappa)}e^{-i\kappa z}d\kappa, \tag{1.6}$$

where C is the boundary curve of the cylinder and the fluid,

$$F(\kappa)=\frac{1}{2\pi}\int_C f(z)e^{-i\kappa z}dz, \tag{1.7}$$

and the bar above the character stands for the complex conjugate value to be taken.

In the far down stream, this potential leaves the regular wave, it is from (1.6)

$$\eta(x)\doteqdot Re.[4\pi g\,\overline{F(g)}e^{-igx}], \qquad \text{for } gx\ll -1, \tag{1.8}$$

If a is the amplitude of the regular wave at a great distance behind the cilinder, the wave resistance R is given by one quarter of $\rho g a^2$.

Accordingly, from the above formula we have

$$R=4\pi^2\rho g^3 F(g)\overline{F(g)}, \tag{1.9}$$ [3]

Now the problem of the wave resistance is to find a function or an amplitude of the wave $F(g)$ by (1.7) with the potential satisfying the condition on the boundary C. But, to satisfy this condition is a difficult problem, and the solutions obtained up to the present are only of a submerged circular cylinder and a hydrofoil.[3),4)]

On the other hand, when we notice to the waveless problem, it seems easier to deal with. That it to say, the system is waveless from (1.9), if we have

$$F(g)=0, \tag{1.10}$$

To deal more generally with this problem, we introduce an auxiliary function $h(z)$ by the next definition,

$$(d/dz-ig)h(z)=f(z), \tag{1.11}$$

we are to note that the operation to $h(z)$ is adjoint to the operation to $f(z)$ in (1.5).

Put (1.11) into (1.7) and integrate by parts, we have

$$2\pi F(\kappa)=[h(x)e^{-i\kappa x}]_{x=x_2}^{x_1}+i(\kappa-g)\int_C h(z)e^{-i\kappa z}dz, \tag{1.12}$$

where x_1 and x_2 are the coordinates of the points on the curve C intersecting the axis Ox.

When κ equals to g, we have

$$2\pi F(g)=h(x_1)e^{-igx_1}-h(x_2)e^{-igx_2}, \tag{1.13}$$

Consequently, if $h(x_1)$ and $h(x_2)$ vanish, the distribution is waveless by (1.10).

Meanwhile, the complex velocity potential is represented by this function in the same manner as before, namely

$$f(z)=(d/dz-ig)m(z)+w(z), \tag{1.14}$$

where

$$m(z)=\frac{1}{2\pi i}\int_C \frac{h(\zeta)}{z-\zeta}d\zeta+\frac{1}{2\pi i}\int_C \frac{\overline{h(\zeta)}}{z-\overline{\zeta}}d\overline{\zeta}, \tag{1.15}$$

it is easy to see that

$$Re.[m(z)]=0, \qquad \text{for } y=0, \tag{1.16}$$

and

$$w(z)=\frac{1}{2\pi i}\left[\frac{h(x)}{z-x}\right]_{x=x_2}^{x_1}+\lim_{\mu\to+0}\int_0^\infty \frac{\kappa+g-\mu i}{\kappa-g-\mu i}e^{-i\kappa z}d\kappa\left[h(x)e^{i\kappa x}\right]_{x=x_2}^{x_1} \tag{1.17}$$

We see thus $f(z)$ consists of two parts. The one is the term with $m(z)$, and this vanishes at infinity by (1.15). The other is $w(z)$, this contains the regular wave term by (1.17).

If the system is waveless, (1.14) reduces to

$$f(z)=(d/dz-ig)m(z), \tag{1.18}$$

where $m(z)$ satisfies (1.16).

F. Ursell's solution for the problem of an oscillating circular cylinder is based on this separation of the velocity potential, so that we may also solve the same problem as it in the steady motion.

2. Pressure distribution, in two dimension.

Consider here the planing surface with infinitely large spect aspect ratio.

The pressure at $y=0$ is given by (1.3) and (1.4) except an arbitrary constant,

$$p(x)/\rho \equiv P(x)=Re.[-(d/dz+ig)f(x)], \qquad \text{for } 1>x>-1, \tag{2.1}$$

We can easily construct the function $\Pi(z)$ having $P(x)$ as the boundary value at $y=0$, that is

$$\Pi(z)=\frac{1}{\pi i}\int_{-1}^{1}\frac{P(x)dx}{z-x}, \tag{2.2}$$

By the analytical continuation of (2.1) into the whole fluid domain, we have

$$(d/dz+ig-\mu)f(z)=-\Pi(z), \tag{2.3}$$

This is a differential equation, and solved as

$$f(z)=\frac{1}{\pi i}\int_{-1}^{1}P(\xi)S[g(z-\xi)]d\xi, \tag{2.4}$$ [4,7]

where

$$S(gz)=\lim_{\mu\to+0}\int_{0}^{\infty}\frac{e^{-i\kappa z}d\kappa}{\kappa-g-\mu i}, \tag{2.5}$$

this function satisfies the next inhomogeneous equation,

$$(d/dz+ig)S(gz)=-1/z, \tag{2.6}$$

Introduce here the function h by (1.11), and put into (2.1), we have

$$P(x)=-Re.[(d^2/dx^2+g^2)h(x)],$$

or writing $\sigma(x)$ for the real part of h changed in the sign,

$$P(x)=(d^2/dx^2+g^2)\sigma(x), \tag{2.7}$$

This is an inhomogeneous differential equation of oscillation for $\sigma(x)$, so that it may be uniquely determined for the given $P(x)$ with an arbitrary boundary condition, say, $\sigma(\pm 1)=0$.

Put (2.7) into (2.4), integrate by parts, and we have with (2.6)

$$f(z)=\frac{1}{\pi i}\left[\frac{d\sigma}{d\xi}S[g(z-\xi)]\right]_{\xi=-1}^{1}+\frac{1}{\pi}\int_{-1}^{1}\frac{\{g\sigma(\xi)+id\sigma/d\xi\}}{z-\xi}d\xi, \tag{2.8}$$

The ship surface condition is (1.4), namely, from the above formula

$$-\eta(x)=Re.\left\{\frac{-1}{\pi}\left[\frac{d\sigma}{d\xi}S[g(x-\xi)]\right]_{\xi=-1}^{1}\right\}+g\sigma(x)+\frac{1}{\pi}\int_{-1}^{1}\frac{d\sigma/d\xi}{x-\xi}d\xi, \tag{2.9}$$

This is in principle the same form as the equation of wing theory, and it is comparatively easy to solve. The method of the solution used by H. Maruo is naturally deduced from this equation.[6]

We consider now the waveless problem, the condition is to be from (2.8)

$$\sigma(\pm 1)=\frac{d\sigma}{dx}(\pm 1)=0, \tag{2.10}$$

and the form of the planing surface is given by (2.9), that is,

$$-\eta(x)=g\sigma(x)+\frac{1}{\pi}\int_{-1}^{1}\frac{d\sigma/d\xi}{x-\xi}d\xi, \tag{2.11}$$

Integrating (2.7) and (2.11) with (2.10), we have

$$\int_{-1}^{1}P(x)\,dx=g^2\int_{-1}^{1}\sigma(x)dx, \tag{2.12}$$

$$-g\int_{-1}^{1}\eta(x)dx=g^2\int_{-1}^{1}\sigma(x)\,dx+\frac{2g}{\pi}\int_{-1}^{1}\frac{\sigma(x)}{1-x^2}dx, \tag{2.13}$$

When the speed is very low or g is very large, we may have an approximate relation from (2.7) and (2.11), that is,

$$P(x)\fallingdotseq -g\eta(x)\fallingdotseq g^2\sigma(x), \tag{2.14}$$

This states that the surface depression nearly equals to the statical pressure head, but this may be not true near the singularity.

For example, we consider the next function,

$$\sigma(x)=(1-x^2)^2, \tag{2.15}$$

Then, we have from above formulae

$$P(x)=g^2(1-x^2)^2+4(3x^2-1), \tag{2.16}$$

$$-g\eta(x)=g^2(1-x^2)^2+\frac{g}{\pi}\left[8(2/3-x^2)+4x(1-x^2)\log\left(\frac{1-x}{1+x}\right)\right], \tag{2.17}$$

$$\int_{-1}^{1}P(x)dx=(16/15)g^2,\ -g\int_{-1}^{1}\eta(x)dx=(16/15)g^2+(8/3\pi)g, \tag{2.18}$$

In this example, the pressure is finite at end points, so that we may not have the splash resistance too.[7]

Lastly, we will note to the following equation. If we carry out the adjoint operation to both sides of (2.3), we have

$$(d^2/dz^2+g^2)f(z)=-(d/dz-ig)\Pi(z), \tag{2.19}$$

It is easily seen that this equation corresponds to (1.14) and gives also the integral equation of the same type as (2.9).

We assume here that the pressure is distributed in the finite range, while Lord Kelvin and Prof. Havelock gave examples of the distribution in an infinite range. It is clear that their examples are included in our theory.

3. Half immersed vertical plate.

F. Ursell solved the two dimensional problem of the wave reflection by a fixed vertical plate half immersed in the water.[8] It is natural in our theory to take such a method he used there, and we will consider the similar problem for an application of our theory.

Consider a plate half immersed in a uniform stream and occupying the segment 0 to -1 on the y-axis.

The complex velocity potential is given by (1.6) and (1.7), but in this case the curve C consists of the face and back of the plate, and the stream function is to take the same value on both sides.

Hence, denoting the difference of the velocity potential between the face and back ϕ, we have from (1.6) and (1.7)

$$f(z)=\frac{1}{2\pi}\int_{-1}^{0}\frac{\phi(\eta)}{z-i\eta}d\eta+\lim_{\mu\to+0}\int_{0}^{\infty}\frac{\kappa+g-\mu i}{\kappa-g-\mu i}\overline{F(\kappa)}e^{-i\kappa z}d\kappa, \tag{3.1}$$

where

$$F(\kappa)=\frac{i}{2\pi}\int_{-1}^{0}\phi(\eta)e^{\kappa\eta}d\eta, \tag{3.2}$$

We can assume naturally

$$\phi(-1)=0, \tag{3.3}$$

The boundary condition is to be

$$\partial\phi/\partial x=\partial\psi/\partial y=-1, \text{ or } \psi=-y-\alpha, \text{ on } 0>y>-1, \tag{3.4}$$

where α is a constant not arbitrary but to be determined in what follows.

We may find the left hand expression of (1.15) is given by this condition and this is from (3.1)

$$(d/dz+ig)f(z)=\frac{1}{2\pi}\int_{-1}^{0}\phi(\eta)d\eta\cdot\left[\frac{ig}{z-i\eta}+\frac{ig}{z+i\eta}+\frac{1}{(z+i\eta)^2}-\frac{1}{(z-i\eta)^2}\right], \tag{3.5}$$

Integrating this equation and after the some calculations, we have their real part as

$$[\phi(y)]_0^y-g\int_0^y\phi(\eta)d\eta=\frac{g}{2\pi}\log(1-y^2)\cdot\int_{-1}^{0}\phi(\eta)d\eta+\frac{1}{\pi}\int_{-1}^{0}m(\eta)\frac{\eta d\eta}{\eta^2-y^2}, \tag{3.6}$$

where

$$m(y)=\phi(y)+g\int_y^0\phi(\eta)d\eta, \tag{3.7}$$

This solution may be found easily but somewhat laborious, and we will consider here only their approximate wave resistance at very low speed.

If we have the solution m by (3.6), ϕ is given from (3.7) as

$$\phi(y)=\frac{d}{dy}\left[e^{gy}\int_0^y m(\eta)e^{-g\eta}d\eta\right], \tag{3.8}$$

Accordingly, we have from (3.2) after the some calculations

$$F(\kappa)=\frac{i}{2\pi(g+\kappa)}\left[\kappa\int_{-1}^{0}e^{\kappa y}m(y)dy+ge^{-(g+\kappa)}\int_{-1}^{0}e^{-gy}m(y)dy\right], \tag{3.9}$$

When g takes a very large value, the greatest term of m, the solution of (3.6), is

$$m(-\cos\theta)\doteqdot(g/2)\sin 2\theta, \tag{3.10}$$

where one of the terms containing the constant α is of the same order, but we omit it merely for simplicity.

Put it into (3.9), after some approximations we have

$$F(g)\doteqdot i/2\pi g, \tag{3.11}$$

and the wave resistance

$$R\doteqdot\rho g, \tag{3.12}$$

On the other hand, the usual approximation for ϕ is

$$\phi(-\cos\theta)\doteqdot 2\sin\theta, \tag{3.13}$$

In the same way as before, we have from (3.2) and (1.9)

$$F(g)\doteqdot i/\pi g, \quad \text{and} \quad R\doteqdot 4\rho g, \tag{3.14}$$

Thus we see that the exact value of the wave resistance is nearly one quarter of the usual approximate one. This is a similar result as that of F. Ursell got.[9]

4. Three dimensional problem, in general.

Consider now the three dimensional problem of the ship fixed on the surface of a uniform stream with great depth.

Take the origin of the right handed Cartesian coordinates on the waterline plane of the ship with Ox horizontal and Oz vertically upwards and suppose the stream to be of unit speed in the negative direction of Ox.

Write the velocity potential of the disturbed fluid motion $\varphi(x, y, z)$ and the surface elevation $\zeta(x, y)$, we have in the same way as (1.2), (1.3) and (1.5) at the free surface, we assume it $z=0$,

$$p/\rho \fallingdotseq \text{const.} - g\zeta - \partial\varphi/\partial x + \mu\varphi = 0, \tag{4.1}$$

$$\partial\zeta/\partial x \fallingdotseq \partial\varphi/\partial z, \tag{4.2}$$

$$(\partial^2/\partial x^2 + g\cdot\partial/\partial z - \mu\cdot\partial/\partial x)\varphi(x, y, 0) = 0, \tag{4.3}$$

For example, the velocity potential at the point $P \equiv (x, y, z)$ with a unit source placed at the point $Q \equiv (x, y, z)$ is well known, that is,

$$S(P, Q) = \frac{1}{r(P, Q)} - \lim_{\mu\to+0} \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_0^{\infty} \frac{\kappa\cos^2\theta + g + \mu i\cos\theta}{\kappa\cos^2\theta - g + \mu i\cos\theta} e^{\kappa(z+z') + i\kappa(\varpi - \varpi')} d\kappa d\theta. \tag{4.4}$$

where $r(P, Q) = \overline{PQ}$, $\varpi = x\cos\theta + y\sin\theta$, $\varpi' = x'\cos\theta + y'\sin\theta$.

Differentiate (4.4) with P, we have

$$\lim_{\mu\to+0}\left[\left(\frac{\partial^2}{\partial x^2} + g\frac{\partial}{\partial z} - \mu\frac{\partial}{\partial x}\right)S(P, Q)\right] = (\partial^2/\partial x^2 + g\,\partial/\partial z)\frac{1}{r(P, Q)} - (\partial^2/\partial x^2 - g\,\partial/\partial z)\frac{1}{r(P, \overline{Q})}, \tag{4.5}$$

where $\overline{Q}$ stands for the mirror image point of Q with respect to the plane $z=0$, and this equation vanishes on the free surface, $z=0$, naturally.

According to the foregoing analysis, we define the auxililary function as

$$(\partial^2/\partial x^2 - g\cdot\partial/\partial z)\mu(x, y, z) = \varphi(x, y, z), \tag{4.6}$$

Then, we may proceed and have results in the same way as in the two dimensional case, but it gives a complicated formula, and so we will not deal with any farther in general, but consider a more concrete case.

5. Pressure distribution, in three dimension.

The velocity potential has been found, making use of the function $S(P, Q)$, to be

$$\varphi(P) = \frac{1}{4\pi g}\iint_{S(z'=0)} P(x', y')\frac{\partial}{\partial x'}S(P, Q)dx'dy', \tag{5.1}$$ [10]

where $P(x, y)$ stands for $p(x, y)/\rho$ and $S(P, Q)$ reduces to

$$S(P, Q) = \lim_{\mu\to+0}\frac{-g}{\pi}\int_{-\pi}^{\pi}\int_0^{\infty}\frac{e^{-\kappa z + i\kappa(\varpi - \varpi')}}{\kappa\cos^2\theta - g + i\mu\cos\theta}d\kappa d\theta, \tag{5.2}$$

The boundary condition over the surface S is (4.1), and this is written as follows,

$$-g\zeta(x, y)=P(x, y)-\frac{1}{4\pi g}\iint_S P(x', y')\left[\frac{\partial^2}{\partial x'^2}S(P, Q)\right]_{z=0}dx'dy', \tag{5.3}$$

The wave resistance R is given by the formula,[10]

$$R=\frac{\rho g^2}{2\pi}\int_{-\frac{2}{\pi}}^{\frac{\pi}{2}}|F(g\sec^2\theta, \theta)|^2\sec^5\theta d\theta, \tag{5.4}$$

where

$$F(\kappa, \theta)=\iint_S P(x, y)e^{-i\kappa\bar{\omega}}dxdy, \tag{5.5}$$

and $|F|$ stands for the absolute value of F.

Now introduce the function $\mu(x, y)$ by the next equation, bearing in mind (4.1) (4.2) and (4.6),

$$P(x, y)=[\partial^4/\partial x^4+g^2(\partial^2/\partial x^2+\partial^2/\partial y^2)]\mu(x, y), \tag{5.6}$$

Putting this into (5.5) and integrating by parts, we have

$$\begin{aligned}F(\kappa, \theta)=&\int e^{-i\kappa\bar{\omega}}dy[\partial^3/\partial x^3+i\kappa\cos\theta\cdot\partial^2/\partial x^2+(g^2-\kappa^2\cos^2\theta)(\partial/\partial x+i\kappa\cos\theta)]\mu(x, y)\\&+g^2\int_C e^{-i\kappa\bar{\omega}}dx(\partial/\partial y+i\kappa\sin\theta)\mu(x, y)+\kappa^2(\kappa^2\cos^4\theta-g^2)\iint_S\mu(x, y)e^{-i\kappa\bar{\omega}}dxdy,\end{aligned} \tag{5.7}$$

where C means the periphery of the surface S.

When κ equals to $g\sec^2\theta$, the last term vanishes and we find the waveless condition as follows,

$$(\partial^3/\partial x^3)\mu=(\partial^2/\partial x^2)\mu=(\partial/\partial x)\mu=(\partial/\partial y)\mu=\mu=0, \text{ on } C. \tag{5.8}$$

We have also the velocity potential as follows,

$$\begin{aligned}\varphi(P)=&\frac{1}{2\pi}\left(\frac{\partial}{\partial z}\right)\left(\frac{\partial^2}{\partial x^2}-g\frac{\partial}{\partial z}\right)\iint_S\mu(x', y')\left(\frac{\partial}{\partial x'}\right)\left[\frac{1}{r(P, Q)}\right]dx'dy'\\&+\frac{1}{4\pi g}\int_C\left[\frac{\partial^3\mu}{\partial x'^3}+g^2\frac{\partial\mu}{\partial x^2}-\left(\frac{\partial^2\mu}{\partial x'^2}+g^2\mu\right)\left(\frac{\partial}{\partial x'}\right)+\frac{\partial\mu}{\partial x'}\left(\frac{\partial}{\partial x'}\right)^2-\mu\left(\frac{\partial}{\partial x'}\right)^3\right]\frac{\partial}{\partial x'}S(P, Q)dy'\\&+\frac{g}{4\pi}\int_C\left(\frac{\partial\mu}{\partial y'}-\mu\frac{\partial}{\partial y'}\right)\frac{\partial}{\partial x'}S(P, Q)dx',\end{aligned} \tag{5.9}$$

The first term is regular and vanishes at infinity, but other terms having the singularity on the curve C leave the wave system in the rear.

Thus, we see that the waveless condition (5.8) is a natural consequence.

For example, put

$$\mu(x, y)=(b^2-y^2)^2(1-x^2)^4, \tag{5.10}$$

for a rectangular plate with the length 2 and the breadth $2b$, we have waveless system, and the pressure is given by (5.6) and the surface elevation by (5.3).

We may have many other examples easily, but all of these distribution have no lift.

That is, integrating (5.6) with (5.8),

$$\iint_S P(x, y)dx\,dy=0, \tag{5.11}$$

This is the different point with (2.12) in the two dimensional case.

Lastly, let us consider the waveless distribution of a slightly different type.

If the pressure distribution has a point symmetry, (5.5) is independent of θ, namely,

$$F(\kappa)=\iint P(r)e^{-i\kappa r\cos(\theta-\varphi)}rdrd\varphi=2\pi\int_0^\infty P(r)J_0(\kappa r)rdr, \tag{5.12}$$

where $r=\sqrt{x^2+y^2}$, $\varphi=\tan^{-1}y/x$ and J is the Bessel function of the first kind.

It is easily seen from (5.4) and (5.5) that the waveless condition in this case is to be

$$F(\kappa)=0, \qquad \text{for } \kappa>g, \tag{5.13}$$

Now we see, F is the Hankel transform of P from (5.12), so that we may have inversely

$$P(r)=\frac{1}{2\pi}\int_0^\infty F(\kappa)J_0(\kappa r)\kappa d\kappa, \tag{5.14}$$

If the function $F(\kappa)$ satisfies (5.13), this reduces to

$$P(r)=\frac{1}{2\pi}\int_0^g F(\kappa)J_0(\kappa r)\kappa d\kappa, \tag{5.15}$$

Inversely, if this equation holds, the condition (5.13) is deduced by the reciprocal transformation, that is to say, the pressure distribution of the form (5.15) is waveless.

For example, put $F(\kappa)$ a constant, we have

$$F(\kappa)=1, \qquad P(r)=\frac{g}{2\pi r}J_1(gr), \tag{5.16}$$

and the total pressure is finite, that is from (5.12).

$$\int_0^\infty P(r)rdr=F(0)=1, \tag{5.17}$$

In this case, however, the range of the distribution extends to infinity.

6. Mitchell type ship.

We mean with this word a ship represented by the singularity distribution over some area, say rectangle, in the $x-z$ plane. The ship of this type is one of the frequently used in the theory and the experiment as the model of the so-called displacement ship.

The velocity potential caused by the doublet distribution $\eta(x, z)$ is written as follows,

$$\varphi(P)=\frac{1}{2\pi}\int_{-t}^{0}\int_{-1}^{1}\eta(x', z')\frac{\partial}{\partial x'}S(P, Q)dx'dz', \tag{6.1}$$ [10]

The function η equals approximately to the half breadth of the ship as usually known.

The wave resistance is given by the next formula,

$$R=\frac{4\rho g^4}{\pi}\int_0^{\frac{\pi}{2}}|F(g\sec^2\theta, \theta)|^2\sec^5\theta\, d\theta, \tag{6.2}$$ [10]

where

$$F(\kappa, \theta)=\int_{-t}^{0}\int_{-1}^{1}\eta(x, z)e^{\kappa z-i\kappa x\cos\theta}dxdz, \tag{6.3}$$

Introduce now the auxiliary function by the next equation,

$$\eta(x, z)=(\partial/\partial z-\partial^2/g\partial x^2)\sigma(x, z), \tag{6.4}$$

and we see that this is a generalised equation of heat conduction.

This equation may be easily solved, and σ is uniquely determined except suitable boundary and initial conditions, say,

$$\sigma(x, -t)=0, \qquad (\partial/\partial x)\sigma(\pm 1, z)=0, \tag{6.5}$$

Put (6.4) into (6.3) with (6.5), and integrate by parts, we have

$$F(\kappa, \theta)=\frac{\kappa}{g}(\kappa\cos^2\theta-g)\int_{-t}^{0}\int_{-1}^{1}\sigma(x, z)e^{\kappa z-i\kappa x\cos\theta}dxdz$$
$$+\int_{-1}^{1}\sigma(x, 0)e^{-i\kappa x\cos\theta}dx-\int_{-t}^{0}\Big[\sigma(x, z)e^{\kappa z-i\kappa x\cos\theta}\Big]_{x=-1}^{1}dz. \tag{6.6}$$

By the similar calculation, we have from (6.1),

$$\varphi(P)=\frac{1}{2\pi}(\partial/\partial z-\partial^2/g\partial x^2)\int_{-t}^{1}\int_{-1}^{1}\sigma(x', z')\frac{\partial}{\partial x'}[1/r(P, Q)-1/r(P, \bar{Q})]dx'dz'$$
$$+\frac{1}{2\pi}\int_{-1}^{1}\sigma(x', 0)\frac{\partial}{\partial x'}S(P, Q)\Big|_{z'=0}dx'+\frac{1}{2\pi g}\int_{-t}^{0}\Big[\sigma\frac{\partial^2}{\partial x'^2}S\Big]_{x'=-1}^{1}dz'. \tag{6.7}$$

Consequently, the regular wave system is determined by the values of σ on the fore and aft perpendiculars and the load water line.

If these values vanish on those lines, we have a waveless distribution, that is, when

$$\sigma=0, \text{ on } z=0,\ z=-t \text{ and } x=\pm 1;\ \partial\sigma/\partial x=0, \text{ on } x=\pm 1, \tag{6.8}$$

The total displacement of these distributions vanishes as the same way as in (5.11), namely, by the integration of (6.4) with (6.8) we have

$$\int_{-t}^{0}\int_{-1}^{1}\eta(x,z)dxdz=\int_{-1}^{1}\sigma(x,0)dx=0\,, \tag{6.9}$$

For example, we have a waveless system by putting

$$\sigma(x,z)=(1-x^2)^2z(1+z/t), \tag{6.10}$$

and $\eta(x, z)$ is given by (6.4).

When the speed is very low and g is very large, we have the approximate relation except near the end points $x=\pm1$,

$$\eta(x,z)\doteqdot(\partial/\partial z)\sigma(x,z), \tag{6.11}$$

These distributions are very usefull in application, allthough we have no waveless ship with displacement. Namely, we have had a very wide arbitrarity to determine the ship form with some resistance, because these waveless distributions do not change their wave resistance, if we add it to or subtract from any distribution.

Meanwhile, if we deal with the distribution in an infinitely long range, we have the waveless one with a finite displacement as follows.

Consider the function $\eta(x, z)$ is of the type,

$$\eta(x,z)=T(z)H(x), \tag{6.15}$$

and write

$$f(p)=\int_{-\infty}^{\infty}H(\kappa)e^{-ipx}dx. \tag{6.16}$$

Then, we have also the waveless condition as before,

$$f(p)=0, \qquad \text{for } p>g. \tag{6.17}$$

The relation (6.16) is of the Fourier transform, so that we may have its inverse transform as

$$H(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(p)e^{ipx}dp\,. \tag{6.18}$$

In the consequence, the condition (6.17) is satisfied, if we have a represetation of the next form,

$$H(x)=\frac{1}{2\pi}\int_{-g}^{g}f(p)e^{ipx}dp. \tag{6.19}$$

For example, put

$$f(p)=2\sqrt{\pi}\,\frac{\Gamma(\nu+1)}{\Gamma(\nu+1/2)}[1-(p/g)^2]^{\nu-\frac{1}{2}}, \tag{6.20}$$

Then, we have

$$H(x)=g\frac{\Gamma(\nu+1)}{(gx/2)^{\nu}}J_{\nu}(gx), \qquad \int_{-\infty}^{\infty}H(x)dx=2\sqrt{\pi}\frac{\Gamma(\nu+1)}{\Gamma(\nu+1/2)}, \tag{6.21}$$

In the foregoing discussion, we have assumed that the distribution is represented by an integrable function, but it is used to assumed that it is succesively differentiable in almost all cases up to the present.

This is a severe restriction especially for the mathematical point of view,[11)] but it is also true that the half breadth curve of the actual ship surface seems to be represented by such a function. Accordingly, we will give some formulas of such case in addition.

At first, all of the foregoing conclusions, of course, hold good.

Secondly, we have an interested formula with its wave resistance as follows.

Integrate (6.3) by parts assuming the differentiability, we have

$$F(g\sec^2\theta,\theta)=\frac{-1}{2g\kappa}\int_{-t}^{0}\int_{-1}^{1}f_1(x,z)e^{\kappa z-ipx}dx\,dz+\frac{1}{2\kappa}\int_{-1}^{1}[\eta(x,z)e^{\kappa z}]_{z=-1}^{1}e^{-ipx}dx$$
$$+\frac{1}{2g\kappa}\int_{-t}^{0}e^{\kappa z}[e^{-ipx}(\partial/\partial x+ip)\eta(x,z)]_{x=-1}^{1}dz, \tag{6.22}$$

where $\kappa=g\sec^2\theta$, $p=g\sec\theta$ and

$$f_n(x,z)=(\partial^2/\partial x^2+g\cdot\partial/\partial z)^n\eta(x,z), \qquad \text{for } n: \text{ positive integer}, \tag{6.23}$$

Continuing this process, assumed the function $f_n(x,z)$ tends to zero, we have finally in symbolic writing,

$$F(g\sec^2\theta,\theta)=\left[\left[e^{\kappa z-ipx}\left(\frac{1}{\kappa+\partial/\partial z}+\frac{g}{p^2+\partial^2/\partial x^2}\right)\left(\frac{ip+\partial/\partial x}{2g\kappa+g\cdot\partial/\partial z+\partial^2/\partial x^2}\right)\eta(x,z)\right]_{x=-1}^{1}\right]_{z=-t}^{0}, \tag{6.24}$$

Now, introduce a function μ by the equation.

$$\eta(x,z)=(\partial^2/\partial x^2+g^2)(\partial/\partial z+g)\mu(x,z), \tag{6.25}$$

put it into (6.24) and tend θ to zero, and we have

$$F(g,0)=\frac{g}{2}\left[[e^{gz-igx}(\partial/\partial x+ig)\mu(x,z)]_{x=-1}^{1}\right]_{z=-t}^{0}, \tag{6.26}$$

Hence, if μ and $\partial\mu/\partial x$ vanish at four end points, we get

$$F(g,0)=0, \tag{6.27}$$

If the speed is very low and g is very large, the wave resistance is determined by the value of F near the point $\theta=0$ approximately, so that we may hope to get a distribution having a small wave resistance by the method as the above.

It is a simple matter to get a function satifying the above condition, for an example, we have of the following types

$$\mu(x,z)=(1-x^2)^2\text{func.}(z), \qquad \mu(x,z)=z(t+z)\text{func.}(x), \tag{6.28}$$

Conclusion

We havě considered a waveless problem of the ship in steady motion on calm sea, and concluded as follows,

1. The velocity potential could be separated into two parts by the aid of the auxiliary function. The one is the part regular and vanishing at infinity, and the other the part containing the wave system in the rear.,
2. Consequently, we have been able to construct a waveless system considerably at ease.
3. It may also propose a usefull method to solve wave problems. For an example, we get the integral equation for solving the problem of a half immered vertical plate in two dimensional case.
4. The waveless pressure system is shown with examples in two and three dimension, and it is found that the total pressure is finite in the former case but vanishes in the latter.
5. The total displacement of a Mitchell type waveless ship is shown to be zero too.
6. Meanwhile, if the range of the singularity distribution extends to infinity, we can get a waveless system with a finite total pressure or displacement in three dimensional case.
7. For the Mitchell type ship represented by a successively differentiable doublet distribution, the formula of the wave resistance is deformed in a neat expression, and we have a simple method to find a distribution with a comparatively small wave resistance.

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