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# An Essay to the Theory of Ship Forms in View of the Wave-Making Resistance Theory

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**Abstract**

The author tries to verify two leading principles of Taylor's method of the series models experiment in research work of the residual, that is wave making, resistance of ships in view of the wave making resistance theory.

The one is the principle to deform the midship section keeping the sectional area curve unchanged without sacrifice of the wave resistance, and deduced theoretically from the invariant character of the wave resistance.

The other is the one to deform the shape of the sectional area curve with least sacrifice of the wave resistance, so that this one may be the same as to obtain the optimum ship form, and deduced from the solution of the minimum problem of the wave resistance mathematically.

**1. Introduction** The theory of the wave making resistance has been able to estimate unsatisfactorily the experimental values in their quantity except few cases by now. These defects of our theory, of course, might be caused merely from a mathematical incompleteness to be conquered in near future, but the theory which we have now used in itself has an approximate character so that we may not proceed any farther from some stage.

In spite of these defects, the theory has thrown bright light on characters of the wave making resistance of ships.

In the other hand, the experimental method of ship resistance research had established by D.W. Taylor in his famous book[1], and this has been almost only one guide to the practical problems, but none of its fundamental ideas has been examined by the theory since then.

The object of this paper is a trial to this problem, so that we may have better understandings and expect farther progress in this science.

For this reason and the theoretical difficulties well known, we consider only ships of the so-called displacement type by doublet distributions proportional to the breadth over the longitudinal vertical center plane of the ship. Accordingly, the results we will obtain in the following have merely the relative character and no quantitative basis.

**2. Taylor's method of experiment** First of all, we must remember the Taylor's idea[1]. He gives five factors affecting the resistance of a ship which has a given displacement and speed, namely,

1 length
2 area of midship section or prismatic coefficient
3 the ratio beam to draft
4 the shape of midship section
5 the details of shape toward the extremities

The first factor relates to relative importance between frictional and residual resistance, then we assume the length fixed too, because we consider only the residual or wave making resistance.

Now, he states that the third to fifth factor are of minor importance compared with the second factor which is most important as regards the residual resistance.

This idea, saying in more detail the longitudinal distribution of the displacement is dominant factor for the residual resistance, has been a leading principle in practice.

We call this hereafter the sectional area curve hypothesis, because we have not had proper theoretical basis for this idea.

On the ground based upon these experiences, he proposes the method of experiment by the so-called series models. He introduces a parent ship form which might be an optimum practically and derives from it various forms in two ways.

The one is to change the proportion of breadth and draft, but to keep its shape and fullness coefficient, so that the residual resistance may not change so much by the above hypothesis.

The other is inversely to change its shape and fullness coefficient by relocating ordinates of the sectional area curve. This might cause any change of the wave resistance, but it might be done naturally as the way in which the resistanc would not change so much as far as possible.

We will call these laws to deform the shape in these ways the deformation principle, then it will be easy to see that these may be deduced from variational principle mathematically. Moreover, the parent form may also be deduced from the variational calculus.

Namely, his method is essentially a variational one, so that we may have some contributions to these problems from the recent two papers[5),6)].

Before treating the variational problems, we will treat them by the classical way in two succeeding paragraphs.

**3. Sectional area curve and water line curve** Suppose a doublet distribution $\eta(x, z)$ over a rectangle on the $x$-$z$ plane, then its wave resistance is given by the formula,

$$R=\frac{4\rho g^4}{\pi}\int_0^{\frac{\pi}{2}} |F(g\sec^2\theta, \theta)|^2 \sec^5\theta\, d\theta\,, \tag{3.1}$$

where

$$F(\kappa,\theta)=\int_{-t}^{0}\int_{-1}^{1}\eta(x,z)\,e^{\kappa z-i\kappa x\cos\theta}\,dx\,dz\,, \tag{3.2}$$

we assume the velocity and the half length of the distribution as the unit, and $\rho$ is the density of the water, $g$ is the gravity constant in this unit system, then Froude number equals to $1/\sqrt{2g}$, and $t$ is the draft to the half length.

As usually done, we assume hereafter that the doublet strength $\eta(x,z)$ equals to the half breadth of ship considered approximately.

Now, assume the expansion

$$\left.\begin{aligned}\eta(x,z)&=P_0(1+2z/t)\,\eta_0(x)+P_1(1+2z/t)\,\eta_1(x)+\cdots\,,\\ \eta_n(x)&=\frac{2}{(2n+1)}\int_{-t}^{0}\eta(x,z)P_n(1+2z/t)\,dz/t\,,\end{aligned}\right\} \tag{3.3}$$

where $P_n$ means Legendre polynomial.

Put it into (3.2), then we have

$$F(\kappa,\theta)=\frac{1}{\kappa}[T_0(\kappa t)F_0(\kappa\cos\theta)+T_1(\kappa t)F_1(\kappa\cos\theta)+\cdots]\,, \tag{3.4}$$

where

$$F_n(\kappa\cos\theta)=\int_{-1}^{1}\eta_n(x)\,e^{-i\kappa x\cos\theta}\,dx\,, \tag{3.5}$$

and

$$T_n(\kappa t)=\kappa\int_{-t}^{0}P_n(1+2z/t)\,e^{\kappa z}dz=\sqrt{\pi\kappa t}\,e^{-\frac{\kappa t}{2}}I_{n+\frac{1}{2}}(\kappa t/2)\,, \tag{3.6}$$

because we have the expansion

$$\exp.\,(pu)=\sqrt{\frac{\pi}{2p}}\sum_{m=0}^{\infty}(2m+1)I_{m+\frac{1}{2}}(p)P_m(u)\,,$$

and the orthogonality of Legendre function, where $I_{m+\frac{1}{2}}$ means Bessel function with imaginary argument.

Then, we have the next approximations,

$$T_0(z)\doteqdot z\,,\quad T_n(z)\doteqdot 2(z/2)^{n+1}/[(2n+1)(2n-1)\cdots 1]\,,\qquad \text{for } z\doteqdot 0\,, \tag{3.7}$$

and

$$T_0(z)\doteqdot T_n(z)\doteqdot 1\,,\qquad \text{for } z\gg 1 \text{ and } n, \tag{3.8}$$

Now, if the velocity is high and the draft is small, the value of $g$ and $gt$ is small, so that we may neglect $T_1$ compared with $T_0$, and we have an approximation

$$F(\kappa,\theta)\doteqdot\frac{1}{\kappa}T_0(\kappa t)F_0(\kappa\cos\theta)\,, \tag{3.9}$$

with

$$F_0(\kappa\cos\theta)=\int_{-1}^{1}\eta_0(x)\,e^{-\kappa x\cos\theta}\,dx\,,\qquad \eta_0(x)=\frac{2}{t}\int_{-t}^{0}\eta(x,z)\,dz\,, \tag{3.10}$$

and

$$R\doteqdot\frac{4\rho g^2}{\pi}\int_0^{\frac{\pi}{2}} T_0^2(gt\sec^2\theta)|F_0(g\sec\theta)|^2\sec\theta\,d\theta. \tag{3.11}$$

Here, the function $\eta_0(x)$ is the sectional area divided by the draft, so that we may say that the wave sesistance at very high speed and of small draft may be determined approximately by the sectional area curve and the draft. Moreover, put (3.7) into (3.11), we have

$$R\doteqdot\frac{4\rho g^4t^2}{\pi}\int_0^{\frac{\pi}{2}}|F_0(g\sec\theta)|^2\sec^5\theta\,d\theta\,, \tag{3.12}$$

This integral diverges unless $\eta_0(x)$ and its derivatives at both end points vanish, but it has no more relation to the draft, and has been proposed by H. Maruo as the so-called slender ship theory[4].

On the contrary, if the velocity is very low and the draft is sufficiently large, that is, $gt$ is much larger than the unit, making use of (3.8), we have

$$F(\kappa,\theta)\doteqdot\frac{1}{\kappa}\int_{-1}^{1}\eta(x,0)\,e^{-i\kappa x\cos\theta}\,dx\equiv\frac{1}{\kappa}F_w(\kappa\cos\theta)\,, \tag{3.13}$$

because we see

$$\eta(x,0)=\sum_{n=0}^{\infty}P_n(1)\,\eta_n(x)=\sum_{n=0}^{\infty}\eta_n(x)\,,$$

and then

$$R\doteqdot\frac{4\rho g^2}{\pi}\int_0^{\frac{\pi}{2}}|F_w(g\sec\theta)|^2\sec\theta\,d\theta\,, \tag{3.14}$$

Hence, we have the well known character that the wave resistance in low speed is depend on the load water line curve $\eta(x,0)$ and not on the draft approximately.

In practical cases, the value of $gt$ at the service speed nearly equals to unit, so that we may apply the above approximations in the sense that the speed is high when it is higher than the service and vice versa.

Meanwhile, the former approximation, say the sectional area curve approximation, is very similar to the sectional area curve hypothesis, but its validity is confined itself in the limited speed range as we saw.

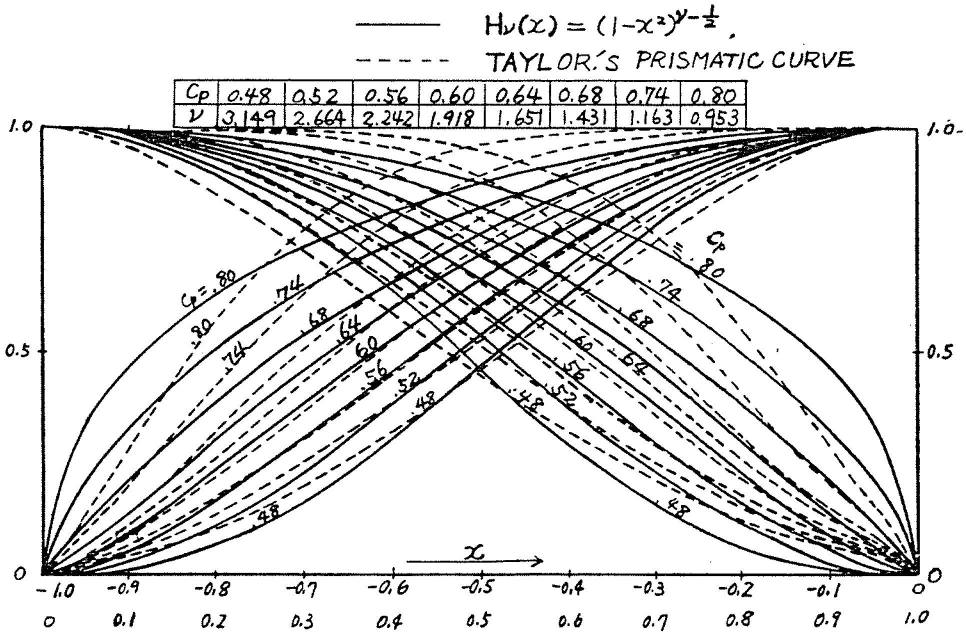
**4. Pseudo-Taylor series distribution** Let us see the other interesting example which is an direct application of the theory.

Consider the distribution

$$\left.\begin{aligned} \eta(x, z) &= \frac{A_0}{2t}(1-x^2)^{\nu-\frac{1}{2}}, \quad \nu > -1/2, && \text{for } 0 > z > -t, \\ &= 0, && \text{for } z < -t, \end{aligned}\right\} \tag{4.1}$$

where $A_0$ is the midship sectional area, and $\nu$ is a parameter.

This group of curves with various $\nu$ values is similar to Taylor's prismatic curves especially in medium prismatic coefficient as we see in Fig. 1, so that we would be able to deduce analogical characters of the wave resistance from this group.




| Cp | 0.48 | 0.52 | 0.56 | 0.60 | 0.64 | 0.68 | 0.74 | 0.80 |
|---|---|---|---|---|---|---|---|---|
| ν | 3.149 | 2.664 | 2.242 | 1.918 | 1.651 | 1.431 | 1.163 | 0.953 |

Fig. 1.

Now, put (4.1) in (3.2), we have

$$F(\kappa, \theta) = c_p \frac{A_0 T_0(\kappa t)}{\kappa t} \Lambda_\nu(\kappa \cos\theta), \tag{4.2}$$

where

$$\begin{aligned} \Lambda_\nu(x) &= \frac{\Gamma(\nu+1)}{(x/2)^\nu} J_\nu(x), \\ c_p &= \nabla/2A_0 = \frac{1}{2}\int_{-1}^{1}(1-x^2)^{\nu-\frac{1}{2}}dx = \frac{\sqrt{\pi}\,\Gamma(\nu+1/2)}{2\Gamma(\nu+1)}, \end{aligned} \tag{4.3}$$

$J_\nu$ means Bessel function, $\nabla$ the displacement and $c_p$ the prismatic coefficient.

Then, we have the resistance integral in the next form,

$$R/[\rho g\nabla(\nabla/L^3)]=\frac{8g^3}{\pi}\int_0^{\frac{\pi}{2}}\left[\frac{1}{gt}T_0(gt\sec^2\theta)\Lambda_\nu(g\sec\theta)\right]^2\sec\theta\,d\theta\,, \tag{4.4}$$

where $L$ means the length of ship.

In the practical cases, the value of $g$ is greater than $\nu$ and unit, so that we may have the approximation from the property of Bessel function, that is,

$$\Lambda_\nu(x)\doteqdot\frac{2\Gamma(\nu+1)}{\sqrt{\pi}\,(x/2)^{\nu+\frac{1}{2}}}\cos\left(x-\nu\frac{\pi}{2}-\frac{\pi}{4}\right),$$

using this formula and integrating (4.4) by the stationary phase method, we have

$$R/[\rho g\nabla(\nabla/L^3)]\doteqdot\frac{8}{\pi}\frac{\Gamma(2\nu+1)}{g^{2\nu-2}}\left[\frac{1-e^{-gt}}{gt}\right]^2\left[1+\frac{\sqrt{\pi}}{2\sqrt{g}\,c_p}\sin\left(2g+\frac{\pi}{4}-\nu\pi\right)\right],$$

$$\text{for } g\gg1 \text{ and } \nu, \tag{4.5}$$

This formula suggests characteristic properties of the wave resistance obtained by the series experiment.

Firstly, using the approximation (3.7) at sufficiently high speed, we see the wave resistance per unit displacement is nearly proportional to the displacement length ratio.

Secondly, the first factor in the above formula has a minimum at some value of $\nu$ in any fixed speed, namely, there is an optimum $\nu$, or $c_p$.

However, its value will be extremely small compared with the experiment, but this seems to be clear if we compare two groups with smaller value of $c_p$ in Fig. 1.

Thirdly, we see from the last factor that the amplitude of hump and hollow in the resistance is nearly proportional to Froude number and the inverse of $c_p$, and the speed of its hump and hollow depends on the speed and the prismatic coefficient.

**5. Deformation principle** As considered in paragraph 2, the first deformation principle which is a synonym of the sectional area curve hypothesis should be based upon the invariant character of the wave resistance.

We find it in the author's paper[5], so that we may follow and apply its results.

Suppose the function $\sigma(x,z)$ defined as the solution of the next differential equation,

$$\eta(x,z)=\left(\frac{\partial}{\partial z}-\frac{1}{g}\frac{\partial^2}{\partial x^2}\right)\sigma(x,z)\,, \tag{5.1}$$

with the boundary values

$$\sigma(x,-t)=\sigma(\pm1,z)=\sigma(x,0)=\frac{\partial}{\partial x}\sigma(\pm1,z)=0\,, \tag{5.2}$$

then the distribution does not leave the regular wave system and consequently has not the wave resistance, namely, the function $F$ defined by (3.2) vanishes.

Hence, the wave resistance of some distribution is invariant, however we add to or subtract from it such distribution multiplied by any constant.

We call this procedure the invariant deformation, and let us consider its property.

Suppose $\sigma(x, z)$ which has the boundary conditions (5.2), and represented as the next form,

$$\sigma(x, z)=X(x)T(\zeta), \qquad \zeta=(z+t)/t, \tag{5.3}$$

where we may not loose the generality by such partition of the function, and the conditions (5.2) are satisfied, whenever we have

$$X(\pm 1)=(d/dx)X(\pm 1)=T(1)=T(0)=0. \tag{5.4}$$

It is very easy to pick up functions satisfying the above conditions, but the function $\eta$ should be confined itself in somewhat narrow class.

Now, integrating (5.1) and using the above conditions, we have simply the sectional area

$$A_s(x)=\int_{-t}^{0}\eta(x, z)dz=-\frac{t}{g}X''(x)\int_{0}^{1}T(\zeta)d\zeta, \tag{5.5}$$

and the total displacement

$$\int_{-1}^{1}A_s(x)dx=0, \tag{5.6}$$

namely, we can not change the total displacement by this deformation, but can do the sectional area curve, and that this change of the area will be small if the speed is low and the draft is small.

Nextly, the moment of the sectional area about midship is also

$$\int_{-1}^{1}xA_s(x)dx=0, \tag{5.7}$$

which means that the position of the center of buoyancy can not be removable.

However, the water plane area is given as

$$A_w(z)=\int_{-1}^{1}\eta(x, z)dx=\frac{T'(\zeta)}{t}\int_{-1}^{1}X(x)dx, \tag{5.8}$$

and its moment about midship as

$$\int_{-1}^{1}x\eta(x, z)dx=\frac{T'(\zeta)}{t}\int_{-1}^{1}xX(x)dx. \tag{5.9}$$

The latter does not always vanish, that is, the position of the center of floatation can be removable.

Now, keeping these properties in mind, let us see examples.

For our purpose, consider here the functions

$$\left.\begin{aligned} X_0(x)&=\xi^4(1-\xi)^2\,,\\ X_1(x)&=1-4\xi^3+3\xi^4\,,\\ X_2(x)&=1-10\xi^3+15\xi^4-6\xi^5\,, \end{aligned}\right\} \tag{5.10}$$

where

$$\xi=\frac{x-b}{1-b}\,,\ \text{for}\ x>b,\quad \text{and}\quad \xi=\frac{-a-x}{1-a}\,,\ \text{for}\ x<-a,$$

$a$ and $b$ are some positive constants smaller than unit.

Moreover, let $T_1(\zeta)$ be the function of which derivatives are composed of three segments and have the next values

$$\left.\begin{aligned} &T_1(0)=T_1(1)=0\,,\qquad T_1'(0)=5\,,\\ &T_0'(0.2)=1\quad\text{and}\quad T_1'(0.4)=T_1'(1)=-1\,, \end{aligned}\right\} \tag{5.11}$$

and also

$$T_2(\zeta)=\zeta(1-\zeta)\,,$$

**Exampe 1)** Put $b=0.5$ and let us deform the quarter fore body using $X_0$ and $T_1$ at Froude number 0.212. We may deform a ship partly as far as the function as a whole

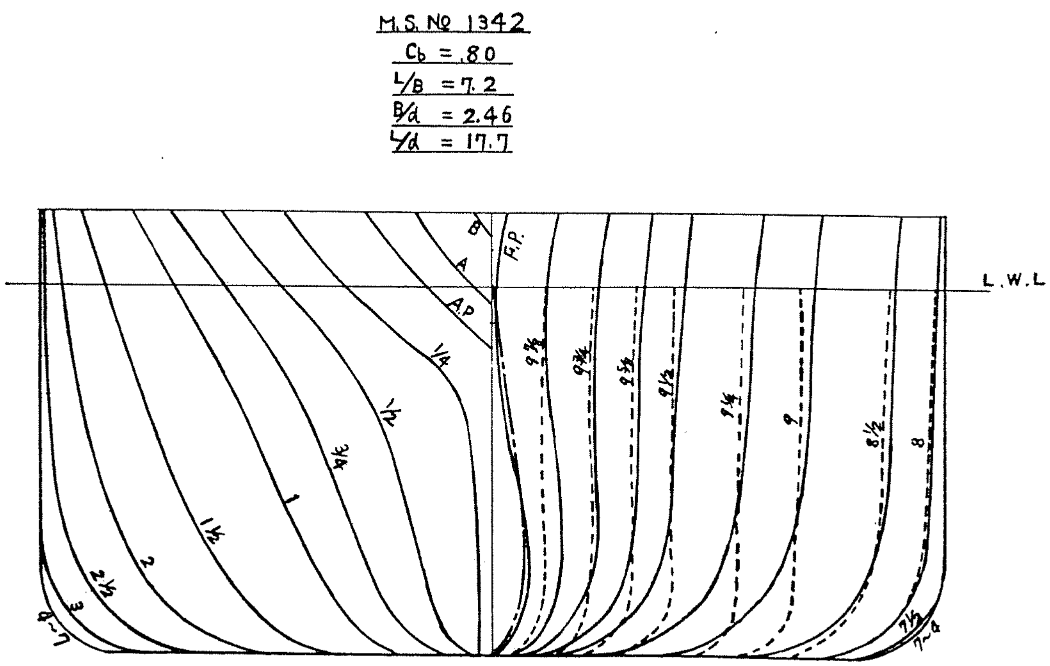



Fig. 2.

preserves the conditions (5.2) or (5.4).

The ship form showed in Fig. 2 by the full lines is of an oil tanker with the block coefficient 0.80[3], and the object is to deform this bulbous bow to an ordinary. Here, we replace the section form at F. P. to the chain line form which is similar to that in the area and the form and also represented by $T_1(\zeta)$, merely because the full line is too complex to be represented by a simple formula, and we may not loose its essential character by this substitution.

Now, calculate $\eta(x, z)$ by (5.1), and multiply an appropriate constant and add to the off-sets of the full lines as the way in which the area bounded by the chain line at F. P. vanishes, then we have a new body plan represented by the dotted lines which should have the same wave resistance as the full lines form in the range of our approximate theory.

This has an ordinary bow, as we expect, and that extremery U-shape frames and somewhat of Youkevitch form.

In a similar way as this procedure, we may deform a bulbous bow to an ordinary, and inversely we may draw a bulbous bow from the ordinary bow form as far as its area is not so much larger.

**Example 2)** Put $a=b=0.5$, and let us deform another oil tanker form[3] shown by the full lines in Fig. 3, using $X_1$, $T_1$ for the fore part and $X_2$, $T_1$ for the after part at Froude number 0.184, as the way in which the breadth is broadend about 10%.

The dotted lines give the deformed body plan as the same way as the above, and it

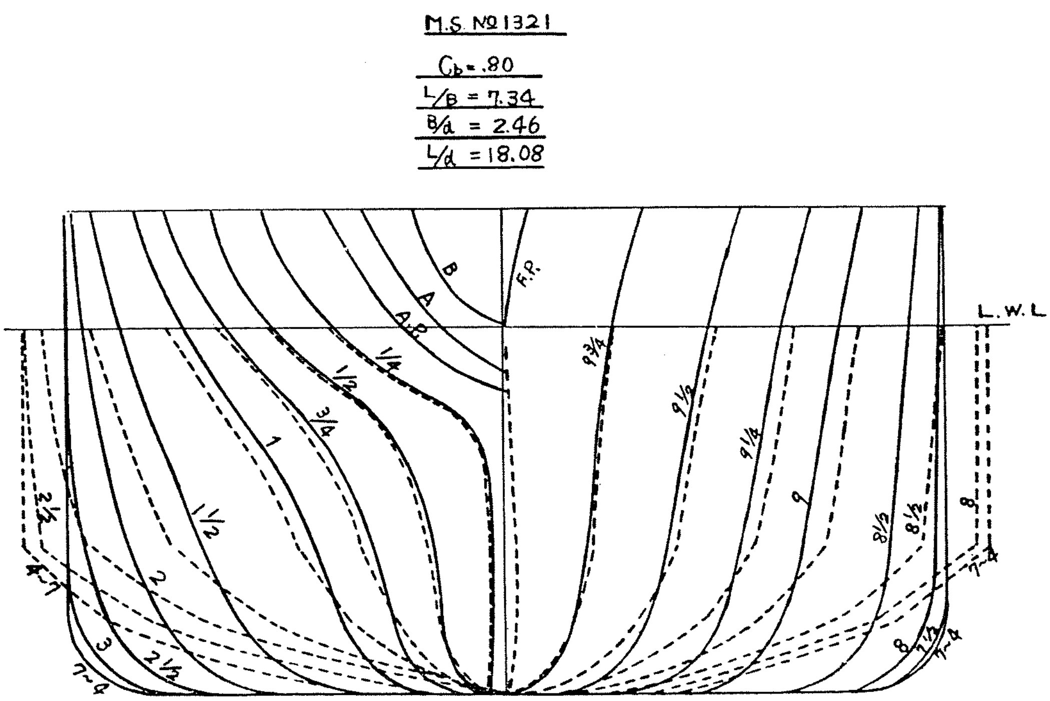



Fig. 3.

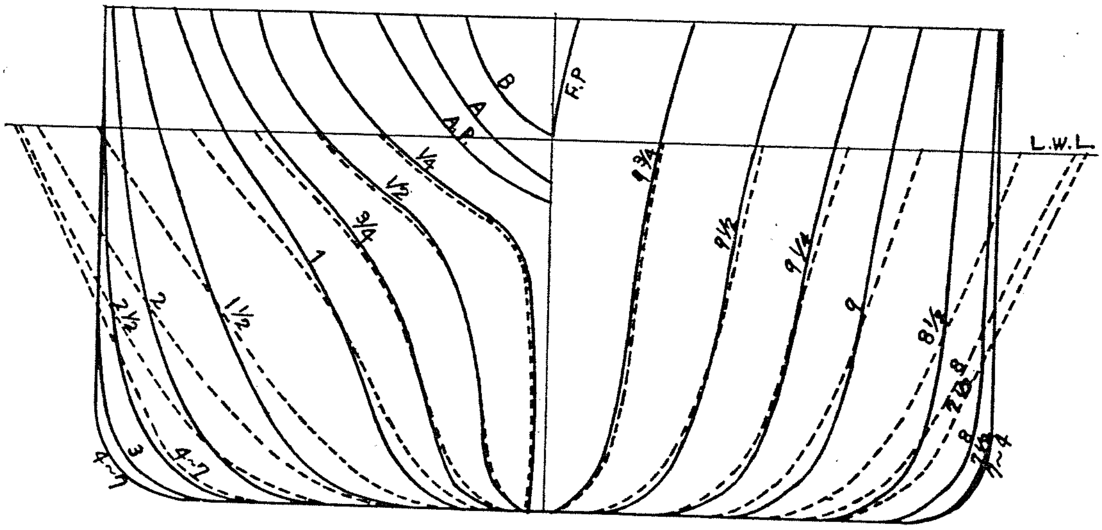



Fig. 4.

has a slight bulbous bow and the breadth broadend by about 10% but keeping the midship sectional area. This midship section with knuckles is choosed merely for simplicity of the computations, and so, of course, we may choose more practical fare curve at our disposal.

**Example 3)** Lastly, put $a=b=0.5$ and use $X_2$, $T_2$ for the both fore and after body of the same ship as the above at the same speed.

Then, we have dotted frame lines in Fig. 4, which are all inclined to the vertical and have the maximum beam broadend about 20% at the load water line.

In the last two examples, the deviation of the sectional area curve to the older amounts less than about 1% to the midship sectional area, so that we may appreciate the sound foundation of the so-called sectional area curve hypothesis in these theory.

Lastly, we must notice that these deformations are depend on the speed and the greater the speed the greater becomes the deformation, but it is not so serious defect as we conceive, because we have another foundation for this idea in high speed as we saw in paragraph 3.

In thus way, the displacement and the center of buoyancy are preserved in the first deformation principle, but they must be both free in the second principle. Accordingly, the problem of the second principle is to minimize the wave resistance variation for this removed volume, so that it is the same one as of the optimum ship form essentially.

**6. Optimum doublet distribution** Let us remember the properties of the optimum doublet distributions.

By the results of the author's paper[6)], the optimum doublet distribution which is distributed draftwise uniformly and tends to infinite depth, is given nearly by Mathieu function of the lowest order and moreover the larger the order of the function the more the wave resistance increases rapidly, and that this character would not change so much if the draft is not deep but rather very shallow.

Namely, this character of the optimum distribution states directly the second deformation principle.

Other writers[2)] write also that the difference of two optimum distribution suffices also a minimum condition, so that we may construct easily any other optimum one by simple interpolation or extrapolation.

Hence, the method of deformation used in practice which depends merely on the idea of the fairness of curve or the experiences would find a powerfull orientation from these theory, and that it should be noticed that its theory is the same as the one of the parent form which would be also an optimum.

Now, define the doublet distributions as follows,

$$\left.\begin{array}{ll} H_i(x)=\varphi_i(\theta)/\sin\theta\,, & x=-\cos\theta\,, \quad i=0,1,2,3,4\,. \\ \varphi_i(\theta)=\sum\limits_n \alpha_n^{(i)}\, ce_n(\theta,q)\,, & q=g^2/4\,, \end{array}\right\} \tag{6.1}$$

where

$$\left.\begin{array}{ll} \alpha_{2n}^{(0)}=a_{2n}^*\,, & \alpha_{2n+1}^{(0)}=0\,, \\ \alpha_{2n}^{(1)}=0\,, & \alpha_{2n+1}^{(1)}=a_{2n+1}^*\,, \\ \alpha_{2n}^{(2)}=b_{2n}^*\,, & \alpha_{2n+1}^{(2)}=0\,, \\ \alpha_{2n}^{(3)}=a_{2n}^*+\gamma b_{2n}^*\,, & \alpha_{2n+1}^{(3)}=0\,, \\ \alpha_{2n}^{(4)}=\alpha_{2n}^{(3)}\,, & \alpha_{2n+1}^{(4)}=-\dfrac{\varphi_3(0)}{\varphi_1(0)}a_{2n+1}^*\,, \end{array}\right| \tag{6.2}$$

the coefficients $a_n^*$, $b_n^*$ and $\gamma$ are defined and given in the former paper[6)], in which $a_{2n+1}^*$ are given here in the table.

For the practical purposes, it is preferable to represent by simple trigonometrical series,

Table

| g | Fr. | $c_{w_1}$ | $a_1^*$ | $a_3^*$ | $a_5^*$ | $a_7^*$ |
|---|---|---|---|---|---|---|
| 1 | 0.7071 | 5.4095 | 0.61656 | 0.63268 | 0.00053 | — |
| 2 | 0.5000 | 14.670 | 0.61952 | 0.16586 | 0.00595 | 0.00006 |
| $\sqrt{10}$ | 0.3976 | 10.849 | 0.64548 | 0.10145 | 0.01083 | 0.00032 |
| 4 | 0.3536 | 5.4129 | 0.74218 | 0.04071 | 0.00835 | 0.00041 |
| $\sqrt{24}$ | 0.3195 | 2.0412 | 0.88398 | 0.01451 | 0.00436 | 0.00034 |
| 6 | 0.2887 | 0.50936 | 1.06089 | 0.00589 | 0.00131 | 0.00018 |
| $\sqrt{50}$ | 0.2659 | 0.11541 | 1.22267 | 0.00379 | 0.00030 | 0.00007 |
| 8 | 0.2500 | 0.029499 | 1.35534 | 0.00310 | 0.00008 | 0.00003 |
| $\sqrt{80}$ | 0.2364 | 0.0069759 | 1.48492 | 0.00266 | 0.00003 | 0.00001 |
| 10 | 0.2236 | 0.0013192 | 1.62408 | 0.00229 | 0.00002 | 0.00000 |

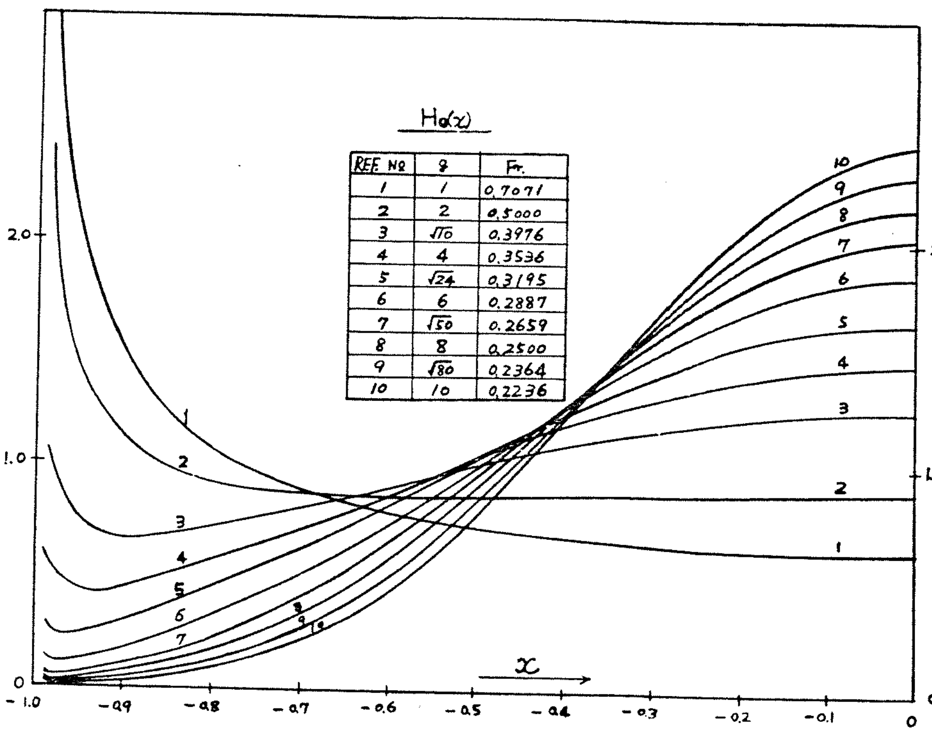
then we may rewrite (6.1) as follows,

$$\left.\begin{aligned} \varphi_i(\theta) &= \sum_m a_m^{(i)} \cos m\theta\,, \\ a_m^{(i)} &= \sum_n \alpha_n^{(i)} A_m^{(n)}\,, \end{aligned}\right\} \tag{6.3}$$

where $A_n^{(m)}$ is the coefficient of $\cos m\theta$ in trigonometrical expansion of Mathieu function of $n$-th order.

Thence, it is a matter of simple computation to obtain these distributions, and they are shown in figures. Let us see them.

**The case $i=0$)** $H_0$ shown in Fig. 5 are optimum at each speed, when the area of these curve, that is approximately the water plane area, is given. Its coefficient of area is given as $\delta \doteqdot \sqrt{\pi}$ Fr. in usual speed, namely the lower the smaller this optimum value becomes, or in other words, the finer the smaller the wave resistance in low speed.




| REF. No | g | Fr. |
|---|---|---|
| 1 | 1 | 0.7071 |
| 2 | 2 | 0.5000 |
| 3 | √10 | 0.3976 |
| 4 | 4 | 0.3536 |
| 5 | √24 | 0.3195 |
| 6 | 6 | 0.2887 |
| 7 | √50 | 0.2659 |
| 8 | 8 | 0.2500 |
| 9 | √80 | 0.2364 |
| 10 | 10 | 0.2236 |

Fig. 5.

**The case $i=1$)** In general, the optimum distribution should be symmetrical about midship, but if we hope to obtain the distribution not symmetrical, it is optimum to add this anti-symmetrical distribution $H_1$ shown in Fig. 6.

For example, add this to $H_0$ so as to vanish at A. P., then we have $H_0^*$ shown in Fig. 12. These curves have fine aft shape except in high speed, and may be efficient for the frictional resistance, but the wave resistance increases nearly twice of the former.

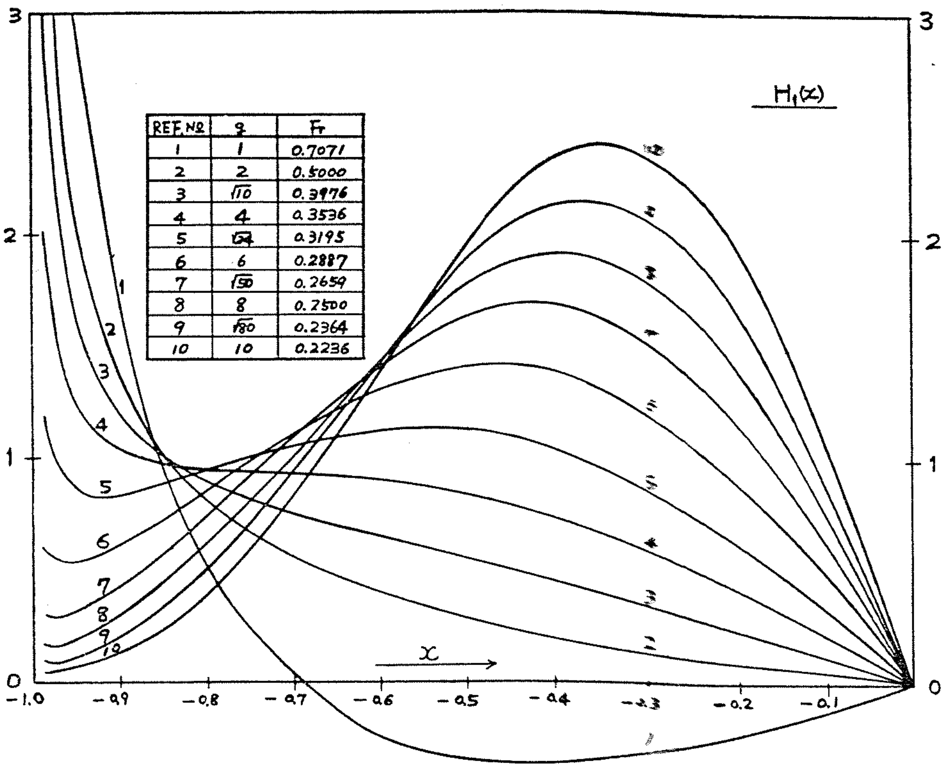

H₁(x)
REF.No | q | Fr
1 | 1 | 0.7071
2 | 2 | 0.5000
3 | √10 | 0.3976
4 | 4 | 0.3536
5 | √24 | 0.3195
6 | 6 | 0.2887
7 | √50 | 0.2659
8 | 8 | 0.2500
9 | √80 | 0.2364
10 | 10 | 0.2236
x
3
2
1
0
-1.0
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1

Fig. 6.

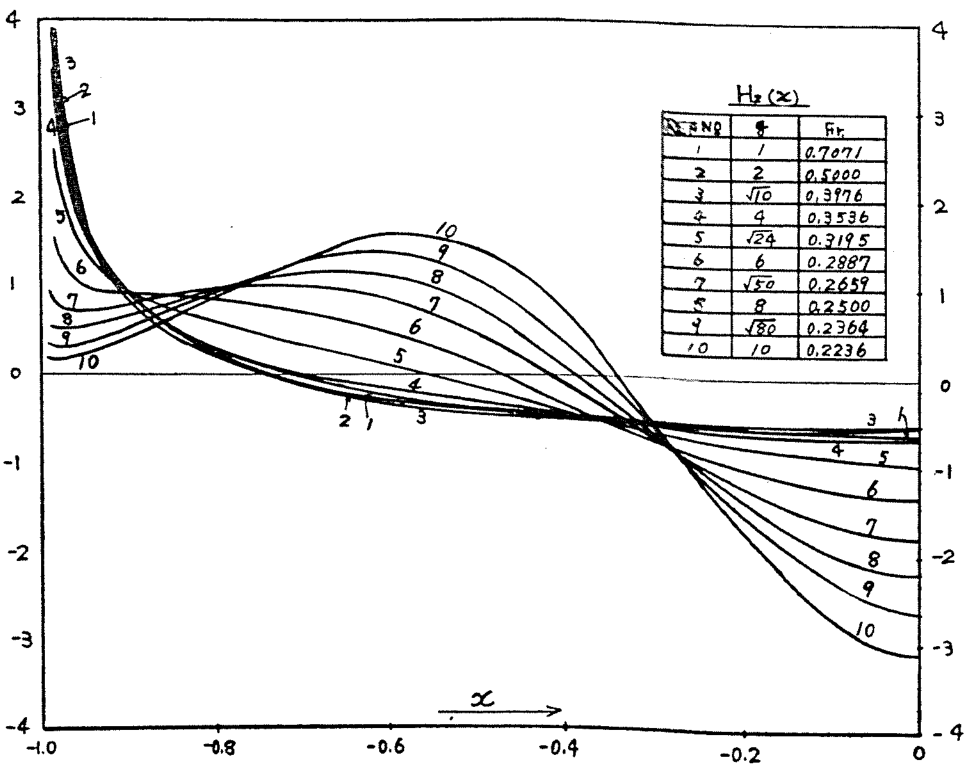
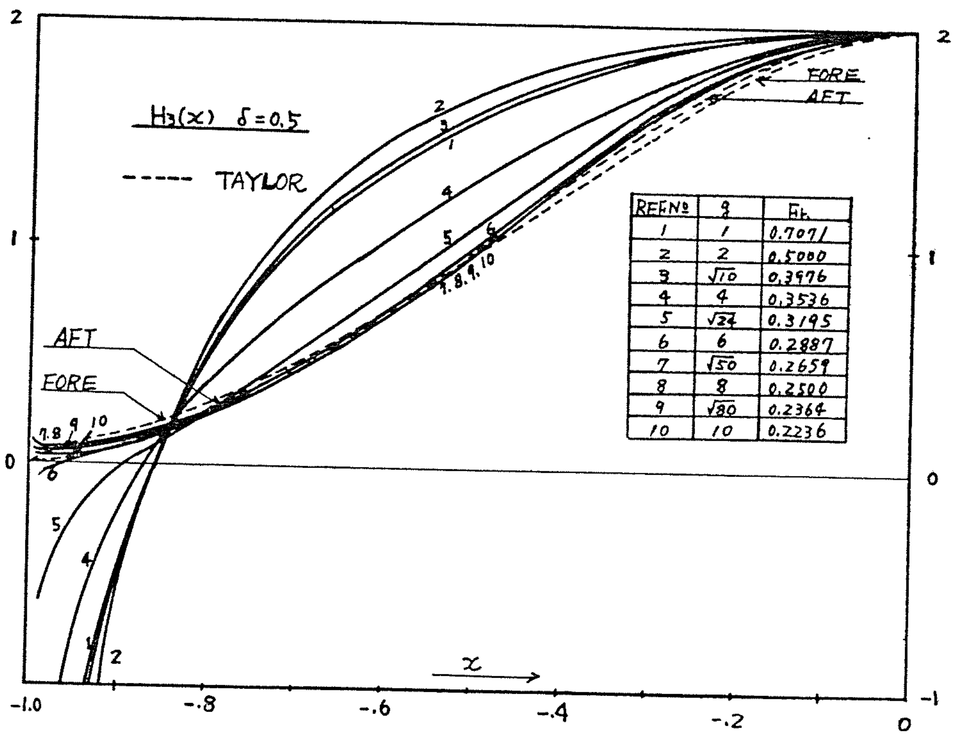
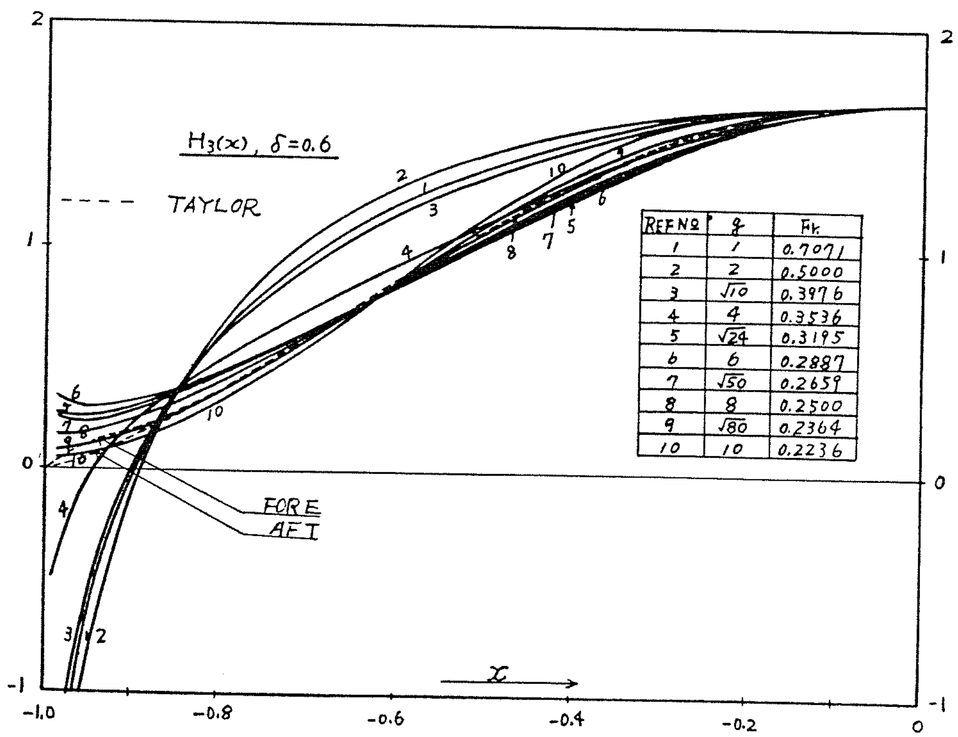

H₂(x)
REF.No | q | Fr
1 | 1 | 0.7071
2 | 2 | 0.5000
3 | √10 | 0.3976
4 | 4 | 0.3536
5 | √24 | 0.3195
6 | 6 | 0.2887
7 | √50 | 0.2659
8 | 8 | 0.2500
9 | √80 | 0.2364
10 | 10 | 0.2236
x
4
3
2
1
0
-1
-2
-3
-4
-1.0
-0.8
-0.6
-0.4
-0.2

Fig. 7.




| REF No | g | Fr. |
|---|---|---|
| 1 | 1 | 0.7071 |
| 2 | 2 | 0.5000 |
| 3 | √10 | 0.3976 |
| 4 | 4 | 0.3536 |
| 5 | √24 | 0.3195 |
| 6 | 6 | 0.2887 |
| 7 | √50 | 0.2659 |
| 8 | 8 | 0.2500 |
| 9 | √80 | 0.2364 |
| 10 | 10 | 0.2236 |

Fig. 8.




| REF No | g | Fr. |
|---|---|---|
| 1 | 1 | 0.7071 |
| 2 | 2 | 0.5000 |
| 3 | √10 | 0.3976 |
| 4 | 4 | 0.3536 |
| 5 | √24 | 0.3195 |
| 6 | 6 | 0.2887 |
| 7 | √50 | 0.2659 |
| 8 | 8 | 0.2500 |
| 9 | √80 | 0.2364 |
| 10 | 10 | 0.2236 |

Fig. 9.

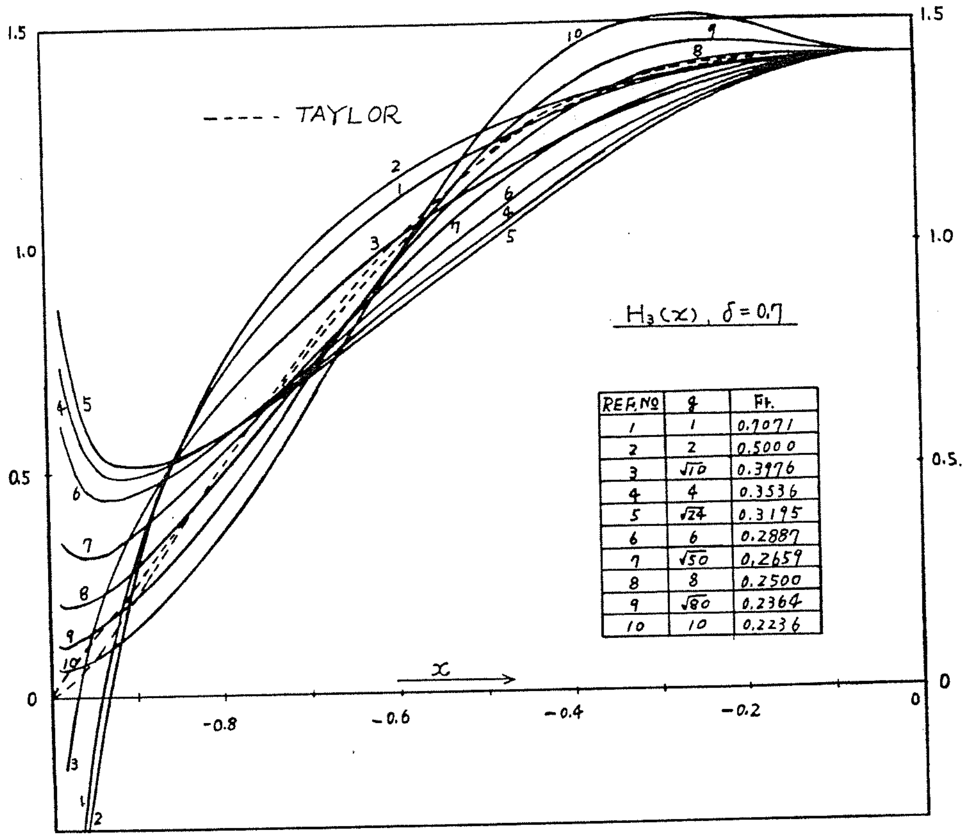

TAYLOR
$H_3(x)$, $\delta = 0.7$
REF.No | q | Fr.
1 | 1 | 0.7071
2 | 2 | 0.5000
3 | √10 | 0.3976
4 | 4 | 0.3536
5 | √24 | 0.3195
6 | 6 | 0.2887
7 | √50 | 0.2659
8 | 8 | 0.2500
9 | √80 | 0.2364
10 | 10 | 0.2236
x
1.5
1.0
0.5
0
-0.8
-0.6
-0.4
-0.2

Fig. 10.

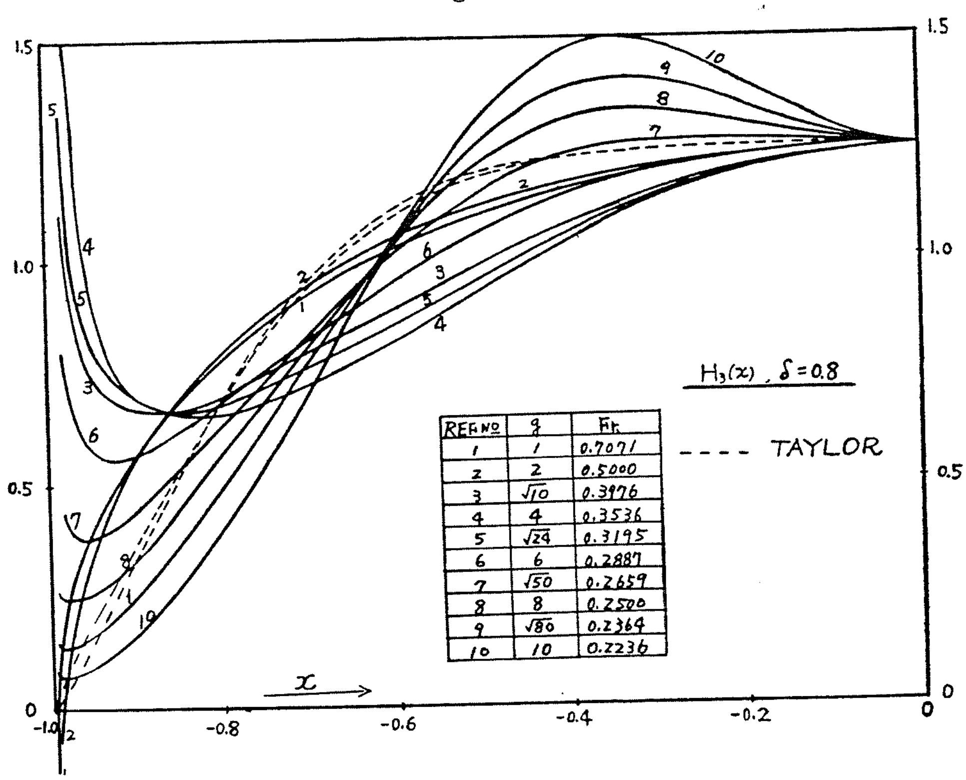
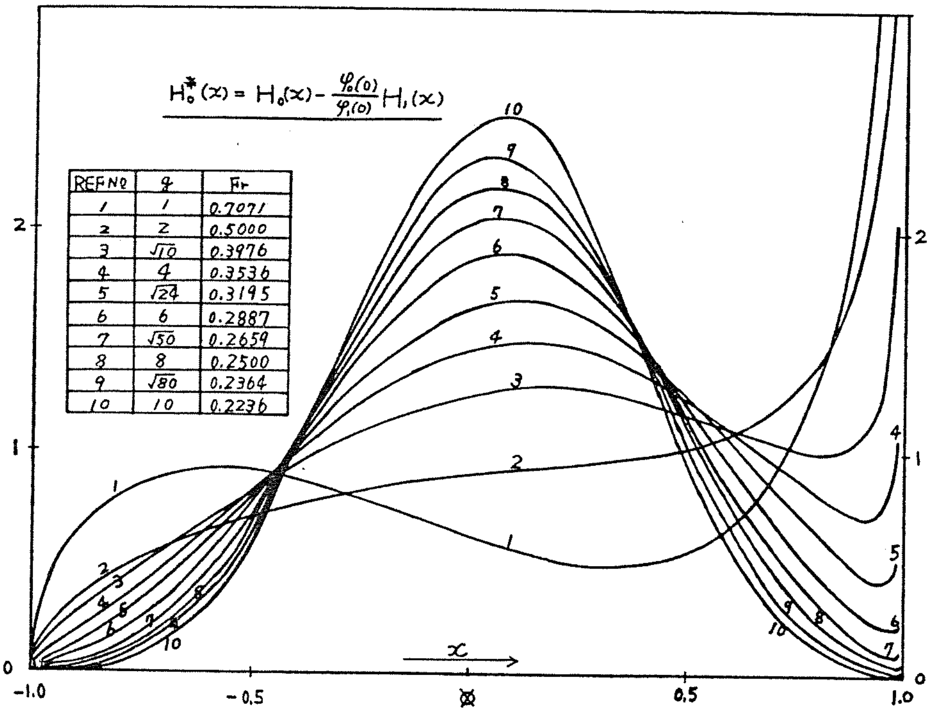
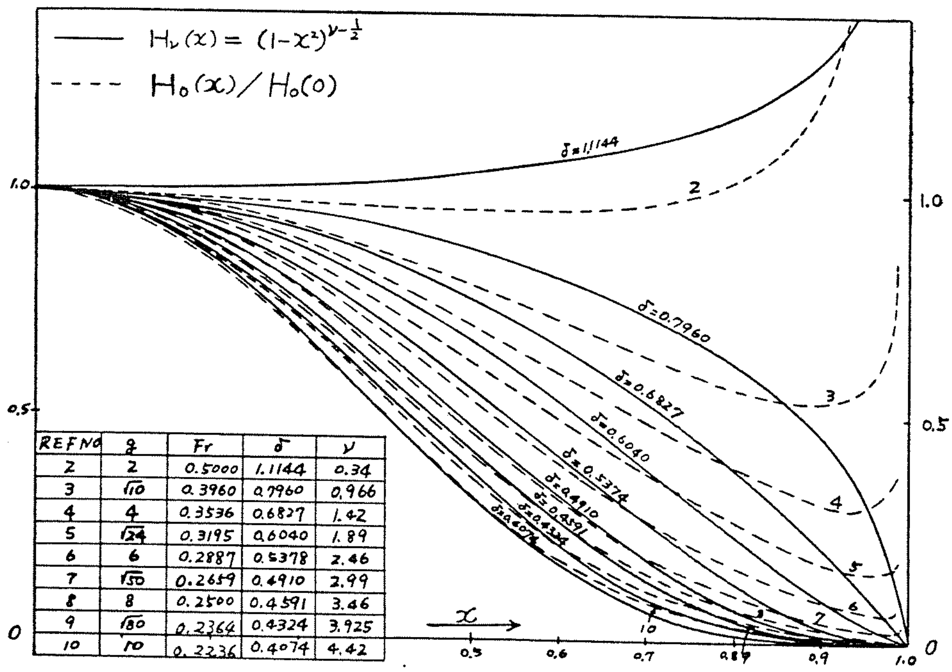

$H_3(x)$, $\delta = 0.8$
TAYLOR
REF.No | q | Fr.
1 | 1 | 0.7071
2 | 2 | 0.5000
3 | √10 | 0.3976
4 | 4 | 0.3536
5 | √24 | 0.3195
6 | 6 | 0.2887
7 | √50 | 0.2659
8 | 8 | 0.2500
9 | √80 | 0.2364
10 | 10 | 0.2236
x
1.5
1.0
0.5
0
-1.0
-0.8
-0.6
-0.4
-0.2

Fig. 11.




| REF NO | q | Fr |
|---|---|---|
| 1 | 1 | 0.7071 |
| 2 | 2 | 0.5000 |
| 3 | $\sqrt{10}$ | 0.3976 |
| 4 | 4 | 0.3536 |
| 5 | $\sqrt{24}$ | 0.3195 |
| 6 | 6 | 0.2887 |
| 7 | $\sqrt{50}$ | 0.2659 |
| 8 | 8 | 0.2500 |
| 9 | $\sqrt{80}$ | 0.2364 |
| 10 | 10 | 0.2236 |

Fig. 12.




| REF NO | q | Fr | δ | ν |
|---|---|---|---|---|
| 2 | 2 | 0.5000 | 1.1144 | 0.34 |
| 3 | $\sqrt{10}$ | 0.3960 | 0.7960 | 0.966 |
| 4 | 4 | 0.3536 | 0.6827 | 1.42 |
| 5 | $\sqrt{24}$ | 0.3195 | 0.6040 | 1.89 |
| 6 | 6 | 0.2887 | 0.5378 | 2.46 |
| 7 | $\sqrt{50}$ | 0.2659 | 0.4910 | 2.99 |
| 8 | 8 | 0.2500 | 0.4591 | 3.46 |
| 9 | $\sqrt{80}$ | 0.2364 | 0.4324 | 3.925 |
| 10 | 10 | 0.2236 | 0.4074 | 4.42 |

Fig. 13.

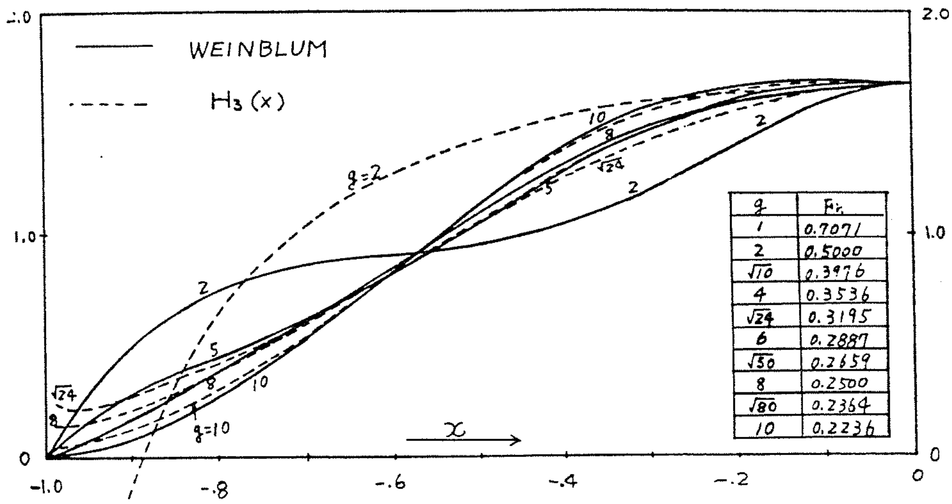

WEINBLUM
H3 (x)
x →
g | Fr.
1 | 0.7071
2 | 0.5000
√10 | 0.3976
4 | 0.3536
√24 | 0.3195
6 | 0.2887
√50 | 0.2659
8 | 0.2500
√80 | 0.2364
10 | 0.2236
g=2
g=10

Fig. 14.

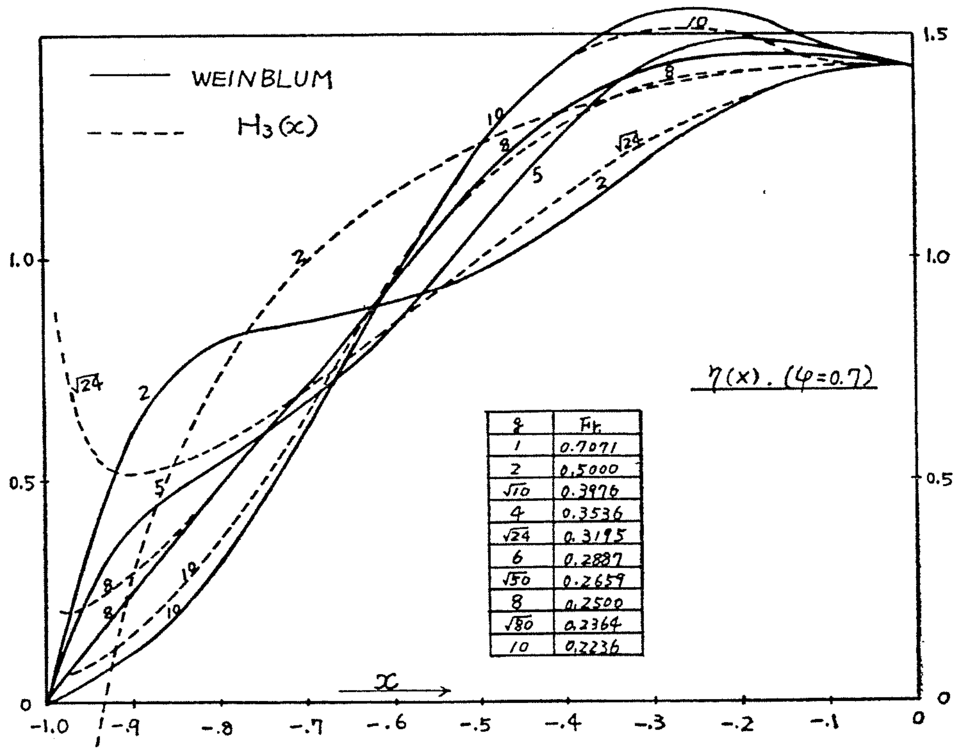

WEINBLUM
H3 (x)
η(x). (φ=0.7)
x →
g | Fr.
1 | 0.7071
2 | 0.5000
√10 | 0.3976
4 | 0.3536
√24 | 0.3195
6 | 0.2887
√50 | 0.2659
8 | 0.2500
√80 | 0.2364
10 | 0.2236

Fig. 15.

**The case $i=2$)** Since $H_0$ are too fine to the practical application in low speed, it will be necessary to shift the displacement to the fore and aft from the middle part.

The distributions $H_2$ shown in Fig. 7 give the mean for this purpose with the minimum sacrifice of the wave resistance.

**The case $i=3$)** $H_3$ shown in Fig. 8 to 11 are computed so as to have a given area coefficient by the above method. Here, we see in these figures that some of them take negative value in some portion.

Although the distribution having negative value has no practical application, yet we may also find that such cases are all when the given area coefficient is smaller than the optimum one at the very speed, so that it may be no serious defect. Namely, if we can choose larger $\delta$ with smaller resistance, why we will choose smaller $\delta$ with larger resistance?

**The case $i=4$)** Add $H_1$ to $H_3$ so as to vanish at A. P., then we have $H_4$, but it is found that the distribution thus obtained has almost always negative value but only in the case when the given $\delta$ equals nearly to the optimum. Accordingly, we do not shown them at all, but $H_0^*$ already described in Fig. 12.

Now, let us see Fig. 14 and 15, in which we see G. Weinblum's results of similar computation[2)] compared with our results.

In his computations, the draft is assumed one twentieth of the length, but both distributions are very similar except when his curve has a distinguished swan neck, that is the symptom that taking fuller form the wave resistance may be smaller.

Nextly, we show Taylor's prismatic curves in Fig. 8 to 10 compared with $H_3$. These all curves show the similarity between both curves especially in medium fullness coefficient, in such cases we find a remarkable correspondence in spite of our assumptions unreasonable to compare.

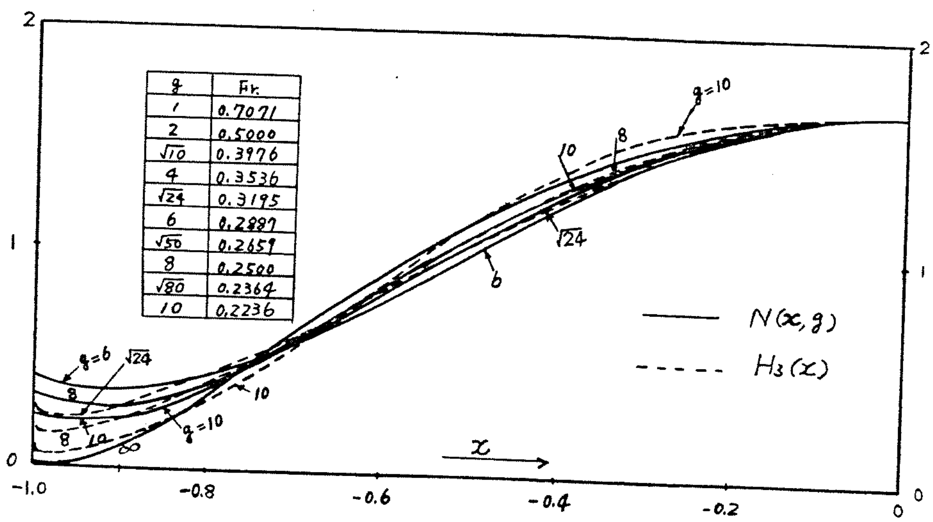



Fig. 16.

Lastly, it is interesting here to remember that the distributions defined in (4.1) are also similar to the optimum ones in very low speed as we see in Fig. 13, and also the distributions defined in the following and shown in Fig. 16, which are similar to the optimum but not corresponding speed, where we should remember that these ones have vanishing transverse wave at the given speed[5].

$$\eta(x, g)=a_0\left[M(x)+\frac{1}{g^2}M''(x)\right]+a_2\left[N(x)+\frac{1}{g^2}N''(x)\right], \tag{6.4}$$

where

$$M(x)=(1-x^2)^2 \quad \text{and} \quad N(x)=x^2(1-x^2)^2, \tag{6.5}$$

and $a_0$ and $a_2$ are determined so as to $\eta(0, g)=1/0.6$.

Thus, we may have a clear concept of the optimum distribution and the second deformation principle, and that may conclude that such properties correspond to the experimental results characteristically in spite of our assumptions.

**7. Conclusions** We may summary the conclusions as follows.

1) We may say that the principles of Taylor's method of the series model experiment will be founded on the means to deform a ship shape so as to keep its wave resistance invariant or least variation.

2) The first principle to keep the wave resistance invariant, of which Taylor says that the resistance is determined nearly by the midship sectional area, may be verified by the theorem which gives wave free distribution except the shape in the extremities, but this detailed shape affects also to the resistance as Taylor says, so that we may have more deep understanding into the wave resistance including the last factor.

3) The second principle to keep the wave resistance least variation may be deduced from the solution of the minimum problem as a matter of course, so that it will be the same problem as the optimum ship form.

The theory of the optimum doublet distribution may give a simple method to deform the ship shape in such a way, and at the same time to obtain the optimum form, but its results should be understand characteristically because of its approximation with regard to the correspondence between the ship form and the doublet distribution.

Thus, the theory of the wave resistance seems very usefull when we are combining it to the method of the experiment, and we may expect any future progress in this field.

Concluding this short essay, the author must thank with his heart to Prof. Inui, Maruo and Yamasaki for their kind persuations and discussions.

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