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"On the Boundary Value Problem in the Theory of of Wave-Making Resistance"

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Abstract

The author proposes a stand point to formulate the boundary value problem of a floating ship as a limit of a submerged body in the theory of the wave-making resistance, because there is an analytical difficulty on the cross curve around the ship surface with the water surface in the former but not in the latter, and he says that the solution of this problem may not be unique.

In this stand point, he tries to study the second order correction to the ship surface condition.

He introduces the reverse flow potential to write down the integral equation to determine the ship surface condition in a symmetrical way, and also does the diffraction potential to obtain the equivalent formula to Haskind-Hanaoka's.

Lastly, he shows the asymptotic expansion of Neumann function at infinity making use of this diffraction potential.

1. Introduction

There are two boundary conditions in the theory of ship waves, that is, the water surface and the ship surface condition, and we have satisfied with linearized condition because we apply the linearized water surface condition in usual.

Meanwhile, the second order corretion, especially for the water surface condition, is purely non-linear and very difficult to estimate it theoretically and also numerically, but for the ship surface condition it is not non-linear mathematically but it is a problem of linear integral equations for the boundary value problem of Laplace equation.

Hence, there is only a numerical difficulty in the latter case. This paper deals with the formulation of this problem.

In the other hand, there must be a parallelism between the theory of ship waves of the uniform motion and the oscillating motion. For example, the diffraction potential plays an important role in the oscillating motion.

How we can state its equivalent in the uniform motion problem? This is also the aim of this paper.

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2. Floating Ship

Let us consider a water motion around a ship in a uniform flow which has a unit velocity and is flowing down to the negative direction of the x-axis.

Here, take the origin on the mean water level and the z-axis vertically upwards.

Let $\phi(x, y, z)$ be the velocity potential of the disturbed water motion except the uniform flow, then it must satisfy the water surface condition

$$\left(\frac{\partial^2}{\partial x^2}+g\frac{\partial}{\partial z}-\mu\frac{\partial}{\partial x}\right)\phi(x, y, 0)=0 \tag{2.1}$$

where g means the gravity constant in this unit system and μ Rayleigh's frictional coefficient which must be tend to zero after the operation, and the ship surface condition

$$\frac{\partial}{\partial n}\phi(x, y, z)=-\frac{\partial x}{\partial n} \quad \text{on } S. \tag{2.2}$$

where n means the outward normal of the surface and S the ship surface.

This problem is very difficult to solve, but, in the other hand, the potential which satisfies the water surface condition and has a unit source is well known, say the fundamental singularity, that is,

$$4\pi S(P, Q)=\frac{1}{r}-\frac{1}{\bar{r}}-\frac{g}{\pi}\lim_{\mu\to+0}\int_{-\pi}^{\pi}\int_0^{\infty}\frac{e^{k(z+z')+ik(\tilde{\omega}-\tilde{\omega}')}dkd\theta}{k\cos^2\theta-g+\mu i\cos\theta}, \tag{2.3}$$

where

$$P\equiv(x, y, z),\ Q\equiv(x', y' z'),\ r=\overline{PQ},\ \bar{r}=\overline{P\bar{Q}},\ \bar{Q}\equiv(x', y', -z').$$

and $\tilde{\omega}=x\cos\theta+y\sin\theta$, $\tilde{\omega}'=x'\cos\theta+y'\sin\theta$.

At infinity, it has next asympototic characters,

$$S(P, Q)\to\frac{1}{4\pi}\left(\frac{1}{r}+\frac{1}{\bar{r}}\right) \qquad \text{for} \quad x\gg x', \tag{2.4}$$

$$S(P, Q)\to 2S_s(P, Q) \qquad \text{for} \quad x\ll x', \tag{2.5}$$

where

$$S_s(P, Q)=\frac{g}{4\pi i}\int_{-\pi/2}^{\pi/2}e^{g(z+z')\sec^2\theta}\left(e^{ig\sec^2\theta(\tilde{\omega}-\tilde{\omega}')}-e^{-ig\sec^2\theta(\tilde{\omega}-\tilde{\omega}')}\right)\sec^2\theta d\theta, \tag{2.6}$$

Making use of this function, by Green's theorem, it can be shown that

$$\phi(P)=\iint_{S+\bar{F}}\left\{\phi(Q)\frac{\partial}{\partial n}S(P, Q)-\frac{\partial\phi}{\partial n}S\right\}dS_Q,$$

where $\bar{F}$ means the water surface out of the ship, by the way, F means the water plane area of the ship.

The integral over $\bar{F}$ is deformed the one on the line C which is the cross curve of the ship surface S with the water surface $\bar{F}$, making use of (2.1) it becomes

$$\phi(P)=\iint_S\left\{\phi(Q)\frac{\partial}{\partial n}S(P,Q)-\frac{\partial\phi}{\partial n}S\right\}dS_Q +\frac{1}{g}\int_C\left\{\phi(Q)\frac{\partial}{\partial x'}S(P,Q)-S\frac{\partial}{\partial x'}\phi\right\}dy', \tag{2.7}$$

The second term of the right hand side does not appear in the case of the submeged body, and is considered as a special difficulty with the floating ship.[1)2)3)3)]
Now, since the surface elevation is

$$\zeta(x,y)=-\frac{1}{g}\frac{\partial}{\partial x}\phi(x,y,0), \tag{2.8}$$

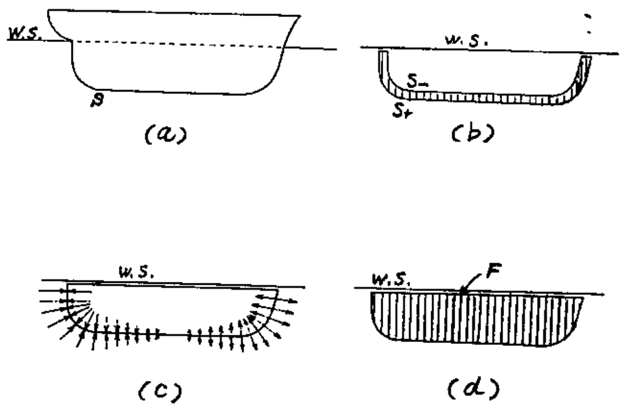
this term may be interpreted as the correction term for the change of the wetted surface of the ship from the mean one.[1)2)] In fact, that change can not be estimated before solving this problem.

There must be also some indeterminancy like as in the case of the surface glider which is very much like the wing.

Thence, this term seems to show such indeterminancy, that is, the present boundary value problem may be not sufficient with (2.2).

A ways to detour this difficulty is to assume the ship as a limit of the submerged one as showing in Figures.

Figure (a) is a real ship, (b) is a doublet layer over the ship surface under the mean level, (c) is a source-sink layer and (d) is a limiting submerged body with a rigid water plane which tends to the mean water level.



FIGURES

The boundary condition becomes respectively

$$\left.\frac{\partial\phi}{\partial n}\right|_{S+}=-\left.\frac{\partial\phi}{\partial n}\right|_{S-},\ \phi|_{S+}-\phi|_{S-}=\mu \qquad \text{for } (b), \tag{2.9}$$

$$\left.\begin{aligned}&\phi|_S=\phi|_{S-},\ \left.\frac{\partial\phi}{\partial n}\right|_{S+}-\left.\frac{\partial\phi}{\partial n}\right|_{S-}=-\sigma\\ &\text{and}\quad \iint_S\sigma dS=0.\end{aligned}\right\}\quad \text{for } (c) \tag{2.10}$$

$$\frac{\partial\phi}{\partial n}=-\frac{\partial x}{\partial n} \qquad \text{on } S \text{ and } F, \text{ for } (d) \tag{2.11}$$

where S_+ means S itself or the outer surface of the ship and S_- the inner surface.

These problems may have unique solutions as a limit of the submerged body, but it is not evident that these solutions are the same one.

However, it may be correct at least to say that the difference between them and a correct solution, if it exists, has a representation as like as (2.7) and the boundary condition

$$\frac{\partial\phi}{\partial n}=0 \qquad \text{on } S, \tag{2.12}$$

Thus, if the analogy to the gliding surface may be permitted, the question which is a compatible solution may be answered by inspecting whether the weigt and moment of the ship will be balanced to the hydrodynamical buoynancy or the water flow around a water line will be physically acceptable.

Namely, all of these problems are of the second order[1] and it is difficult in general to treat it as a whole.

Fortunately for the usual displacement ship at a moderate speed, this weight unbalance could be considered as small numerically and the water flow condition would not raise a serious difficulties.

Hence, the case (d) is taken up for the following treatment but the results obtained will be applied for other cases.

Then, the potential can be represented as

$$\phi(P)=\iint_{S+F}\left\{\phi(Q)\frac{\partial}{\partial n}S(P,Q)-\frac{\partial\phi}{\partial n}S\right\}dS_Q \tag{2.13}$$

3. Boundary Value Problem

Let us consider the artificial water motion when the trailing wave direction is reversed and ϕ^* be its velocity potential, the reverse flow potential,[5][6] then it can be represented as follows:

$$\phi^*(x,y,z)=\iint_{S+F}\left\{\phi^*(Q)\frac{\partial}{\partial n}S^*(P,Q)-\frac{\partial\phi^*}{\partial n}S^*\right\}dS_Q, \tag{3.1}$$

where S^* means the unit source potential and its regular wave must be lie in the upper stream, then it may be taken as

$$S^*(P,Q)=S(Q,P), \tag{3.2}$$

where S will be defined by (2.3), because it has the necessary conditions from (2.4) and (2.5).

Clearly, the difference between them becomes to

$$S(P, Q)-S^*(P, Q)=2S_s(P, Q), \tag{3.3}$$

or they can be written as

$$\left.\begin{matrix} S\ (P, Q) \\ S^*(P, Q) \end{matrix}\right\}=S_c(P, Q)\pm S_s(P, Q) \tag{3.4}$$

and S_c is an even function in $(x\text{-}x')$ and S_s an odd.

The boundary condition of ϕ^* is assumed to be

$$\frac{\partial}{\partial n}\phi^*=\frac{\partial}{\partial n}\phi=-\frac{\partial x}{\partial n} \qquad \text{on } S \text{ and } F, \tag{3.5}$$

Corresponding to (3.4), the next partition is possible.

$$\left.\begin{matrix} \phi \\ \phi^* \end{matrix}\right\}=\pm\phi_c+\phi_s, \tag{3.6}$$

and their boundary conditions must be

$$\frac{\partial\phi_c}{\partial n}=0,\ \frac{\partial\phi_s}{\partial n}=-\frac{\partial x}{\partial n} \qquad \text{on } S \text{ and } F, \tag{3.7}$$

where ϕ_c is an even function in x and ϕ_s is an odd when S is symmetric with respect to the y-z plane, but it has not always necessary to have such symmetry.

The explicit expressions of ϕ_c and ϕ_s are as follows:

$$\phi_c=\iint_{S+F}\phi_c\frac{\partial}{\partial n}S_c\,dS+\iint_{S+F}\left(\phi_s\frac{\partial}{\partial n}S_s-\frac{\partial\phi_s}{\partial n}S_s\right)dS, \tag{3.8}$$

$$\phi_s=\iint_{S+F}\left(\phi_s\frac{\partial}{\partial n}S_c-\frac{\partial\phi_s}{\partial n}S_c\right)dS+\iint_{S+F}\phi_c\frac{\partial}{\partial n}S_s dS, \tag{3.9}$$

Putting (3.7) into these formula, and tending the argument point on the surface S and F, they compose the system of Fredholm's integral equation of the second type. Then it must have a unique solution.

In the other hand, the potentials have a regular wave system in the far down stream, that is, from (2.5), (3.2) and (2.13), (3.1),

$$\phi(P)\xrightarrow[x\ll 0]{}2\iint_{S+F}\left(\phi\frac{\partial}{\partial n}S_s-\frac{\partial\phi}{\partial n}S_s\right)dS, \tag{3.10}$$

$$\phi^*(P)\xrightarrow[x\gg 0]{}-2\iint_{S+F}\left(\phi^*\frac{\partial}{\partial n}S_s-\frac{\partial\phi^*}{\partial n}S_s\right)dS, \tag{3.11}$$

or

$$\phi(P)\xrightarrow[x\ll 0]{}\frac{g}{2\pi i}\int_{-\pi/2}^{\pi/2}\{\phi_0(P, \theta)\bar{H}(\theta)-\bar{\phi}_0(P,\theta)H(\theta)\}\sec^2\theta d\theta, \tag{3.12}$$

$$\phi^*(P)\xrightarrow[x\gg 0]{}\frac{ig}{2\pi}\int_{-\pi/2}^{\pi/2}\phi_0(P,\theta)\bar{H}^*(\theta)-\bar{\phi}_0(P,\theta)H^*(\theta)\}\sec^2\theta d\theta, \tag{3.13}$$

where

$$\phi_0(P,\theta)=e^{gz\sec^2\theta+ig\sec^2\theta\tilde{\omega}}, \tag{3.14}$$

$$\left.\begin{matrix}H(\theta)\\ H^*(\theta)\end{matrix}\right\}=\iint_{S+F}\left\{\begin{pmatrix}\phi\\ \phi^*\end{pmatrix}\frac{\partial}{\partial n}\phi_0-\phi_0\begin{pmatrix}\frac{\partial}{\partial n}\phi\\ \frac{\partial}{\partial n}\phi^*\end{pmatrix}\right\}dS, \tag{3.15}$$

Dividing these into two parts, it can be written by (3.6),

$$\left.\begin{matrix}H(\theta)\\ H^*(\theta)\end{matrix}\right\}=\pm H_c(\theta)+H_s(\theta), \tag{3.16}$$

$$H_c(\theta)=\iint_{S+F}\left(\phi_c\frac{\partial}{\partial n}\phi_0-\phi_0\frac{\partial\phi_c}{\partial n}\right)dS=\iint_{S+F}\phi\frac{\partial}{\partial n}\phi_0 dS, \tag{3.17}$$

$$H_s(\theta)=\iint_{S+F}\left(\phi_s\frac{\partial}{\partial n}\phi_0-\phi_0\frac{\partial}{\partial n}\phi_s\right)dS, \tag{3.18}$$

Especially, since ϕ_c is even in x and ϕ_s odd if S is symmetric in fore and aft then H_c is real and H_s is imaginary, so that

$$H^*(\theta)=-\bar{H}(\theta)=-H(\theta\pm\pi), \tag{3.19}$$

4. Diffraction Potential[11]

To proceed further, it is convenient to introduce here potentials ϕ_d and $\phi_d{}^*$ as follows:

$$\frac{\partial}{\partial n}\phi_d(P,\theta)=\frac{\partial}{\partial n}\phi_d{}^*(P,\theta)=-\frac{\partial}{\partial n}\phi_0(P,\theta)\qquad \text{on } S \text{ and } F, \tag{4.1}$$

Since ϕ_0 stands for the plane wave advancing with the uniform flow, we call it the diffraction potential in analogy with the oscillating problem.

ϕ_d has a regular wave in the negative side of the x-axis but $\phi_d{}^*$ in the positive side.

Firstly, it can be easily seen to be, by Green's theorem,

$$\iint_{S+F}\left(\phi\frac{\partial}{\partial n}\phi_d{}^*-\frac{\partial\phi}{\partial n}\phi_d{}^*\right)dS=\iint_{S+F}\left(\phi^*\frac{\partial}{\partial n}\phi_d-\frac{\partial\phi^*}{\partial n}\phi_d\right)dS=0, \tag{4.2}$$

because ϕ and $\phi_d{}^*$ have their wave in the different side.

Then, putting (4.1) and (4.2) into (3.15), it can be shown to be

$$\left.\begin{matrix}H(\theta)\\ H^*(\theta)\end{matrix}\right\}=\iint_{S+F}\frac{\partial x}{\partial n}\left[\phi_0(\theta)+\left\{\begin{matrix}\phi_d{}^*(\theta)\\ \phi_d(\theta)\end{matrix}\right\}\right]dS, \tag{4.3}$$

In the other hand, since

$$\iint_{S+F}\left(x\frac{\partial}{\partial n}\phi_0-\phi_0\frac{\partial x}{\partial n}\right)dS=0, \tag{4.4}$$

(3.15) also may be written in the form

$$\left.\begin{matrix}H(\theta)\\ H^*(\theta)\end{matrix}\right\}=\iint_{S+F}\left\{x+\begin{pmatrix}\phi\\ \phi^*\end{pmatrix}\right\}\frac{\partial}{\partial n}\phi_0(\theta)dS, \tag{4.5}$$

Moreover, since these are equivalent to, for example,

$$H(\theta)=\iint_{S+F}\left\{(x+\phi)\frac{\partial}{\partial n}\phi_0-\phi_0\frac{\partial}{\partial n}(x+\phi)\right\}dS\ ,$$

if the potentials can be represented by the source-sink distribution, that is,

$$\left.\begin{matrix}\text{the difference of } \left\{\dfrac{\partial}{\partial n}\phi\right\}=-\sigma\\ \text{〃} \qquad \left\{\dfrac{\partial}{\partial n}\phi_d^*\right\}=-\sigma_d^*\end{matrix}\right\}\text{ on } S \text{ and } F \tag{4.6}$$

the integral can be taken around the source, and we may obtain

$$H(\theta)=\iint_{S+F}\sigma(P)\phi_0(P,\theta)dS_P=\iint_{S+F}\sigma_d^*(P,\theta)xdS_P\ , \tag{4.7}$$

In fact, for the thin ship, the first approximation is well known to be

$$\left.\begin{matrix}\sigma=\dfrac{\partial x}{\partial n}\\ \sigma_d^*=\sigma_d=\dfrac{\partial}{\partial n}\phi_0\end{matrix}\right\} \tag{4.8}$$

then, putting this into (4.7), these formula are clearly consistent with (4.4), and also it is equivalent to the next volume distribution by Green's theorem:

$$H(\theta)=\iiint_D\frac{\partial}{\partial x}\phi_0 dxdydz=2ig\sec\theta\iint_{\bar{S}}\eta(x,z)e^{gz\sec^2\theta+igx\sec\theta}dxdz\ , \tag{4.9}$$

where D means the displaced volume of the ship, $\bar{S}$ the projection of S on the $x-z$ plane and $\eta(x,z)$ the half breadth of the ship surface.

Secondly, making use of (4.1) and (4.2), the sum of two formula of (4.5) can be shown to become

$$2H_c(\theta)=H(\theta)-H^*(\theta)=\iint_{S+F}\left(\phi_d\frac{\partial\phi}{\partial n}-\phi\frac{\partial}{\partial n}\phi_d\right)dS\ , \tag{4.10}$$

Deforming the surface of the integration, it is equal to

$$H_c(\theta)=\frac{1}{2}\iint_{-\infty}^{+\infty}\left(\phi_d\frac{\partial}{\partial z}\phi-\phi\frac{\partial}{\partial z}\phi_d\right)dxdy\ .$$

Since ϕ_d has also an asymptotic expansion like (3.12) as

$$\phi_d(P,\theta)\xrightarrow[x\ll 0]{}\frac{g}{2\pi i}\int_{-\pi/2}^{\pi/2}\{\phi_0(P,\theta')\overline{H}_d(\theta,\theta')-\overline{\phi_0}(P,\theta')H_d(\theta,\theta')\}\sec^2\theta' d\theta' , \tag{4.11}$$

where

$$H_d(\theta',\theta)=H_d(\theta,\theta')=\iint_{S+F}\left\{\phi_d(P,\theta)\frac{\partial}{\partial n}\phi_0(P,\theta')-\phi_0(P,\theta')\frac{\partial}{\partial n}\phi_d(P,\theta)\right\}dS , \tag{4.12}$$

After a lengthy manipulation, it can be shown that

$$H_c(\theta)=\frac{ig}{4\pi}\int_{-\pi/2}^{\pi/2}\{H(\theta')\overline{H}_d(\theta,\theta')-H_d(\theta,\theta')\overline{H}(\theta')\}\sec^2\theta' d\theta' \tag{4.13}$$

For the thin ship, the approximation (4.8) gives

$$H_d(\theta,\theta')=-2g^2\sec\theta\sec\theta'\iint_{\bar{S}}\eta(x,z)e^{gz(\sec^2\theta+\sec^2\theta')+igx(\sec\theta+\sec\theta')}dxdz , \tag{4.14}$$

Then, if we consider a function such that

$$\left.\begin{aligned}P^*_{2n}(x,t)&=(-1)^n\int_0^{\pi/2}e^{-t\sec^2\theta}\cos(x\sec\theta)\cos^{2n}\theta d\theta ,\\ P^*_{2n+1}(x,t)&=(-1)^n\int_0^{\pi/2}e^{-t\sec^2\theta}\sin(x\sec\theta)\cos^{2n+1}\theta d\theta ,\end{aligned}\right\} \tag{4.15}$$

(4.13) can be written in the form:

$$\begin{aligned}H_c(\theta)=\frac{2}{\pi}g^4\sec\theta\iint_{\bar{S}}\iint_{\bar{S}}&\eta(x,z)\eta(x',z')P^*_{-4}\{g(x-x'),g\,|z+z'|\}\\ &\times e^{gz\sec^2\theta}\cos(gx\sec\theta)dxdzdx'dz' ,\end{aligned} \tag{4.16}$$

Since H_c is clearly zero for the first approximation (4.9), this gives the second approximation and is the different phase component with H_s of (4.9).

Lastly, the function $P_n{}^*$ is a conjugate to P_n function[7] with respect to x, that is,

$$\left.\begin{aligned}P_{2n}(x,t)&=(-1)^n\int_0^{\pi/2}e^{-t\sec^2\theta}\sin(x\sec\theta)\cos^{2n}\theta d\theta ,\\ P_{2n+1}(x,t)&=(-1)^{n+1}\int_0^{\pi/2}e^{-t\sec^2\theta}\cos(x\sec\theta)\cos^{2n+1}\theta d\theta ,\end{aligned}\right\} \tag{4.17}$$

so that there exist close expressions with each other, for example,[7]

$$P^*_{-1}(x,0)=-\frac{\pi}{2}J_0(x) , \tag{4.18}$$

$$P^*_{-2}(0,t)=-\frac{1}{2}e^{-t/2}K_{1/2}\left(\frac{t}{2}\right) , \tag{4.19}$$

and

$$P_{-1}^{*}(x,t)=-\frac{\sqrt{\pi}}{4\sqrt{t}}\int_{-\infty}^{\infty}e^{-v^2/4t}J_0(|x-v|)dv\ , \qquad (4.20)$$

Lastly, as explained at the end of § 2, the second order effect is very much complicated, but it is easy to obtain the correction term from the preceding formulas. For example, (4.16) is also such one.

More simpler formula may be deduced from (4.5).

Namely, deforming the integral of (4.5) by Green's theorem, it becomes the volume integral as

$$H(\theta)=\iiint_D\left[\left(1+\frac{\partial\phi}{\partial x}\right)\frac{\partial}{\partial x}\phi_0+\frac{\partial\phi}{\partial y}\frac{\partial\phi_0}{\partial y}+\frac{\partial\phi}{\partial z}\frac{\partial\phi_0}{\partial z}\right]dxdydz\ , \qquad (4.17)$$

for the thin ship,

$$H(\theta)=2\iint_{\bar{S}}\left[\left(1+\frac{\partial\phi}{\partial x}\right)\frac{\partial\phi_0}{\partial x}+\frac{\partial\phi}{\partial z}\frac{\partial\phi_0}{\partial z}\right]\eta(x,z)dxdz\ , \qquad (4.18)$$

and, for the slender ship,

$$H(\theta)=\int\left[\left(1+\frac{\partial\phi}{\partial x}\right)\frac{\partial\phi_0}{\partial x}+\frac{\partial\phi}{\partial z}\frac{\partial\phi_0}{\partial z}\right]A(x)dx\ , \qquad (4.19)$$

where $A(x)$ means the sectional area.

Comparing (4.18) with (4.9), the first term says that the doublet strength η in x direction is to be multiplied by $(1+\partial\phi/\partial x)$, that is, the local velocity.

This permits us an intuitive understanding.

The effect of this term is well examined and it gives a good correction for the deep draft ship.[3)8)]

The last term in the integrand gives the correction to the induced vertical velocity component and its phase is different with the above.

It is already shown in the case of the submerged body that the contribution of this term is very large,[9)] but such trial in the case of floating ships can hardly be seen.

5. Neumann Function[10)]

If there exists a Green function $N(P,Q)$ such that

$$\frac{\partial}{\partial n_Q}N(P,Q)=0\ , \qquad \text{on } S \text{ and } F \qquad (5.1)$$

then all the velocity potentials may be written by their normal derivatives as

$$\phi(P)=-\iint_{S+F}\frac{\partial\phi}{\partial n}(Q)N(P,Q)dS_Q\ , \qquad (5.2)$$

Such function as N is called a Neumann function.[6][10]

Since the singularity of this function at the origin is the same as $S(P, Q)$, it may be represented as

$$N(P, Q)=S(P, Q)+A(P, Q)\ , \tag{5.3}$$

with a regular function A.

Then, it must be by (5.1)

$$\frac{\partial}{\partial n_Q}A(P, Q)=-\frac{\partial}{\partial n_Q}S(P, Q) \qquad \text{on } S \text{ and } F\ , \tag{5.4}$$

In the other hand, it may be assumed because of (3.2) that

$$N(Q, P)=N^*(P, Q),\ A(Q, P)=A^*(P, Q)\ , \tag{5.5}$$

and

$$\phi^*(P)=-\iint_{S+F}\frac{\partial\phi}{\partial n}(Q)N(Q, P)dS_Q\ , \tag{5.6}$$

where it is assumed that

$$\frac{\partial}{\partial n}\phi^*=\frac{\partial}{\partial n}\phi\ , \tag{5.7}$$

Since A is assumed as regular in the domain and its boundary value is given by (5.4), it can be represented by the same Neumann function as

$$A(P, Q)=\iint_{S+F}\frac{\partial}{\partial n_R}S(P, R)N(R, Q)dS_R\ , \tag{5.8}$$

Thence, putting (2.4) and (2.5) into the above, it has next asymptotic expansions,

$$A(P, Q)\doteqdot\frac{1}{2\pi r_0^3}\sum_{j=1}^{2}x_j\phi_j^*(Q)\ , \qquad \text{for } x\equiv x_P\gg x_R\ , \tag{5.9}$$

where $r_0^2=x^2+y^2+z^2$, $x_1\equiv x$, $x_2\equiv y$ and

$$\phi_j^*(Q)=\iint_{S+F}\frac{\partial x_j}{\partial n_R}N(R, Q)dS_R\ , \tag{5.10}$$

and

$$A(P, Q)\doteqdot\frac{ig}{2\pi}\int_{-\pi/2}^{\pi/2}\{\phi_0(P, \theta)\overline{\phi_d^*}(Q, \theta)-\overline{\phi_0}(P, \theta)\phi_d^*(Q, \theta)\}\sec^2\theta d\theta\ , \qquad \text{for } x_P\ll x_R, \tag{5.11}$$

Hence, N is also written as

$$N(P, Q)\doteqdot\frac{1}{2\pi r_0^3}\sum_{j=1}^{2}x_j\{x_j'+\phi_j^*(Q)\}\ , \qquad \text{for } x_P\gg x_R\ , \tag{5.12}$$

$$N(P, Q) \doteqdot \frac{ig}{2\pi}\int_{-\pi/2}^{\pi/2}[\phi_0(P, \theta)\{\overline{\phi_0}(Q, \theta)+\overline{\phi_d}^*(Q, \theta)\}-\overline{\phi_0}(P, \theta)\{\phi_0(Q, \theta)+\phi_d^*(Q, \theta)\}] \sec^2\theta d\theta \,, \quad \text{for } x_P \ll x_R \,, \tag{5.13}$$

Putting these into (5.2), the asymptotic character of the velocity potential can be easily deduced, namely,

$$\phi(P) \doteqdot -\frac{1}{2\pi r_0^3}\sum_{j=1}^{2} x_j B_j \qquad \text{for } x \gg 0 \,, \tag{5.14}$$

where

$$B_j = -\iint_{S+F} \frac{\partial\phi}{\partial n}(x_j+\phi_j^*)dS \,, \tag{5.15}$$

and

$$\phi(P) \doteqdot \frac{g}{2\pi i}\int_{-\pi/2}^{\pi/2}\{\phi_0(P, \theta)\overline{H}(\theta)-\overline{\phi_0}(P, \theta)H(\theta)\} \sec^2\theta d\theta \,, \qquad \text{for } x \ll 0 \,, \tag{5.16}$$

where

$$H(\theta) = -\iint_{S+F} \frac{\partial\phi}{\partial n}\{\phi_0(\theta)+\phi_d^*(\theta)\}dS \,, \tag{5.17}$$

which is the same but more general one as (4.3).

For the slender ship,

$$B_j \doteqdot -\iint_{s+F} \frac{\partial\phi}{\partial n} x_j ds, \tag{5.18}$$

and especially, if

$$\frac{\partial\phi}{\partial n} = -\frac{\partial x}{\partial n}$$

then

$$\left.\begin{array}{l} B_1 \doteqdot V\text{: the displacement volume} \\ B_2 \doteqdot 0\text{:} \end{array}\right\} \tag{5.19}$$

and

$$\phi(P) \doteqdot \frac{(2V)}{4\pi}\frac{\partial}{\partial x}\left(\frac{1}{r_0}\right), \quad \text{for} \quad x \gg 0, \tag{5.20}$$ [2]

6. Conclusion

The second order correction to the boundary value problem of the wave-resistance theory contains many difficulties.

In this paper the author tries to formulate its boundary value problem for a floating ship with respect to the ship surface condition as a limit of a submerged ship.

But there is no promise that the solution of this problem represents the water motion around an actual floating ship, or it seems preferable to consider as that there may be many potentials satisfying the under water ship surface condition except on the a cross line of ships water line with the mean water level surface.

Nextly, the velocity potential consists of two parts by introducing the reverse flow potential which has the same boundary condition as the above but leaves the wave system in the reverse direction, the one of which is the same as the one of the reverse flow but the other of which is different only in the sign.

Thus, the wave system of both potentials is the same except the phase.

Lastly, introducing the diffraction potential, that is a potential diffracting an elementary plane wave by the ship, the equivalent of Haskind-Hanaoka's relation in the oscillating motion can be deduced. Namely, Kotchin function or the amplitude function can be calculated by the knowledge of the diffraction potential. Moreover, there is also the equivalent of the energy integral in the oscillating motion and this gives the difference between two amplitude functions of the ordinary and reverse flow potential.

For this formula, it may be possible to estimate the out of phase component to the first approximation.

These important characters of the diffraction potential come from the fact that Neumann function of this boundary value problem can be represented asympotitically at infinity by the diffraction potential.

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