ON STREAM LINES

by M. Bessho (*Defense Academy*)

The flow observation and measurement around a model ship is one of the most favourable role of the circulating water tank, but the measurement of the velocity or pressure is very difficult because the size of model is usually very small. We have a circulating tank and sometimes trace stream lines over the model ship surface by various methods but suffer from this difficulty.

This is a short note how much we may learn from the streamline observation.

1. Geometry of Streamline[1)]

Let s be the length measured along a stream line in three dimensional space, t the binormal, n the principal normal and x, y, z the Cartesian co-ordinates. Besides, define q as the velocity along the stream line and u, v, w the components of q in the x, y, z directions respectively.

The stream line is a curve having the direction cosines as

$$x_s = u/q, \quad y_s = v/q, \quad z_s = w/q \qquad (1)$$

where the suffix stands for the partial differentiation as in the following.

Then, the direction cosines of its principal normal are

$$x_n = \kappa x_{ss}, \quad y_n = \kappa y_{ss}, \quad z_n = \kappa z_{ss} \qquad (2)$$

where κ is the curvature and calculated by the formula:

$$1/\kappa = \sqrt{x_{ss}^2 + y_{ss}^2 + z_{ss}^2} \qquad (3)$$

Since

$$x_{ss} = u_s/q - uq_s/q^2, \cdots, \qquad (4)$$

by differentiating the equations (1), and

$$q_t = q_x x_t + q_y y_t + q_z x_t = u_s x_t + v_s y_t + w_s z_t = 0 \quad (5)$$

by the orthogonality between s, n and t, the curvature becomes

$$1/\kappa = \sqrt{q_x^2 + q_y^2 + q_z^2 - q_s^2/q} = |q_n|/q \qquad (6)$$

or

$$q^2/\kappa = \frac{d}{dn}(q^2/2).$$

This means that the centrifugal force balances to the gradient of the velocity head.

Nextly, the torsion $1/\lambda$ of the stream line becomes

$$1/\lambda = -\kappa^2 \begin{vmatrix} x_s & y_s & z_s \\ x_{ss} & y_{ss} & z_{ss} \\ x_{sss} & y_{sss} & z_{sss} \end{vmatrix} = -\frac{\kappa^2}{q^3} \begin{vmatrix} q & 0 & 0 \\ q_s & 0 & q_n \\ q_{ss} & q_{st} & q_{sn} \end{vmatrix} = q_{st}/q_n \quad (7)$$

Now, consider a surface covered by stream lines, for example, a ship surface, free water surface, and let ν be the normal of the surface. Then, the curvature of the surface along the streamline $1/\kappa'$ is

$$1/\kappa' = x_\nu x_{ss} + y_\nu y_{ss} + z_\nu z_{ss} = q_\nu/q \qquad (8)$$

Hence, if n and ν', that is, the principal normal of the stream line and the normal of the surface become coincident with each other, which means that the stream line is a geodesic, κ is equal to κ'. For example, q being constant over the free surface without gravity, the binormal of the stream line on the surface lies on the tangent plane, by the formula (5), that is ,both normals are coincident with, and that the torsion is zero by the formula (7).

Namely, a stream line on the free surface without gravity is a geodesic without torsion.

Since a geodesic line is a shortest path between two points over the surface, Maier form is a ship shape so designed as to be covered by geodesic stream lines.

Fixing the idea, suppose the center line of a long strip of paper as a curve in space curved and twisted. The binormal lies on the paper plane perpendicular to the center line and the principal normal is perpendicular to the paper plane. If this strip of paper is put softly on the surface, that is, as the paper plane becomes its tangent plane everywhere, its center line represents a geodesic line of the surface.

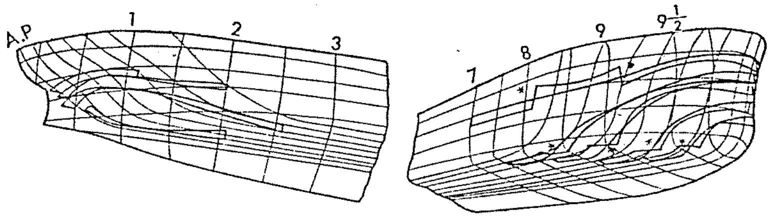
Now, draw a stream line on the ship surface with any means and put the one side of a paper strip along the stream line, then the other side does not touch the surface generally because a stream line is not always a geodesic. From the curvature and the torsion of the strip the derivatives of q may be calculated by the formulas (6) and (7).

Fig. 1 shows a sketch of such paper strip attached on a ship surface. From this experiment, it is found that the paper strip is strongly twisted near the bilge and can hardly touch on the stream line there, where the separation of the flow from the ship surface can be observed. This is considered as a result of large q_{st} by the formula (7).

2. Velocity and Pressure over the Ship Surface

Bernoulli's theorem is that

$$p/\rho + q^2/2 + gz = V^2/2 \qquad (9)$$



* Where attached side reversed
Model Dimensions: L_{pp}=2.00M, B=0.3005M, d=0.115M, C_b=0.81
Fig. 1. Torsion of streamline

where p is the gauge pressure, ρ the water density, g the gravity constant, V the ship speed and z taken upwards as positive.

Let Z be the surface elevation of the water on the ship surface, applying the theorem,

$$q^2/2+gZ=V^2/2, \text{ or } q=\sqrt{2g(H-Z)},\ H=V^2/2g \quad (10)$$

When the velocity q at the point z equals of the water surface point, the pressure is

$$p=\rho g(Z-z) \quad (11)$$

If the velocity potential ϕ exists, the velocity is its gradient and especially

$$q=d\phi/ds \quad (12)$$

and equi-potential surfaces or lines are orthogonal to streamlines.

Thence, if streamlines including the wave profile are given, the velocity and so the pressure is calculated as follows[2]:

a) Calculate q on the wave profile by the formula (10)
b) Integrate q so calculated to obtain ϕ
c) Draw equi-potential lines so as to be orthogonal to stream lines
d) Measure the gradient of ϕ on the stream line to obtain the velocity
e) The pressure is calculated from the formula (9) putting the above obtained velocity.

Figs. 2 and 3 are the result of such calculations for the fore body of a full shaped ship model with a round cylindrical stem[3]. Figs. 4 and 5 are for the aft body.

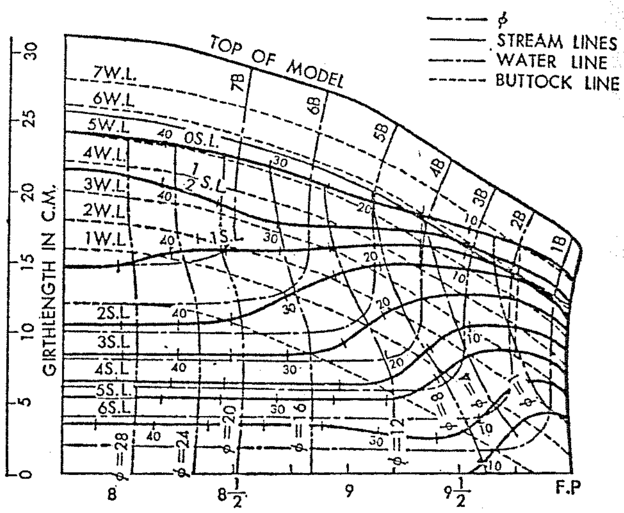
In the latter case, although the existence of the velocity potential is doubtful, assuming it to exist at first, define it by (12) and put $\tilde{\phi}$ and also the velocity $\tilde{q}$ calculated from $\tilde{\phi}$.

Then $\tilde{q}$ at the aft end does not become zero as shown by the dotted line in Fig. 5, but there is no speed by the observation. Hence, assuming the head loss ΔH in Bernoulli's equation (9), it becomes

$$p/\rho+q^2/2+gz=g(H-\Delta H) \quad (13)$$

$$q^2/2+gZ=g(H-\Delta H), \text{ at the water surface} \quad (14)$$

and assuming $q=0$ at the aft end where Z is Z_a,



Number along streamline means length in C.M. from F.P.
Fig. 2. Streamlines of forebody

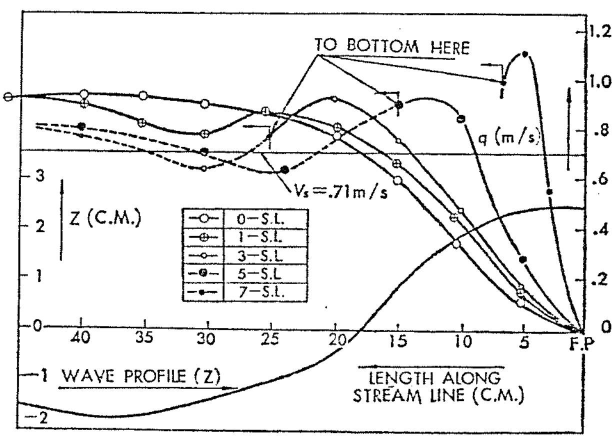
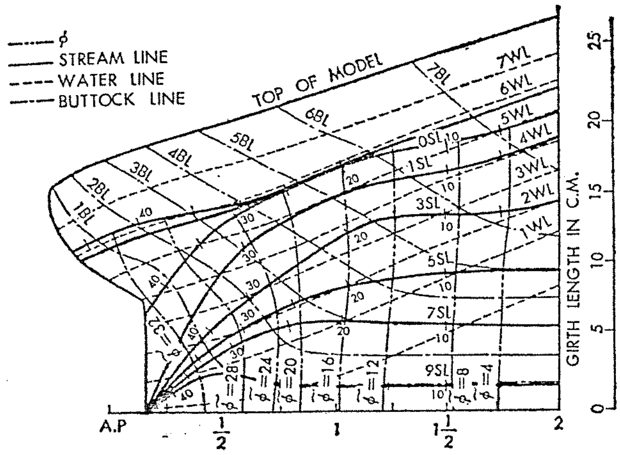


Fig. 3. Velocity distribution of fore body

$$\Delta H=H-Z_a \quad (15)$$

Making use of these formula, the calculation is carried out forwards from the aft end and arrives at near Sq. Station $1^1/_2$ where the phenomena similar to the hydraulic jump can be observed, which is the swell of the water diverging from the ship surface like the wave crest.

Since the existence of the velocity potential will not



Number along streamline means length in C.M. from SQ. ST. 2.

Fig. 4. Streamlines of AFT body

be doubtful forwards of this station and the values thus calculated seems to be reasonable, it seems to be reliable that the total head of the fluid is lost abruptly by some amount near this station like as in the hydraulic jump.

Lastly, it may be remarked that the calculated velocity on the bottom part may not be reliable because there are discontinuous lines of streamlines and equipotential lines may not be extended across these lines.

It will be clear that, from these considerations, if only the traces of streamlines over the ship surface are known, many flow characteristics are read from them qualitatively and even quantitatively.

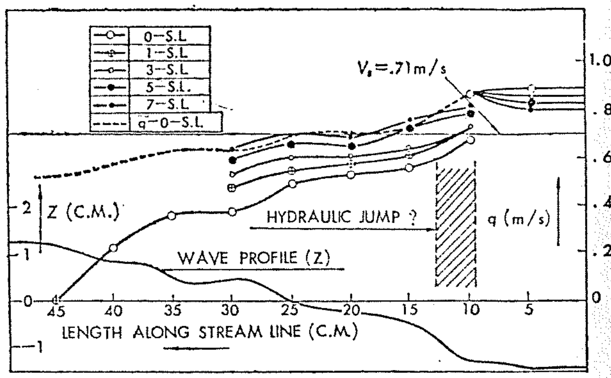


Fig. 5. Velocity distribution of AET body

References

1) K. Terazawa: "Sūgaku Gairon" 3rd ed., Iwanami, Tokyo, 1934.
2) M. Bessho: Journal of Zosen Kyokai Vol. 110, 1961.
3) M. Bessho: Journal of Zosen Kyokai Vol. 115, 1964.

INVESTIGATIONS ON THE FLOW AROUND THE ENTRANCES OF FULL HULL FORMS

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1. Introduction

At the present time bulbous bows of considerable dimensions have been quite commonly used for low speed ships of large block coefficients, such as mammoth oil tankers and bulk carriers. Suitably designed bulbous bows achieve a considerable reduction of residuary resistance for the entire speed range in the full load condition, while in the ballast condition a predominant reduction is obtained at the characteristic Froude number of about 0.2, as shown in Fig. 1.

In this connection the author presented a tentative explanation of how the bulbs work to reduce the residuary resistance. The bulb has a double function, smoothing the flow and canceling the bow wave. The primary function in the full load condition is the former. The flow is smooth and undisturbed around the entrances due to the bulbs. On the other hand the primary function in the ballast condition is wave cancellation due to the bulbs. These explanations, based on the author's streamline tracing on five models in the wind tunnel as well as the resistance test results at towing tanks and theoretical interpretations, accounted for some of the superficial effects of the bulbs fitted to the blunt entrances of the full hull forms. However, the hydrodynamic functions of the bulbs can hardly be said to have been revealed yet.

As the ship continues to become larger and takes smaller length–breadth ratio aind larger block coefficients, the main hull itself has presented various important ship-hydrodynamic problems. A poor correlation between models and ships is one of the most serious concerns among the people who are involved in ship propulsive power estimation.

One of the sources of difficulty lies in the extrapolation of model resistance results to full scale. Some