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2. Study into Frame Line Configuration

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Summary

By making use of the slender body theory, the author studies into the optimum configuration of the frame line form of ships.

Firstly, fixing his eyes upon the energy of the secondary flow around the frame line, he finds out that there are ship forms which have no secondary flow as like as rotational bodies, the stream lines on such body become approximately geodesic and, if so, there may be no cross flow in the boundary layer by Squire's theorem. He calls them Approximate Geodesic Stream Line Ship Forms and shows some examples represented by Lewis' conformal mapping function.

Secondly, he deduces some typical ship forms by setting simple characters upon the secondary flow and they contain the so-called U-frame, V-frame and bulbous bow form.

Lastly, he analyses three representative practical ship forms and finds out that the aft-bodies of three ship forms are all nearly the forms of which the secondary flow is the minimum.

Although the theory developed here does not foretell the resistance quantitatively, it seems very usefull to design the optimum frame line configuration of the ship.

Introduction

Recently, progressing the theory of wave-making resistance, a turning point would have been appeared in investigation of ship forms. Since the theory of minimum wave-making resistance could concretely determine ship forms and qualitatively coincides with experiments well, it seems to be no wonder in this situation that by developing this theory ship forms fitted with given purposes could be determined theoretically. Wave-making resistance, however, as verified theoretically, determined mainly by the sectional area curve and this theory is powerfull for determination of the sectional area curve but powerless for design of the frame line configuration except the special case such as bulbous bow. The theory is not also valid in the range where phenomena caused by viscosity are thought to play a principal role as the sterns of low speed ships. Although, in consideration of these facts and in order to design more desirable ship forms it is of course necessary for us to study on viscous resistance or boundary layer, it seems necessary before doing it for us to grasp and interpret hydrodynamically phenomena of flow around ship hull and, if possible, to set up definite object to arrive at after investigation. Considering full ship form especially, author has made a few attempts from the thought on the possible dependency of sectional area curve on the form effects of viscous resistance, but practically speaking there exists no theoretical background for determination of the frame line configurations and we find great difficulty in determination of ship forms[1)2)12)].

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Now it is not strict but natural thought that, ships being generally slender, the section shape uniquely determines flow in the plane normal to the direction of the advance almost independently of the fore and after sections. These intuitive consideration had been extended to the so-called slender body theory in aerodynamics. Making use of this theory, Jinnaka showed a method to calculate stream lines around ship of which framelines are approximately represented by Lewis form and found that they agree well with the experiment[4]. In theory of ship motion (especially in sea way) this theory has been made use of and developed as strip theory. Considering fairly good coincidence between experiment and theory and contributions to the progress of research in that field it should be natural for this Jinnaka's method to be extended further. In fact the direct numerical calculation by the computers recently coming to have great capacity has made it comparatively easy to solve flow field around three-dimentional bodies, but for it is still complicated, this method which tells change of section shape along ship's longitudinal direction has no effect on the secondary flow makes intuitive consideration easy and so it is approximate but practical. In short, in view point to give the possibility to study on the phenomena, by leaving the interference between each section out of consideration, method is excellently superior, qualitatively at least but not to say quantitatively, to the accurate theory which gives complicated results. On the back ground of the above discussion, problem of wave-making resistance being not considered in this paper, considerations are carried on the flow around so-called double model in the infinite fluid by making use of the slender body theory. Here interesting subjects are relationships between a frameline configuration and flow around ship hull, characteristics of three-dimentional boundary layer and resistance caused by the vortices observed near the bilge.

1. Slender Body Theory[3)4)]

Let's take rectangular coordinate as shown in Fig. 1, ξ-axis directed longitudinally and

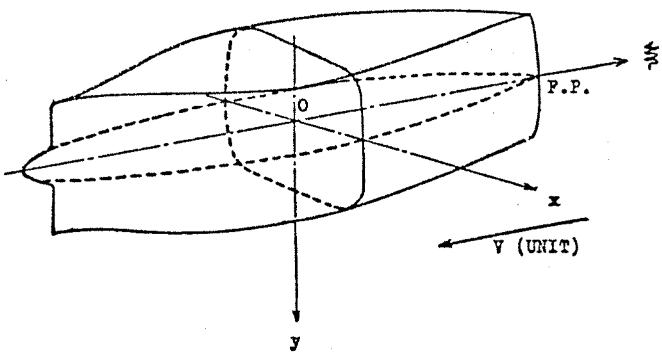


Fig. 1 Co-ordinate system

y-axis downwards vertically. Ship hull form is symmetric with respect to $y\xi$-plane and is so-called double model with its reflection with respect to $x\xi$-plane. We neglect viscosity, assume the motion of incompressible fluid and introduce velocity potential. If a body is very slender, then perturbed motion of the fluid caused by the body, being observed in the sectional plane ξ=const, is supposed to be nearly two-dimensional near this plane, and

$$\frac{\partial^2}{\partial \xi^2}\phi(x, y; \xi)=0 \tag{1·1}$$

$$\frac{\partial^2}{\partial x^2}\phi(x, y; \xi)+\frac{\partial^2}{\partial y^2}\phi(x, y; \xi)=0 \tag{1·2}$$

may be assumed. Since ϕ is two-dimensional harmonic function under this assumption, it can be obtained easily by making use of conformal mapping. A boundary condition for ϕ is given by the projection of the exact normal velocity onto the body plan, which may be got by drawing but here we devise a method of applying mapping functions for analytical convenience. First draw the section of ship on $Z(=x+iy)$-plane, and suppose that it is mapped on unit circle of ζ-plane. Take two sections apart by $\delta\xi$ and let normal distance between the two sections $\delta\nu$, then normal velocity v_ν on the boundary is by

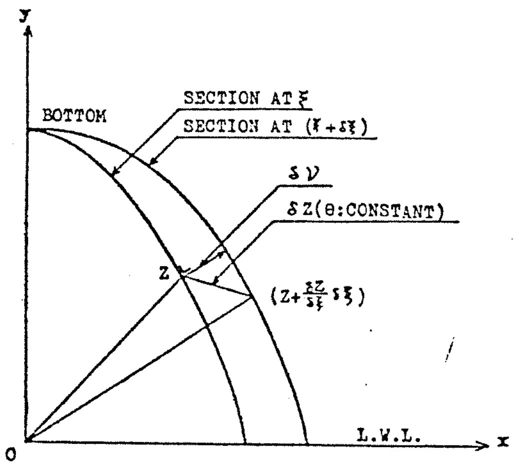


Fig. 2 Variation of Frame Line

Jinnaka (Fig. 2)

$$v_\nu = \frac{\delta\nu}{\delta\xi} \,. \tag{1·3}$$

Meanwhile as both sections are mapped on the same unit circle, two points on z-plane corresponding to the same position and ζ-plane are apart by δz. If $\delta\xi$ becomes very small and both sections approach infinitesimally each other, clearly $\delta\nu$ might be obtained by the normal component of δz. Therefore, take girth length s which is coordinate along the curve of ξ-section, then direction cosines of the normal is

$$\left(\frac{\partial y}{\partial s},\ -\frac{\partial x}{\partial s}\right)$$

and so $\delta\nu$ or v_ν is

$$v_\nu = \mathfrak{R}\left\{\frac{\overline{\partial z}}{\partial \xi}\cdot\frac{\partial z}{i\partial s}\right\} \,. \tag{1·4}$$

Because $\partial z/\partial\xi$ means partial differential with respect to ξ of mapping function, v_ν may be calculated if mapping function of each section is given by this formula. If normal velocity is given, it is easy to look for velocity potential and it is obtained by setting a source of

$$Q(\xi) = \int v_\nu(\xi)\,ds \tag{1·5}$$

on the origin of ζ-plane and sink-and source-distribution

$$q(\theta) = 2\left(v_\nu \frac{ds}{d\theta} - \frac{Q}{2\pi}\right) \tag{1·6}$$

on the unit circle. The complex potential f is

$$f = f_1 + f_2 = (\phi_1 + \phi_2) + i(\phi_1 + \phi_2)\,, \tag{1·7}$$

$$f_1 = \frac{Q}{2\pi}\log\zeta\,, \tag{1·8}$$

and $$f_2 = \frac{1}{2\pi}\int_0^{2\pi} q(\theta')\log(\zeta - e^{i\theta'})\,d\theta' \tag{1.9}$$

where f_2 is regular at infinity. From the above formula, tangential component of velocity is[3)4)]

$$v_\tau = \frac{\partial\phi_2}{\partial s} = -\frac{\partial\psi_2}{\partial\nu} = \mathfrak{R}\left\{\frac{d\theta}{\partial s}\ \frac{df}{d\theta}\right\}_{\zeta=e^{i\theta}} . \tag{1·10}$$

On the other hand, as this term is equal to normal derivative of a function ψ_2, by putting

$$\Gamma(\theta) = 2v_\tau \frac{ds}{d\theta}\,, \tag{1·11}$$

f_2 is also represeted as the following by distributing circulation.

$$f_2 = \frac{1}{2\pi i}\int_0^{2\pi} \Gamma(\theta')\log(\zeta - e^{i\theta'})\,d\theta' \tag{1·12}$$

Now we think about the projection of stream lines onto body plan. Let the stream line element between two sections distant by $\delta\xi$ be δS, then from Fig. 3,

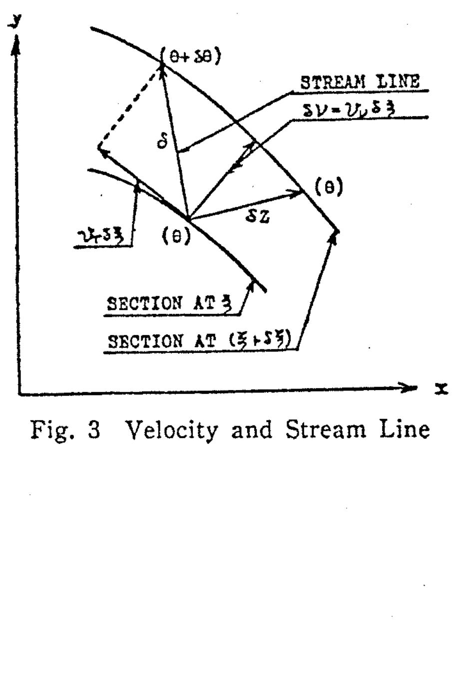


Fig. 3 Velocity and Stream Line

$$\frac{\delta S}{\delta \xi}=\frac{\overline{df}}{dz}. \qquad (1\cdot13)$$

This is not always convenient for calculation. As every point on the frame line may be represented one-to-one by argument θ on ζ-plane, it is convenient, as the case may be, to represent these stream lines with parameter θ. From the figure variance of θ may be calculated as the following.

$$\frac{\delta S}{\delta \xi}=\frac{dS}{d\theta}\frac{\delta\theta}{\partial\xi}=v_\tau-\mathcal{J}\left\{\frac{1}{i}\frac{\partial z}{\partial s}\frac{\delta \bar{z}}{\delta \xi}\right\} \qquad (1\cdot14)$$

At last we think about the kinetic energy T of this system.

$$T=\frac{\rho}{2}\iint\left\{\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2 dx\,dy=T_1+T_2\right. \qquad (1\cdot15)$$

where

$$T_1=\frac{\rho}{2}\int_R \phi\frac{\partial\phi}{\partial\nu}dS, \qquad (1\cdot16)$$

$$\text{and} \quad T_2=-\frac{\rho}{2}\int_C \phi\frac{\partial\phi}{\partial\nu}dS. \qquad (1\cdot17)$$

R is infinitely large circle whose radius is R and C is the frame line curve. By refering to Eqs. (1·7), (1·8) and (1·9),

$$T_1=\frac{\rho}{2}\int_R \phi_1\frac{\partial\phi_1}{\partial R}Rd\theta=\frac{\rho}{4\pi}Q^2\log R, \qquad (1\cdot18)$$

$$\text{and} \quad T_2=-\frac{\rho}{2}\int_C \phi_2\frac{\partial\phi_2}{\partial\theta}d\theta. \qquad (1\cdot19)$$

If R becomes infinite, T also becomes infinite and this is meaningless. But if a body is closed, generally

$$\int_{F.P.}^{A.P.} Q(\xi)d\xi=0 \qquad (1\cdot20)$$

and kinetic energy is finite. This is one of the contradictions due to the assumption of slender body. As the term of T_1 contains no factors representing frame line configuration and depends only on the longitudinal variance of displacement, we may well leave it out of consideration on discussing the secondary flow. Therefore T_2 is a characteristic quantity of the secondary flow, as this is explicit from Eq. (1·19).

Similarly pressure distribution is distinguished between its longitudinal variance and that of secondary flow. Let ξ-component of velocity be $1+v_\xi$ and let pressure at infinity be zero then Bernoulli's theorem is

$$\begin{aligned}\frac{2}{\rho}p&=1-(1+v_\xi)^2-(v_\nu^2+v_\tau^2)\\&=-2v_\xi-(v_\xi^2+v_\nu^2+v_\tau^2).\end{aligned}$$

Therefore, putting

$$p=p_1+p_2, \qquad (1\cdot21)$$

$$\frac{2p_1}{\rho}=-2v_\xi-v_\xi^2\doteqdot 2v_\xi \qquad (1\cdot22)$$

and

$$\frac{2p_2}{\rho}=-(v_\nu{}^2+v_\tau{}^2) \qquad (1\cdot23)$$

and on the assumption of the theory v_ξ is to be function of ξ only and p_2 is thought to represent the variance of the pressure of the secondary flow on the frame line. Such a pressure distribution gives rise to torsion of stream lines. Because in this case the direction of stream lines nearly coincides with ξ-axis, principal normal and binormal may be supposed to lie on the xy-plane by the assumption[5]. Then principal curveture $1/\kappa$ is

$$\frac{1}{\kappa}=\frac{q_n}{q}=\frac{1}{2q^2}\frac{\partial}{\partial n}(q^2)=\frac{p_n}{2p} \qquad (1\cdot24)$$

where q is absolute value of velocity and the suffix means partial differential with respect to its direction. Torsion $1/\lambda$ is

$$\frac{\kappa}{\lambda}\doteqdot\kappa\frac{q_{\xi t}}{q}=\frac{\kappa q_{\xi t}}{2p}=\frac{p_{\xi t}}{p_n}, \qquad (1\cdot25)$$

since $q_t=0$ by the definition and

$$\frac{2p}{\rho}=1-q^2 \quad \text{and} \quad \left(\frac{2p}{\rho}\right)_{\xi t}=-2qq_{\xi t}.$$

Then the assumption of slender body would allow p in Eqs. (1·24) and (1·25) to be taken

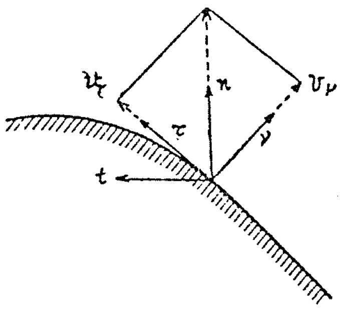


Fig. 4 Normal and Tangent

as p_2 in Eq. (1·23). If v_τ vanishes, then from Fig. 4 q_t vanishes, from Eq. (1·25) torsion vanishes, and n coincides with ν. In other words the principal normal coinsides obviously with the normal of the body and the stream lines become approximately geodesic[5].

2. Lewis Frame Ship[4]

Secondary flow can be calculated by the formulae in the previous section if a function mapping the frameline to a unit circle is found. It is, however, not always easy to find a mapping function suited for actual ship forms. Meanwhile, by making use of so-called Lewis form, frame lines similar to the practical ship form are obtained as Jinnaka calculated and the calculation becomes very simple. We take our stand on optimal frame line configuration if possible rather than on finding actual flow by calculation, therefore simpler formula makes the outlook more plain. Then we shall call the whole ship form of which frame line curve is represented by Lewis form Lewis frame ship and restrict our thought on such ship forms. So the mapping function is

$$z=c\zeta+\frac{a}{\zeta}+\frac{b}{\zeta^3} \tag{2·1}$$

Hence half breadth β, draft τ sectional area (of double model) $A_m=\pi\gamma$ and section coefficent C_m are

$$\left.\begin{aligned}\beta&=c+a+b\,,\\ \tau&=c-a+b\,,\\ \gamma&=\frac{1}{\pi}A_m=c^2-a^2-3b^2\\ \text{and}\quad C_m&=\frac{\pi\gamma}{4\beta\tau}\end{aligned}\right\} \tag{2·2}$$

To approximate the actual ship forms by Lewis frame ships, β and τ may be chosen to coincide with those of the actual ships respectively. If they are given as functions of ξ, then a, b and c are obtained by the above formulae. (In the case when they don't give the actual frame line curves, we don't, of cource, persue the problem any further) If a, b and c are prescribed as functions of ξ as the above, firstly from Eq. (1·4) the following are calculated.

$$v_\nu\frac{ds}{d\theta}=\Re\left\{\frac{\delta\bar{z}}{\delta\xi}\cdot\frac{dz}{id\theta}\right\}=C+A\cos 2\theta+B\cos 4\theta\,, \tag{2·3}$$

$$\left.\begin{aligned}C&=cc'-aa'-3bb'\\ A&=ca'-ac'-ab'-3a'b\\ B&=ab'-3c'b\end{aligned}\right\} \tag{2·4}$$

where the prime means differential with respect to ξ. Since these are also written from Eq. (2·2) as

$$\left.\begin{aligned}\beta'&=c'+a'+b'\,,\\ \tau'&=c'-a'+b'\,,\\ \text{and}\quad \gamma'&=2(cc'-aa'-3bb')\end{aligned}\right\} \tag{2·5}$$

they are followed by

$$\left.\begin{aligned}C&=\frac{\gamma'}{2}\\ A&=\frac{\beta'}{2}(c-a-3b)-\frac{\tau'}{2}(c+a-3b)\,,\\ \text{and}\quad B&=\frac{\beta'}{2}(c-a-3b)+\frac{\tau'}{2}(c+a-3b)-\frac{\gamma'}{2}\end{aligned}\right\} \tag{2·6}$$

Especially in $b=0$ or in the case of circular and elliptic sections, paying regard to Eq. (2·2) too,

$$\left.\begin{aligned} C&=\frac{\gamma'}{2}, \\ A&=\frac{1}{2}(\beta'\tau-\beta\tau'), \\ \text{and}\quad B&=0. \end{aligned}\right\} \qquad (2\cdot7)$$

When draft $\tau=$const, or $\tau'=0$,

$$\left.\begin{aligned} C&=\frac{\gamma'}{2}, \\ A&=\frac{\beta'}{2}(c-a-3b), \\ \text{and}\quad B&=A-C. \end{aligned}\right\} \qquad (2\cdot8)$$

Besides

$$\left(\frac{ds}{d\theta}\right)^2=c^2+a^2+9b^2-2a(c-3b)\cos 2\theta -6bc\cos 4\theta \qquad (2\cdot9)$$

and especially

$$\left.\begin{aligned} \frac{ds}{d\theta}\bigg|_{\theta=0}&=c-a-3b \\ \frac{ds}{d\theta}\bigg|_{\theta=\pi/2}&=c+a-3b \end{aligned}\right\} \qquad (2\cdot10)$$

Now substitute Eq. (2·3) in Eq. (1·5) and

$$Q=2\pi C=\pi\gamma' \qquad (2\cdot11)$$

and f_1 in Eq. (1·8) is obtained easily. Moreover substitute Eq. (2·3) in Eq. (1·6) and

$$q(\theta)=2A\cos 2\theta+2B\cos 4\theta \qquad (2\cdot12)$$

and substitute Eq. (2·3) in Eq. (1·9), then

$$f_2=\frac{A}{2\zeta^2}+\frac{B}{4\zeta^4} \qquad (2\cdot13)$$

Therefore from Eq. (1·10) tangential component of velocity is

$$v_\tau\frac{ds}{d\theta}=A\sin 2\theta+B\sin 4\theta \qquad (2\cdot14)$$

Then main component v_τ of secondary flow is determined only by A and B independently of C. If potential is represented by circulation distribtion in Eq. (1·12), substitution of the above equation in (1·11) indicates

$$\Gamma(\theta)=2A\sin 2\theta+2B\sin 4\theta \qquad (2\cdot15)$$

Now if stream lines are calculated by Eq. (1·13), then

$$\frac{\delta S}{\delta\xi}=\frac{d\bar{f}}{d\theta}\bigg/\frac{d\bar{z}}{d\theta}=i\left(\frac{C+Ae^{2i\theta}+Be^{4i\theta}}{c-ae^{2i\theta}-3be^{4i\theta}}\right)e^{i\theta} \qquad (2\cdot16)$$

and this being represented by parameter in Eq. (1·14) as

$$\frac{\delta\theta}{\delta\xi}=-(A^*\sin 2\theta+B^*\sin 4\theta)\left(\frac{d\theta}{ds}\right)^2 \qquad (2\cdot17)$$

$$\left.\begin{aligned} A^*&=2(ac'+3a'b), \\ \text{and}\quad B^*&=6bc' \end{aligned}\right\} \qquad (2\cdot18)$$

and thinking graphically, the following formula will be convinient for calculation.

$$\frac{v_\tau}{v_\nu}=\frac{A\sin 2\theta+B\sin 4\theta}{C+A\cos 2\theta+B\cos 4\theta} \qquad (2\cdot19)$$

By substitution of Eqs. (2·3) and (2·14) in Eq. (1·23), p_2 of secondary flow is

$$-\frac{2}{\rho}p_2=\{(C^2+A^2+B^2)+2A(C+B)\cos 2\theta+2CB\cos 4\theta\}\left(\frac{d\theta}{ds}\right)^2 \qquad (2\cdot20)$$

Substitution of Eq. (2·13) in Eq. (1·19) indicates that kinetic energy T_2 of secondary flow is

$$T_2=\frac{\pi}{4}\rho\left(A^2+\frac{B^2}{2}\right) \qquad (2\cdot21)$$

Although each value can be calculated easily as the above, yet the problem is what these values mean hydrodynamically and what values are optimum. In the following we shall consider these section by section.

3. Trefftz Plane[3)]

Firstly let's consider bilge vortex of the recent topics. Slender body theory has been developed from R. T. Jones' theory of small aspect wing and it is assumed in this theory that, in the case of slender wing, velocity induced by bound vortices perpendicular to main flow is negligible. As the result, vortex distribution (of which axis is parallel to main flow) of each section of the wing is brought backwards as it is. That is, the vortex distribution of each section forms Trefftz plane. In our case, potential of secondary flow may be represented by circulation as Eq. (1·12), and it means that the ship is considered as a cylindrical wing. Induced velocity is determined by the boundary condition as Fig. 5 with consideration of velocity

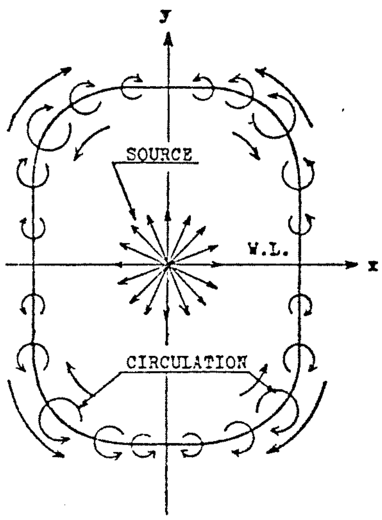


Fig. 5 Source and Circulation at Fore Body seen forewards from aft

by the source representing displacement effect, and vortex distribution is given by Eqs. (1·10) and (1·11). Kinetic energy of Trefftz plane is clearly equal to T_2[3]. In the case of Lewis frame line ship the vortex distribution is given by Eqs. (2·15) and T_2 by Eq. (2·29). Distributions of T_2 of actual ships are shown in Fig. 17. Since T_2 at any section means work done by the ship from stem to the section, force of ξ-direction on the section is (all forces in the xy-plane may well be neglected for they cancell out each other.)

$$F=\frac{dT_2}{d\xi} \qquad (3\cdot1)$$

Accordingly it turns out to be thrust force in the fore body, drag force in the aft body, concentrated drag force at the fore end peak and concentrated thrust force at the aft end peak. If Trefftz plane, however, is once settled, vorticity is to be thought brought backwards invariably, and if so, it is not supposed reasonable that Trefftz plane is independent on other sections. Hence it is to be solved by integral-equation[6], but in this case it would become very complex in comparison with the plane wings and, even if it is solved, it should be difficult to grasp how it relates to the practical.

Through Jones' theory induced resistance is simply to be given as follows[3][6];

$$D_J=Max.\ T_2 \qquad (3\cdot2)$$

Although this formula is known to give a good appoximation in the case of wings with very small aspect ratio, yet the accuracy is not predicted for our problem because the conditions are very much different. If there is, however, resistance induced by bilge vortex,[7] T_2 is thought from the above consideration to turn out a criterion of its magnitude, and in order to compare with residual resistance let's no-dimensionalize it.

$$C_{DJ}=\frac{(T_2/2)}{\frac{\rho}{2}\nabla^{2/3}V^2} \qquad (3.3)$$

where ∇ is the volume of displacement of the ordinary ship and not of the double-model. For instance for the elliptical section, substitute Eq. (2·7) in Eq. (2·22), and we get

$$\frac{2}{\rho}T_2=\frac{\pi}{8}(\tau\beta)^2\left(\frac{\beta'}{\beta}-\frac{\tau'}{\tau}\right)^2 \qquad (3\cdot4)$$

but clearly, from the above formula, T_2 become small, if both β' and τ' are positive, and resistance would be small. Especially if $(\beta/\tau)'=0$, then

$$T_2=0\ . \qquad (3\cdot5)$$

That is, when breadth-draft-ratio is constant, resistance turns out to be minimum. This corresponds well to the fact that resistance of ships, of which keel hangs down like Inuid, is small.[8]

On the other hand, since T_2 is kinetic energy of secondary flow, it is anyhow favourable to be small, and it may be proved T_2 to vanish as minimum. The condition when it vanishes is from Eq. (2·29)

$$A=B=0 \tag{3·6}$$

and substitution of Eq. (2·4) gives the following conditions,

$$\left.\begin{aligned} cb'&=3bc' \\ (c-3b)a'&=a(c'+b') \end{aligned}\right\} \tag{3·7}$$

If they can be solved under the given initial condition regarded as the simultaneous partial differential equations with respect to ξ, then T_2 vanishes along whole ship length. This solution, being solved easily, is

$$\left.\begin{aligned} c&=c_0\lambda\,, \\ a&=a_0\lambda\frac{\left(1-\dfrac{3b_0}{c_0}\right)}{\left(1-\dfrac{3b_0}{c_0}\lambda^2\right)}\,, \\ \text{and}\quad b&=b_0\lambda^3 \end{aligned}\right\} \tag{3·8}$$

where c_0, a_0 and b_0 are initial values and λ is an arbitrary parameter. Example of Eq. (3·5) is the case $b=0$, and further Fig. 6 shows some examples of ship forms like the practical ship forms. Such ship forms have no twist of stream lines through Eqs. (3·6) to (2·14). That is, the stream lines become approximately geodesic. Since geodesic curve is the shortest path between two points, a ship form is obtained, which corresponds to the idea of Maier form (i. e. a ship form of which stream lines arrive at midship along the shortest path). In the following, we call these ship forms as A.G.S. Approximate Geodesic Stream Line Ship Form). Since λ in Eq. (3·8) is an arbitrary function of ξ, distribution of displacement is, too, arbitrary. In other words, the nature is invariable with elongation in longitudinal coordinate.

4. Limiting Stream Line

Stream lines considered above is in fact regarded as those of the exterior to the boundery layer, but generally inside the boundary layer cross flow crossing to them is induced, and especially stream line along

λ	c	b
1.0	1.5000	-0.5000
0.8	1.2000	-0.2560
0.6	0.9000	-0.1080
0.4	0.6000	-0.0320
0.2	0.3000	-0.0040
a = 0		

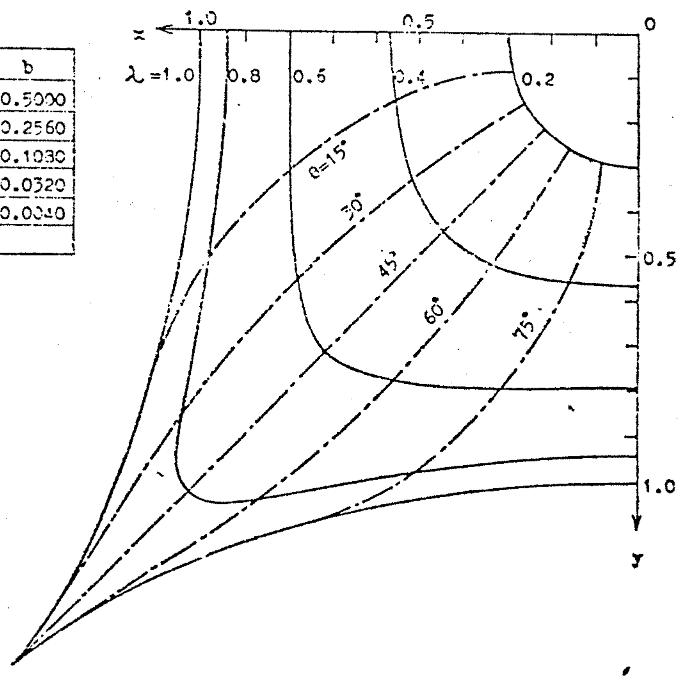


Fig. 6 A. G. S. Form (1)

	c	b
1.0	0.7500	0.2500
0.8	0.6000	0.1280
0.6	0.4500	0.0540
0.4	0.3000	0.0160
0.2	0.1500	0.0020
	a=0	

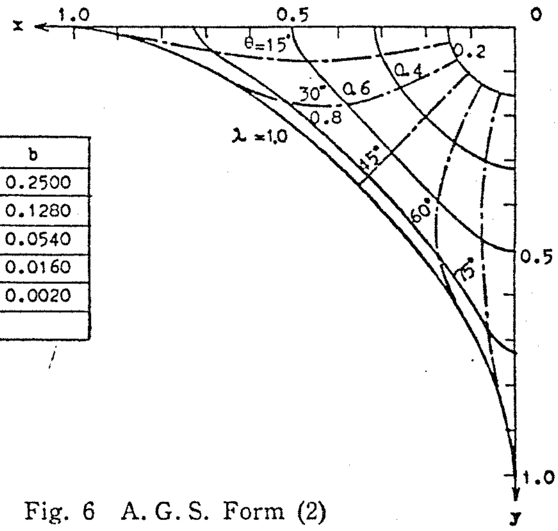


Fig. 6 A. G. S. Form (2)

λ	c	a	b
1.0	1.5	0.2500	-0.250
0.8	1.2	0.2273	-0.128
0.6	0.9	0.1907	-0.054
0.4	0.6	0.1389	-0.016
0.2	0.3	0.0735	-0.002

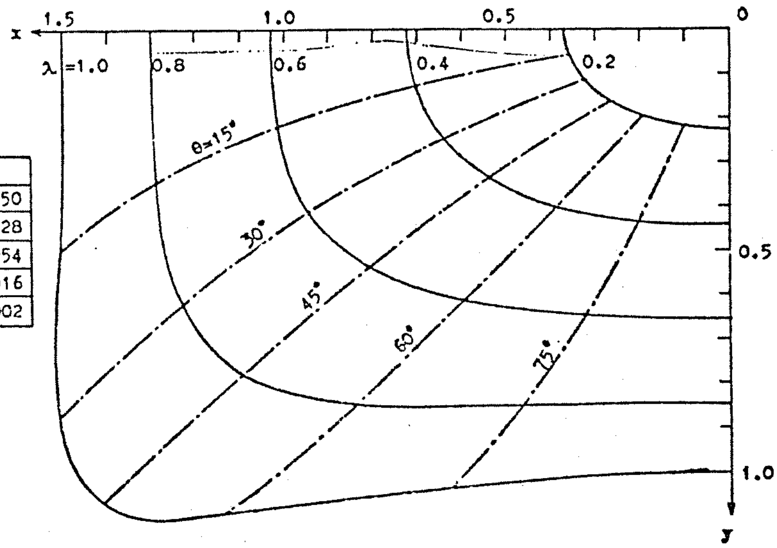


Fig. 6 A. G. S. Form (3)

λ	c	a	b
1.0	1.2500	0.4000	0.2000
0.8	1.0000	0.2600	0.1280
0.6	0.7500	0.1532	0.0540
0.4	0.5000	0.0885	0.0160
0.2	0.2500	0.0410	0.0020

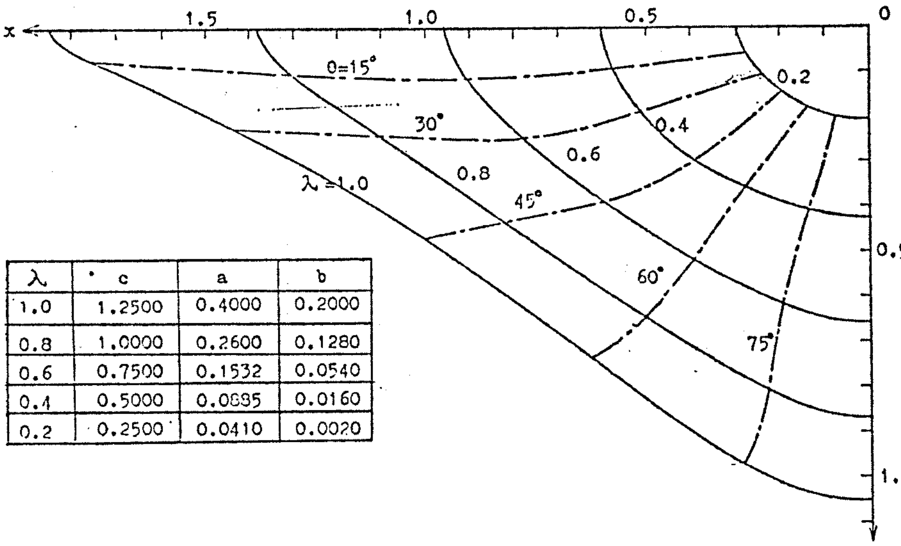


Fig. 6 A. G. S. Form (4)

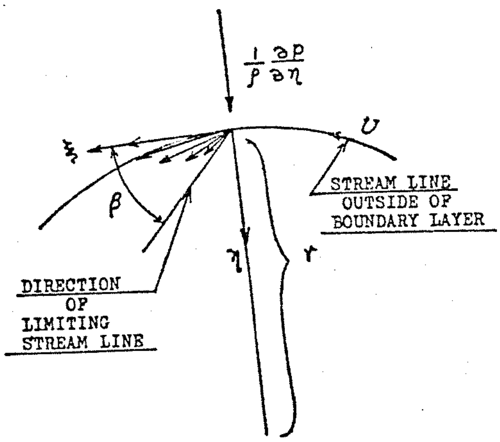


Fig. 7 Limiting Stream Line

the ship surface is called limiting stream line. As shown in Fig. 7, let's take ξ along stream line, η normal to it, ζ-axis upwards vertically, u and v components of velocity, r radius of curvature of the stream line at the points considered and β an angle between stream line outside the boundary layer and limiting stream line. Curvature of the surface and pressure gradient are to be small in direction of main flow. Then since centrifugal force of a particle of fluid is equivalent to the pressure gradient in the exterior of boundary layer,

$$-\frac{1}{\rho}\frac{\partial p}{\partial \eta}=\frac{U^2}{2r} \qquad (4\cdot1)$$

where U is velocity outside the boundary layer. It is to be thought that velocity on the surface of the body vanishes and that all inertia forces vanish, therefore pressure gradient of the above formula must be equivalent to the viscous force. That is

$$\frac{1}{\rho}\frac{\partial p}{\partial \eta}=\nu\frac{\partial^2 v}{\partial \zeta^2}\bigg|_{\zeta=0} \qquad (4\cdot2)$$

where ν is kinetic viscosity. Let there be the following representation of velocity distribution[10],

$$v=U\tan\beta\left(1-\frac{\zeta^2}{\delta^2}\right)f\left(\frac{\zeta}{\delta}\right) \qquad (4\cdot3)$$

$$u=Uf\left(\frac{\zeta}{\delta}\right) \qquad (4\cdot4)$$

where δ is thickness of boundary layer. Then

$$\frac{d^2v}{d\zeta^2}\bigg|_{\zeta=0}=\tan\beta\frac{d^2u}{d\zeta^2}\bigg|_{\zeta=0}=\frac{U}{\delta^2}\tan\beta f''(0). \qquad (4\cdot5)$$

Substitute this in Eq. (4·2) and solve with respect to $\tan\beta$, and

$$\tan\beta=\frac{\delta^2}{\nu Uf''(0)}\left(\frac{1}{\rho}\frac{\partial p}{\partial \eta}\right)=-\frac{\delta^2 U}{2\nu rf''(0)} \qquad (4\cdot6)$$

is obtained. The assumption on which the above formula is deduced seems too rough actually, but several results are given by them. The larger is the pressure gradient normal to the stream line or the larger is curveture of the stream line, the larger is β. In this case stream lines in the boundary layer curve in the direction of the center of curvature of main flow. Since β becomes very large rapidly as δ enlarges, near the stern gets much larger generally than the fore body. Also with the curve of velocity distribution of the boundary layer in the direction of main flow β will vary greatly. It is not always certain that, by the grawth of cross flow in the boundary layer, viscous resistance increases rather than without it. Because of the assumption of the present three-dimensional boundary layer theory velocity distribution in the direction of main flow is determined independently on the cross flow (p. 468 of (9)). Growth of the cross flow means that of vortex of which axis is parallel to main flow, therefore it flows out of the boundary layer when it grows to a certain size. It is near the bilge that there is such a risk. Because the limiting stream lines, gathering from ship side and bottom, come to cross together at an acute angle, such deviation of vortex of three-dimensional separation of boundary layer is thought to be promoted. For the above reason, establishment of quantitative limits within which β is harmless depends on future investigation, but at least it is certainly desirable that it is small as possible and does not vary discontinuously. From Eq. (4·6) β vanishes as pressure gradient vanishes, and this means there is no twist of stream line approximately. That is, if stream line becomes geodesic, there is no cross flow (This is Squire's theorem[11]. Accordingly A. G. S. form derived in the previous section turns out also in these means to be a ship form which hardly cause bilge vortex[7] and of which stream lines outside of boundary layer coincide nearly with limiting stream lines. Accordingly viscous resistance of A. G. S. is considered to agree almost with calculation of boundary layer theory, and expected to be equal to that of the bodies of revolution with almost same sectional area.[9]

5. U shape-Frame and V shape-Frame

Although A. G. S. form of which secondary flow is minimum is thought to be suitable as the the above to keep viscous resistance small, yet on the other hand it seems disadvantageous in view of wave-making resistance, interference with propeller and course stability, and one of the advantages applied to the practical ship forms is found merely in Maier ship forms. Whether U-shape or V-shape is a criterion of distinction of the practical ship forms and it is said that U-frame is advantageous with respect to wave-making resistance and propulsive efficiency, while V-frame is advantageous in point of view of viscous resistance[16].

In this section let's study on the relation between the characteristics of secondary flow and geometrical feature of the frame line about the Lewis frame ship form. We take the keel line horizontal or $\tau'=0$ as the usage of practical ship forms and let's take the section of parallel body be nearly rectangular. On the condition of $\tau'=0$, from Eqs. (2·8) and (2·2)

$$\left.\begin{aligned} C&=\frac{\gamma'}{2}=\frac{2}{\pi}(C'_m\beta\tau+C_m\beta'\tau)\,,\\ A&=\frac{\beta'}{2}(\tau-4b)\,,\\ \text{and}\quad B&=A-C\,. \end{aligned}\right\} \qquad (5\cdot1)$$

For the distinction between U-frame and V-frame is rather vague, here we would once call a frame line whose section coefficient is larger than ellipse as U-frame, and smaller as V-frame. Therefore, because in the case of ellipse $b=0$ in Lewis form section, above distinction means the followings,

$$\left.\begin{aligned} C_m&\leq\frac{\pi}{4},\quad b\geq0,\quad \text{V-frame}\\ C_m&\geq\frac{\pi}{4},\quad b\leq0,\quad \textit{U}\text{-frame} \end{aligned}\right\}. \qquad (5\cdot2)$$

The quantities of secondary flow are represented by those of the tangential component of velocity or T_2. Substitute $B=A-C$ of Eqs. (5·1) in Eqs. (2·14) and (2·21), and

$$v_\tau\frac{ds}{d\theta}=\{A(1+2\cos2\theta)-2C\cos2\theta\}\sin2\theta, \quad (5\cdot3)$$

and

$$\frac{2}{\rho}T_2=\frac{\pi}{2}\left\{\frac{1}{3}C^3+\frac{3}{2}\left(A-\frac{C}{3}\right)^2\right\}. \quad (5\cdot4)$$

C is determined by the first of Eqs. (5·1) if the sectional area curve is given, therefore only A depends on frame line configuration. A depends, however, on only b from the second of Eqs. (5·1) once β' is given, and, from the classification of Eqs. (5·2), A of V-frame turns out small comparing with that of U-frame. This is, however, the matter of the case when γ' and β' are given, and, if they are arbitrary, whether U-frame or V-frame has larger secondary flow is not determined.

That is, quantities of the secondary flow depend on displacement distribution in longitudinal direction and shapes of water lines. Consequently for instance in the case of constant γ', β' of U-frame ship needs to be taken less than V-frame ship in order that A has the same value. It does not always, however, affect to the quantity of T_2. As a result of Eq. (5·4), clearly T_2 is minimum when $A=C/3$. Substitution of this condition in Eq. (2·4) and $\tau'=0$ indicates that ordinary differential equations with

REF. NO.	c	a	b
1	0.5484	-0.4916	-0.04
2	0.5943	-0.4857	-0.08
3	0.6384	-0.4816	-0.12
4	0.6812	-0.4768	-0.16
5	0.7129	-0.1471	0.14
6	0.7732	-0.1068	0.12
7	0.8641	-0.0559	0.08
8	1.0000	0	0
9	1.1112	0.0312	-0.08
10	1.2108	0.0508	-0.16

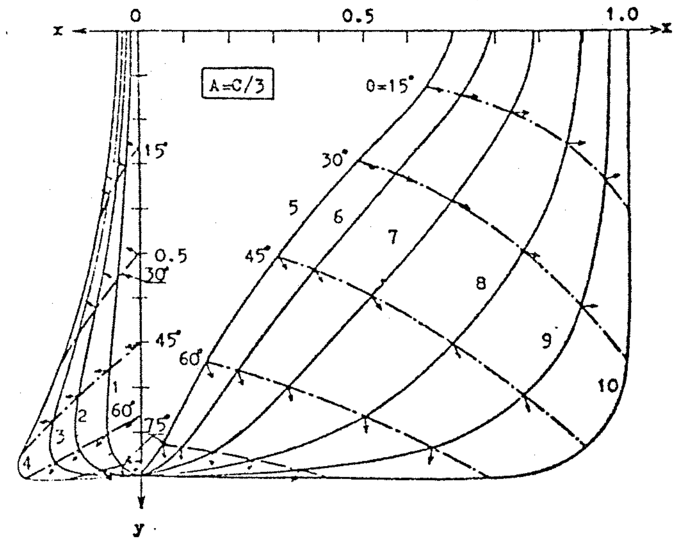


Fig. 8 Ship Form for $A=C/3$

respect to ζ are given as follows;

$$\left.\begin{aligned} &A=\frac{C}{3}, \ 2A=-B, \\ &a'(3c-2a-6b)=(c+3b)c', \\ \text{and} \quad &a'=b'+c'. \end{aligned}\right\} \qquad (5\cdot5)$$

That is, in the case of the ship form which has the multipliers containing the above equations, T_2 is minimum at each section and

$$\frac{2}{\rho}T_2=\frac{\pi}{6}C^2=\frac{\pi}{6}\left(\frac{\gamma'}{2}\right)^2 \quad \text{for} \quad A=\frac{C}{3} \qquad (5\cdot6)$$

Solutions of Eqs. (5·5) (cf. Appendix) are shown in Fig. 8. The right hand side of this figure shows typical so-called V-frame ship form and the left bulbous bow. In this ship form tangential velocity is, by substitution $A=C/3$ in Eq. (5·3),

$$v_\tau\frac{ds}{d\theta}=\frac{\gamma'}{6}(1-4\cos 2\theta)\sin 2\theta \qquad (5\cdot7)$$

and by substitution of it in Eq. (2·3)

$$v_\nu\frac{ds}{d\theta}=\frac{\gamma'}{6}(3+\cos 2\theta-2\cos 4\theta), \qquad (5\cdot8)$$

consequently

$$\frac{v_\tau}{v_\nu}=\frac{\sin 2\theta-2\sin 4\theta}{3+\cos 2\theta-2\cos 4\theta}\,. \qquad (5.9)$$

These are shown roughly in Fig. 8 by arrows. In order to compare with this let's consider U-frame ship form of which sectional area coefficient is nearly constant. For this purpose, take $B=0$ in Eq. (5·1), and

$$C=A=\frac{\gamma'}{2} \qquad (5\cdot10)$$

therefore

$$\frac{2}{\rho}T_2=\frac{\pi}{2}C^2, \qquad (5\cdot11)$$

$$\left.\begin{aligned} v_\tau\frac{ds}{d\theta}&=\gamma'\sin\theta\cos\theta \\ v_\nu\frac{ds}{d\theta}&=\gamma'\cos^2\theta \end{aligned}\right\}, \qquad (5\cdot12)$$

$$\text{or} \quad \frac{v_\tau}{v_\nu}=\tan\theta\,. \qquad (5\cdot13)$$

Accordingly directions of the stream lines at θ as shown in Fig. 9, for example, incline by θ more from the normal. Then stream lines as shown in the figure run first outwards to the bilge and further go around to the bottom. The comparison to Fig. 8 in Jinnaka's paper shows that ship forms of which sectional area coefficient are nearly constant along longitudinal direction have the common features in this aspect. Mean-

REF.NO.	c	a	b
1	1.2750	0.1250	-0.15
2	1.1636	0.0636	-0.12
3	1.0340	-0.0460	-0.08
4	0.8207	-0.2193	-0.04
5	0.6514	-0.3686	-0.02
6	0.5170	-0.4930	-0.01

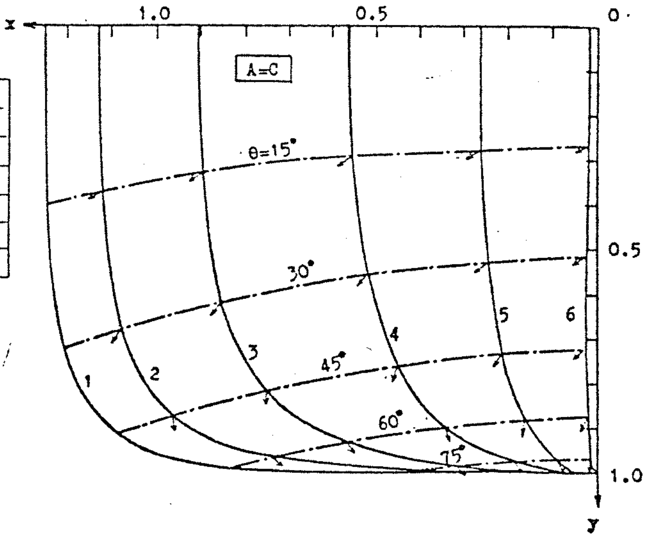


Fig. 9 Ship Form for $A=C$

while, as explained in section 3, when curvatures of the stream lines are large as like as these, the limiting stream lines are thought to curve more than these and it seems disadvantageous with respect to resistance. In this meaning the stream lines of the ship form in Fig. 8 turn out not so unnaturally as compared with those in Fig. 9. As shown by this example disadvantage of so-called U-frame ship is thought to be caused not by values of sectional area coefficient but by maintaining it almost constant. Therefore definition of (5·2) is to be not so suitable. To the contrary, advantage of V-frame ship is thought due to the small secondary flow of these kinds of ship forms of which rate of decrease of sectional area is designed as like as Fig. 8 to be larger than the rate of increase of breadth as going to the peaks. As the above, the frame line configuration of itself does not determine secondary flow but a rate of its longitudinal variation determines it. Let us see some examples.

i) Case of $A=0$ and $B=-C$

On the above condition the relation between the coefficients is through Eqs. (2·8)

$$c=a+3b\,, \tag{5·14}$$

and by adding the condition that τ is constant the coefficents are determined. In this case

$$\frac{2}{\rho}T_2=\frac{\pi}{4}C^2 \tag{5·15}$$

and

$$\frac{v_\tau}{v_\nu}=\frac{-\sin 4\theta}{1-\cos 4\theta}=\tan\left(2\theta-\frac{\pi}{2}\right), \tag{5·16}$$

then as shown in Fig. 10 these are V-frame. Similarly to the ship form in Fig. 9, v_τ changes sign at the half way from bottom to water surface and v_τ/v_ν does not become so large at the bottom.

ii) Case of $A=-2B$ or $A=\frac{2}{3}C$

The relation between the coefficients is

$$b'(3c-a-3b)+c'(c-a-9b)=0 \tag{5·17}$$

and

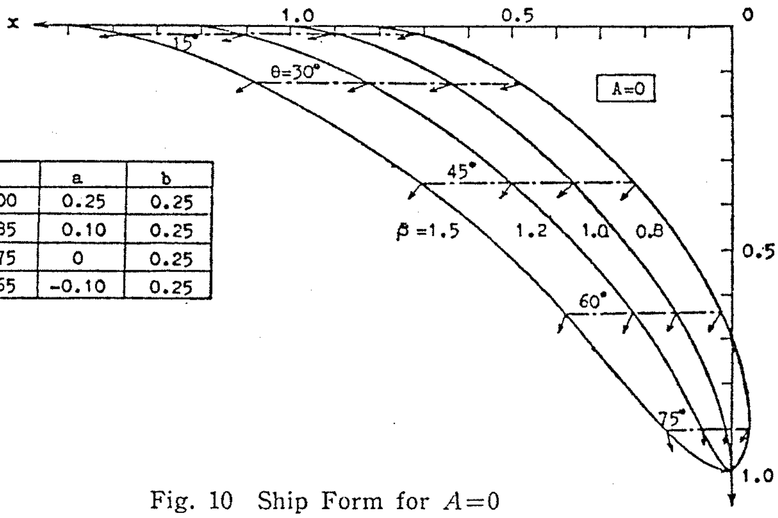
$$\frac{v_\tau}{v_\nu}=\frac{2\sin 2\theta-\sin 4\theta}{3+2\cos 2\theta-\cos 4\theta}, \tag{5·18}$$

$$\frac{2}{\rho}T_2=\frac{\pi}{4}C^2\,. \tag{5·19}$$

This ship form is deduced so that v_τ vanishes at $\theta=0°$, and resembles[12] closely that deduced by the same thought of slender thin body theory.[3] As shown in Fig. 11, this is V-frame ship form similar to Fig. 8.

iii) Case of $A=2B$ or $A=2C$

The relation between the coefficients is



β	c	a	b
1.5	1.00	0.25	0.25
1.2	0.85	0.10	0.25
1.0	0.75	0	0.25
0.8	0.65	-0.10	0.25

Fig. 10 Ship Form for $A=0$

REF.NO	c	a	b
1	0.7258	-0.4242	-0.15
2	0.6615	-0.4385	-0.10
3	0.5884	-0.4616	-0.05
4	0.6160	-0.3440	0.08
5	0.7607	-0.1793	0.06
6	0.8586	-0.1014	0.04
7	1.0000	0	0
8	1.1306	0.0806	-0.05
9	1.2359	0.1359	-0.10
10	1.3264	0.1764	-0.15

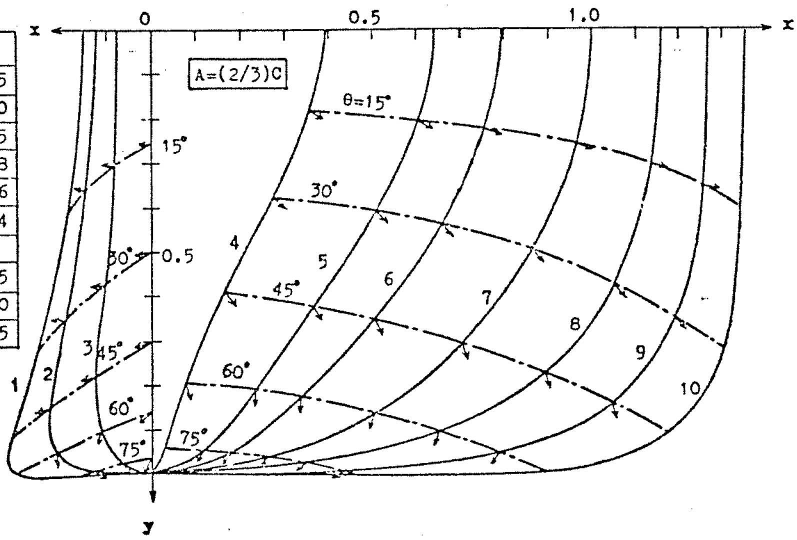


Fig. 11 Ship Form for $A=C/3$

$$b'(c+a+3b)=c'(c-a+3b) \qquad (5\cdot20)$$

and

$$\frac{v_\tau}{v_\nu}=\tan 2\theta\,, \qquad (5\cdot21)$$

$$\frac{2}{\rho}T_2=\frac{3\pi}{2}C^2\,. \qquad (5\cdot22)$$

This ship form is designed as $v_\tau=0$ at $\theta=\pi/2$ or on the keel line[12].

iv) Case of $A=B$, $C=0$

$$b'(a+3b)=(c-a)c' \qquad (5\cdot23)$$

$$\frac{v_\tau}{v_\nu}=\tan 3\theta \qquad (5\cdot24)$$

$$\frac{2}{\rho}T_2=\frac{3\pi}{4}C^2 \qquad (5\cdot25)$$

$v_\tau=0$ at $\theta=60°$ (near by the bilge) in this ship form and as shown in Fig. 12 it is similar to the previous example. Since T_2 value of this example is, together with previous example much Jarger than that of V-frame ship form, this case is not favourable when γ' or β' is large.

Although by prescribing characters of secondary flow and giving initial conditions frame line configuration is determined, in the case of constant depth, the frames near the fore and aft ends of ships, as like as the right hand side figures of Fig. 8 and 10 of V-frame

REF.NO.	c	a	b
1	0.7228	-0.1972	0.08
2	0.6815	-0.3185	0
3	0.6660	-0.4140	-0.08
4	0.6812	-0.4788	-0.16
5	0.6812	-0.4788	-0.16
6	0.6220	-0.4580	-0.08
7	0.5790	-0.4210	0
8	0.5598	-0.3802	0.06
REF. NO. 1～4 : A=2C			
REF. NO. 5～8 : C=0			

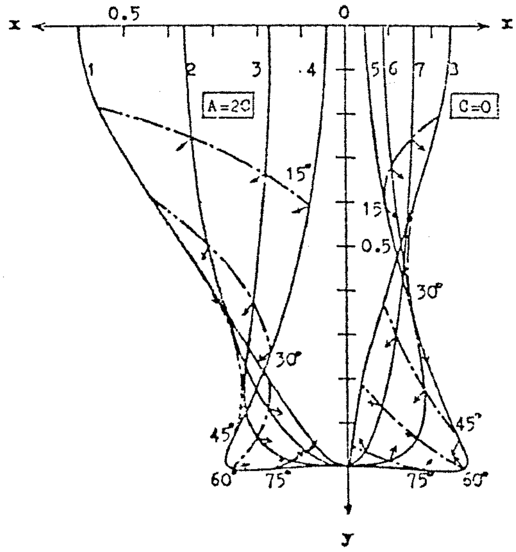


Fig. 12 Ship Form for $A=2C$ and $C=0$

ship form favourable in the sense of the studies in the previous sections can not be connected by bulbous bow as like as the left hand side figure of Fig. 8 directly. For the sake of it, one ideas is to adopt Maier form or cruiser stern by introducing the idea of A.G.S. form giving up to keep the condition of constant depth and the other is to adopt bulbous bow by inserting the connecting part like as Fig. 12[12].

6. Slender Thin Body Theory

What will occur if the consideration in the previous sections are adopted to more complicated ship form than Lewis form? Firstly in general through the form of two-dimensional potential (1·9) in reference to Eqs. (1·3) and (1·6), an expansion

$$v_\nu \frac{ds}{d\theta} = \frac{\delta\nu}{\delta\xi}\frac{ds}{d\theta} = \sum_{n=0}^{\infty} A_{2n}\cos 2n\theta \qquad (6\cdot1)$$

is possible. These coefficients are defined by the integrals like

$$\left.\begin{aligned} A_0 &= \frac{\gamma'}{2} = \frac{1}{2\pi}\int_C \frac{\delta\nu}{\delta\xi} ds, \\ \text{and} \quad A_{2n} &= \frac{1}{\pi}\int_C \frac{\delta\nu}{\delta\xi}\cos 2n\theta\, ds \end{aligned}\right\} \qquad (6\cdot2)$$

By extending Eq. (2·21), energy T_2 of secondary flow is given by

$$T_2 = \frac{\pi}{2}\rho\sum_{n=1}^{\infty}\frac{A_{2n}^2}{2n} \qquad (6\cdot3)$$

Accordingly if mapping function is given, these quantities can be calculated by the same way as that of section 2. Let's consider minimum value problem of T_2 with the background of studies in section 3. In the simplest case when γ' or slope of sectional area curve is only given, the solutions are clearly

$$\left.\begin{aligned} A_{2n} &= 0 \quad \text{for} \quad n \geq 1 \\ \text{and} \quad \text{Min. } T_2 &= 0 \end{aligned}\right\} \qquad (6\cdot4)$$

and this is A. G. S. form named before. Thinking of this through Eq. (6·1) and since

$$v_\nu = \frac{\delta\nu}{\delta\xi} = \frac{\gamma'}{2}\frac{d\theta}{ds}, \qquad (6\cdot5)$$

by making use of this formula A. G. S. form is drawn easily by the method of successive approximation if $ds/d\theta$ is known. Namely, it is certain generally that A. G. S. form which starts from arbitrary frame line configuration exists. Further let a restriction of constant draft set like the case of previous section. Now in order to clarify the relation to the practical ship form let's adopt slender ship theory[3)12)]. That is, let half breadth curve of the ship form be $\eta(\xi, y)$ and approximate as follows;

$$v_\nu = \frac{\delta\nu}{\delta\xi} \doteqdot \frac{\delta\eta(\xi, y)}{\delta\xi}, \qquad s \doteqdot y = \sin\theta, \quad \text{and} \quad \frac{ds}{d\theta} \doteqdot \cos\theta, \tag{6·6}$$

and it becomes intuitively easy to consider variation of half breadth in ξ-direction because on any water line θ is constant. Because, even after this approximate estimation, above formulae are invariable essentially, it is easy to rewrite them in accurate forms. On this approximation Eq. (6·1) is written as

$$\frac{\delta\eta}{\delta\xi} = \frac{1}{\cos\theta} \sum_{n=0}^{\infty} A_{2n} \cos 2n\theta \tag{6·7}$$

and Eq. (6·5)

$$\frac{\delta\eta}{\delta\xi} = \frac{\gamma'}{2\cos\theta} \tag{6·8}$$

then this is a law of variation of water lines of A. G. S. form. Since the condition of constant depth is, in other words, that η should vanish when $\theta = \pi/2$, from Eq. (6·7)

$$\sum_{n=0}^{\infty} (-1)^n A_{2n} = 0 \tag{6·9}$$

Take N coefficients A_2 to A_{2n}, and solve the minimum value problem of T_2 under the condition of above formula by the method of indeterminate coefficients, and

$$A_{2n} = (-1)^{n+1} \frac{2nA_0}{N(N+1)}, \qquad T_2 = \left(\frac{\pi\rho}{2}\right) \frac{A_0^2}{N(N+1)} \tag{6·10}$$

are obtained. The case when $N=2$ corresponds to Lewis form and above formulae are the same as Eqs. (5·5) and (5·6). As N is taken larger, A_{2n} and T_2 tend to zero and clearly they tend to the solution (6·4) as the limit. That is, generally if we increase the number of coefficients and look for the frame line configuration which minimumizes T_2 we may obtain the A. G. S. form as the limit.

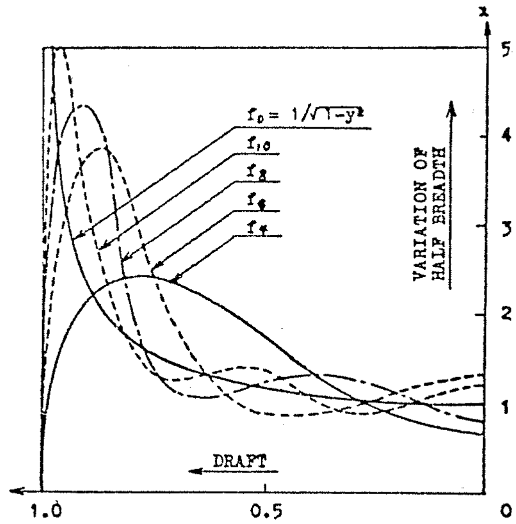


Fig. 13 Optimum Variation of Half Breadth

About A. G. S. form, however, draft can not be constant, therefore, as N increses, uneveness of frame line becomes conspicious. Fig. 13 is drawn in order to show these circumstances. The notations are as the followings and the figure illustrates the solutions of Eq. (6·10)

$$f_0 = \frac{1}{\cos\theta}, \qquad f_{2N} = \frac{1}{\cos\theta}\left\{1 - \frac{2}{N(N+1)} \sum_{n=1}^{N} (-1)^n n \cos 2n\theta\right\} \tag{6·11}$$

In thin ship theory minimum T_2 is obtained by adding to or subtracting from the given initial frame line and especially f_4 corresponds to Lewis form. To do more correctly, substitute this into Eq. (6·1) and solve it as a partial differential equation with respect to $\delta\nu/\delta\xi$ and then the ship form can be drawn as a previous section.

7. Practical Ship Form

As shown in section 5, if the characters of secondary flow are appointed, there appear fairly distinct features of the ship form and exist many groups of curves similar to the practical ship form. Then in this section let's take up the typical and practical ship form and, from a point of view as metioned above, examine what features they have. Although the method of making the practical ship forms approximated by Lewis form seems to be a very rough treatment as shown in the later figure, yet from now we will continue the discussion considering the equivalent Lewis form ship of which breadth, draft and sectional area are the same. Because more accurate approximation makes calculation complicate exceptionally and, by the practical examination in part, it does not prove to have so effect as to deny the results. Firstly, as the general standard of practical ship form, Taylor's parent ship form is shown by real lines of Fig. 14. Dotted lines, showing the equivalent Lewis

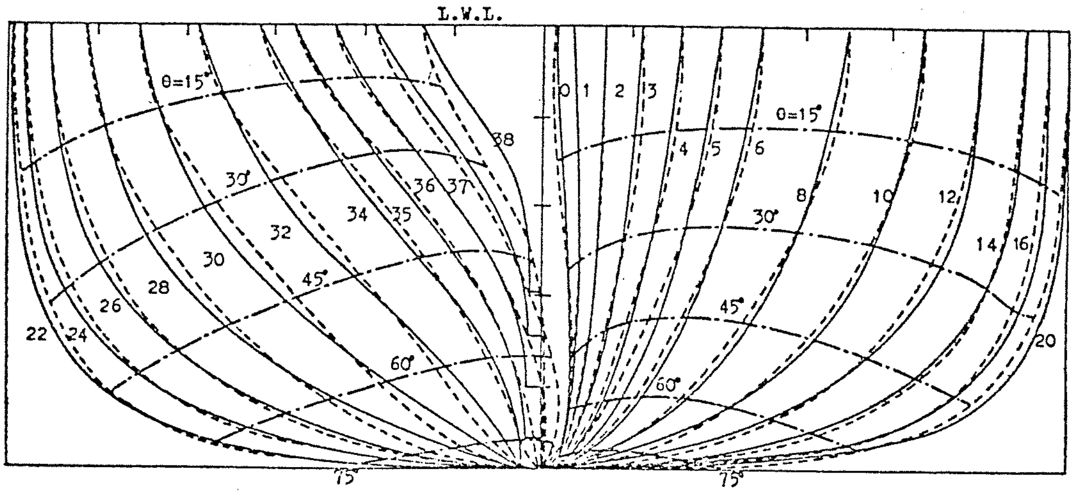


Fig. 14 Body Plan of Taylor's Parent Model.

TABLE 1

MODEL: TAYLOR'S PARENT FORM

L/B	B/d	C_b	C_p	$C_{\otimes}$	∇/L^3
8	2.4	.507	.553	.916	3.298 $\times 10^{-3}$

SQ. STATION		γ/τ^2	β/τ	c/τ	a/τ	b/τ	$C(L/\tau)$	$A(L/\tau)$	$B(L/\tau)$	C_{DJ} $\times 10^{-3}$	Min.C_{DJ} $\times 10^{-3}$
38	1/2	.0829	.2700	.5116	-.3650	.1234	—	—	—	—	—
37	3/4	.1430	.3768	.5474	-.3116	.1410	1.35	.83	-.52	.22	.16
36	1	.2154	.4668	.5900	-.2666	.1434	1.54	.76	-.78	.23	.21
35	1 1/4	.2948	.5544	.6359	-.2228	.1413	1.68	.71	-.97	.25	.25
34	1 1/2	.3871	.6324	.6865	-.1838	.1297	1.80	.73	-1.07	.29	.28
32	2	.5723	.7776	.7853	-.1112	.1035	1.96	.80	-1.16	.34	.33
30	2 1/2	.7745	.9060	.8884	-.0470	.0646	2.02	.91	-1.11	.38	.36
28	3	.9722	1.0212	.9869	.0106	.0237	1.87	.91	-.96	.34	.31
26	3 1/2	1.1436	1.1052	1.0712	.0526	-.0186	1.53	.75	-.78	.23	.20
24	4	1.2777	1.1604	1.1375	.0802	-.0573	1.08	.52	-.56	.11	.10
22	4 1/2	1.3615	1.1880	1.1801	.0940	-.0861	.62	.24	-.38	.03	.03
20	5	1.3995	1.2000	1.1997	.1000	-.0997	.16	.01	-.15	.00	.00
18	5 1/2	1.3877	1.1856	1.1948	.0928	-.1020	-.35	-.30	.05	.02	.01
16	6	1.3289	1.1568	1.1652	.0784	-.0868	-.84	-.55	.29	.09	.06
14	6 1/2	1.2163	1.1016	1.1066	.0508	-.0578	-1.40	-.90	.50	.25	.17
12	7	1.0532	1.0092	1.0270	.0046	-.0224	-1.81	-1.18	.63	.42	.28
10	7 1/2	.8602	.8748	.9296	-.0626	.0078	-2.02	-1.42	.60	.57	.36
8	8	.6542	.7020	.8238	-.1490	.0272	-2.04	-1.68	.36	.75	.36
6	8 1/2	.4565	.5064	.7214	-.2468	.0318	-1.86	-1.71	.15	.76	.30
5	8 3/4	.3606	.4068	.6720	-.2966	.0314	-1.71	-1.66	.05	.72	.25
4	9	.2847	.3144	.6353	-.3428	.0219	-1.54	-1.65	-.11	.71	.21
3	9 1/4	.2106	.2256	.6003	-.3872	.0120	-1.41	-1.64	-.23	.71	.17
2	9 1/2	.1444	.1428	.5728	-.4286	-.0014	-1.22	-1.59	-.37	.68	.13
1	9 3/4	.0910	.0684	.5563	-.4658	-.0221	-.88	-1.41	-.53	.56	.07
F.P.	F.P.	.0619	.0276	.5497	-.4862	-.0359	—	—	—	—	—

form, coincide comparatively with practical frame line as shown in the figure. Calculated values of C, A and B are shown in Table 1 and at the aft body they prove to correspond to the case when $A=C/3$ or the case of Fig. 8 of minimum T_2 and at the fore body to the case when $A=C$ or the case of Fig. 9. In the Table, C_{DJ} is defined by Eq. (3·3) and Min. C_{DJ} shows the values corresponding to Eq. (5·6). Secondly the example of high speed cargo ships, a parent ship form of 45th committee of Japan Ship Research Association,[14] is shown in Fig. 15 and Table 2. Although the equivalent Lewis form differs considerably from it in this case, at the aft body still $A \fallingdotseq C/3$ and at the fore body which may be so-called semi-Maier form $A \fallingdotseq 2/3C$ and it is a form whose v_r is small near the

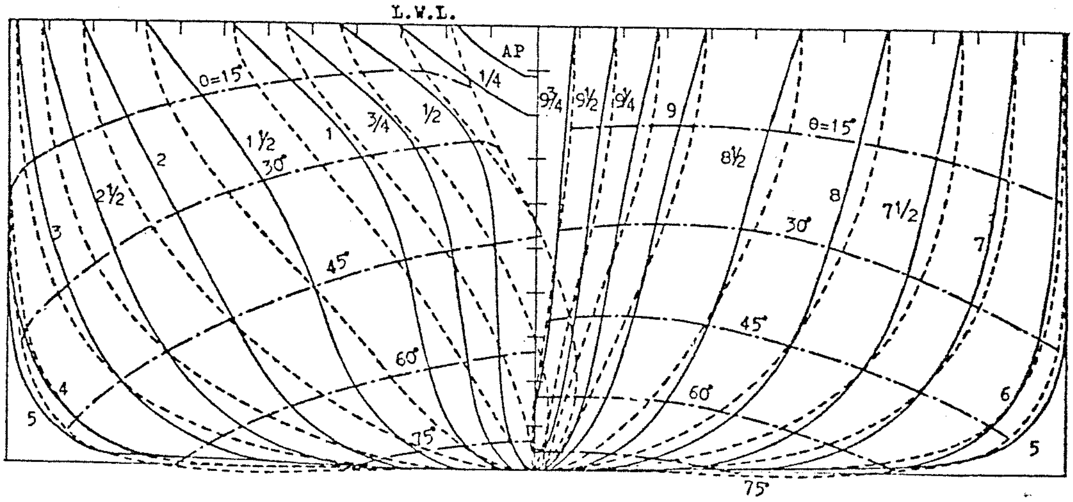


Fig. 15 Body Plan of SR 45 M. S. 1382

TABLE 2

MODEL : SR45 M.S. 1382

L/B	B/d	C_b	C_p	$C_\otimes$	∇/L^3	l.c.b.
7	2.4	.625	.642	.974	5.313×10^{-3}	+1.29%

Sq.St.	γ/τ^2	β/τ	c/τ	a/τ	b/τ	$C(L/\tau^2)$	$A(L/\tau^4)$	$B(L/\tau^3)$	C_{DJ} $\times10^{-3}$	Min.C_{DJ} $\times10^{-3}$
A.P.	.0179	.1908	.4746	-.4046	.1208	——	1.28	——		
1/4	.0476	.3180	.4948	-.3410	.1642	2.37	.88	-1.49	.61	.61
1/2	.1801	.4464	.5712	-.2768	.1520	2.55	.99	-1.56	.71	.70
3/4	.3110	.5700	.6450	-.2150	.1400	2.68	1.06	-1.62	.79	.78
1	.4479	.6864	.7199	-.1568	.1233	2.76	1.13	-1.63	.84	.82
1 1/2	.7233	.8856	.8633	-.0572	.0795	2.66	1.19	-1.47	.81	.76
2	.9748	1.0332	.9887	.0166	.0279	2.30	1.08	-1.22	.62	.57
2 1/2	1.1831	1.1292	1.0906	.0646	-.0260	1.79	.84	- .95	.38	.35
3	1.3349	1.1856	1.1659	.0928	-.0731	1.21	.45	- .76	.16	.16
4	1.4659	1.2000	1.2382	.1000	-.1382	.30	.02	- .28	.01	.01
5	1.4882	1.2000	1.2520	.1000	-.1520	-.02	0	.02	.00	.00
6	1.4450	1.1928	1.2269	.0964	-.1305	-.54	-.38	.16	.05	.03
7	1.2382	1.0824	1.1225	.0412	-.0813	-1.63	-1.26	.37	.54	.29
7 1/2	1.0477	.9648	1.0265	-.0176	-.0441	-2.20	-1.66	.54	.94	.52
8	.8066	.8004	.9038	-.0998	-.0036	-2.55	-1.89	.66	1.23	.70
8 1/2	.5491	.5976	.7694	-.2012	.0294	-2.49	-1.88	.61	1.21	.67
9	.3185	.3804	.6480	-.3098	.0422	-2.09	-1.76	.33	1.02	.47
9 1/4	.2203	.2772	.5968	-.3614	.0418	-1.88	-1.67	.21	.91	.38
9 1/2	.1310	.1800	.5511	-.4100	.0389	-1.68	-1.58	.10	.81	.30
9 3/4	.0521	.0888	.5125	-.4556	.0319	-1.45	-1.56	-.11	.79	.23
F.P.	0	0	.5000	-.5000	0	——	-1.73	——	——	——

water surface as shown through Eq. (5·18). For the example of full ship forms, a parent ship form of U-frame of 41th committee of Japan Ship Research Association,[15] is shown in Fig. 16 and Table 3. Also in this case $A \doteqdot C/3$ at the aft body and this law seems to be applicable generally to the aft bodies of practical ships. At the fore body $A \doteqdot C$ similarly to Taylor's parent form and then it may be said that general feature of U-frame is $A=C$. Accordingly if equi-θ curve is given, stream lines can be drawn by Eq. (5·13). The stream lines given by such ways are, however, not thought to be desirable from the study of section 3 and 4. To consider this further, we compare Max$\{C_{DJ}\}$ and Max$\{$Min. $C_{DJ}\}$ defined by Eq. (3·3) of these ships with residual resistance

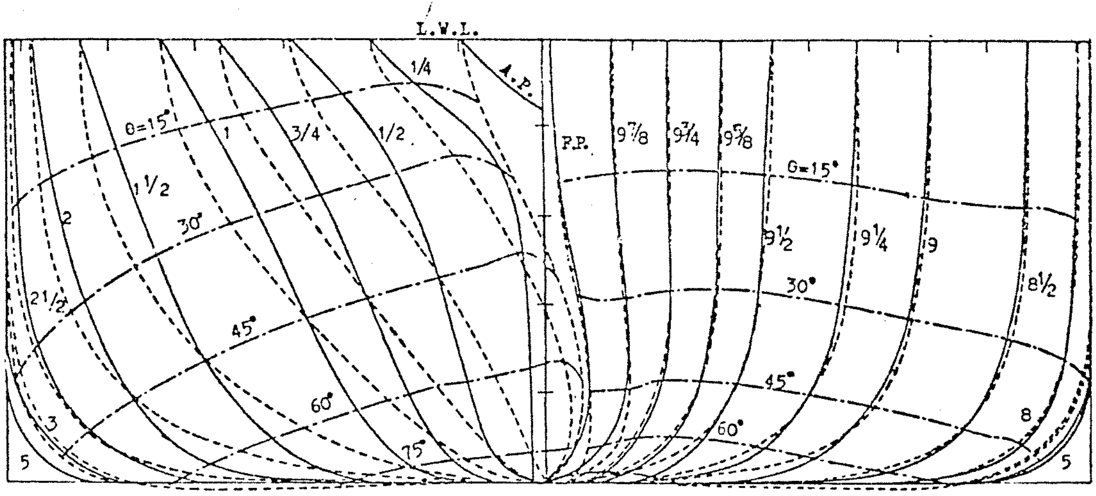


Fig. 16 Body Plan of SR 41 M. S. 1342

TABLE 3

MODEL: SR41 M.S. 1342

L/B	B/d	C_b	C_p	C_ϕ	∇/L^3	l.c.b
7.2	2.463	.799	.807	.9905	6.253 $\times 10^{-3}$	-1.574%

Sq.S_T	γ/τ^2	β/τ	c/τ	a/τ	b/τ	$C(L/\tau^2)$	$A(L/\tau^2)$	$B(L/\tau^2)$	C_{DJ} $\times 10^{-3}$	Min.(C_{DJ}) $\times 10^{-3}$
A.P.	.0228	.1894	.4776	-.4053	.1171	——	2.53	——		
1/4	.1295	.4055	.5427	-.2973	.1600	4.88	1.42	-3.46	1.87	1.85
1/2	.3667	.5864	.6730	-.2068	.1202	4.49	1.74	-2.75	1.59	1.57
3/4	.5779	.7433	.7853	-.1284	.0863	4.01	1.88	-2.13	1.36	1.25
1	.7686	.8746	.8838	-.0627	.0535	3.60	1.89	-1.71	1.18	1.01
1 1/2	1.0893	1.0667	1.0445	.0334	-.0111	2.80	1.56	-1.24	.75	.61
2	1.3263	1.1751	1.1622	.0876	-.0746	1.94	.96	-.98	.33	.29
2 1/2	1.4745	1.2228	1.2394	.1114	-.1280	1.01	.36	-.65	.08	.08
3	1.5391	1.2315	1.2769	.1158	-.1611	.30	0	-.30	.01	.01
4～7	1.5531	1.2315	1.2858	.1158	-.1700	0	0	0	0	0
7 1/2	1.5491	1.2315	1.2833	.1158	-.1675	-.23	-.08	.15	.00	.00
8	1.4932	1.2097	1.2531	.1049	-.1482	-1.07	-1.00	.06	.24	.09
8 1/2	1.3228	1.0959	1.1706	.0480	-.1226	-2.33	-2.53	-.20	1.51	.42
9	1.0269	.8742	1.0272	-.0629	-.0901	-3.69	-3.85	-.16	3.47	1.06
9 1/4	.8262	.7165	.9277	-.1418	-.0695	-4.38	-4.50	-.12	4.73	1.49
9 1/2	.5928	.5219	.8108	-.2391	-.0499	-4.93	-5.21	-.28	6.34	1.89
9 5/8	.4693	.4087	.7503	-.2957	-.0460	-5.10	-5.62	-.52	7.43	2.03
9 3/4	.3380	.2858	.6864	-.3571	-.0435	-5.22	-5.99	-.77	8.45	2.12
9 7/8	.2061	.1537	.6362	-.4232	-.0594	-5.32	-6.78	-1.46	10.98	2.21
(F.P.)	.0700	.0124	.5715	-.4938	-.0653	-5.70	-7.38	-1.68	13.06	2.53

coefficients

$$C_r = \frac{\text{residual resistance}}{\frac{\rho}{2} \nabla^{2/3} V^2} \tag{7·1}$$

of each model at low speed, then following table is obtained.

MODEL	$C_r \times 10^3$	Max $\{C_{DJ}\} \times 10^3$	Max $\{\text{Min}.C_{DJ}\} \times 10^3$
TAYLOR	2.4 at F_r=.15	0.8 at Ord. $8\frac{1}{4}$	0.37 at Ord. $7\frac{3}{4}$
SR 45. MS 1382	3.8 at F_r=.12	1.26 at Ord. $8\frac{1}{4}$	0.83 at Ord. 1
SR 41. MS 1342	5.0 at F_r=.12	(13.0) at F. P.	(2.5) at F. P.

It is shown in this table that C_{DJ} is the same order quantitatively as C_r and if C_{DJ} is large, C_r is large, further C_{DJ} is especially large in full ship. In Fig. 17 longitudinal variations of C_{DJ} are shown. It is worth to note that C_{DJ} of the full ship form, differing from the other two ship forms, varies monotonously and rapidly becomes larger at F. P. Accordingly, if Jones' theory is adopted intactly to full ship form, Trefftz plane which has largest vortex is set up at F. P., brought backwards as it is and this vortex would be observed as so-called bilge vortices.[7] It is clearly seen through Fig. 17 why the bilge vortex is taken up as a problem firstly in the full ship forms. Remenbering that accuracy of approximation of this theory, however, is not satisfactory especially at the stem and the effect of induced velocity of vortices perpendicular to the main

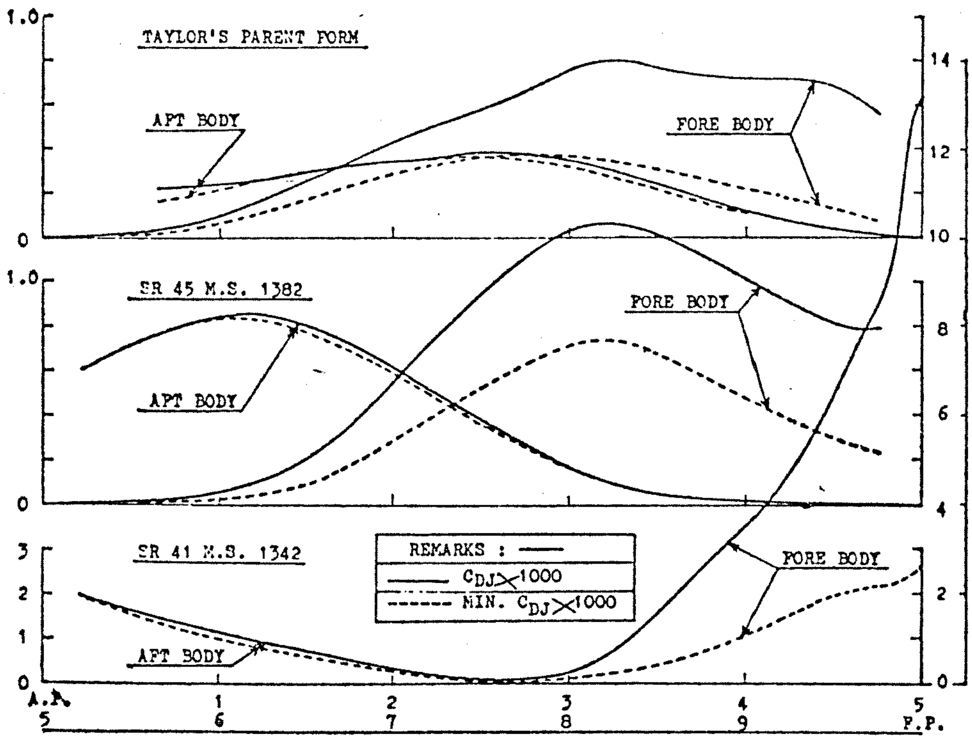


Fig. 17 Energy of Secondary Flow

flow, resistance from such kinds of cause, if it exists, is thought to be much smaller than Max. C_{DJ}.[6] Therefore we might imagine a fairly clear image about the relation between C_{DJ} and C_r through the above table, and it would be not necessarily unreasonable to expect that, by making C_{DJ} small, residual resistance could be reduced. It is, however, difficult in this stage to find more suitable relation between C_{DJ} and C_r rather than this and accordingly there are no data to conclude whether the longitudinal distribution of C_{DJ} of full ship forms is good or not. Anyhow C_{DJ} is proportional to the square of slope of prismatic curve, therefore this problem returns to how the longitudinal distribution of displacement effects on the viscous resistance and how the optimal distribution is in this point of view.

Conclusions

Summarizing the preceding discussions,

1. By making use of slender body theory, flow field around the section of the double models was analyzed.
2. Sections of ships are approximated by Lewis form.
3. Velocity potential is represented by a source of displacement effect and circulation on the frame line.
4. The strength of secondary flow so obtained is studied hydrodynamically from the two points of view of kinetic energy T_2 of Trefftz plane and inclination of stream lines in boundary layer.
5. As the result, the ship forms of which secondary flow becomes small, is firstly obtained. T_2 of such a flow vanishes and the streamlines are approximately geodesic, and hence cross flow inside the boundary layer also approximately vanishes.
6. As draft is constant in most of the pratical ships, so a few examples of the ship forms which processes the characteristic secondary flows are drawn on this condition.
7. As the result, the ship forms of which T_2 is minimum are V-frame ship forms and it is seen that the aft body of practical ships are almost similar to them.
8. The ship form which T_2 is minimum forms a bulbous bow if it starts from vertical plated stem, and can not be connected usually with V-shaped frame line.
9. Practical ship forms seems to correspond well to some of these clasifications, therefore the feature of their secondary flow may possibly be grasped.
10. Longitudinal distribution of T_2 of full ship form differs greatly from that of the usual thin ship. One of the causes, in which the unusual flow recently noted appears, seems also to be these points.

Thus, the definite co-relation between feature of the secondary flow and the ship form is found and some of the ship forms are proved to be realized in practical ship forms, therefore, if the ship forms used in practice showed best efficiency in the conceivable sense at present, and if there were not great contradiction, in the present theory it could not be doubted that by drawing the frame line by means of this theory the efficent ship form as far as conceivable now should be obtained. On the other hand, if this theory is valid, the fore half body of practical ship forms, particularly of U-frame ship form, on the points of view of this paper seems to be necessarily reexamined from now including the co-relation with the wave-making resistance. At last, as the theory developed here can not predict the resistance quantitatively and it has many unknown points of the optimum longitudinal distribution of displacement, to continue further deduction seems to be less efficient. It seems to be desirable to study directly from the points of view of both wave-making resistance theory and viscous resistance theory.

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Appendix

Solutions of the partial differential equations

Substitute $\tau=c-a+b=1$ in Eq. (2·4) and eliminate a, then

$$\left.\begin{aligned} C&=c'(1-b)-b'(c+4b-1)\,,\\ A&=(b'-c)(1-4b)\,,\\ \text{and}\quad B&=A-C \end{aligned}\right\} \tag{A·1}$$

Because all partial differential equations appeared in the paper are written as the form

$$mA=C \tag{A·2}$$

after rearrangement by substituting Eq. (A·1) into Eq. (A·2) it becomes

$$c'\{(4m-1)b-m+1\}=b'\{c+(4b-1)(1-m)\} \tag{A·3}$$

and this is a first order homogeneous partial differential equation. Then put

$$b-\frac{m-1}{4m-1}=y\,,$$

and

$$c+\frac{3(m-1)}{4m-1}=x \tag{A·4}$$

and Eq. (A·3) becomes

$$(4m-1)yx'=y'\{x-4(m-1)y\} \tag{A·5}$$

and further put

$$x=vy \tag{A·6}$$

and

$$x'=v'y+vy'$$

Hence, substituting this in Eq. (A·5), it becomes

$$\frac{(4m-1)v'}{(2-4m)v-4(m-1)}=\frac{y'}{y}\,. \tag{A·7}$$

Integrate this and substitute it in Eq. (A·6) and

$$y=K\left(x+\frac{2m-2}{2m-1}y\right)^{4m-1} \quad (A\cdot 8)$$

where K is integration constant. Further substitute Eq. (A·4) in the above, and if $m\neq 1/2,\ \infty$

$$b-\frac{m-1}{4m-1}=K\left(c+\frac{2m-2}{2m-1}b+\frac{m-1}{2m-1}\right)^{4m-1}$$

or

$$c=K\left(b-\frac{m-1}{4m-1}\right)^{\frac{1}{4m-1}}-\frac{2m-2}{2m-1}b-\frac{m-1}{2m-1} \quad (A\cdot 9)$$

The cases apeared in the paper are shown in the following table.

CASE		m	$4m-1$	$\frac{m-1}{4m-1}$	$\frac{2(m-1)}{2m-1}$
1	$A=0$	∞	∞	1/4	1
2	$A=C/3$	3	11	2/11	4/5
3	$A=2/3C$	3/2	5	1/10	1/2
4	$A=C$	1	3	0	0
5	$A=2C$	1/2	1	$-1/2$	$-\infty$
6	$C=0$	0	-1	1	2

Accordingly in the case of 2), 3), 4) and 6) the solutions are at once obtained as follows;

2) $A=\frac{C}{3},\ m=3$

$$c=K\left(b-\frac{2}{11}\right)^{1/11}-\frac{4}{5}b-\frac{2}{5} \quad (A\cdot 10)$$

3) $A=\frac{2}{3}C,\ m=\frac{3}{2}$

$$c=K(b-0.1)^{1/5}-\frac{b}{2}-\frac{1}{4} \quad (A\cdot 11)$$

4) $A=C,\ B=0,\ m=1$

$$b=Kc^3 \quad (A\cdot 12)$$

6) $C=0,\ m=0$

$$c=\frac{K}{b-1}-2b-1 \quad (A\cdot 13)$$

The solution in the case of 1) is already given by Eq. (5·15) in the paper. In the case of 5), re-integration at the stage of Eq. (A·7) shows the result

$$c=\frac{3}{2}+(1+2b)\log\left(\frac{1+2b}{K}\right) \quad (A\cdot 14)$$

K in these formulae is to be determined by the initial values of c, a and b. Actually, by giving suitable values to b, the values of c are calculated with above formulae.