37

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THE MINIMUM PROBLEM OF THE WAVE RESISTANCE OF THE SURFACE PRESSURE DISTRIBUTION

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INTRODUCTION

The minimum problem of the wave resistance has no solution in thin ship theory, and this means that singularity distributions exist which have no wave resistance. On the other hand, the wave-free distribution belonging to the usual functional class has no displacement, but wave-free distributions with a finite displacement exist in the theory of the slender ship, although the wave-resistance integral has no finite value in such case (1,2). This apparent contradiction is caused by the confusion of the functional class of the distribution, but the introduction of the function of the wider class or the higher order singularity makes the theory more fruitful (3).

This paper explains such a situation of the problem with respect to the surface pressure distribution (1,4,5). The theory is very similar to the thin and slender ship theory.

By the way, this theory is the case in which the ship surface and the pressure are given in the framework of linearized theory, so that it may be interesting to compare it with the so-called second-order theory.

PRESSURE DISTRIBUTION

Consider a uniform stream with unit velocity, and Cartesian coordinates, taking the origin at the water surface, the x axis as positive toward the upstream side, and the z axis as positive upward. If a pressure $p(x,y)$ acts over the surface S at the water surface, some wave motion occurs. Let $\phi(x,y,z)$ be the velocity potential of this motion; then it must satisfy the conditions (6)

$$\frac{1}{\rho} p(x,y) \approx -\frac{\partial}{\partial x}\phi(x,y,0) - gZ(x,y) , \tag{1}$$

$$p(x,y) = 0 \quad \text{outside of } S , \tag{2}$$

$$\frac{\partial}{\partial x} Z(x,y) = \frac{\partial}{\partial z} \phi(x,y,0) , \tag{3}$$

and

$$\frac{\partial^2}{\partial x^2} \phi(x,y,0) = -g \frac{\partial}{\partial z} \phi(x,y,0) \text{ outside of } S , \tag{4}$$

where ρ is the water density, g is the gravity constant, and $Z(x,y)$ is the water surface elevation.

Then, it has the well-known (6) representation

$$\phi(x,y,z) = \frac{1}{4\pi\rho g} \iint_S p(x',y') \frac{\partial}{\partial x'} S(x,y,z;\, x',y',0)\, dx'dy' , \tag{5}$$

where (7)

$$S = - \lim_{\mu \to +0} \frac{g}{\pi} \int_{-\pi}^{\pi} \int_0^{\infty} \frac{\exp[kz + ik(x-x') \cos\theta + ik(y-y') \sin\theta]}{k \cos^2\theta - g + i\mu \cos\theta} dk d\theta$$

$$= 4gO_{-2}^{(1)} [g(x-x'),\ g(y-y'),\ -z] , \tag{6}$$

and where

$$\frac{\partial^2}{\partial x^2} + g \frac{\partial}{\partial z} \quad S = 2g \frac{\partial(1/r)}{\partial z} , \quad r^2 = (x-x')^2 + (y-y')^2 + z^2 , \tag{7}$$

Then the condition on the ship surface S becomes

$$\frac{\partial}{\partial x} Z = \frac{\partial}{\partial z}\phi = -\frac{1}{\rho g} \frac{\partial}{\partial x} p - \frac{1}{g} \frac{\partial^2}{\partial x^2} \phi , \tag{8}$$

or

$$Z(x,y) = -\frac{1}{\rho g} p(x,y) + \frac{1}{4\pi\rho g} \iint_S p(x',y') \frac{\partial^2}{\partial x'^2} S\, dx'dy' . \tag{9}$$

The solution of this integral equation has been examined by Maruo for small values (7,8).

For large g values, it is well known that the second term of the right-hand side of Eq. (9) is small and

$$Z(x,y) \approx -p(x,y)/\rho g \tag{10}$$

except near the periphery of S.

If the auxiliary function $m(x,y)$ is defined by the partial differential equation

$$p(x,y) = \left[\frac{\partial^4}{\partial x^4} + g^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right] m(x,y)\ , \qquad (11)$$

then it will be determined uniquely except for some arbitrary boundary conditions, say,

$$m(\pm 1,y) = \frac{\partial}{\partial x} m(\pm 1,y) = 0, \qquad m(x,\pm b) = 0\ , \qquad (12)$$

where S is assumed as a rectangle with length 2 and breadth $2b$.

Putting Eqs. (11) and (12) into Eq. (5), and integrating partially, yields (10)

$$\phi(x,y,z) = f(x,y,z) + \frac{1}{4\pi\rho g}\int_{-1}^{1}\left(\frac{\partial}{\partial y'}\ m\ \frac{\partial}{\partial x'}\ S\right)_{y'=-b}^{b} dx'$$

$$+ \frac{1}{4\pi\rho g}\int_{-b}^{b}\left(\frac{\partial^3}{\partial x'^3}\ m\ \frac{\partial}{\partial x'}\ S - \frac{\partial^2}{\partial x'^2}\ m\ \frac{\partial^2}{\partial x'^2}\ S\right)_{x'=-1}^{1} dy'\ , \qquad (13)$$

where (10)

$$f(x,y,z) = \frac{1}{2\pi\rho}\left(\frac{\partial}{\partial z}\right)\left(\frac{\partial^2}{\partial x^2} - g\frac{\partial}{\partial z}\right)\iint_S m(x',y')\ \frac{\partial}{\partial x'}\left(\frac{1}{r}\right)dx'dy'\ . \qquad (14)$$

Since the first term has no trailing wave, this formula shows that the potential consists of two parts; one is, say, the wave-free potential, and the other is the part having the trailing wave, which is a sum of singularity distributions along its periphery.

If $m(x,y)$ satisfies also the conditions

$$\frac{\partial^2}{\partial x^2} m(\pm 1,y) = \frac{\partial^3}{\partial x^3} m(\pm 1,y) = 0\ , \qquad \frac{\partial}{\partial y} m(x,\pm b) = 0\ , \qquad (15)$$

along with Eq. (12), then the potential is wave-free.

In this case, integrating Eq. (11) and imposing the conditions of Eqs. (12) and (15), it is found that the total pressure is zero:

$$\iint_S p(x,y)\ dxdy = 0\ . \qquad (16)$$

This is similar to the conclusion in thin ship theory (10).

For a simple example, the pressure distribution deduced from

$$m(x,y) = (b^2 - y^2)^2 (1 - x^2)^4 \tag{17}$$

is shown in Fig. 1.

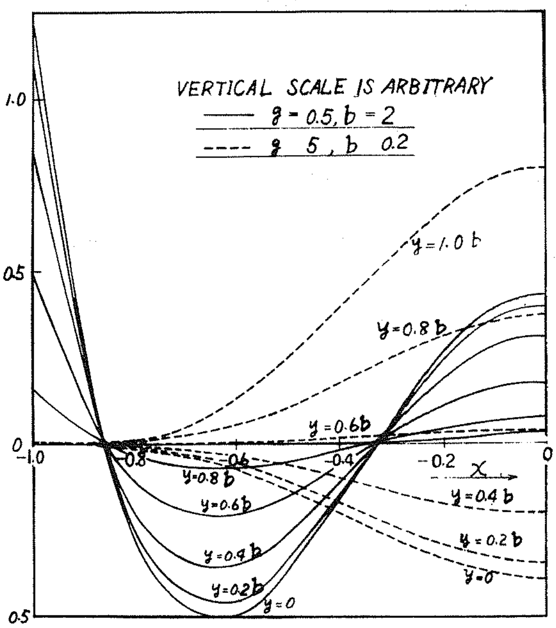

Fig. 1 - Pressure distribution of a wave-free potential

By the way, there are wave-free distributions with finite displacement (payload) in the two-dimensional problem, but their displacement is very small compared with their static buoyancy for high speed. This means that there is a large negative lift at high speed with wave-free distributions (11).

In another way, the potential $f(x,y,z)$ of Eq. (14) is always wave-free for an arbitrary function $m(x,y)$ without the conditions of Eqs (12) and (15), because it satisfies the surface condition of Eq. (4), but higher order singularities than the doublet must be introduced.

Thus, an arbitrary large number of pressure distributions exist with the same displacement and wave resistance as the following simple cases:

1. The longitudinal line distribution along the two segments $|y| = b$, $|x| \leqq 1$, which may be called the twin hull ship type.

2. The slender ship as the limiting case $b = 0$ of the above.

3. The transversal line distribution along one or two segments $|y| \leqq b$, which may be called the planing surface type.

For example, the pressure distribution deduced from

$$m(x,y) = (b^2 - y^2)(1 - x^2)^4 \tag{18}$$

belongs to case 1, and the pressure distribution deduced from

$$m(x,y) = (b^2 - y^2)^2 (1 - x)^4 (1 + x)^3 \tag{19}$$

belongs to case 2. Figures 2 and 3 show these examples.

When g is very small, the first term of Eq. (11) is dominant, but as g becomes larger and b smaller, the third term becomes dominant, where Eq. (10) is to be remembered.

The twin hull ship type for large g (Fig. 2) is especially interesting, for it may be considered as a model of a broad flat stern of a displacement ship (12).

Finally, the wave resistance R is (6)

$$R = \frac{g^2}{2\pi\rho} \int_{-\pi/2}^{\pi/2} |F(g \sec^2 \theta, \theta)|^2 \sec^5 \theta \, d\theta , \tag{20}$$

where

$$F(k,\theta) = \iint_S p(x,y) \exp[-ik(x \cos \theta + y \sin \theta)] \, dxdy , \tag{21}$$

or, interchanging the order of integration, the wave resistance can be written

$$R = \frac{g^2}{\pi} \iint_S p(x,y) \, G'(x,y) \, dxdy , \tag{22}$$

where

$$G'(x,y) = \frac{1}{\rho} \iint_S p(x',y') \, P_{-5}[g(x - x'), g(y - y'), 0] \, dx'dy' , \tag{23}$$

in which (13)

$$\left.\begin{matrix} P_{2n+1} \\ P_{2n} \end{matrix}\right\} (x,y,z) = (-1)^n \int_0^{\pi/2} \cos (y \sec^2 \theta \sin \theta) \left\{ \begin{matrix} -\cos (x \sec \theta) \cos^{2n+1} \theta \\ \sin (x \sec \theta) \cos^{2n} \theta \end{matrix} \right\} e^{-z \sec^2 \theta} \, d\theta . \tag{24}$$

This function G' may be called the influence function.

Putting Eq. (11) into Eq. (20), integrating partially, and making use of Eqs. (12) and (15), it is found that the wave-free distribution has no wave resistance.

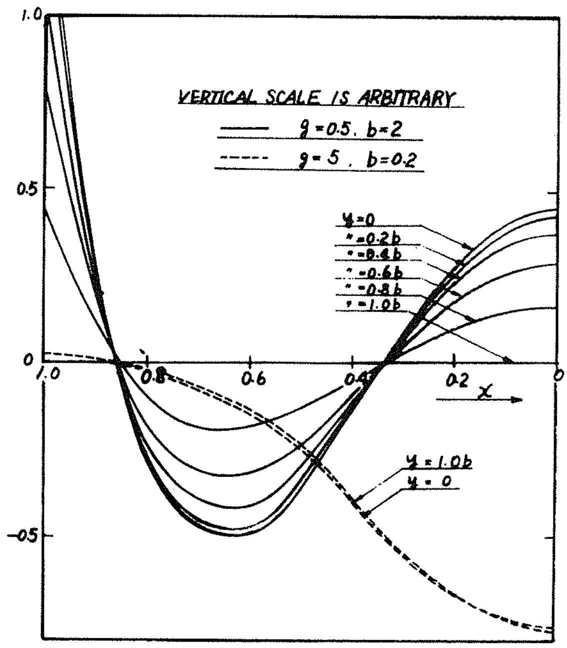

Fig. 2 - Pressure distribution of the twin hull ship type

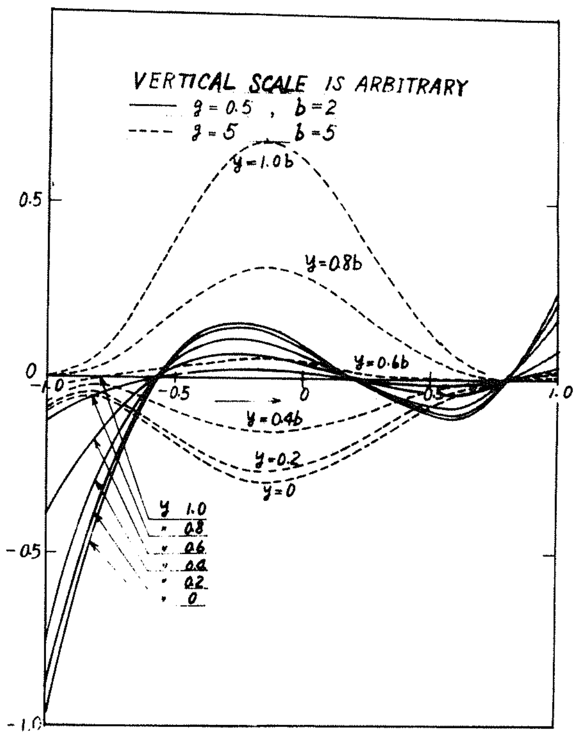

Fig. 3 - Pressure distribution of the planing surface type

MINIMUM PROBLEM

When the total pressure is given, namely,

$$\iint_S p(x,y)\ dxdy = \Delta\ , \tag{25}$$

if the influence function becomes constant over S, that is, if

$$G'(x,y) = C > 0\ , \tag{26}$$

then the wave resistance is minimum.

On the other hand, differentiating partially, G' satisfies the differential equation

$$\left[\frac{\partial^4}{\partial x^4} + g^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right] G'(x,y) = 0\ , \tag{27}$$

assuming the existence of the integral.

Accordingly, G' may be represented uniquely by some boundary conditions. Hence, the integral equation given by Eq. (26) is regular in the domain, so that it may have a unique solution.

Since the existence of its derivative is assumed, the integral equation obtained by the differentiation of both sides of Eq. (26) may also have a unique solution, and this solution must be identically zero, because the right-hand side is zero.

Hence, the present minimum problem has no definite solution and no minimum value exists for the wave resistance (1,14). This fact may mean that its least value will be zero, because it may be possible to reduce the total wave amplitude as small as necessary, adding the longitudinal and transversal distribution to each other appropriately. However, the minimum solution exists in elementary cases such as thin ship theory.

Thus, the problem may be classified as follows:

1. Twin hull ship type distribution. When the speed is very high and the breadth narrow, this case nearly equals the next. As seen from the preceding section, it is also interesting at low speed, but usually it seems more useful to consider it in combination with case 3 (12), which means case 4.

2. Slender ship. In this well-known case there exists a unique solution except for arbitrary wave-free distributions which have no wave resistance but a finite displacement (1,9).

3. Transversal line distribution. This is the case to be studied in the following section (1,4,5,7).

4. Problem to reduce the wave resistance. This is also a practical problem, that is, how and how much the wave resistance can be reduced by the adequate combination of the elementary distributions, keeping the practical restrictions of the actual ship.

There is another case which has a unique solution, that is, a symmetric distribution about the origin over a circular disc, but it is nearly equal to case 3 for large velocity (1).

TRANSVERSAL LINE DISTRIBUTION

Case 3 above will now be studied, namely, a transversal line distribution (1,5,7-9). Suppose the wave-source doublet is along the segment $|y| < 1$, of which the total is given as Eq. (25). Introducing a normalized distribution H as

$$H(y) = \frac{2}{\Delta}\int p(x,y)\,dx\,, \tag{28}$$

this is written as

$$\int_{-1}^{1} H(y)\,dy = 2\,. \tag{29}$$

The wave resistance (Eq. (22)) can be written as

$$r^* \equiv R\left(\frac{\Delta^2}{\rho g B^3}\right) = R\Big/\left(\frac{\Delta^2}{8\rho g}\right) = 2\int_{-1}^{1} H(y)\,G(y)\,dy\,, \tag{30}$$

where

$$G(y) = \frac{g^3}{2}\left(1 - \frac{2}{g^2}\frac{d^2}{dy^2}\right)G^*(y) \tag{31}$$

and also

$$G^*(y) = \frac{1}{\pi}\int_{-1}^{1} H(y')\,P_{-1}[0,\ g(y-y'),\ 0]\,dy' \tag{32}$$

or

$$G^*(y) = \frac{1}{2\pi}\int_{-1}^{1} H(y')\,K_0\left(\frac{g}{2}|y-y'|\right)dy'\,, \tag{33}$$

where B is the breadth in the usual unit system and K_0 is a modified Bessel function.

Equation (31) can be deduced from the following relations. Since the function P_{-1} has an expansion (13)

$$P_{-1}(x, y, 0) = \sum_{n=0}^{\infty} \frac{\epsilon_n}{2} K_{2n}(y/2)\, J_{4n}(x) \,, \tag{34}$$

and since

$$P_{-5}(x, y, 0) = \frac{\partial^4}{\partial x^4} P_{-1}(x, y, 0) \,,$$

then

$$P_{-5}(0, y, 0) = \frac{3}{16} K_0(y/2) - \frac{1}{16} K_2(y/2) = \frac{1}{4}\left(1 - 2\frac{d^2}{dy^2}\right) K_0(y/2) \,. \tag{35}$$

To solve the integral equation (33), assume the next expansion in Mathieu functions (15)

$$H(y) = \phi(\theta)/\sin\theta \,, \qquad y = \cos\theta \,, \tag{36a}$$

$$\phi(\theta) = \sum_{n=0}^{\infty} a_{2n}\, ce_{2n}(\theta, -q) \,, \qquad q = g^2/16 \,. \tag{36b}$$

Then, since there is the integral

$$\frac{1}{2\pi}\int_0^{\pi} K_0\left(\frac{g}{2}\,|\cos\theta - \cos\theta'|\right) ce_{2n}(\theta', -q)\, d\theta' = \lambda_{2n}\, ce_{2n}(\theta, -q) \,, \tag{37}$$

where

$$\lambda_{2n} = \frac{\pi}{2}\left[\frac{A_0^{(2n)}}{ce_{2n}(0, q)}\right]^2 \frac{Fek_{2n}(0, -q)}{ce_{2n}(0, -q)} \,, \tag{38}$$

because of the representations

$$\frac{1}{\pi}\int_0^{\pi} \cos(2k\cos\theta\sinh u)\, ce_{2n}(\theta, -q)\, d\theta = \frac{(-1)^n A_0^{(2n)}\, Ce_{2n}(u, q)}{ce_{2n}(0, q)} \,, \tag{39a}$$

$$\frac{1}{\pi}\int_0^{\infty} \cos(2k\cosh z\sinh u)\, Ce_{2n}(u, q)\, du = \frac{(-1)^n A_0^{(2n)}\, Fek_{2n}(z, -q)}{ce_{2n}(0, q)} \,, \tag{39b}$$

and the relation

$$\mathrm{Re}\,[Fek_{2n}(-i\theta, -q)] = \frac{Fek_{2n}(0, -q)}{ce_{2n}(0, -q)}\, ce_{2n}(\theta, -q) \,, \tag{40}$$

G^* can be integrated as (1,15)

$$G^*(\cos\theta) = \sum_{n=0}^{\infty} \lambda_{2n}\, a_{2n}\, ce_{2n}(\theta,-q)\,. \tag{41}$$

The minimum solution is a solution such that

$$G(y) = \text{constant}\,, \tag{42}$$

but, putting this into Eq. (31) results in a differential equation, so that

$$G^*(y) = \text{constant} \tag{43}$$

may be a special solution, but there is also a homogeneous solution:

$$G^*(y) = C \cosh\,(gy/\sqrt{2})\,, \tag{44}$$

where C is an arbitrary constant, for which

$$G(y) = 0 \quad \text{and} \quad r^* = 0\,. \tag{45}$$

This solution will be called a wave-free solution (1).

Since there are expansions (15)

$$1 = 2\sum_{n=0}^{\infty} (-1)^n A_0^{(2n)}\, ce_{2n}(\theta,-q)\,, \tag{46}$$

$$\cosh\,(g\cos\theta/\sqrt{2}) = \sum_{n=0}^{\infty} (-1)^n C_{2n}\, ce_{2n}(\theta,-q)\,, \tag{47}$$

where

$$C_{2n} = \frac{2(-1)^n A_0^{(2n)}\, Ce_{2n}(\sinh^{-1} 1,-q)}{ce_{2n}(0,q)}\,,$$

the general solution can be written in the form

$$H(y) = aH_a(y) - bH_b(y)\,, \quad a - b = 1\,, \tag{48}$$

where

$$H_a(y) = \phi_a(\theta)/\sin\theta\,, \quad \phi_a(\theta) = \sum_{n=0}^{\infty} a_{2n}^*\, ce_{2n}(\theta,-q)\,, \tag{49}$$

$$H_b(y) = \phi_b(\theta)/\sin\theta\ , \qquad \phi_b(\theta) = \sum_{n=0}^{\infty} b_{2n}^* \, ce_{2n}(\theta,-q)\ , \tag{50}$$

$$a_{2n}^* = \frac{2A_0^{(2n)}}{\pi A\lambda_{2n}}\ , \qquad A = \sum_{n=0}^{\infty} A_0^{(2n)\,2}/\lambda_{2n}\ , \tag{51}$$

and

$$b_{2n}^* = \frac{2(-1)^n C_{2n}}{\pi D\lambda_{2n}}\ , \qquad D = \sum_{n=0}^{\infty} C_{2n} A_0^{(2n)} \lambda_{2n}\ . \tag{52}$$

Then

$$G(y) = ag^3/(2\pi A)\ , \qquad r^* = 2ag^3/(\pi A)\ . \tag{53}$$

Although this solution becomes infinite at both ends $y = \pm 1$ in general, taking as a the value

$$a = \phi_b(0)/[\phi_b(0) - \phi_a(0)]\ , \tag{54}$$

this solution becomes zero there, in which case it will be called H_c.

For the numerical computation at high speed, it is more convenient to expand in the series of trigonometric functions as follows:

$$\phi_a(\theta) = \sum_{n=0}^{\infty} \alpha_{2n} \cos 2n\theta\ , \qquad \phi_b(\theta) = \sum_{n=0}^{\infty} \beta_{2n} \cos 2n\theta\ . \tag{55}$$

Figures 4, 5, and 6 and Tables 1 and 2 show the results (4). When the velocity is very large, that is, when g is very small, the functions become approximately

$$\phi_a(\theta) \approx \frac{2}{\pi}\left[1 - \frac{g^2}{16}\log(8/\gamma g)\cos 2\theta\right]\ , \tag{56a}$$

$$\phi_b(\theta) \approx \frac{2}{\pi}\left[1 + \frac{3g^2}{16}\log(8/\gamma g)\cos 2\theta\right]\ , \tag{56b}$$

$$\phi_c(\theta) \approx \frac{4}{\pi}\sin^2\theta\ , \tag{56c}$$

and

$$H_c(y) = \frac{4}{\pi}\sqrt{1-y^2}\ , \tag{56d}$$

where γ means Euler's constant. The wave resistance is, respectively (1,4)

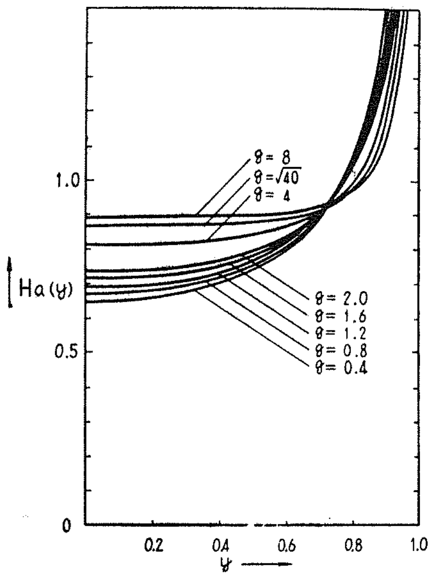

Fig. 4 - Minimum solution $H_a(y)$

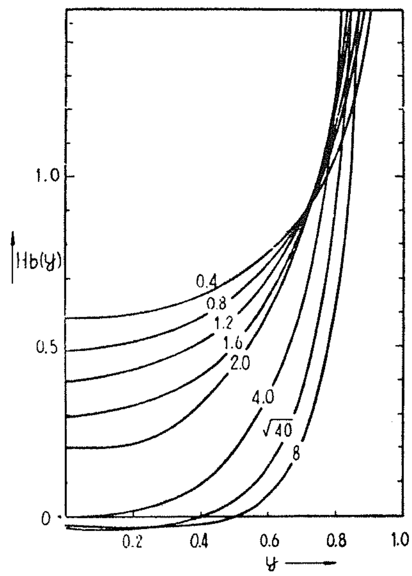

Fig. 5 - Wave-free solution $H_b(y)$

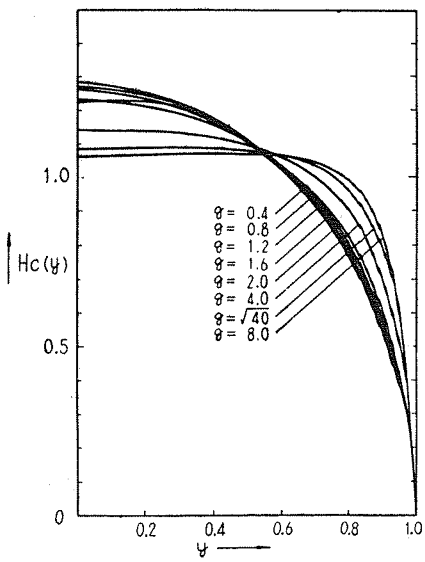

Fig. 6 - Minimum solution $H_c(y)$

Table 1
Fourier Coefficients of Trigonometrical Expansion

g	F_r	$2/\pi A$	r_c^*	a	b	α_0	α_2	α_4	α_6	α_8	$\phi_a(0)$	β_0	β_2	β_4	β_6	β_8	β_{10}	$\phi_b(0)$
0.4	1.118	1.5551	1.1051	11.10	10.10	0.6366	-0.0124	0	0	0	0.6242	0.6366	0.0492	0.0002	0	0	0	0.6860
0.8	0.7906	1.1431	2.570	4.392	3.392	0.6366	-0.0334	0	0	0	0.6032	0.6366	0.1428	0.0017	0	0	0	0.7811
1.2	0.6455	0.9196	4.696	2.955	1.955	0.6366	-0.0565	0	0	0	0.5801	0.6366	0.2377	0.0065	0.0001	0	0	0.8769
1.6	0.5590	0.7732	7.052	2.227	1.227	0.6366	-0.0788	-0.0003	0	0	0.5575	0.6366	0.3553	0.0199	0.0003	0	0	1.0120
2.0	0.5000	0.6683	10.25	1.919	0.919	0.6366	-0.0997	-0.0007	0	0	0.5362	0.6366	0.4522	0.0301	0.0008	0	0	1.1196
4	0.3536	0.4000	35.74	1.396	0.396	0.6366	-0.1810	-0.0040	0.0001	0	0.4515	0.6366	0.7808	0.1612	0.0137	0.0006	0	1.5923
$\sqrt{40}$	0.2812	0.2731	85.61	1.239	0.239	0.6366	-0.2389	-0.0098	-0.0004	0	0.3874	0.6366	0.9509	0.3489	0.0632	0.0068	0.0001	2.0065
8	0.2500	0.2222	135.0	1.187	0.187	0.6366	-0.2667	-0.0144	-0.0008	0	0.3546	0.6366	1.0164	0.4668	0.1177	0.0175	0.0003	2.2553

Table 2
Fourier Coefficients in Mathieu Functions

q	g	F_r	λ_0	λ_2	λ_4	λ_6	α_0^*	α_2^*	α_4^*	α_6^*	α_8^*	b_0^*	b_2^*	b_4^*	b_6^*	b_8^*	b_{10}^*
0.25	2	0.5000	0.53006	0.23202	0.12297	0.084	0.88814	0.17815	0.00177	0.00002	0	0.93649	0.37250	0.37250	0.02102	0	0
1	4	0.3536	0.33596	0.19364	0.11737	0.0814	0.80137	0.44815	0.01776	0.00021	0	1.09883	0.47674	0.10260	0.00684	0.00021	0
2.5	$\sqrt{40}$	0.2812	0.25608	0.15043	0.10793	0.07789	0.64228	0.66966	0.08237	0.00238	0.00002	1.28569	0.38995	0.18623	0.02797	0.00259	0.00007
4	8	0.2500	0.22568	0.12770	0.10006	0.07503	0.55116	0.73834	0.18163	0.00821	0.00019	1.37913	0.35853	0.20719	0.04676	0.00547	0.00025

$$r_0^* \approx (2g^3/\pi) \log (8/\gamma g) \quad \text{for } H_a(y) , \tag{57a}$$

$$r^* = 0 \quad \text{for } H_b(y) , \tag{57b}$$

and

$$r_c^* \approx 8g/\pi \quad \text{for } H_c(y) . \tag{57c}$$

Thus, H_c represents the elliptic loading of which resistance and minimum character are found by Maruo. He explains that the wave resistance is similar to the induced drag of a wing physically and theoretically (7).

In this respect, H_b corresponds to the load distribution of a wing:

$$H(y) = 1/\sqrt{1-y^2} . \tag{58}$$

Since the induced velocity of this distribution is zero, there might be no induced drag for such a wing, if such a flow could be realized (16). For the planing surface, however, there may be a possibility to realize such a flow by adding floats at both ends (17).

On the other hand, the similarity of H_b (Fig. 5) to the wave-free distribution (Fig. 1), especially at low speed, is also to be remarked.

Generally speaking, the situation with respect to the wave-free solution may be similar to that of slender ship theory, in which case there also exist wave-free distributions having a finite displacement, and they correspond to another class of the distribution which has smaller resistance than the slender ship (2).

CONCLUSION

As explained above, there is a close similarity between the theory of the thin ship and the pressure distribution. Thus, any pressure distribution is composed of line wave sources and wave-free distributions which have no displacement.

A typical elementary wave source is the twin hull ship type, that is, the longitudinal line distribution of the pressure on two parallel lines. Another is the planing surface type, that is, the transversal line distribution of the pressure.

Generally speaking, the minimum problem of the wave resistance has no solution and the least wave resistance may be zero, because some elementary wave sources could be summed up so as to cancel out their amplitude functions with each other. However, there are special cases when the solutions exist.

In this paper, the transversal line distribution of the pressure is treated, and it is found that there exists a unique minimum solution except for the wave-free distribution with a finite displacement. This wave-free distribution corresponds to the inverse elliptic load distribution of a wing and seems difficult to realize practically but suggests that distributions of another class may exist which have smaller wave resistance than the one considered here.

Finally, the larger the velocity, the smaller the wave resistance of the transversal line distribution; namely, this type is essentially preferable for very high speed, but it may be also possible to apply this theory to, say, the design of the destroyer stern.

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DISCUSSION

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Science is subject to fashion as much as other human activities. Recently the thin ship and surrogates have completely dominated the field, but in the twenties (and earlier) the pressure system has been considered as being an equally important hydrodynamic model (at least in principle) as the Michell ship, especially suitable for picturing fast shallow-draft and planing vessels. By Dr. Bessho's paper a sound equilibrium has been established. The present speaker had emphasized the similarity of the Hogner and the Michell integral (Zamm, 1930) and thus inspired Sir Thomas Havelock to derive the simple relation between source-sink distributions σ and pressure systems p

$$4\pi\rho g\sigma = c\,\frac{\partial p}{\partial x}$$

with the usual notations (c = speed of advance) (Havelock, collected papers, p. 373). The line distribution established by Dr. Bessho for a rectangular pressure domain follows from this equation.

Somewhat later (1935) von Karman has shown that the induced resistance of a finite-span wing can be derived from Hogner's integral for vanishing gL/c^2 or vanishing g in Bessho's formulation, thus anticipating Maruo's result in his splendid papers on planing surfaces. His analysis of pressure systems and corresponding form of planing hulls should be developed. A useful scheme had been developed by H. Wagner, who connected planing surface and wing phenomena.

Dr. Bessho has clarified the conditions of minimum resistance for pressure systems in a similar far-reaching manner, as he and Krein have succeeded in doing it for the Michell ship — now a classical problem which caused so much discussion. Obviously, further work should be done on nonrectangular domains of pressure distributions and on combinations of such domains. Further, his remarks on ship forms and singularity (source-line) distributions open the field for a much needed treatment of the resistance of moderately fast and high-speed forms, including fast displacement ships with transom sterns.

I have two questions: Dr. Bessho's g equals my speed parameter $\gamma_0 = 1/2F^2$. How is this g defined, and how is the condition obtained given by Eq. (27)?

* * *

REPLY TO DISCUSSION

Masatoshi Bessho

I thank Prof. Weinblum for his kind remarks and will clarify his questions. The definition of g in my paper is as follows: Suppose the uniform velocity V is unity and take the unit length to be a half of the ship length L; then the gravity constant g in this unit system is

$$g = \frac{g^*}{V^2}\left(\frac{L}{2}\right),$$

where g^* is the gravity constant in the usual unit system, so that it equals Prof. Weinblum's γ. In the latter part, the breadth of the planing surface is taken as twice unity, so that

$$g = \frac{g^*}{V^2}\left(\frac{B}{2}\right).$$

Equation (11) is derived as follows: We can introduce a regular function π such that

$$\frac{\partial}{\partial x}\pi(x,y,z) = -\left(\frac{\partial^2}{\partial x^2} + g\frac{\partial}{\partial z}\right)\phi(x,y,z)$$

having the surface value

$$\pi(x,y,-0) = \frac{1}{\rho}P(x,y).$$

On the other hand, if ϕ can be calculated from a regular function M by the equation

$$\phi = -\left(\frac{\partial^2}{\partial x^2} - g\frac{\partial}{\partial z}\right)\frac{\partial}{\partial x} M(x,y,z) \, ,$$

where $M(x,y,z) = -M(x,y,z)$, then ϕ has no radiating wave. Hence, putting the above into the first equation, we have

$$\pi(x,y,-0) = \frac{1}{\rho} P(x,y) = \left(\frac{\partial^4}{\partial x^4} - g^2\frac{\partial^2}{\partial z^2}\right) m(x,y)$$

$$= \left[\frac{\partial^4}{\partial x^4} - g^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right] m(x,y) \, ,$$

where $m(x,y) = M(x,y,-0)$. By the assumption, this is a wave-free pressure distribution.

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