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ON THE MINIMUM WAVE RESISTANCE OF SHIPS WITH INFINITE DRAFT

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1. Introduction

The minimum problem of the wave resistance of ships has generally no solution or at least no unique one analytically except special cases.[8]

One of such special cases is of ships with infinite draft represented by draftwise uniform doublet distribution over the vertical center line plane.[7]

In this case, the existence, the uniqueness and characters of the solution were given by Professor Karp and others.[7]

They solved the integral equation numerically, and this was an easy way but difficult to secure a necessary accuracy especially in low speed where the wave resistance becomes very small.

In the other way, the analytical treatment is not only simple too but supplies very many interesting knowledges so far as we have complete tables of necessary functions.

In this paper, the author presents such treatment of the problem with the aid of the text and tables of Mathieu functions which he has been able to use.[4,5]

2. Wave Resistance Formula

Consider the uniform flow with unit velocity flowing from the positive direction of x-axis down to the negative and the doublet of which axis directs to the positive x-axis and distributing draftwise uniformly over the plane $|x| \leqq 1$.

Then, the wave making resistance of this doublet distribution is given as follows.[7,8]

$$R = \rho g^2 \bar{B}^2/\pi \int_0^{\pi/2} |F(g \sec\theta)|^2 \sec\theta \, d\theta \, , \tag{2.1}$$

with

$$F(p) = \int_{-1}^{1} H(x) \exp(-ipx) \, dx \, , \tag{2.2}$$

where ρ means the water density, g the gravity constant in this unit system and $|F|$ the absolute value of F .

The function $\bar{B}H(x)$ represents the doublet strength except the constant multiplier and equals nearly to the breadth of the ship considered by well-known approximation.

Moreover, $\bar{B}$ stands for the mean breadth, so that the mean value of $H(x)$ should be unit, namely,

$$(1/2) \int_{-1}^{1} H(x)\,dx = 1 . \tag{2.3}$$

Changing the order of integration, the formula (2.1) can be written as

$$Cw = 8R/(\rho\bar{B}^2) = 8g^2 \int_{-1}^{1} H(x)\Gamma(x)\,dx , \tag{2.4}$$

with

$$\Gamma(x) = -(1/2) \int_{-1}^{1} H(x')Y_o(g|x-x'|)\,dx' , \tag{2.5}$$

where Y_o is the Bessel function of the second species.

Taking the variation with respect to $H(x)$ in this formula and neglecting higher order terms, we have

$$\Delta Cw = 16g^2\Gamma(x)[\Delta H(x)\Delta x] . \tag{2.6}$$

Namely, the variation of the wave resistance with small deformation of the water plane area is proportional to the function $\Gamma(x)$.

For this property, we name it the influence function following to E. Hogner.(3)

Now, taking the class of functions considered by S. Karp and others, assume the next expansion in series of Mathieu functions, hereafter we follow the notations of McLachlan's text,(4)

$$H(x) = \varphi(\Theta)/\sin\Theta , \quad x = -\cos\Theta , \tag{2.7}$$

and

$$\varphi(\Theta) = \sum_{n=o}^{\infty} a_n ce_n(\Theta,q), \quad q = g^2/4 . \tag{2.8}$$

Putting this expansion in (2.5) and (2.4), we have

$$\Gamma(x) = \sum_{n=0}^{\infty} \mu_n a_n ce_n(\Theta, q) \ , \tag{2.9}$$

and

$$Cw = 4\pi g^2 \sum_{n=0}^{\infty} \mu_n, a_n^2 \ , \tag{2.10}$$

where

$$\mu_{2n} = -(\pi/2)[A_0^{(2n)}/ce_{2n}(\pi/2,q)]^2 Fey_{2n}(0,q)/ce_{2n}(0,q),$$

$$\mu_{2n+1} = -(\pi/2)[\sqrt{q}A_0^{(2n+1)}/ce'_{2n+1}(\pi/2,q)]^2 Fey_{2n+1}(0,q)/ce_{2n+1}(0,q), \tag{2.11}$$

because of the orthogonality of functions.(4,8)

The form of the right hand side of (2.10) reduces the problem to simple calculation.

Some value of μ_n are shown in Table 1.

3. Minimum Problem I(8)

Firstly, consider the problem to minimize the wave resistance when the ship length, speed and water plane area are given, or in other words Froude number ($Fr. = 1/\sqrt{2g}$) and the mean breadth $\overline{B}$ are given.

This means to minimize Cw of (2.4) or (2.10) under the condition (2.3), which is rewritten as follows, putting the expansion (2.7) and (2.8),

$$\sum_{n=0}^{\infty} a_{2n} A_0^{(2n)} = 2/\pi \ , \tag{3.1}$$

Thence, making use of Lagrange's method, we may have the solution as

$$a_{2n} = 2\lambda A_0^{(2n)}/\mu_{2n} = a_{2n}^{*} \ , \quad a_{2n+1} = 0 \ , \tag{3.2}$$

TABLE 1

		1	4	10	16	24	36	50	64	80	100
	g	1	2	$\sqrt{10}$	4	$\sqrt{24}$	6	$\sqrt{50}$	8	$\sqrt{80}$	10
	Fr.	0.7071	0.5000	0.3976	0.3536	0.3195	0.2887	0.2659	0.2500	0.2364	0.2236
	μ_0	0.644094	0.998125(1)	0.814806(2)	0.132126(2)	0.193168(3)	0.189457(4)	0.202547(5)	0.294995(6)	0.419816(7)	0.478425(8)
	μ_1	1.096150	0.732746	0.194796	0.470898(1)	0.851693(2)	0.992818(3)	0.122314(3)	0.199029(4)	0.313925(5)	0.397238(6)
	μ_2	0.547239	0.686380	0.662995	0.397181	0.138960	0.234523(1)	0.342655(2)	0.624663(3)	0.109448(3)	0.154365(4)
	μ_3	0.344356	0.385909	0.500315	0.582640	0.510346	0.227603	0.547913(1)	0.120592(1)	0.237545(2)	0.373999(3)
	μ_4	0.254270	0.268533	0.306237	0.360255	0.448653	0.495383	0.320119	0.128702	0.344321(1)	0.634796(2)
	μ_5	0.202113	0.208842	0.224475	0.244169	0.279845	0.358126	0.442317	0.402119	0.230739	0.712211(1)
	μ_6	0.167868	0.171617	0.179844	0.189269	0.204374	0.235323	0.290597	0.362817	0.406359	0.305693
	μ_7	-	0.145917	0.150846	0.156245	0.164335	0.179028	0.202103	01235452	0.290416	0.363293
	μ_8	-	0.127026	0.130230	0.133654	0.138613	0.147071	0.159004	0.174202	0.197839	0.241970
	C_{w0}	6.5518	4.4226	1.1384	0.34299	0.85324(1)	0.14156(1)	0.23083(2)	0.46094(3)	0.87171(4)	0.13191(4)
	C_{w1}	5.410	14.670	10.849	5.413	2.041	0.5094	0.1154	0.2950(1)	0.6976(2)	1.319(3)
	C_{w2}	2.790	12.73	23.60	20.36	12.02	4.521	1.441	0.4719	0.1398	0.0330
	δ_0	1.606	1.114	0.7960	0.6827	0.6039	0.5378	0.4910	0.4591	0.4323	0.4074
	m_0	0.7109	0.6358	0.5371	0.4837	0.4401	0.3997	0.3694	0.3480	0.3298	0.3123
C_w	δ = 0.5	20.26	53.46	46.35	12.26	1.639	0.6213(1)	0.2905(2)	0.3574(2)	0.2083(2)	0.7188(3)
	0.6	14.43	28.29	14.90	2.038	0.8688(1)	0.1046	0.6283(1)	0.2610(1)	0.8602(2)	0.2134(2)
	0.7	11.24	15.80	3.563	0.3974	0.7623	0.4658	0.1647	0.5553(1)	0.1603(1)	0.3609(2)
	0.8	9.395	9.438	1.142	2.261	2.224	0.9176	0.2758	0.8488(1)	0.2312(1)	0.4970(2)
γ	δ = 0.5	-2.217	-1.963	-1.384	-0.7648	-0.3590	-0.1025	0.0204	0.0812	0.1196	0.1462
	0.6	-1.680	-1.369	-0.7638	-0.2885	-0.0114	0.1407	0.2049	0.2331	0.2471	0.2534
	0.7	-1.297	-0.9456	-0.3206	0.0517	0.2374	0.3145	0.3367	0.3416	0.3381	0.3300
	0.8	-1.010	-0.6277	0.0118	0.3069	0.4240	0.4448	0.4356	0.4230	0.4064	0.3874

The numbers (n) in parentheses indicate that the results must be multiplied by 10^{-n}, for example, 0.3441(2) means 0.003441.

TABLE 2

4q Fr.	1 0.7071	4 0.5000	10 0.3976	16 0.3436	24 0.3195	36 0.2887	50 0.2659	64 0.2500	80 0.2364	100 0.2236
a*o	0.89565	0.93186	1.05195	1.13513	1.20958	1.28477	1.34651	1.39366	1.43700	1.48113
a*1	0.61656	0.61952	0.64548	0.74218	0.88398	1.06089	1.22267	1.35534	1.48492	1.62408
a*2	0.09256	0.04368	0.00792	0.00286	0.00141	0.00088	0.00065	0.00052	0.00043	0.00035
a*3	0.63268	0.16586	0.10145	0.04071	0.01451	0.00589	0.00379	0.00310	0.00266	0.00229
a*4	0.00105	0.00268	0.00151	0.00061	0.00017	0.00003	0.00001	0.00000	0.00000	0.00000
a*5	0.00053	0.00595	0.01083	0.00835	0.00436	0.00131	0.00030	0.00008	0.00003	0.00002
a*6	0.00001	0.00003	0.00005	0.00039	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000
a*7	-	0.00006	0.00032	0.00041	0.00034	0.00018	0.00007	0.00003	0.00001	0.00000
b*0	-0.05567	-0.19362	-0.32323	-0.38490	-0.45604	-0.55957	-0.67128	-0.77320	-0.88073	-1.00420
b*2	0.63372	0.59764	0.50740	0.47106	0.50503	0.63798	0.80724	0.96082	1.11919	1.29827
b*4	0.02856	0.12674	0.23008	0.18878	0.09742	0.03211	0.01299	0.00828	0.00669	0.00591
b*6	0.00021	0.00266	0.01698	0.04590	0.02089	0.01157	0.00455	0.00157	0.00047	0.00014
b*8	0.00000	0.00004	0.00006	0.00101	0.00135	0.00116	0.00058	0.00037	0.00016	-

TABLE 3

	4q	1	4	10	16	24	36	50	64	80	100
$a_m^{(o)} \times 10^5$	m=0	63662	63662	63661	63662	63661	63662	63663	63662	63662	63662
	2	1387	-24378	-53883	-67762	-78015	-86653	-92584	-96488	-99649	-102496
	4	34	1648	7570	13647	20743	29333	37053	43100	48625	54119
	6	-1	-47	-499	-1327	-2898	-5586	-8853	-12026	-15431	-19321
	8	0	1	19	75	244	665	1367	2230	3344	4843
	10	0	0	0	-3	-13	-54	-146	-290	-519	-884
	12	0	0	0	0	0	3	11	28	60	121
	14	0	0	0	0	0	0	0	-1	-6	-13
	16	0	0	0	0	0	0	0	0	0	1
$a_m^{(1)} \times 10^5$	1	63662	63662	63662	63661	63662	63662	63662	63662	63662	63662
	3	61242	7785	-14585	-37683	-59286	-79624	-94411	-104445	-112736	-120321
	5	-914	-64	2230	7327	15650	28888	43289	56033	68695	82155
	7	6	-1	-128	-632	-1986	-5246	-10308	-16117	-23155	-32057
	9	0	1	4	31	148	575	1506	2910	4994	8175
	11	0	0	0	-1	-8	-43	-150	-360	-748	-1468
	13	0	0	0	0	0	2	11	32	82	195
	15	0	0	0	0	0	0	0	-3	-7	-19
	17	0	0	0	0	0	0	0	0	0	2
$a_m^{(2)} \times 10^5$	0	0	0	0	0	0	-1	-3	1	-1	3
	2	63662	63662	63676	63718	63660	63663	63657	63650	63669	63667
	4	1529	7384	9842	-2338	-25293	-53553	-76853	-93503	-107710	-120977
	6	-26	-102	-153	3496	6060	16883	31896	47462	64645	84522
	8	0	2	-45	-422	-657	-2668	-6893	-12897	-20744	-32294
	10	0	0	3	23	38	252	922	2093	4190	7877
	12	0	0	0	-1	-1	-15	-84	-238	-585	-1338
	14	0	0	0	0	0	1	5	19	60	168
	16	0	0	0	0	0	0	0	-1	-4	-16
	18	0	0	0	0	0	0	0	0	0	1

where λ is Lagrange's constant and determined by (3.1), that is,

$$\lambda = 1/(\pi C_{o,o}), \quad C_{o,o} = \sum_{n=o}^{\infty} (A_o^{(2n)})^2/\mu_{2n} \tag{3.3}$$

Putting these values into (2.10), we have

$$Cw = 16g^2\lambda = 16g^2/(\pi C_{o,o}) = Cwo \ . \tag{3.4}$$

Moreover, the influence functions (2.9) becomes

$$\Gamma(x) = 2\lambda \cdot \sum_{n=o}^{\infty} A_o^{(2n)} ce_{2n}(\Theta,q) = \lambda = Cwo/(16g^2) \ , \tag{3.5}$$

by the expansion of a constant in Mathieu functions.

Secondly, consider the case when the area and the moment or center of floatation of the water plane area are given.

This latter condition is given as, say,

$$(1/4) \int_{-1}^{1} H(x)\ x\ dx = \alpha \ , \tag{3.6}$$

which means that the center of floatation is aft of midship by α times of the ship length, and this determines one more relation between the coefficients other than (3.1), namely,

$$\sum_{n=o}^{\infty} a_{2n+1} A_1^{(2n+1)} = 8\ \alpha/\pi \ . \tag{3.7}$$

In the same way as the above, we have easily found the following result.

$$a_{2n+1} = 8\ \alpha\ A_1^{(2n+1)}/(\pi C_{1,1}\mu_{2n+1}) = \alpha\ a^*_{2n+1} \ , \tag{3.8}$$

$$Cw = Cwo + 16\ \alpha^2 Cw1 \ , \tag{3.9}$$

$$\Gamma(x) = Cwo/(16g^2) + \alpha\ Cw1\ \cos\Theta/(2g^2) \ , \tag{3.10}$$

a_{2n} and Cwo are the same as (3.2) and (3.4) and

$$Cw1 = 16g^2/(\pi\ C_{1,1}) \ , \quad C_{1,1} = \sum_{n=o}^{\infty} A_1^{(2n+1)})^2/\mu_{2n+1} \ , \tag{3.11}$$

Thirdly and lastly, let us add to the first problem the condition that the second moment of the area is given arbitrarily.

That is given as

$$(1/8) \int_{-1}^{1} H(x)\, x^2\, dx = m^2$$

$$m^2 = 1/8 + (\pi/32) \sum_{n=0}^{\infty} a_{2n} A_2^{(2n)} \tag{3.12}$$

Then, we find easily the next solution in the same way as the above.

$$a_{2n} = a_{2n}^* + \gamma\, b_{2n}^* \,, \quad a_{2n+1} = 0 \,,$$

$$b_{2n}^* = 2[C_{o,o}\, A_2^{(2n)} - C_{o,2}\, A_o^{(2n)}]/(\pi\, \Delta\, \mu_{2n}) \,, \tag{3.13}$$

$$Cw = Cwo + \gamma^2 Cw2 \,, \tag{3.14}$$

$$\Gamma(x) = Cwo\, (16g^2) - 2\gamma[C_{o,2} = C_{o,o} \cos 2\theta]/(\pi\, \Delta) \tag{3.15}$$

where

$$\Delta = C_{o,o}\, C_{2,2} - C_{o,2}^2 \,,$$

$$C_{o,2} = \sum_{n=0}^{\infty} A_o^{(2n)} A_2^{(2n)}/\mu_{2n} \,, \quad C_{2,2} = \sum_{n=0}^{\infty} (A_2^{(2n)})^2/\mu_{2n} \tag{3.16}$$

$$\gamma = 16(m^2 - m^2) \,, \quad m_o^2 = 1/8 + C_{o,2}/(16 C_{o,o}) \,, \tag{3.17}$$

$$Cw2 = 16 g^2 C_{o,o}/(\pi\, \Delta) \,. \tag{3.18}$$

In this place, let us introduce the quantity

$$\delta = 1/H(0) = 1/[\sum_{n=0}^{\infty} a_{2n} ce_{2n}(\pi/2, q)] \,, \tag{3.19}$$

which equals approximately to the usual water plane area coefficient.

In the first problem, this quantity takes the value

$$\delta = \delta_o = \pi C_{o,o}/(2D_o) \,, \quad D_o = \sum_{n=0}^{\infty} A_o^{(2n)} ce_{2n}(\pi/2, q)/\mu_{2n} \,, \tag{3.20}$$

and in this case, using the above notations,

$$1/\delta = 1/\delta_o + 2\gamma(C_{o,o}D_2 - C_{o,2}D_o)/(\pi\ \Delta),$$
$$D_2 = \sum_{n=o}^{\infty} A_2^{(2n)} ce_{2n}(\pi/2,q)/\mu_{2n}\ . \tag{3.21}$$

The quantity m or γ is not familiar than δ, so that the solutions in this case are computed for four given values of δ using this relation.

The numerical values of the above appeared quantities are shown in Table 1 and 2 and Figure 1 to 6, and moreover the expansion coefficients of the solution in Mathieu functions are converted in the ones of trigonometrical functions and shown in Table 3.

Namely, write the solution of the first problem

$$Ho(-\cos\Theta) = \varphi o(\Theta)/\sin\Theta\ ,$$
$$\varphi o(\Theta) = \sum_{n=o}^{\infty} a_{2n}^{*} ce_{2n}(\Theta,q)\ , \tag{3.22}$$

then we may write by the conversion

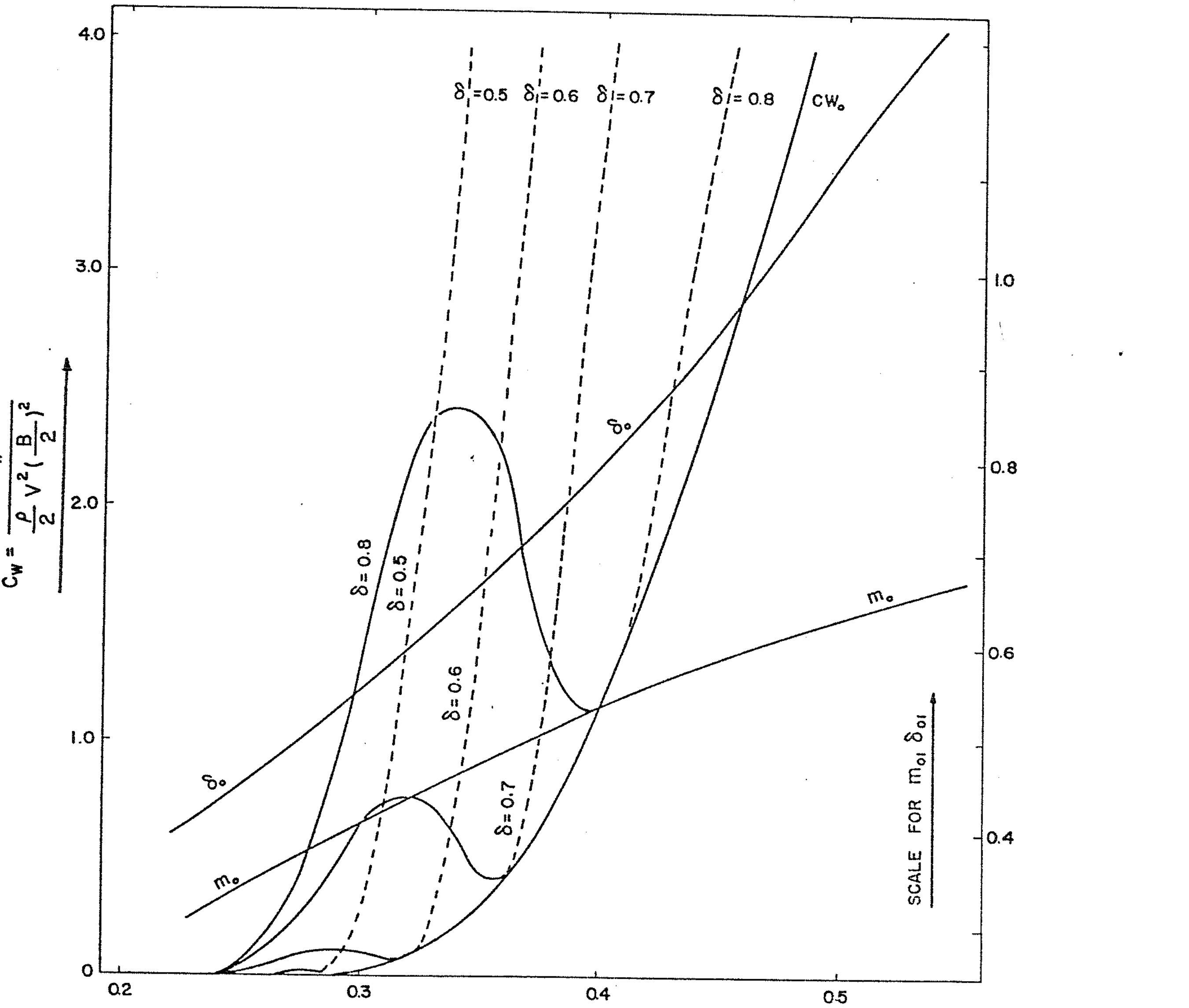
$$\varphi o(\Theta) = \sum_{m=o}^{\infty} a_{2m}^{(o)} \cos 2m\Theta\ ,\ a_{2m}^{(o)} = \sum_{n=o}^{\infty} a_{2n}^{*} A_{2m}^{(2n)}\ . \tag{3.23}$$

In the same way, we define the coefficient $a_m^{(i)}$ as follows,

$$H_1(x)\ \sin\Theta = \varphi_1(\Theta) = \sum_{n=o}^{\infty} a_{2n+1}^{*}\ ce_{2n+1}\ (\Theta,q)\ ,$$

$$\varphi_1(\Theta) = \sum_{m=o}^{\infty} a_{2m+1}^{(1)}\ \cos(2m+1)\Theta\ , \tag{3.24}$$

$$a_{2m+1}^{(1)} = \sum_{n=o}^{\infty} a_{2n+1}^{*} A_{2m+1}^{(2n+1)}$$


4.0
3.0
2.0
1.0
0
0.2
0.3
0.4
0.5
1.0
0.8
0.6
0.4
$\delta_1 = 0.5$
$\delta_1 = 0.6$
$\delta_1 = 0.7$
$\delta_1 = 0.8$
CW_o
δ_o
m_o
$\delta = 0.8$
$\delta = 0.5$
$\delta = 0.6$
$\delta = 0.7$
$C_W = \frac{R_W}{\frac{\rho}{2} V^2 (\frac{B}{2})^2}$
$F_r = V/\sqrt{2g}$
SCALE FOR m_{01} δ_{01}

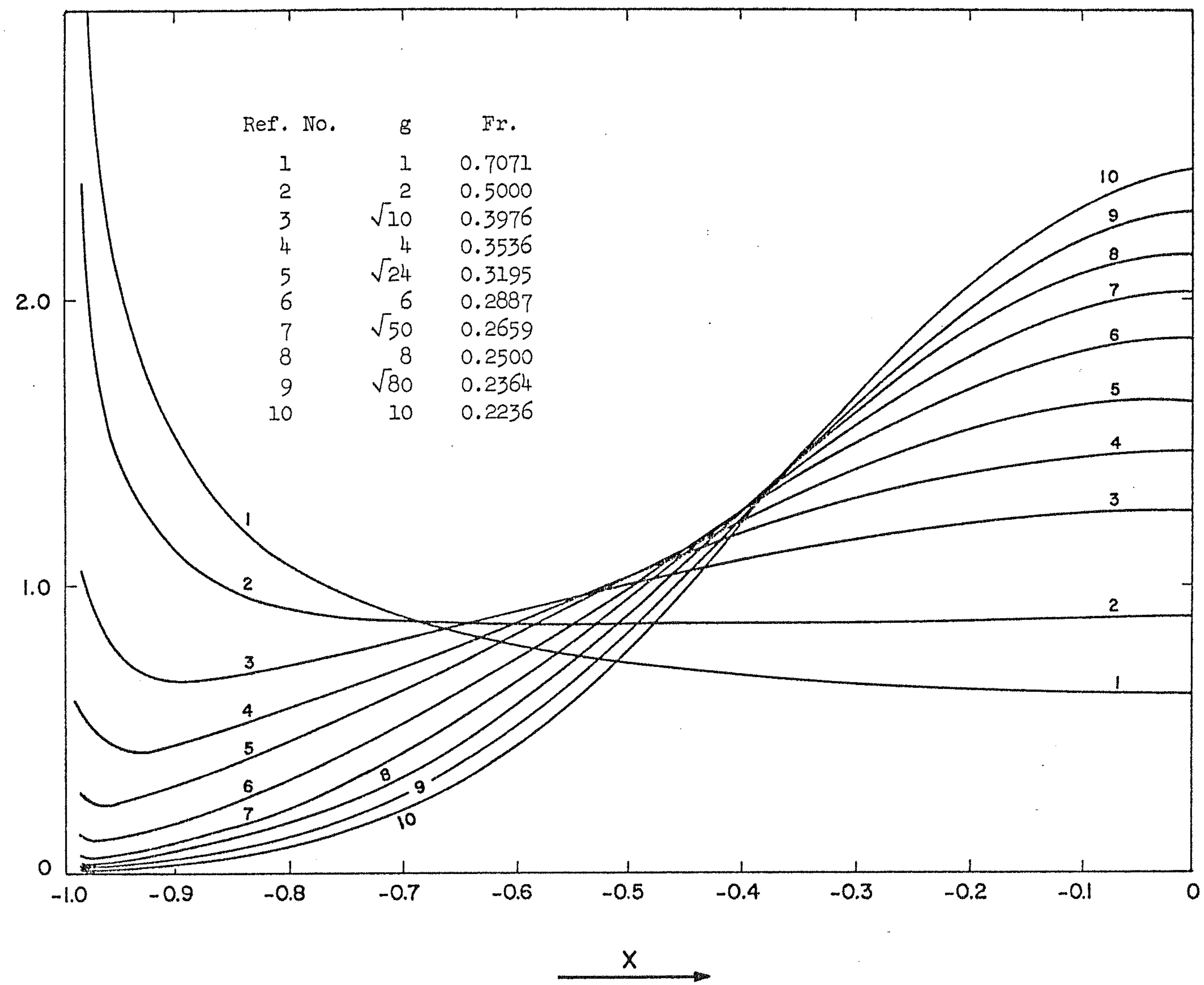

Ref. No. g Fr.
1 1 0.7071
2 2 0.5000
3 √10 0.3976
4 4 0.3536
5 √24 0.3195
6 6 0.2887
7 √50 0.2659
8 8 0.2500
9 √80 0.2364
10 10 0.2236
0
1.0
2.0
-1.0
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
X

Figure 2. $H_o(x)$.

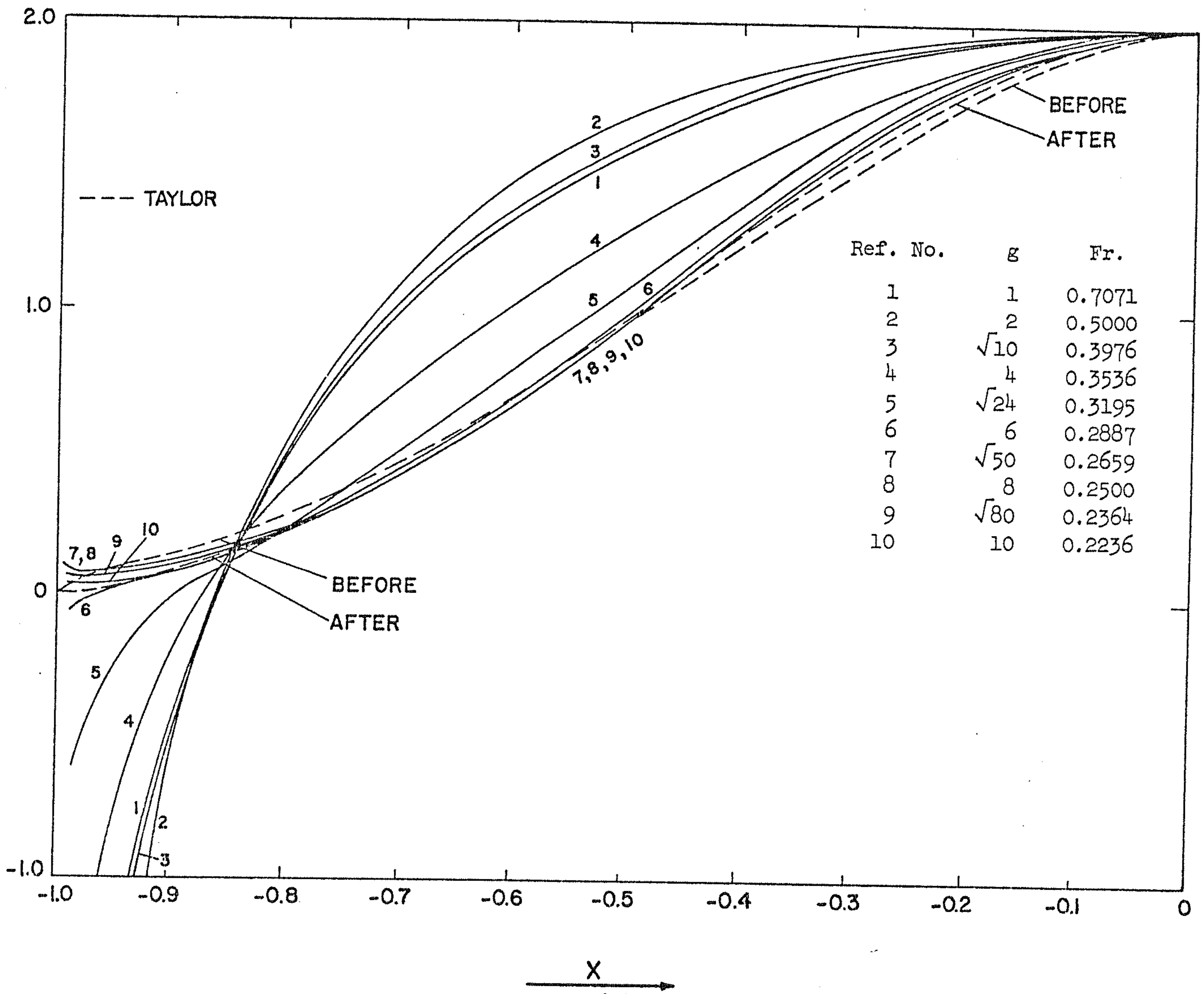


Figure 3. $H_3(x) \cdot \delta = 0.5$

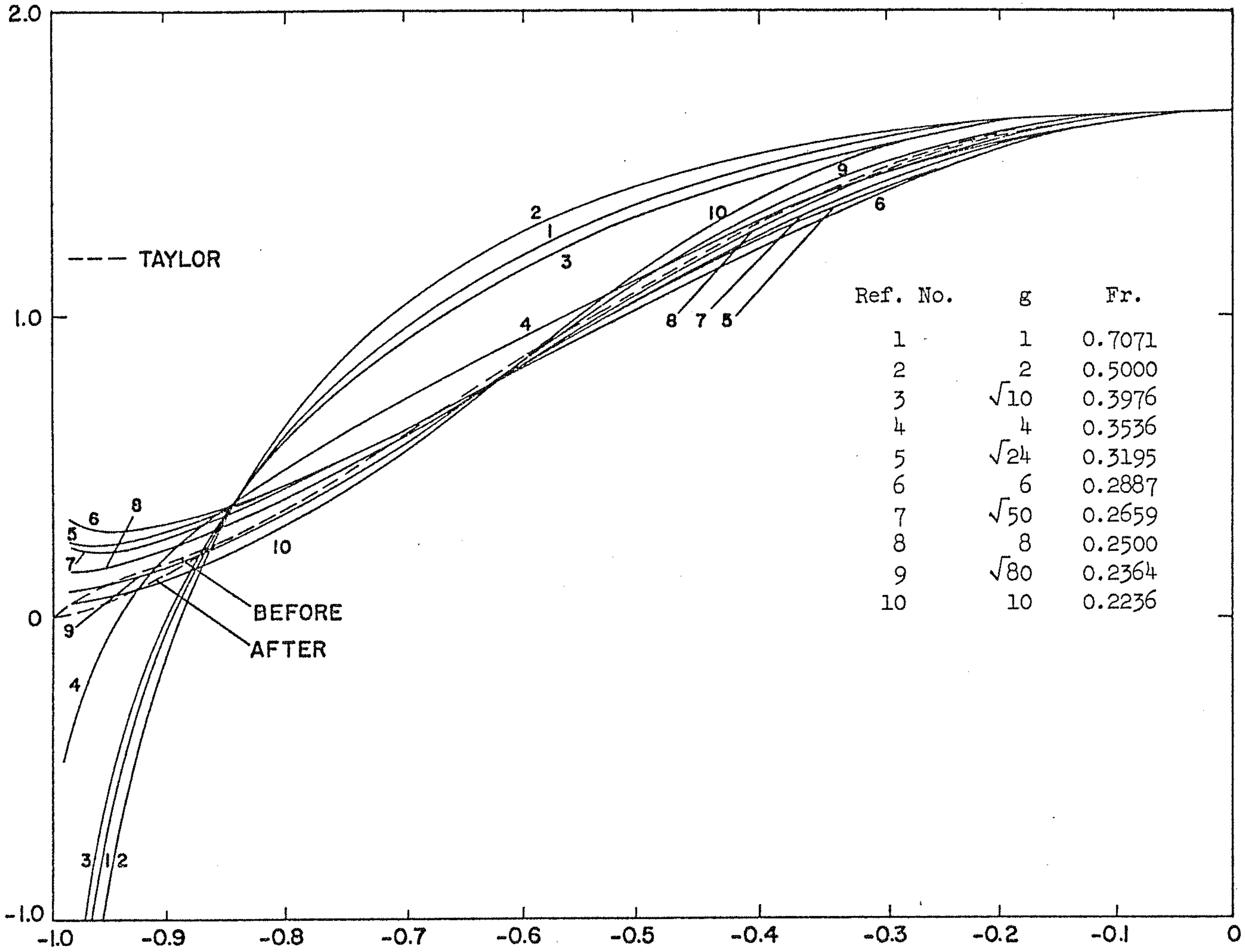


Figure 4. $H_3(x)$, $\delta = 0.6$

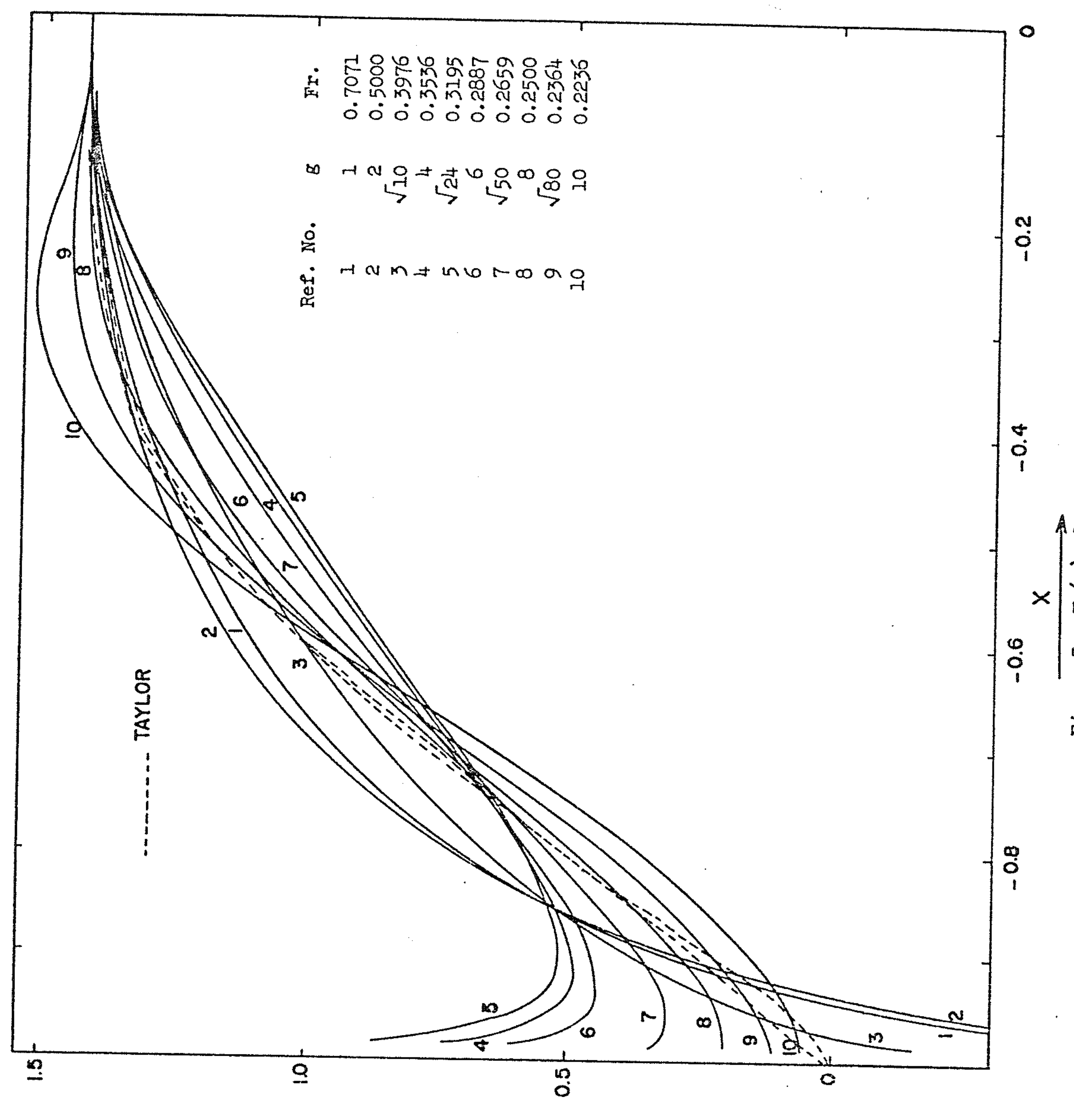


Figure 5. $H_3(x)$, $\delta = 0.7$

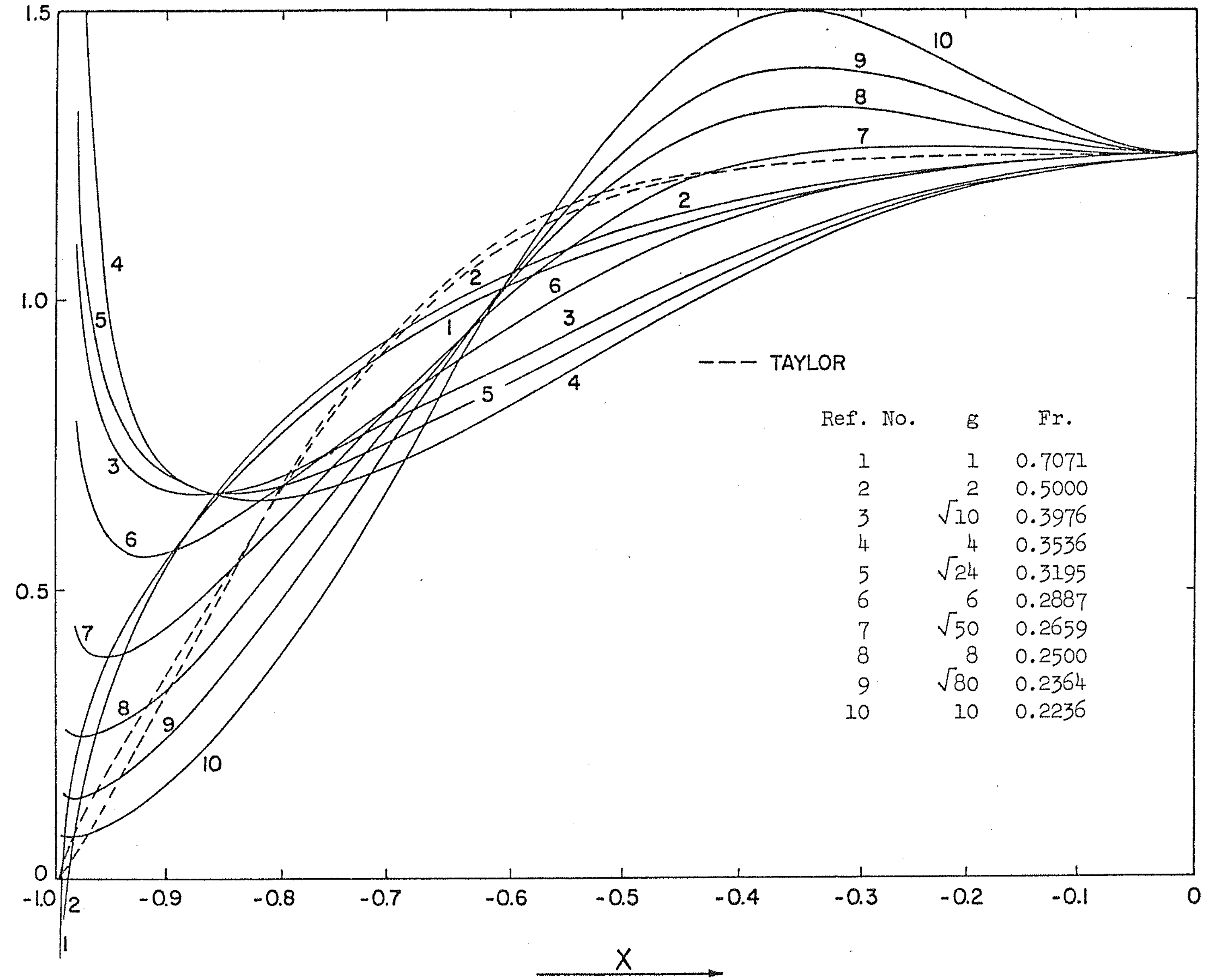


Figure 6. $H_3(x)$, $\delta = 0.8$

$$H_2(x)\sin\Theta = \varphi_2(\Theta) = \sum_{n=0}^{\infty} b^*_{2n} ce_{2n}(\Theta,q) ,$$

$$\varphi_2(\Theta) = \sum_{m=0}^{\infty} a^{(2)}_{2m} \cos 2m\Theta , \qquad (3.25)$$

$$a^{(2)}_{2m} = \sum_{n=0}^{\infty} b^*_{2n} A^{(2n)}_{2m}$$

4. Minimum Problem II[8]

In the preceding paragraph, we do not encounter any theoretical difficulty, but such proposition of the problem is somewhat different from G. Weinblum's.[6]

As widely known, he had considered the problem when the distribution vanished at end points and the area coefficient was given.

It was pointed out by S. Karp and others that this first condition was not adequate analytically.[7]

Here, we will show that the second condition is not adequate too in the same meaning.

The second condition says that $H(0)$ is to be given arbitrarily by (3.19).

By Lagrange's method, introducing two constants, we have the solution

$$a_{2n} = [2\lambda_1 A_0^{(2n)} + \lambda_2 ce_{2n}(\pi/2,q)]/\mu_{2n} . \qquad (4.1)$$

Putting this into (2.9), the influence function in this case becomes

$$\Gamma(x) = \lambda_1 + \lambda_2 \sum_{n=0}^{\infty} ce_{2n}(\pi/2,q) ce_{2n}(\Theta,q) , \qquad (4.2)$$

in which the series of the second term is summed up as the next by interchanging the order of summation and using the relations between Fourier coefficients of Mathieu functions.

$$\sum_{n=0}^{\infty} ce_{2n}(\pi/2,q)ce_{2n}(\theta,q) = \sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\sum_{n=0}^{\infty}(-1)^r A_{2r}^{(2n)} A_{2s}^{(2n)} \cos 2s\theta$$

$$= 1/2 + \sum_{r=0}^{\infty}(-1)^r \cos 2r\theta = (1/2)\lim_{N\to\infty}[\sin(2N+1)(\theta-\pi/2)/\sin(\theta-\pi/2)] , \quad (4.3)$$

This value oscillates the more rapidly as the more N increases, so that we may not find the definite meaning of this problem, that is this problem is not adequate.

As easily seen, we shall meet the same difficulty as in (4.3) when we consider the first condition.

This property of his method may induce an instability of the solution as he wrote.[6]

5. Properties of the Solution

We will notice here some properties especially in low speed, because S. Karp and others gave them in high speed.

One of the most interesting is that the value of μ_n becomes very much smaller by decreasing its order especially in low speed as we see in Table 1.

This means from (2.10) that the lower the order of Mathieu function is, the smaller the wave resistance.

Hence, the optimum distribution consists almost of the first three Mathieu functions as we see in Table 2.

This fact enables next simple asymptotic formulas by making use of asymptotic ones of Mathieu functions, that is,

$$Cwo \fallingdotseq 64g^2 \exp(-2g) , \quad \text{for} \quad g \gg 1 , \quad (5.1)$$

$$\delta_o \fallingdotseq \sqrt{\pi/(2g)} , \quad (5.2)$$

$$m_o^2 \fallingdotseq 1/(4g) , \quad (5.3)$$

$$Cw1 \fallingdotseq 64g^4 \exp(-2g) , \quad (5.4)$$

$$Cw2 \doteq 16g^6 \exp(-2g) \;, \tag{5.5}$$

$$\delta \doteq \delta_0/(1 - \gamma g/4) \;, \tag{5.6}$$

These formulas show also more explicitly the above principle in low speed.

Now, the distributions of the first problem without other restriction are shown in Figure 2 and those of the third problem with the given area coefficient in Figure 3 to 6.

In Figure 7 and 8, G. Weinblum's results are shown compared with our results, and notwithstanding that they were computed for the finite draft, they are similar as ours in general but the case when they have a large swan neck. In these cases, our solution has always a negative part, and so it has no practical meaning, but it should be remembered that the area coefficient of the optimum distribution without restriction is always larger than the value in these cases.

In this meaning, the third solutions corresponding to the dotted branch of the resistance curves in Figure 1 have no meaning.

6. Wave Profile

The wave profile of the optimum distribution must have some character different from the ordinary ones.

For this purpose, consider the surface elevation on the x-axis.

It is given as follows in our case.(2)

$$\zeta(x)/\overline{B} = -(g/4) \int_{-1}^{1} H(x')Z[g(x-x')]\, dx' \;, \tag{6.1}$$

with

$$\begin{aligned} Z(u) &= (d/du)[H_0(u) - Y_0(u)] \;, \quad \text{for} \quad u > 0 \;, \\ &= (d/du)[H_0(u) - 3Y_0(u)] \;, \quad \text{for} \quad u < 0 \;, \end{aligned} \tag{6.2}$$

where H_0 means Struve's function.

Now we may write (6.2) as

$$\begin{aligned} Z(u) &= [H_0'(u) - Y_1(u)] + 2Y_1(u) \;, \quad \text{for} \quad u > 0 \\ &= [H_0'(u) + Y_1(u)] + 2Y_1(u) \;, \quad \text{for} \quad u < 0 \end{aligned} \tag{6.3}$$

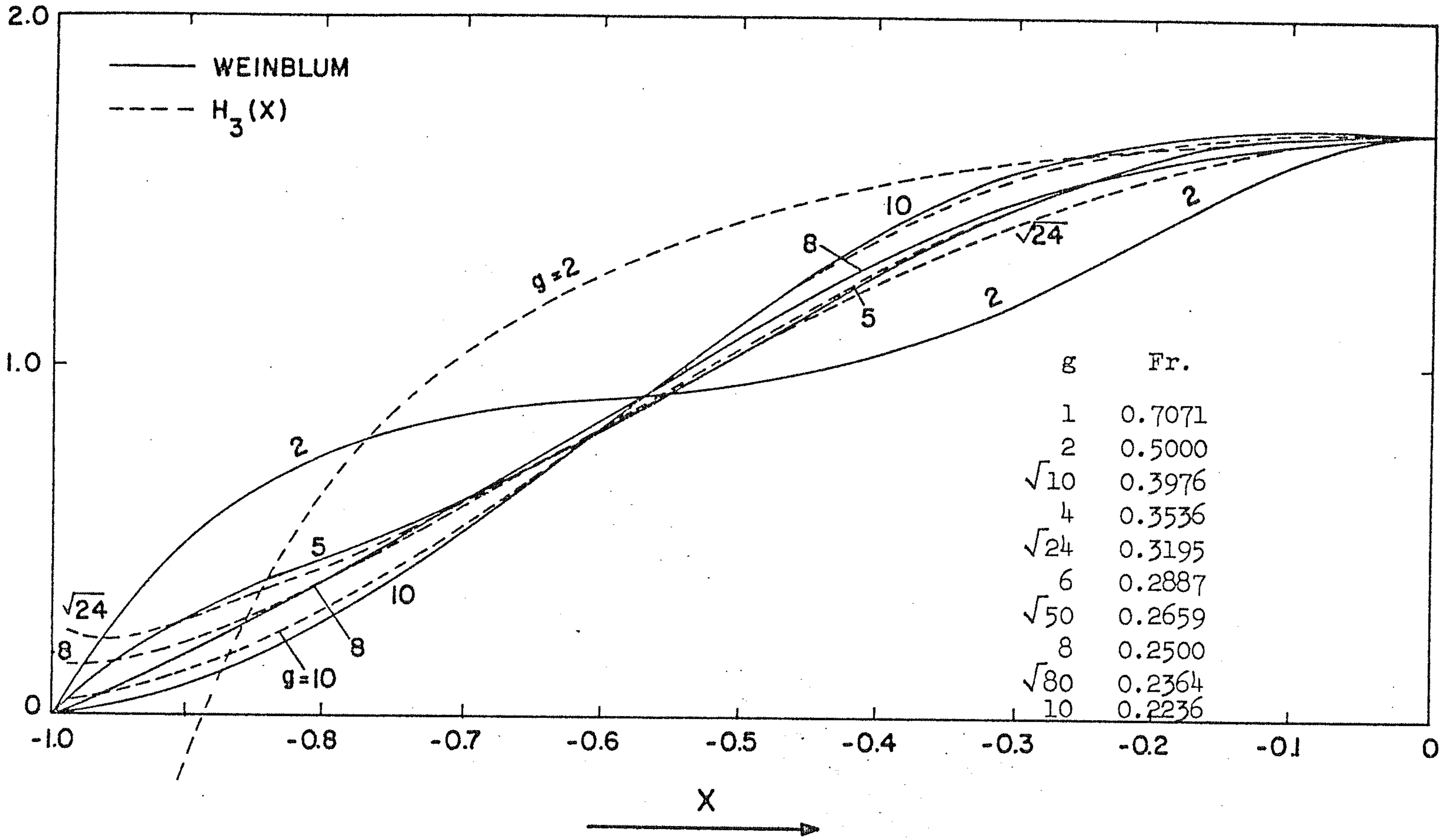


Figure 7. $\eta(x)$, $(\delta = 0.6)$.

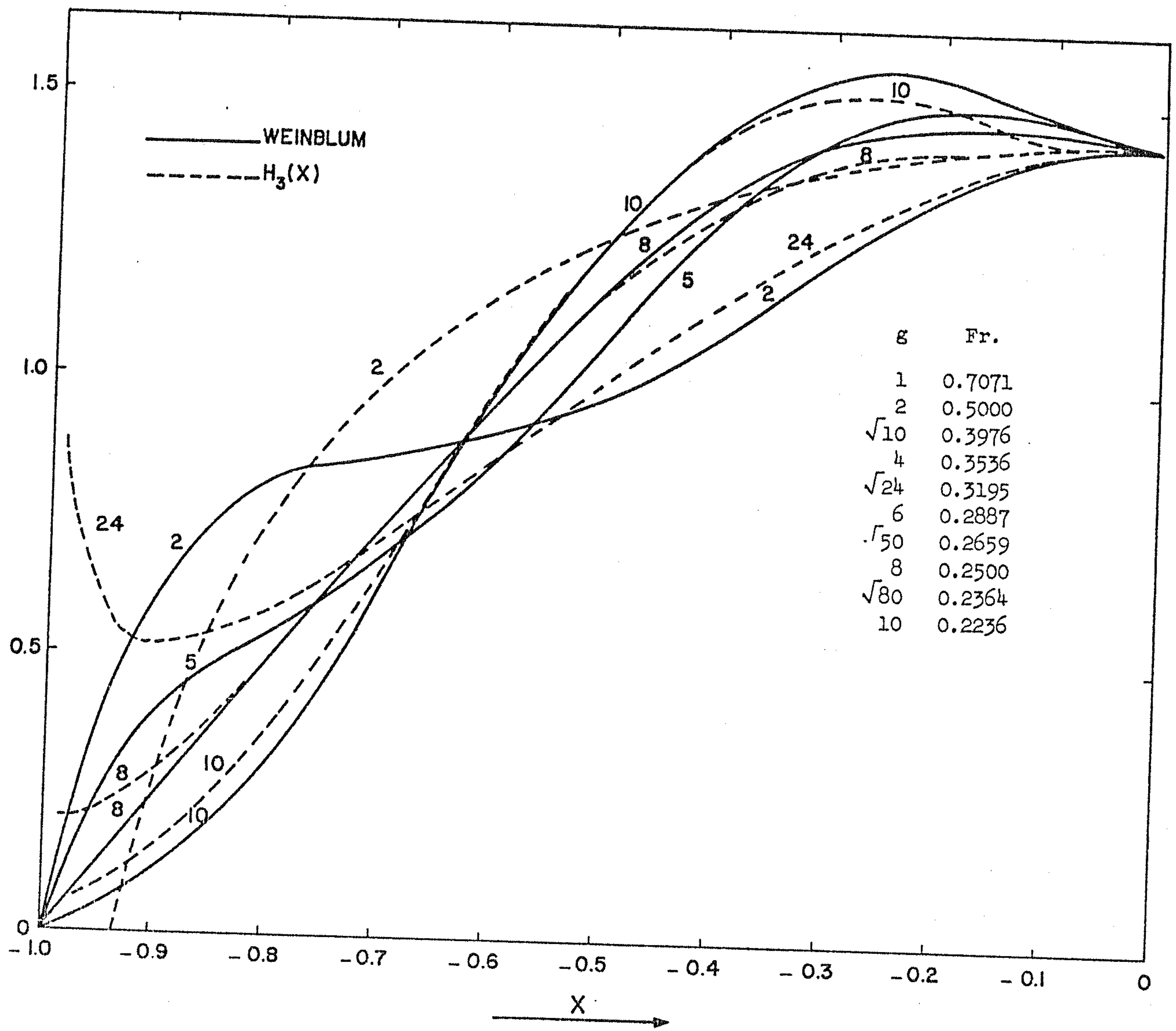


Figure 8. $\eta(x)$, $(\varphi = 0.7)$.

In this right hand side, the function in the parentheses is even, but $Y_1(u)$ odd, and both functions tend infinity at $u = 0$ as $1/u$, so that the integral of the even part diverges but the one of the odd part has a definite value in the sense of Cauchy's principal value for $|x| < 1$.

Namely, if we assume the distribution symmetric about the origin, the even part of the surface elevation tends infinity but not for the odd part. That is, remembering (2.5),

$$\text{Odd part of } [\zeta/\overline{B}] = -(g/2) \int_{-1}^{1} H(x')Y_1[g(x-x')]\,dx'$$

$$= -(d/dx)\,\Gamma(x)\ . \qquad (6.4)$$

Hence, this should vanish in the first problem, because the influence function is constant by (3.5), and means that the surface elevation of this optimum distribution should be symmetric about the midship. Inversely, if the surface elevation be symmetric, then the influence function should be constant by (6.4), so that the distribution would be optimum.

In the same way, the odd part of the surface elevation of the third problem is linear by (3.15) but numerically very small in low speed.

Thence, these are the very character to distinguish the optimum distribution.

Lastly, we will give the surface elevation far upper and down stream.

Since we have an asymptotic expansion[1]

$$H_0(u) - Y_0(u) \doteqdot 2/(\pi u), \quad \text{for} \gg 1 ,$$

putting this into (6.1), we have approximately

$$\zeta(x)/\overline{B} \doteqdot [1/(2\pi g x^2)] \int_{-1}^{1} H(x')dx' = 1/(\pi g x^2)\ , \quad \text{for} \quad x \gg 1\ , \qquad (6.5)$$

When x is negative, the elevation consists of the same symmetrical part and the free wave part.

The latter part takes the next form by (6.1) and (6.2),

$$\zeta_f(x)\ \overline{B} = -g \int_{-1}^{1} H(x')Y_1[g(x-x')]dx'$$

$$= -2(d/dx)\ \Gamma(x)\ , \quad \text{for} \quad x < -1\ , \qquad (6.6)$$

where the influence function takes a different form from the above in this case when $|x| > 1$.

Namely, for the even distribution, we have

$$\Gamma(x) = -(\pi/2) \sum_{n=0}^{\infty} a_{2n}[A_0^{(2n)}/ce_{2n}(\pi/2,q)]^2\ Fey_{2n}(u,q)\ ,$$

$$\text{for} \quad x = -\cosh u\ , \qquad (6.7)^{8)}$$

Especially, for the first problem in low speed, this equals nearly to

$$\Gamma(x) \doteq -(\pi\ 2)a_0^*[A_0^{(o)}\ ce_0(\pi\ 2,q)]\ Fey_0(u,q)$$

$$\doteq [2\exp(-g)\ \sqrt{\pi g|x|}]\ \cos(g|x| + \pi/4)\ , \qquad (6.8)$$

so that we have, by differentiation,

$$\zeta_f\ (x)/\overline{B} = -\ [4\sqrt{g}\ \exp(-g)/\sqrt{\pi|x|}]\ \sin(g|x| + \pi/4)$$

or

$$-\ 1/2)\ \sqrt{Cwo/(\pi g|x|)}\ \ \sin(g|x| + \pi/4)\ , \qquad (6.9)$$

7. Conclusion

Summarizing the above results, we may conclude as follows.

i) The minimum problem of the wave making resistance of the doublet distribution which is draftwise uniform and tends infinitely great depth can be solved most easily by making use of Mathieu functions.

ii) The solution of the first problem when the total sum of the doublet or approximately the water plane area is given, the second problem when the center of floatation is also given and the third problem when the second moment of the area is given are solved and shown in tables and figures.

iii) The third problem is solved instead of the one with the restriction for the area coefficient, because the proposition of the problem for this case is not adequate usually.

iv) In low speed, expanding the distribution in Mathieu functions, the lower its order, the smaller the wave resistance becomes, so that the optimum one consists almost of the one of the lowest order.

v) The wave profile along the distribution becomes infinitely large, but it should be symmetric about the midship at the optimum speed in the first problem.

In the last place, the author thanks heartily to Professor Maruo for his kind suggestions and discussions.

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TABLE 1

4q	1	4	10	16	24	36	50	64	80	100
g	1	2	$\sqrt{10}$	4	$\sqrt{24}$	6	$\sqrt{50}$	8	$\sqrt{80}$	10
Fr.	0.7071	0.5000	0.3976	0.3536	0.3195	0.2887	0.2659	0.2500	0.2364	0.2236
μ_0	0.644094	0.998125(1)	0.814806(2)	0.132128(2)	0.195168(3)	0.189457(4)	0.202547(5)	0.294995(6)	0.419816(7)	0.478425(8)
μ_1	1.096150	0.752746	0.194796	0.470898(1)	0.851693(2)	0.992816(3)	0.122314(3)	0.199029(4)	0.313925(5)	0.397238(6)
μ_2	0.547239	0.686380	0.662995	0.397181	0.138930	0.254523(1)	0.342655(2)	0.624663(3)	0.109448(3)	0.154365(4)
μ_3	0.344556	0.585909	0.500515	0.582640	0.510346	0.227603	0.547913(1)	0.120592(1)	0.237545(2)	0.373999(3)
μ_4	0.254270	0.268533	0.306237	0.360255	0.448653	0.495383	0.320119	0.128702	0.344321(1)	0.634796(2)
μ_5	0.202113	0.208842	0.224475	0.244169	0.279845	0.558128	0.442317	0.402119	0.230739	0.712211(1)
μ_6	0.167868	0.171617	0.179844	0.189269	0.204374	0.255323	0.290597	0.362817	0.406359	0.305693
μ_7	-	0.145917	0.150846	0.158245	0.164335	0.179028	0.202103	0.235452	0.290416	0.363293
μ_8	-	0.127026	0.130230	0.133654	0.138613	0.147071	0.159004	0.174202	0.197839	0.241970
C_{wo}	6.5518	4.4226	1.1584	0.34299	0.85524(1)	0.14156(1)	0.23083(2)	0.46094(3)	0.87171(4)	0.13191(4)
C_{w1}	5.410	14.670	10.849	5.413	2.041	0.5094	0.1154	0.2950(1)	0.6976(2)	1.319(3)
C_{w2}	2.790	12.73	23.60	20.38	12.02	4.521	1.441	0.4719	0.1398	0.0330
δ_0	1.606	1.114	0.7960	0.6827	0.6039	0.5378	0.4910	0.4591	0.4323	0.4074
m_0	0.7109	0.6358	0.5371	0.4857	0.4401	0.3997	0.3694	0.3480	0.3298	0.3123
C_w $\delta=$ 0.5	20.26	53.46	46.35	12.26	1.659	0.6213(1)	0.2905(2)	0.3574(2)	0.2083(2)	0.7168(3)
0.6	14.43	28.29	14.90	2.038	0.8688(1)	0.1046	0.6283(1)	0.2610(1)	0.8602(2)	0.2134(2)
0.7	11.24	15.80	3.563	0.3974	0.7623	0.4658	0.1657	0.5553(1)	0.1603(1)	0.3609(2)
0.8	9.395	9.458	1.142	2.261	2.224	0.9176	0.2758	0.8488(1)	0.2512(1)	0.4970(2)
γ $\delta=$ 0.5	-2.217	-1.963	-1.384	-0.7648	-0.3590	-0.1025	0.0204	0.0812	0.1196	0.1462
0.6	-1.680	-1.369	-0.7638	-0.2885	-0.0114	0.1407	0.2049	0.2331	0.2471	0.2534
0.7	-1.297	-0.9456	-0.3206	0.0517	0.2374	0.3145	0.3367	0.3416	0.3381	0.3300
0.8	-1.010	-0.6277	0.0118	0.3069	0.4240	0.4448	0.4556	0.4230	0.4064	0.3874

The numbers (n) in parentheses indicate that the results must be multiplied by 10^{-n}, for example o.3441(2) means o.003441 .

TABLE 2

4q	1	4	10	16	24	36	50	64	80	100
Fr.	0.7071	0.5000	0.3976	0.3536	0.3195	0.2887	0.2659	0.2500	0.2364	0 2236
a^*_0	0.89565	0.93186	1.05195	1.13513	1.20958	1.28477	1.34651	1.39366	1.43700	1.48113
a^*_1	0.61656	0.61952	0.64548	0.74218	0.88398	1.06089	1.22267	1.35534	1.48492	1.62408
a^*_2	o.09256	0.04368	0.00792	0.00266	0.00141	0.00088	0.00065	0.00052	0.00043	0.00035
a^*_3	0.63268	0.16586	0.10145	0.04071	0.01451	0.00589	0.00379	0.00310	0.00266	0.00229
a^*_4	0.00105	0.00268	0.00151	0.00061	0.00017	0.00003	0.00001	0.00000	0.00000	0.00000
a^*_5	0.00053	0.00595	0.01083	0.00835	0.00436	0.00131	0.00030	0.00008	0.00003	0.00002
a^*_6	0.00001	0.00003	0.00005	0.00039	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000
a^*_7	--	0.00006	0.00032	0.00041	0.00934	0.00018	0.00007	0.00003	0.00001	0.00000
b^*_0	-0.05567	-0.19362	-0.32323	-0.38490	-0.45604	-0.55957	-0.67128	-0.77320	-0.86073	-1.00420
b^*_2	0.63372	0.59764	00.50740	0.47106	0.50503	0.63798	0.80724	0.96062	1.11919	1.29827
b^*_4	0.02656	0.12674	0.23008	0.18878	0.09742	0.03211	0.01299	0.00828	0.00669	0.00591
b^*_6	0.00021	0.00366	0.01698	0.04590	0.02089	0.01157	0.00455	0.00157	0.00047	0.00014
b^*_8	0.00000	0.00004	0.00006	0.00101	0.00135	0.00116	0.00058	0.00037	0.00016	---

TABLE 3

	4q	1	4	10	16	24	36	50	64	80	100
$a_m^{(0)}$ x10[illegible]	m=0	63662	63662	63661	63662	63661	63662	63663	63662	63662	63662
	2	1387	-24378	-53883	-67762	78015	-86653	-92584	-96488	-99649	-102496
	4	34	1648	7570	13647	20743	29335	37033	43100	48625	54119
	6	- 1	- 47	- 499	- 1327	- 2898	- 5586	- 8853	-12026	-15431	-19321
	8	0	1	19	75	244	665	1367	2230	3344	4843
	10	0	0	0	- 3	- 13	- 54	- 146	- 290	- 519	- 884
	12	0	0	0	0	0	3	11	28	60	121
	14	0	0	0	0	0	0	0	- 1	- 6	- 13
	16	0	0	0	0	0	0	0	0	0	1
$a_m^{(1)}$ x10[illegible]	1	63662	63662	63662	63661	63662	63662	63662	63662	63662	63662
	3	61242	7785	-14588	-37683	-59286	-79624	-94411	-104445	-112736	-120321
	5	- 914	- 64	2230	7237	15650	28888	43289	56033	68695	82155
	7	6	- 1	- 128	- 632	- 1986	- 5246	-10308	-16117	-23155	-32057
	9	0	1	4	31	148	575	1506	2910	4994	8175
	11	0	0	0	- 1	- 8	- 43	- 150	- 360	- 748	- 1468
	13	0	0	0	0	0	2	11	32	82	195
	15	0	0	0	0	0	0	0	- 3	- 7	- 19
	17	0	0	0	0	0	0	0	0	0	2
$a_m^{(2)}$ x10[illegible]	0	0	0	0	0	0	- 1	- 3	1	- 1	3
	2	63662	63662	63676	63718	63660	63663	63657	63650	63669	63667
	4	1529	7384	9842	- 2338	-25293	-53553	-76853	-93503	-107710	-120977
	6	- 26	- 102	- 153	3496	6060	16883	31896	47462	64645	84522
	8	0	2	- 45	- 422	- 657	- 2668	- 6893	-12897	-20744	-32294
	10	0	0	3	23	38	252	922	2093	4190	7877
	12	0	0	0	- 1	- 1	- 15	- 84	- 238	- 585	- 1338
	14	0	0	0	0	0	1	5	19	60	168
	16	0	0	0	0	0	0	0	- 1	- 4	- 16
	18	0	0	0	0	0	0	0	0	0	1