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WAVE-FREE DISTRIBUTIONS AND THEIR APPLICATIONS

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1. Introduction

The idea of wave free or waveless distributions is a very old one, and we can find it in Lord Kelvin's work in the early history of the water wave research.(1)

Since then, Professors Havelock, Ursell and Maruo have used its idea in their works explicitly or implicitly.(2,3,4)

On the other hand, Professor Inui has proposed theoretically and experimentally the waveless ship a few years ago.(5)

About this time, the author also took up this problem in general and found some of its general properties.(6)

Here, he will show some of the results and applications.(7)

2. Two Dimensional Pressure Distributions(6)

Consider, at first, two dimensional water motions due to a fixed pressure distribution on the surface of a uniform stream with unit velocity and great depth.

Take the origin at the center of the distribution on the undisturbed surface of the stream, with the x-axis horizontally in up-stream direction and the y-axis vertically upwards.

Then, the wave-making resistance of this distribution is given by the formula,

$$R = \rho g^3 |F(g)|^2 , \qquad (2.1)$$

with

$$F(p) = \int_{-1}^{1} H(x) \exp.(-ipx)\, dx , \qquad (2.2)$$

where ρ means the water density, g the gravity constant in this unit system, $|F|$ the absolute value of F and $H(x)$ the water head of the pressure.

Hence, the necessary and sufficient condition for vanishing the wave resistance is clearly

$$F(p) = 0 \quad \text{for} \quad p = g , \tag{2.3}$$

that is, $F(p)$ should have a zero value at $p = g$.

It is very easy to seek such function, but here we consider it in a different way. Let us introduce an auxiliary function by the next differential equation

$$(d^2/dx^2 + g^2)\,\sigma(x) = H(x) . \tag{2.4}$$

This is a well-known equation, and the function σ is determined uniquely except two boundary conditions when $H(x)$ is given, so that we may use σ instead of H.

Now, putting (2.4) into (2.2) and integrating by parts, we have

$$F(p) = [(d/dx + ip)\,\sigma]\,\exp.(-ipx)\Big|_{x=-1}^{1} + (g^2 - p^2)\int_{-1}^{1}\sigma(x)\,\exp.(-ipx)\,dx, \tag{2.5}$$

so that we may have

$$F(g) = [(d/dx + ip)\,\sigma]\,\exp.(-ipx)\Big|_{x=-1}^{1} \tag{2.6}$$

Thence, if

$$\sigma(\pm 1) = (d/dx)\,\sigma\,(\pm 1) = 0 , \tag{2.7}$$

$F(g)$ vanishes and the wave resistance vanishes by (2.1).

Thus, to find a wave-free distribution we simply need to find a function σ with the boundary conditions (2.7). Then, the pressure is given by (2.4) and the surface elevation

$$-\eta(x) = g^2\sigma(x) + (g/\pi)\int_{-1}^{1}\frac{(d/dx')\sigma(x')}{x - x'}\,dx' . \tag{2.8}$$

It is noteworthy that this surface elevation is symmetric when σ is symmetric. For example, let us consider

$$\sigma(x) = (1/g^2)(1 - x^2)^2 , \tag{2.9}$$

Then, we have

$$H(x) = (1 - x^2)^2 + 4(3x^2 - 1)/g^2 \quad , \tag{2.10}$$

$$-\eta(x) = (1-x^2)^2 + [8(2/3-x^2) + 4x(1-x^2)\log(1-x/1+x)]/\pi g \ , \tag{2.11}$$

$$\int_{-1}^{1} H(x)dx = 16/15, \quad -\int_{-1}^{1} \eta(x)dx = 16/15 + 8/(3\pi g) \quad , \tag{2.12}$$

We see these curves in Figure 1.

In low speed, we may notice the next approximation except near both ends.

$$H(x) \doteqdot -\eta(x) \doteqdot g^2\sigma(x) \quad . \tag{2.13}$$

3. Three Dimensional Case I[6]

Consider the three dimensional flow and take the origin at midship on the undisturbed stream, the x-axis horizontally in upstream direction and the z-axis vertically upwards.

The wave-resistance of the Michell-Havelock type ship is given as

$$R = (\rho g^4/\pi) \int_0^{\pi/2} |F(g\sec^2\theta, \theta)|^2 \sec^5\theta \, d\theta, \tag{3.1}$$

with

$$F(k,\theta) = \int_{-t}^{0} \int_{-1}^{1} H(x,z) \exp.(kz-ik\, x \cos\theta)dxdz, \tag{3.2}$$

where $H(x,z)$ means the breadth of the ship.

First, we will show a type of wave-free distribution, when H has a form

$$H(x,z) = T(z)H(x) \quad . \tag{3.3}$$

Then, the function in (3.2) reduces to

$$F(k,\theta) = I(k)f(k \cos\theta) \quad , \tag{3.4}$$

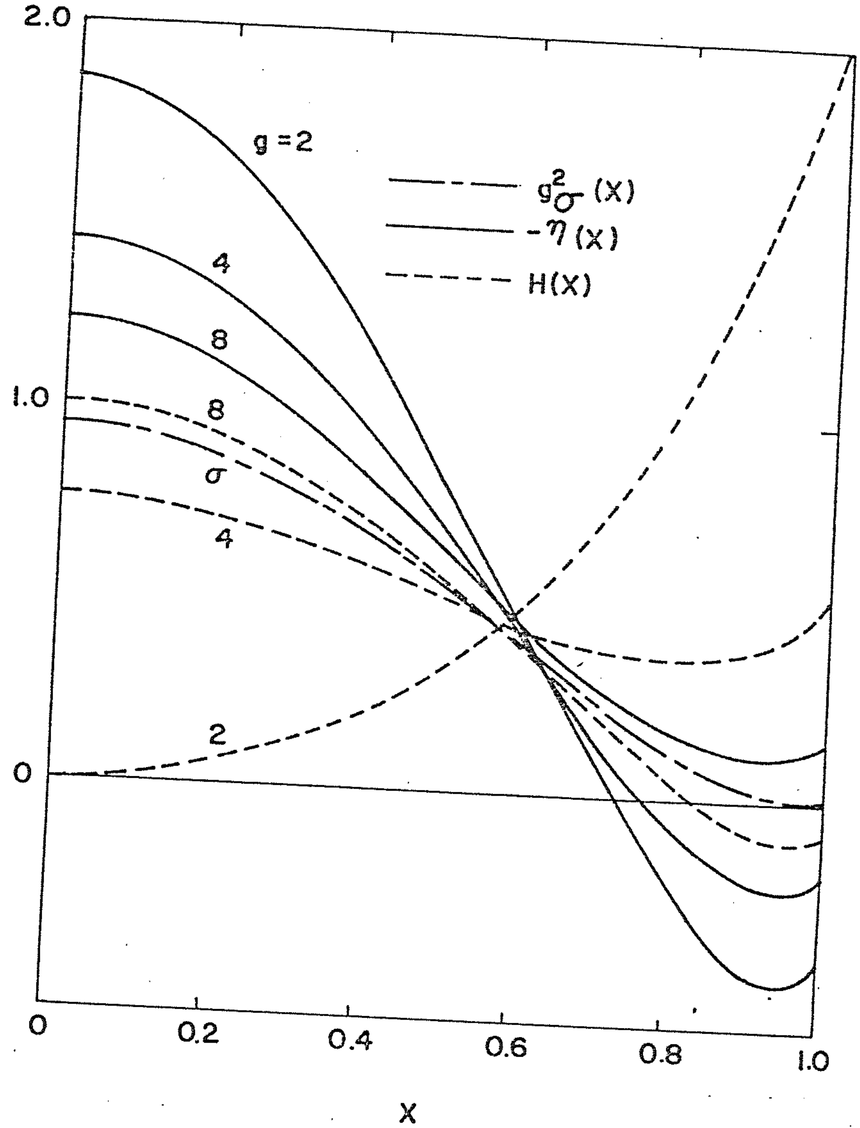
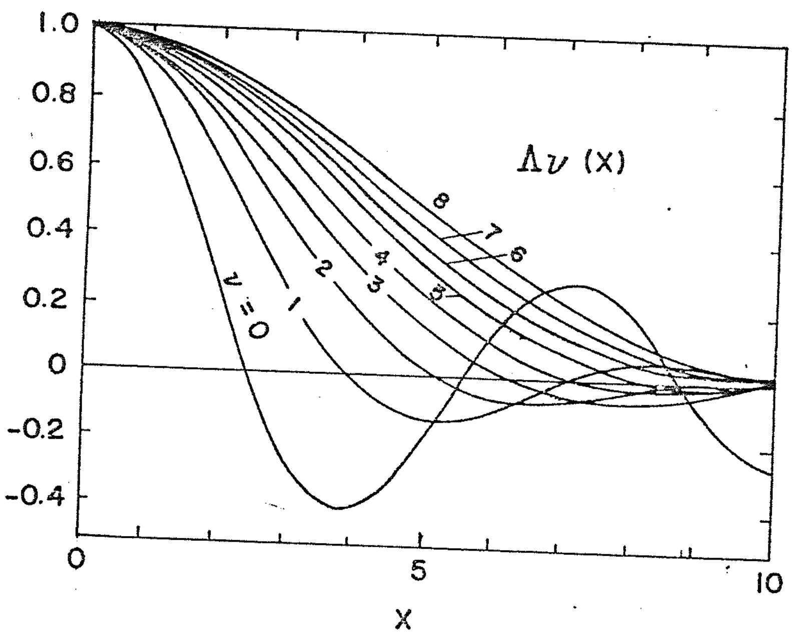


Figure 1.



$$\Lambda_\nu(X) = \frac{\Gamma(\nu+1)}{(gx/2)^\nu} J_\nu(gx)$$

Figure 2.

where

$$I(k) = \int_{-t}^{0} T(z) \exp(kz)\, dz, \tag{3.5}$$

and

$$f(p) = \int_{-1}^{1} H(x) \exp(-ipx)\, dx, \tag{3.6}$$

Now, if $\quad f(p) = 0, \quad \text{for } p = g\sec\theta > g ,$ $\quad$ (3.7)

then we clearly have a type of wave-free distribution.

This condition (3.7) is fulfilled if $H(x)$ is represented as

$$H(x) = \int_{-g}^{g} f(p) \exp(ipx)\, dp, \tag{3.8}$$

and so the range of distribution tends to infinity and generally it takes negative value in some portions.

For example, let us put

$$f(-g\cos\theta) = \cos n\theta/(g\sin\theta) \quad , \tag{3.9}$$

then we have

$$H(x) = 2\pi\, (-i)^n J_n(gx) , \tag{3.10}$$

that is, they are represented by Bessel functions.

For another example, let us put

$$f(-g\cos\theta) = (2\sqrt{\pi}/g)[\Gamma(\nu+1)/\Gamma(\nu+1/2)] \sin^{\nu-1}\theta, \ \nu > 0 \tag{3.11}$$

then, we have

$$H(x) = \Lambda_\nu(x) \equiv \Gamma(\nu+1) J_\nu (gx)/(gx/2)^\nu , \tag{3.12}$$

These curves are shown in Figure 2 and we see that these, especially for small ν, tend rapidly to zero by departing the origin, so that we may obtain the distribution with small wave-resistance by cutting out both ends.

4. Three Dimensional Problem II(6,7)

If the distribution is confined to some finite area, then we may treat it the same way as in paragraph 2.

Introduce an auxiliary function by the next differential equation.

$$[\partial/\partial z - (1/g)\, \partial^2/\partial x^2]\, \sigma(x,z) = H(x,z) \quad . \qquad (4.1)$$

Then, putting this into (3.2) and integrating by parts, we have

$$F(g \sec^2\theta, \theta) = \int_{-1}^{1} [\sigma(x,z) \exp(gz \sec^2\theta - igx \sec\theta)]_{z=-t}^{0} dx$$
$$- \int_{-t}^{0} [(\sigma + ig \sec\theta\, \partial\sigma/\partial x) \exp(gz \sec^2\theta - igx \sec\theta)]_{x=-1}^{1} dz \quad . \qquad (4.2)$$

Hence, if we have

$$\sigma(x,0) = \sigma(x,-t) = \sigma(\pm 1,z) = \partial\sigma/\partial x(\pm 1,z) = 0, \quad (4.3)$$

then F vanishes and we have a wave-free distribution. However, integrating (4.1) and putting (4.3), we have

$$\int_{-t}^{0} \int_{-1}^{1} H(x,z)\, dxdz = 0, \qquad (4.4)$$

namely, the total displacement of this distribution vanishes.

This is a different result from the above obtained, and there is no more interest with this for the object only to obtain the wave-free ship, but this distribution suggests a possibility to deform a ship shape in a certain arbitrary degree without change of the wave-resistance.

Namely, even if we add to or subtract from the given distribution this wave free one multiplied by an arbitrary constant, the wave-resistance is kept unchanged as easily seen.

We will call this method the invariant deformation. Considering in this way, (4.4) is an important property of such deformation, and that we have easily

$$\int_{-t}^{0} \int_{-1}^{1} xH(x,z)\, dxdz = 0 \quad , \qquad (4.5)$$

which says that the center of buoyancy does not also change by such deformations.

For example, consider the function of the next type

$$\sigma(x,z) = X(x)T(\zeta), \quad \zeta = (z+t)/t, \tag{4.6}$$

and put

$$X_0(x) = \xi^4(1-\xi)^2,$$

$$X_1(x) = 1-4\xi^3 + 3\xi^4,$$

$$X_2(x) = 1-10\xi^3 + 15\xi^4 - 6\xi^5, \tag{4.7}$$

where $\xi = (x-b)/(1-b)$ for $1 > x > b$ and $\xi = (-a-x)/(1-a)$ for $-1 < x < -a$, a and b are arbitrary constants, and let $T_1(\zeta)$ be the function of which derivative is composed of three segments and has the values

$$T_1'(0) = 5, \quad T_1'(0.2) = 1 \text{ and } T_1'(0.4) = T_1'(1) = -1 \tag{4.8}$$

and

$$T_2(\zeta) = \zeta(1 - \zeta) . \tag{4.8a}$$

Example 1.

At the speed Fr. = 0.212, put

$$X(x) = X_0(x) \text{ for } 1 > x > b = 0.5,$$

$$= 0 \qquad \text{for } b > x > -1 ,$$

$$T(\zeta) = T_1(\zeta) . \tag{4.9}$$

From these functions, compute $H(x,z)$ by (4.1) and subtract it from the off-sets of the ship showed by full lines in Figure 3, which is of an oil tanker. Then, the bulbous bow may become a conventional one which is shown by dotted lines. Here we replace the section form at F.P. by the chain line which is represented by $T_1(\zeta)$, merely because the full line is too complex to represent by a simple formula.

Example 2.

At the speed Fr. = 0.184 and of another oil tanker shown by full lines in Figure 4, put

Figure 3.

M. S. No. 1342
C_b = .80
L/B = 7.2
B/d = 2.46
L/d = 17.7

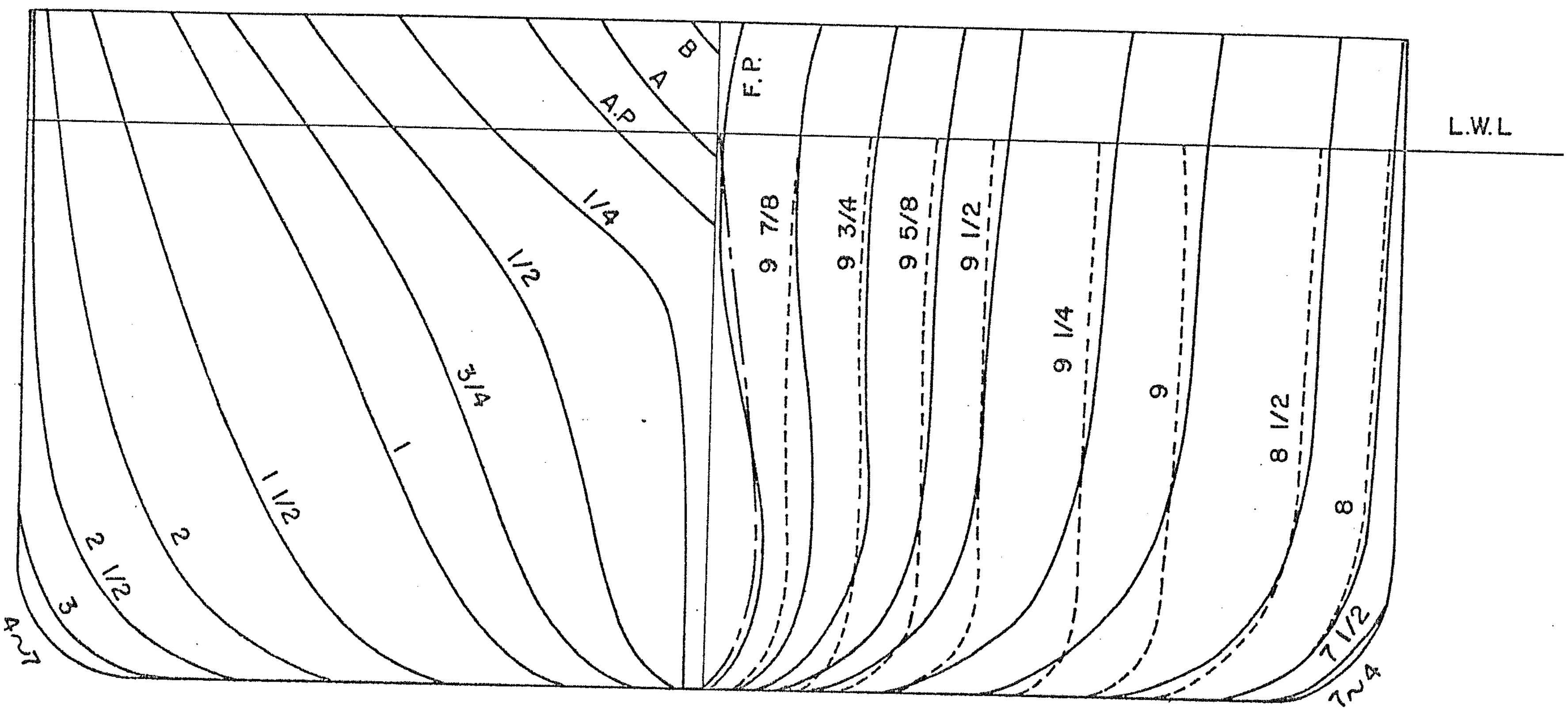
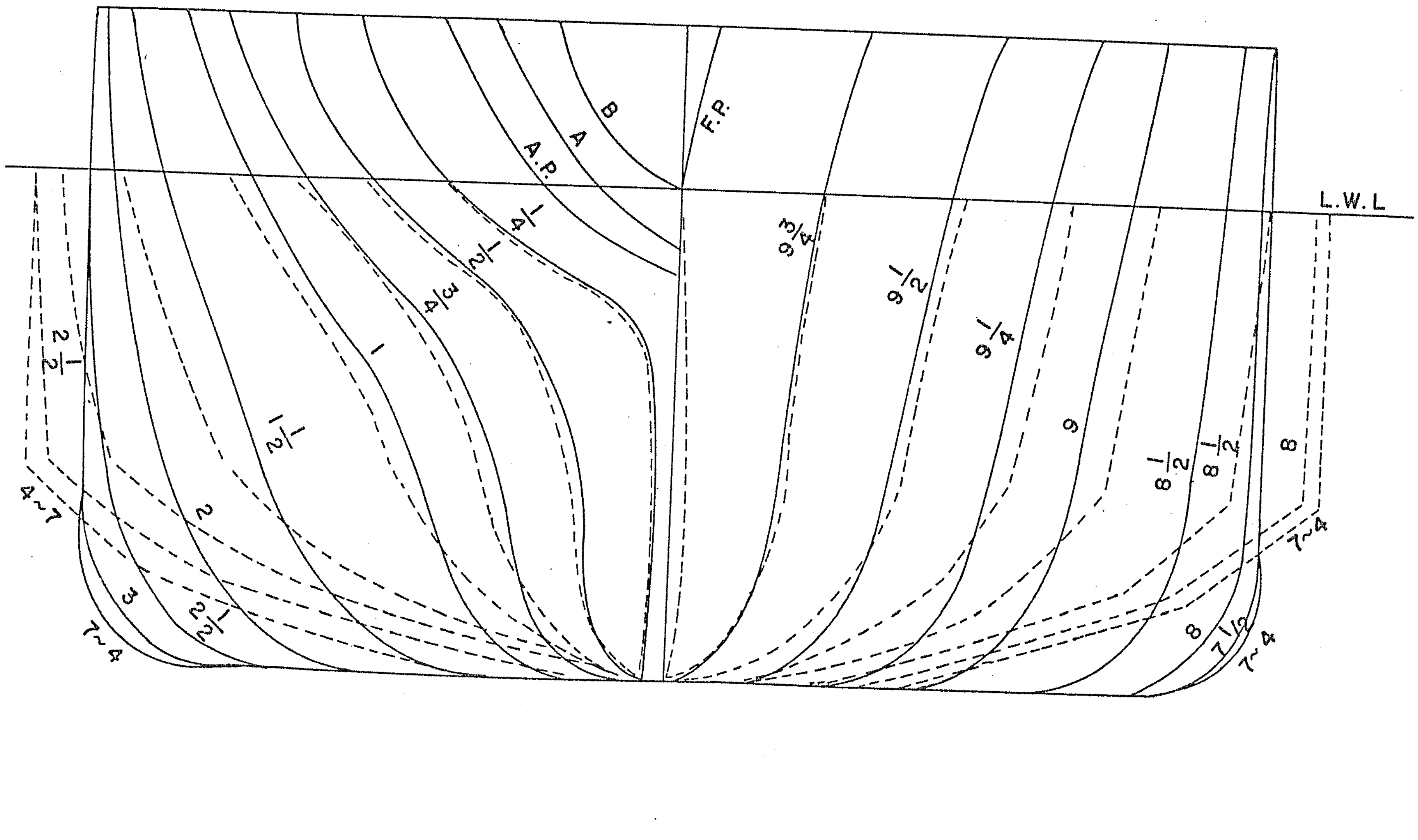


Figure 4.

M. S. No. 1321
C_b = .80
L/B = 7.34
B/d = 2.46
L/d = 18.08



$$
\begin{aligned}
X(x) &= X_1(x) && \text{for} \quad 1 > x > b = 0.5, \\
&= 1 && \text{for} \quad b > x > -a = -0.5, \\
&= X_2(x) && \text{for} \quad -a > x > -1 , \\
T(\zeta) &= T_1(\zeta) ,
\end{aligned}
\tag{4.10}
$$

and proceed as the above, we have dotted lines of which breadth is broadened about 10%.

Example 3.

For the same speed and the same ship as in Example 2, put

$$
\begin{aligned}
X(x) &= X_2(x) && \text{for} \quad 1 \geq x \geq b = 0.5, \\
&= 1 && \text{for} \quad b > x > -a = -0.5, \\
&= X_2(x) && \text{for} \quad -a > x > -1 ,
\end{aligned}
\tag{4.11}
$$

and $$T(\zeta) = T_2(\zeta) .$$

Then, as we see in Figure 5 by dotted lines, we may have the ship shape with inclined side shell.

In the latter two examples, the sectional area does not change so much that we may understand the well-known practice which says that the residual resistance of ships does not change so much in such cases. Namely, this method may propose a theoretical foundation to the problem of deforming the ship shape without sacrifice of the resistance so that we may easily find a suitable solution to the demand of the ship designer.

5. Transverse Wave-Free Distribution[7]

The method used in paragraph 2 is also applicable to making the wave element vanish in three dimensional case.

For example, put $\theta = 0$ and $k = g$ in (3.2), that is, consider the transverse wave only and the distribution of the type as (3.3), then

$$F(g,0) = \int_{-t}^{0} T(z)\exp(gz)dz \cdot \int_{-1}^{1} H(x)\exp(-igx)dx. \tag{5.1}$$

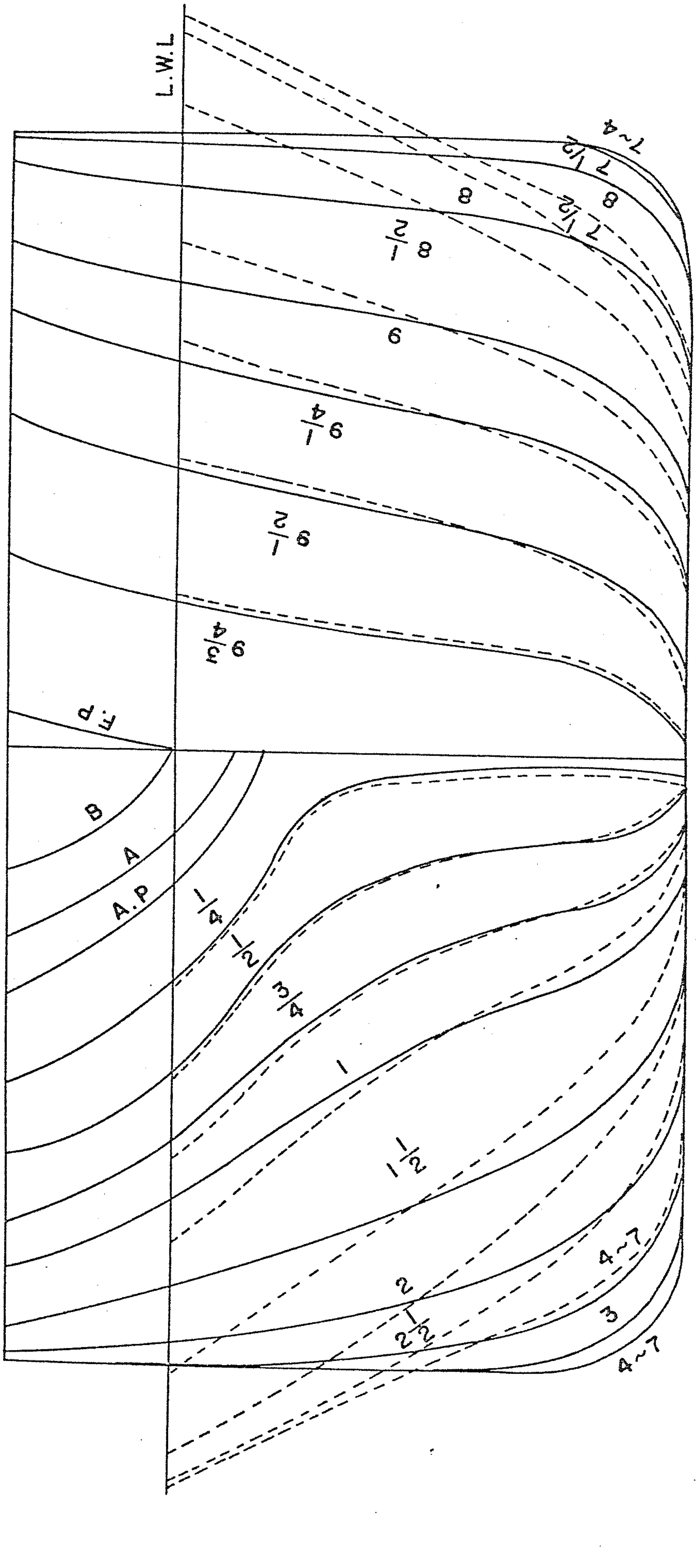


Figure 5. M. S. No. 1321.

Figure 5. M. S. No. 1321.

The integral in x is the same form as in (2.2), so that we may have a wave-free distribution by introducing the function σ defined by (2.4) and having the boundary values (2.7). We have seen an example of such distribution on Figure 1, and this may be a very simple method to obtain a ship form with small wave-resistance.

Let us see one more example. Put

$$H(x) = a_o[M(x) + (1/g^2)M''(x)] + a_2[N(x) + (1/g^2)N''(x)], \quad (5.2)$$

with

$$M(x) = (1-x^2)^2 \quad \text{and} \quad N(x) = x^2(1-x^2)^2 , \quad (5.3)$$

where a_o and a_2 are determined so as to $H(0) = 1/0.6$.

Then, we have curves in Figure 6 compared with the distributions of the minimum wave-resistance which are drawn by dotted lines. These curves show the similarity between both groups, so that we may suppose the wave-resistance of the present one's comparatively small.

Lastly, it is noteworthy that we may have also a transverse wave-free distribution by introducing the next function, considering in (5.1),

$$(d/dz + g)\,\mu(z) = T(z),$$

with

$$\mu(0) = \mu(-t) = 0 . \quad (5.4)$$

6. Conclusion

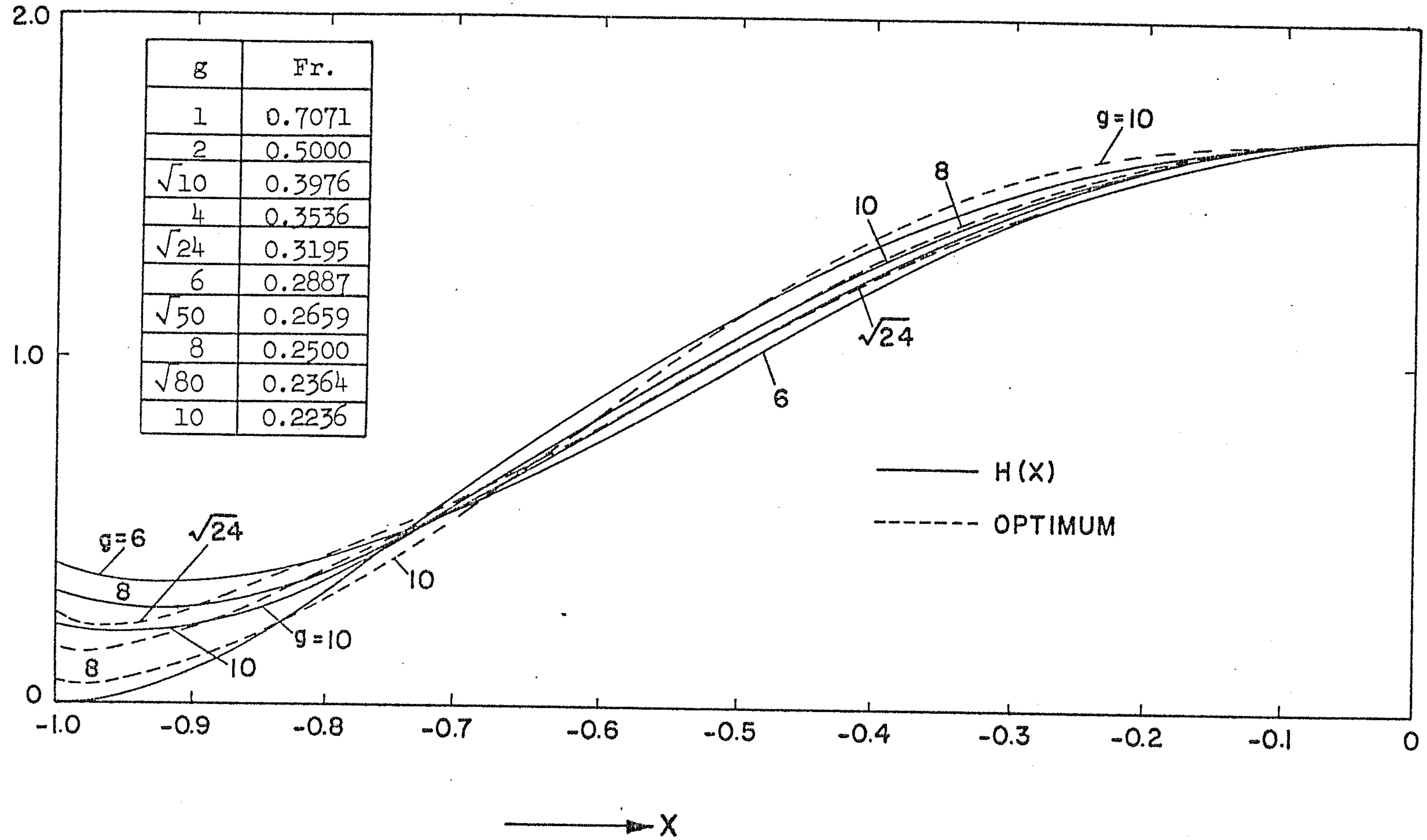
We may summarize the conclusions as follows:

i) The two dimensional wave-free distributions are obtained and applied to obtain ship water lines with small wave-resistance.

ii) The three dimensional ones distributed over an infinitely long range are obtained and may be useful to obtain such water lines as the above.

iii) The three dimensional ones over a finite area are obtained too, but have no displacement and longitudinal moment.

Figure 6. H(x), δ = 0.6.



g	Fr.
1	0.7071
2	0.5000
$\sqrt{10}$	0.3976
4	0.3536
$\sqrt{24}$	0.3195
6	0.2887
$\sqrt{50}$	0.2659
8	0.2500
$\sqrt{80}$	0.2364
10	0.2236

Figure 6. H(x), δ = 0.6.

However, they are useful in the sense that we may deform a ship shape without change of the wave-making resistance.

Lastly, the author wishes to thank Professors Inui, Jinnaka and Maruo for their kind encouragement and discussions and especially to Professor Yamazaki for pointing out large errors in his former manuscript and corrected in the present paper.

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