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"Some Notes on the Theory of the Wave-Resistance in Two-Dimension"

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Abstract

The author shows at first that the large second order effect of the free water surface of the submerged cylinder depends mainly upon its height of the entrained regular wave and that the range in which that effect becomes serious will be contained almost in the cases in which there is no stationary solution or there occurs a phenomenon like as hydraulic jump.

This effect depends upon the form, the submergence and the speed of the cylinder naturally and he shows examples of this effect with respect to the simple wave-free potential and concludes that there may exist no definite general rule.

Lastly, he shows the pressure measurement of some two dimensional wave-free cylinder models and obtains a fairly good correspondence with the linearized theory especially taking the second order term into consideration.

1. Introduction

The present difficulty of the wave-resistance theory is the so-called second order effect problem except the frictional effect and it is natural to try to study in the two-dimensional case, although the wave system is too simple to be analoguous to the three dimensional ship wave. The problem is generally divided into two parts, the accuracy of the ship surface and the water surface condition.

The former is studied by T. H. Havelork (ref. 1) for a circular cylinder firstly and the latter by T. Nishiyama (ref. 2) and M. Bessho (ref. 3) firstly but more correct treatment by E. O. Tuch (ref. 4), and it becomes clear by these works that the second order effect of the water surface is more important than the one of the body surface for a circular cylinder especially in low speed.

From this observation, E. O. Tuck looks with suspicion the validity of the linearized theory.

However, there is also a thought, in the other hand, that the smaller the wave-resistance, the smaller is the error of the first approximation and in fact a circular cylinder entrains large wave.

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Accordingly, it is felt that more things will be left to study.

In this stand point, this paper deals with some examples and characters of the wave-free potential and physical view of the second order potential of a circular cylinder.

2. A submerged cylinder

Consider the uniform water flow with the unit velocity, take the origin at the water surface, the x-axis as positive inversely to the flow direction and the y-axis vertically upwards.

Let the complex perturbation velocity potential be

$$\left.\begin{aligned} &f(z)=\varphi(x, y)+i\phi(x, y), \\ &z=x+iy\,, \qquad \frac{df}{dz}=-u+iv, \end{aligned}\right\} \tag{2.1}$$

where u and v mean the component velocities.

The pressure $p(x, y)$ is given by Bernoulli's theorem.

$$\frac{1}{\rho}\,p(x,y)=-gy-\frac{1}{2}\left|\,1+\frac{df}{dz}\right|^2+\frac{1}{2}, \tag{2.2}$$

where ρ means the water density and g the gravity constant.

If $\eta(x)$ means the water surface elevation, it is given by the equation

$$\eta(x)=-\phi(x, \eta), \tag{2.3}$$

Now, the first order potential satisfies the water surface condition where the pressure is constant, that is,

$$g\phi_1(x, 0)-\frac{\partial}{\partial x}\varphi_1(x_1\, 0)=0, \tag{2.4}$$

and

$$\eta_1(x)=-\phi_1(x, 0). \tag{2.5}$$

Then, the second order pressure excess at $y=0$ by this potential is

$$\frac{1}{\rho}\,p_2(x)=-\frac{1}{2}\left|\frac{df_1}{dz}\right|^2-\phi_1(x, 0)\,\frac{\partial}{\partial y}\left\{g\phi_1(x,0)-\frac{\partial}{\partial x}\varphi_1(x, 0)\right\}, \tag{2.6}$$

Hence, introducing the correction potential $f_2(z)$ with this pressure at $y=0$ as

$$g\phi_2(x,0)-\frac{\partial}{\partial x}\varphi_2(x, 0)+p_2(x)/\rho=0, \tag{2.7}$$

the surface elevation becomes to the second order

$$\eta(x)=-\phi_1(x, 0)-\phi_2(y, 0)+\eta\frac{\partial}{\partial y}\phi_1(x, 0), \tag{2.8}$$

This is the ordinary procedure to obtain the second order potential.
In the far down stream, f_1 will have the wave, that is.

$$f_1(z) \xrightarrow[x\to-\infty]{} ae^{-igz+iz}, \tag{2.6}$$

where a is the amplitude of the wave.
then p_2 will become by (2.6) (ref. 4)

$$p_2(x)/\rho \fallingdotseq -\frac{g^2}{2}a^2, \tag{2.10}$$

In the other hand,

$$\text{D.C. part of } \left[\eta(x) = -\phi_1(x, 0) + y\frac{\partial}{\partial y}\phi_1(x, 0)\right] = \frac{ga^2}{2}, \tag{2.11}$$

that is, there is the mean elevation as in the theory of the finite amplitude regular wave (ref. 7), and the second order pressure (2.10) balances to the mean surface elevation (2.11), so that ultimately ϕ_2 may balance (2.11) and the mean level may be kept unchanged (ref. 8).

In the other way, there exists a mean flow with the wave transmission as well known.

Its total volume is $ga^2/2$ per unit time and equals (2.11). (ref. 7).

In fact, this is explained as follows.

The actual representation of f_2 becomes

$$f_2(z) = \frac{1}{\pi\rho i}\int_{-\infty}^{\infty} p_2(x')S\{g(z-x')\}\,dx', \tag{2.12}$$

where

$$S(gz) = \lim_{\mu\to+0}\int_0^{\infty}\frac{e^{-ikz}}{k-g-\mu i}dk, \tag{2.13}$$

and

$$\left(\frac{d}{dz}+ig\right)S(gz) = -\frac{1}{z} \tag{2.14}$$

and, as seen from Fig. 2 of Tuck's paper, it will be justified in the far down stream to take the next approximation, assuming the pressure as a step function,

$$f_2(z) \fallingdotseq f_2^*(z) = \frac{ig^2a^2}{2\pi\rho}\int_{-\infty}^{0} S\{g(z-x')\}dx', \tag{2.15}$$

that is,

$$f_2^*(z) = \frac{ga^2}{2\pi\rho}\Big[\log z + S(gz)\Big]_{z=-\infty}^{z=z} \tag{2.16}$$

Thus, the excess flow by the mean level up is supplied by this sink at the origin, or near the origin more correctly, this sink concells out the source at infinite down stream, and causes the mean flow with the wave propagation.

Moreover, it is interesting to know that the wave-resistance R is

$$R=\frac{\rho g}{4}a^2=\frac{1}{2}\left(\frac{\rho g}{2}a^2\right), \tag{2.17}$$

which is explained hydraulically such as that mean momentum flow with the regular wave is $\rho\cdot ga^2/2$, but the one half of it balances the pressure rise by the level up to the water surface in the rear and the other half to the external force.

Meanwhile, by Tuck's calculation, it is clear that the contribution of f_2 to the second order effect is the greatest, and this is understood from the form (2.15) and (2.16).

Accordingly, if there is no wave, then this term does not contribute so greatly, because it has no logarithmic term as in (2.16).

Nextly, the wave has its maximum height, by Mithchell's formula, when

$$a/\lambda=ga/2\pi\leq 0.071, \tag{2.18}$$

where λ means the regular wave length (ref. 6 and 7).

For a submerged cirular circular cylinder at the depth h, since

$$f_1(z)=\frac{1}{z+ih}-\frac{1}{z-ih}-2igS\{g(z-ih)\}, \tag{2.19}$$

$$f_1(x)\xrightarrow[x\to-\infty]{}-4\pi ige^{-gh-igx}, \tag{2.20}$$

$$a=4\pi ge^{-gh}, \tag{2.21}$$

and then (2.18) gets to

$$2g^2e^{-gh}\leq 0.071, \tag{2.22}$$

or, putting (2.17) into (2.18) directly and taking its square,

$$\frac{R}{\rho g}\leq\frac{0.05}{g^2}, \tag{2.23}$$

This is a very severe restriction, for example, the immersin for which the wave does not become highest for all speed is

$$h\geq 3.91 \quad \text{at} \quad g=2/h, \tag{2.24}$$

This means moderately a deep immersion and the effect of the free surface is not very large.

If the immersion is shallower than (2.24), there is a range in which the wave-amplitude exceeds the upper limit, that is, for example,

$$\left.\begin{array}{ll} 0.285 \leq Fr = V/\sqrt{gh} \leq 1.80\,, & \text{for} \quad h=1 \\ 0.494 \leq Fr = V/\sqrt{gh} \leq 1.18\,, & \text{for} \quad h=2 \end{array}\right\} \tag{2.25}$$

where V is the uniform velocity.

In this range, there exists no stationary potential flow and occurs a phenomena similar as a hydraulic jump (ref. 8, 9 and 10).

Since a weak jump is a wave motion and a strong one is a state that there exists no stationary wave, this is a close analogy (ref. 6).

The criterion (2.18) can also be written in the usual unit system as

$$a/(v^2/2g) < 0.892, \tag{2.26}$$

Namely, this will mean that the maximum surface depression can not also exceed nearly the velocity head.

By the way, it is worthwhile to note that there exists an equi-pressure line for the first order velocity potential.

Let it describe y, it is given by pressure equation as

$$gy = -\frac{\partial}{\partial x}\varphi_1(x, y) - \frac{1}{2}q_1^2, \quad q_1^2 = \left(\frac{\partial \varphi_1}{\partial x}\right)^2 + \left(\frac{\partial \varphi_1}{\partial y}\right)^2,$$

taking to the second-order term, it becomes

$$gy = -\frac{\partial}{\partial x}\varphi_1(x, 0) - y\frac{\partial^2}{\partial x \partial y}\varphi_1(x, 0) - \frac{1}{2}q_1^2\Big|_{y=0}, \tag{2.27}$$

In the other hand, the surface elevation to the second order is represented by (2.8) so that the difference between them may become

$$g(y-\eta) = \frac{\partial}{\partial x}\varphi_2(x, 0), \tag{2.28}$$

This is a simple relation and the same as in the three dimensional case.

Namely, the difference between the equi-pressure line in the first order potential (2.27) and the free surface line in the theory accurate to the second order is given by the x-component of the second-order velocity.

3. Wave-free potential

The water surface condition can also be written in the form:

$$Re\left[\frac{d}{dz}f(z) + igf(z)\right] = 0\,, \quad \text{for} \quad y=0, \tag{3.1}$$

Hence, if $f(z)$ is represented as

$$f(z)=\frac{d}{dz}m(z)-igm(z), \tag{3.2}$$

by a regular $m(z)$ such that

$$Re[m(z)]=0 \quad \text{for} \quad y=0, \tag{3.3}$$

then $f(z)$ satisfies (3.1) and has no wave (ref. 11, 12).

For example, putting

$$m_n(z)=\frac{i}{g}\left[\frac{e^{i\alpha}}{(z+ih)^n}+\frac{e^{-i\alpha}}{(z-ih)^n}\right], \tag{3.4}$$

$f(z)$ can be calculated as

$$f_n(z)=\frac{e^{i\alpha}}{(z+ih)^n}+\frac{e^{-i\alpha}}{(z-ih)^n}+\frac{n}{ig}\left[\frac{e^{i\alpha}}{(z+ih)^{n+1}}+\frac{e^{-i\alpha}}{(z-ih)^{n+1}}\right], \tag{3.5}$$

Especially, for $n=1$ and $\alpha=0$,

$$f_1(z)=\frac{1}{z+ih}+\frac{1}{z-ih}+\frac{1}{ig}\left[\frac{1}{(z+ih)^2}+\frac{1}{(z-ih)^2}\right], \tag{3.6}$$

and that for $h=0$,

$$f_{10}(z)=\frac{1}{z}+\frac{1}{igz^2}, \tag{3.7}$$

In Fig. 1~4, the stream lines are sketched from rough calculations.

As seen from these figures, if g is very large they represent cylinders closed, although E. O. Tuck reports that a submerged doublet potential gives unclosed stream lines.

Now, velocity potential with a doublet sigularity can be expanded as

$$f(z)=(\alpha+i\beta)z+\frac{1}{z}, \tag{3.8}$$

near the origin, including the uniform flow potential, so that the equation of the stream line may be

$$\beta r\cos\theta+\left(\alpha r-\frac{1}{r}\right)\sin\theta=\text{Const.} \tag{3.9}$$

To obtain a circular stream line it is necessary that β is zero, that is, the flow is symmetric about the origin. (ref 4, 8)

Since the wave source potential can not be symmetric in uptream side and down tream side, there will be no possibility to have a closed stream line in the usual meaning, but this unclosendness character will diminish when the wave becomes smaller and, in fact, there is a closed stream line for a wave-free potential in a limited range.

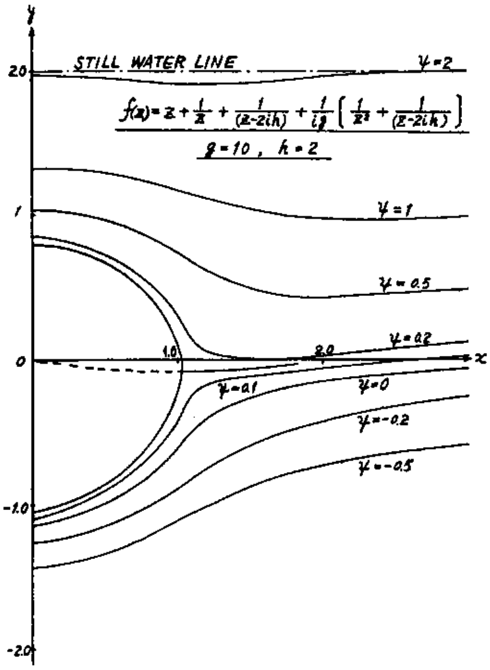

Fig. 1. Stream line of wave-free potential

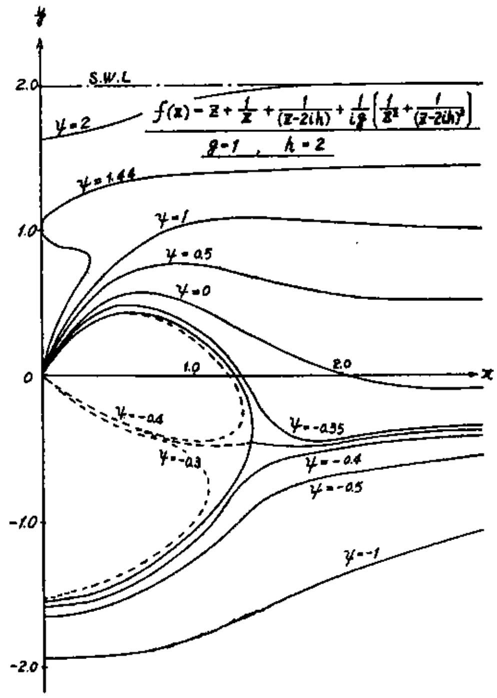

Fig. 2. Stream line of wave-free potential

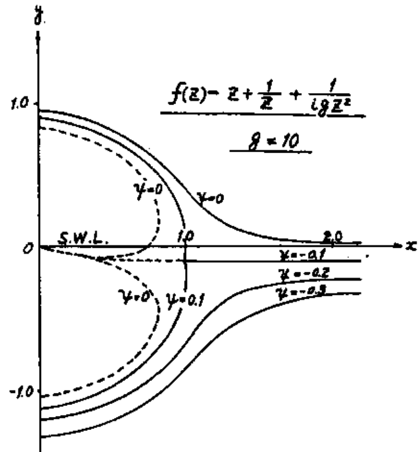

Fig. 3. Stream line of wave-free potential

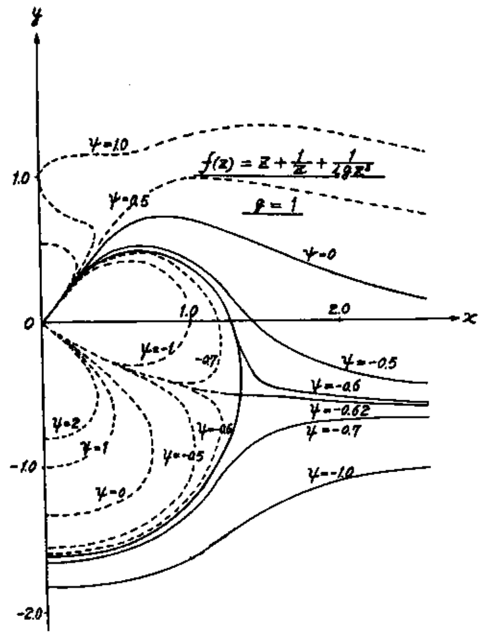

Fig. 4. Stream line of wave-free potential

Lastly, the wave-free potential has of course a wave train in the second order theory and there may not exist an exact wave-free potential in deep water but this will be a future problem.

Since the wave-free potentials (3.6) and (3.7) give nearly an accurate cylinder for very low speed in Figures, they can be served as the first approximation potentials of the circular cylinder.

Then, the second order terms with respect to the water surface will be calculated easily.

For the one example, consider (3.7) for large g, that is, a half immersed cylinder.

The second order potential is given by (2.12) where $p_2(x)$ by (2.6)

$$p_2(x)/\rho=\frac{1}{2x^4}+\frac{4}{g^2x^6}\xrightarrow[x=1]{}\frac{1}{2}+\frac{4}{g^2}, \tag{3.10}$$

Since

$$S(gx)\xrightarrow[x\ll -1]{} 2\pi ie^{-igx}, \tag{3.11}$$

The wave amplitude by (2.12) is

$$a_2=\frac{2}{\rho}\int_{-\infty}^{\infty} p_2(x)e^{-igx}dx, \tag{3.12}$$

Putting (3.10) into (3.12) and leaving without integration the segment $|x|\leq 1$, this is

$$a_2=2\int_1^\infty\left(\frac{1}{x^2}+\frac{8}{g^2x^6}\right)\cos gxdx\xrightarrow[g\gg 1]{}-\frac{2}{g}\sin g+, \tag{3.13}$$

This is very small compared with (2.12), but, since the wave-resitance is

$$R=\frac{\rho g}{4}a_2{}^2\doteqdot\frac{\rho}{2g}(1-\cos 2g)+\cdots, \tag{3.14}$$

it is always larger than the critical resistance (2.23) except narrow range near the hollow point $\cos 2g=1$.

For the other example, considering (3.6) for a submerged cylinder with large g, $p_2(x)$ becomes

$$p_2(x)/\rho=\frac{1}{r^4}(1+\cos 4\theta)+\frac{4}{g^2r^6}(2\cos 6\theta+3\cos 2\theta-1), \tag{3.15}$$

where $re^{i\theta}=x-ih$.

Then, a_2 by (3.12) is evaluated by the contour integration and its largest term becomes

$$a^2\xrightarrow[g\gg 1]{}\frac{4\pi}{15}g^3e^{-gh}, \tag{3.16}$$

This is much larger than (2.21) in contrary with the above case but consistent with Tuck's and Salvesen's results.

The wave-resistance will be

$$R \doteqdot \frac{\rho}{4}\left(\frac{4\pi}{15}\right)^2 g^7 e^{-2gh}, \tag{3.17}$$

As seen from these examples, the second order effect is very difierent in character with between a submerged and floating body.

Moreover, since there are also cases in which $p_2(x)$ has no integral as like as in the next paragraph's examples, it can not be treated in a straight manner.

Especially, for the floating body case, it is very complicated because the body surface condition too will contribute so much (ref. 13) and at least there may be two cases different in nature such that the intersection angle between the body and the water surface is 90° or not.

4. **Wave free pressure distribution** (ref. 11, 14)

Let $\sigma(x)$ be a real function such that

$$\sigma(\pm 1) = \frac{d}{dx}\sigma(\pm 1) = 0, \tag{4.1}$$

and take $m(z)$ in (3.2) and (3.3) as

$$m(z) = \frac{i}{\pi g^2}\int_{-1}^{1}\frac{\sigma(x')}{z-x'}dx', \tag{4.2}$$

then $f(z)$ becmes by (3.2) and using (4.1)

$$f(z) = \frac{1}{\pi g}\int_{-1}^{1}\frac{dx'}{z-x'}\left[\sigma(x') + \frac{i}{g}\frac{d}{dx'}\sigma(x')\right], \tag{4.3}$$

and the pressure by (2.7)

$$P(x) \equiv p(x)/\rho = -Re\left[\frac{df}{dz} + igf\right]_{y=0} = \sigma(x) + \frac{1}{g^2}\sigma''(x), \tag{4.4}$$

The surface elevation is calculated from (4.3) as

$$H(x) \equiv -g\eta(x) = \sigma(x) + \frac{1}{\pi g}\int_{-1}^{1}\frac{d\sigma(x')}{x-x'} \tag{4.5}$$

Consider a function which satisfies (4.1), namely,

$$\sigma_n(x) = \frac{1}{2n}\left[\frac{\sin(n-1)\theta}{n-1} - \frac{\sin(n+1)\theta}{n+1}\right], \quad n \geq 2, \quad x = -\cos\theta \tag{4.6}$$

Then, by the integration and differentiation, (4.4) and (4.5) become

$$P_n(x) = \sigma_n(x) + \frac{1}{g^2}\frac{\cos n\theta}{\sin\theta}, \tag{4.7}$$

$$H_n(x)=\sigma_n(x)-\frac{1}{gn}\cos n\theta, \tag{4.8}$$

Fig. 5 shows an example for $n=2$.

The pressure becomes infinite at both ends, but this is not favourable for the experiment.

Hence, the offset of models are drawn as follows:

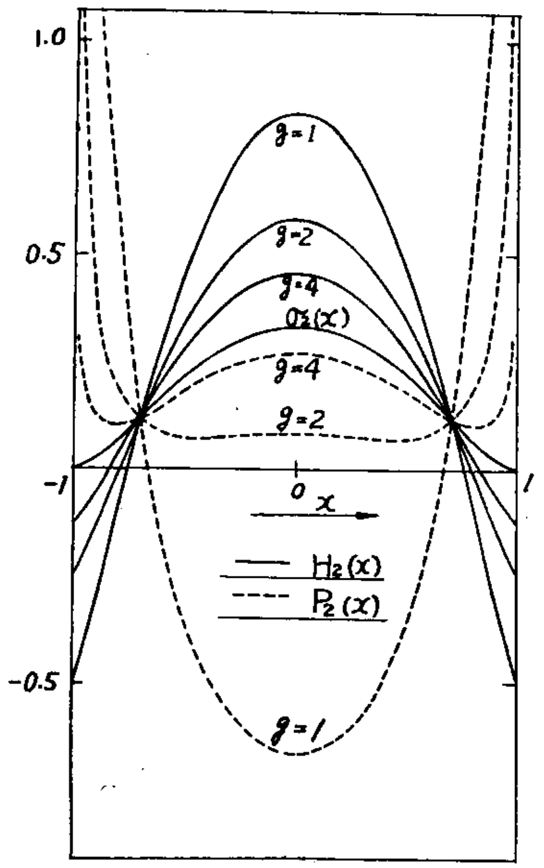

Fig. 5. Wave-free pressure distribution

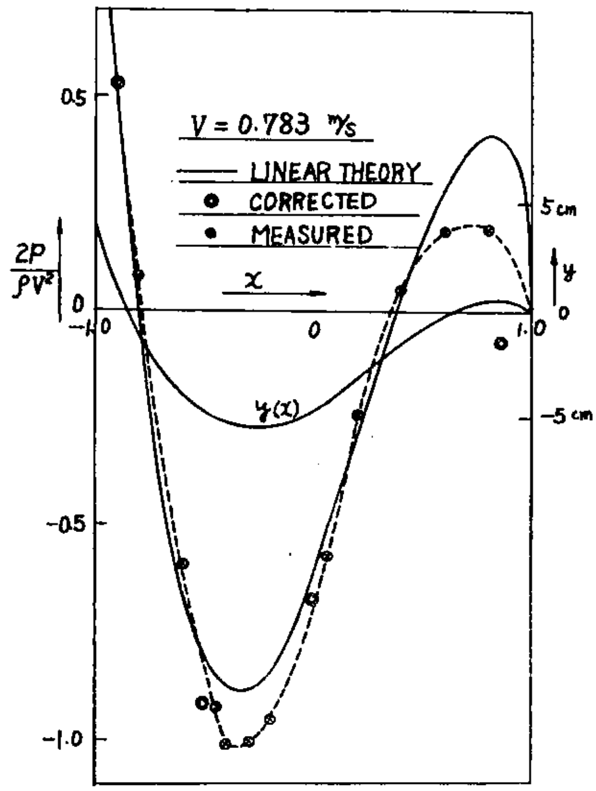

Fig. 6. Pressure distr of. M23G4

Table 1. Model Particulars.

Model	M23G4	M23G2	M23G1	M24G2	M24G1
Length (m)	0.56	0.40	0.30	0.40	0.30
Breadth (m)	1.18	1.20	1.19	1.20	1.20
Design Vel. (m/s)	0.783	0.990	1.212	0.990	1.212
Froude No. $V/\sqrt{gL}$	0.354	0.500	0.707	0.500	0.707
$1/\lambda$	2.5	3	4	3	4
Pay Load (Kg)	11.57	6.26	2.63	6.26	2.65

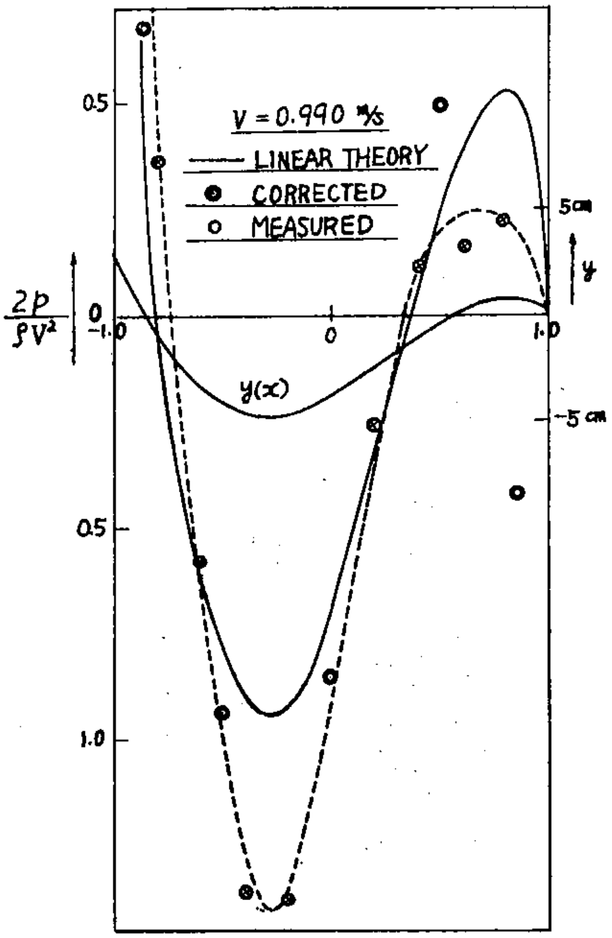

Fig. 7. Pressure distr. of M23G2

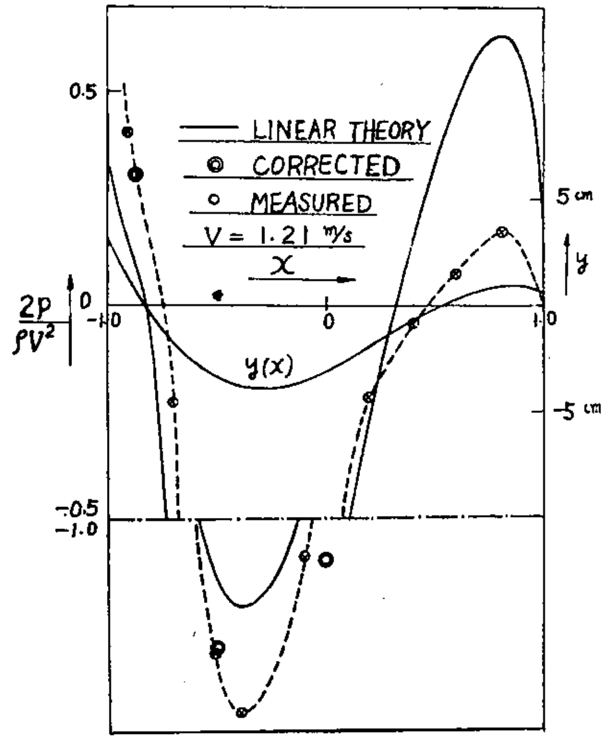

Fig. 8. Pressure distr. of M23G1

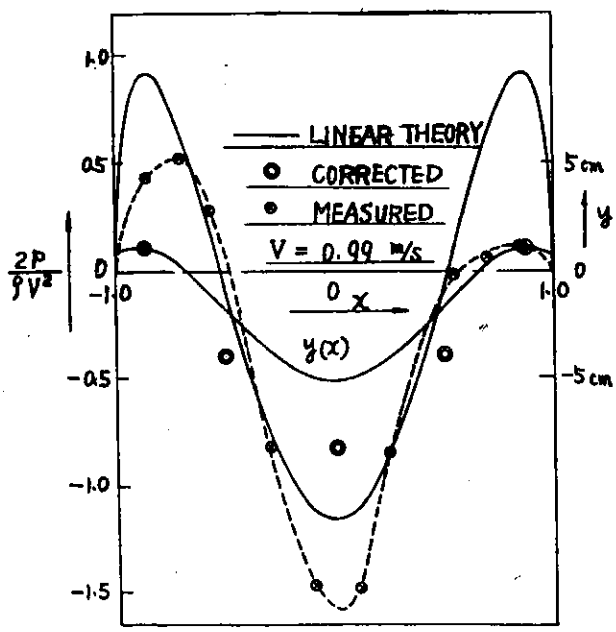

Fig. 9. Pressure distr. of M24G2

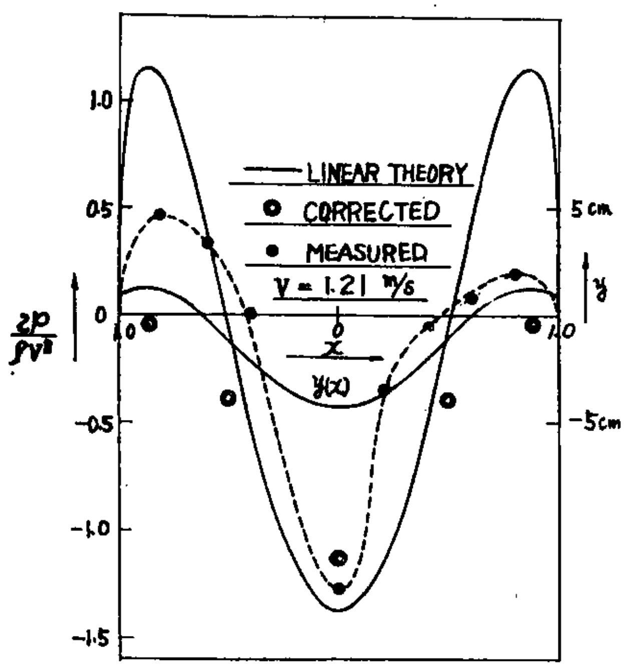

Fig. 10. Pressure distr. of M24G1

$$-y(x)=\lambda\{H_2(x)-H_3(x)\}, \tag{4.9}$$

which for $g=a$ are called *M23Ga.*

$$-y(x)=\lambda\{H_2(x)-H_4(x)\}, \tag{4.10}$$

which are called *M24Ga.*

Model dimensions are shown in the table and the experimental results in Fig. 6~10.

Here, the manometer head rise above the still water level is the difference of (4.4) and (4.5), that is,

$$\textit{Manometer pressure} = \frac{p}{\rho} + gy \fallingdotseq -\left.\frac{\partial\varphi}{\partial x}\right|_{y=0}, \tag{4.11}$$

but the pressure in these figures is the sum of this and the statical head gy so as to be convenient to compare with the linearized pressure.

The theoretical values are not bad, (ref. 15) if it takes account into consideration that the model depth is greater than the velocity head and the linearization assumption is hardly consistent.

Thus, the second order pressure must be considered.

From Bernoulli's equation, the pressure is given as

$$\frac{1}{\rho}p(x,y) = -gy - \frac{\partial\varphi}{\partial x} - \frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2\right], \tag{4.12}$$

Accordingly, subtracting the first order pressure (4.4) from the above, the correction term is

$$\frac{1}{\rho}p_c(x) = -y\frac{\partial^2}{\partial x\partial y}\varphi(x,0) - \frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2\right], \tag{4.13}$$

or introducing the boundary condition,

$$\frac{1}{\rho}p_c(x) = -y\frac{d^2}{dx^2}y - \frac{1}{2}\left(\frac{dy}{dx}\right)^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2, \tag{4.14}$$

The corrected values are plotted with the doulble circle marks at some points.

They seem approach the experimental values qualitatively except the concave part and M24-type models.

Of course, there may occur the boundary layer separation in the concave part and also in fact M24-type models was observed that they were unstable in trimming.

At the design speed, the wave-train could hardly be observed.

Lastly, these wave-free cyliders have few pay-load, that is, their statical buoyancy is very much larger than the pay-load.

As seen from the figures, the statical buoyancy $\rho g\nabla_s$ equals nearly

$$\rho g\nabla_s \doteqdot \rho g\lambda \int_{-1}^{1}(H_n-H_m)dx, \tag{4.15}$$

and the pay-load $\rho g\nabla_d$ is

$$\rho g\nabla_d = \rho g\lambda \int_{-1}^{1}(p_n-p_m)dx, \tag{4.16}$$

Hence, by the integration, it is shown that

$$\left.\begin{aligned} \nabla_d/\nabla_s &\doteqdot \frac{g}{g+8/(3\pi)}, \quad \text{for } M\text{-23 Type} \\ \nabla_d/\nabla_s &\doteqdot \frac{g}{g+8/(15\pi)}, \quad \text{for } M\text{-24 Type} \end{aligned}\right\} \tag{4.17}$$

Thus, the pay-load tends to zero with g.

5. Conclusion

The second order effect problem of the wave-theory is very much complicated but at least the following conclusions are deduced from the present considerations:

1) The second order effect with respect to the water surface depends upon the magnitude of the wave for a fairly large portion.
2) The speed range in which the effect occurs at the largest grade will be contained in the range when there exists no stationary wave and occurs a hydraulic jump. This means that the wave height has its maximum and accordingly the wave-resistance does also.
3) In very low speed, the second order effect of the water surface is very large for a submerged body but does not seem to be large for a floating body and that there may be various cases.
4) The wave-free cylinder models representing pressure distributions were made and measured the pressure. The results show a fairly good correspondence with the linear theory, if the pressure are corrected by the higher order terms in Bernoulli's equation.

Finally, it seems very important how much and to which functional class the wave-resistance which is a function of geometrical properties of the body belongs.

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