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"On the Theory of Wave-Free Ship Forms"

(To Prof. Y. Ikeda this paper is dedicated.)

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Abstract

There are ship forms having no radiating wave when they oscillate on the water surface, which are called wave-free. If a ship is wave-free, then it has no wave-damping so that it may be unfavourable to the practical application but, on the other hand, it receives no exciting force of the incident wave by Haskind-Hanaoka's relation. The latter character is very much interesting with the application to the theory of ship motions among waves.

The author developes the various methods to obtain such wave-free ship forms. In the two-dimensional case, there are wave-free sections for the heaving-dipping and rolling oscillations but not for the swaying. In the three-dimensional case, the conclusion is not only almost the same, but there are many interesting cases.

1. Introduction

The water waves produced by a ship are almost always unfavourable phenomena for the naval architects. In this point of view, the problem is of variation rather than of boundary value, and at the beginning of the 20-th century Lord Kelvin[1] shew examples of a wave-less pontoon which has no canal wave by the interference and just after him Prof. Yokota[2] proposed the possibility of the wave-less, rolling-less, pitching-less and heaving-less ship.

Since then, after about a half century, Prof. Inui[3] had a success in the realization of a wave-less ship although it is not wave-less in the mathematical sense. His success encourages us to research this idea in another problems.
For this aim, it will be most effective to study the property of the wave-free potential, because the velocity potential consists of two parts, that is, it and the wave-source one.

Namely, at first, does there exist the wave-free potential which describes a motion around an actual ship-like body? Secondly, does the wave correspond to the body producing it in one-to-one manner?

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The answers are affirmative for the problem of the uniform motion except the first question in three dimension and this theory seems very usefull to the practical application, especially methodologically[4].

The present note deals with the problem of the oscillation of ships with no advance speed.[5] In this case, what there is no wave means that there is no wave-damping, so that it may be not preferable in this respect but do in the sense that the exciting force of the on-coming wave vanishes if the radiating wave by the motion in the direction of that force vanishes. The last relation is a corollary of Haskind-Hanaoka's theorem[6][7][8], which is easily understood from the asymptotic character of Neumann function described in the Appendix.

2. Wave-Free Potential[10][11]

Let us consider a Cartesian co-ordinate, taking the origin on the mean water surface and at the center of the body, x-axis vertically upwards and x-axis length-wise forwards. Assume the water motion as a harmonic oscillation by the same motion of the body with circular frequency $\omega=2\pi/T$. Then its wave number K is ω^2/g.

Let the real part of $\phi e^{i\omega t}$ its velocity potetial, then ϕ must satisfy the water surface condition[9]:

$$K\phi(x, y, 0)-\frac{\partial}{\partial z}\phi(x, y, 0)=0, \tag{2.1}$$

Now, if ϕ is deduced from the function m by the operation as follows;

$$\phi=Km+\frac{\partial}{\partial z}m, \tag{2.2}$$

then the surface condition (2.1) for m gets to

$$K^2m-\frac{\partial^2}{\partial z^2}m=0, \qquad \text{for} \quad z=0, \tag{2.3}$$

Hence, if m has a symmetry:

$$m(x, y, z)=-m(x, y, -z), \tag{2.4}$$

and has no wave, ϕ of (2.2) satisfies the water surface condition and clearly has no wave[5].

For example, suppose simple functions as m

$$m=\frac{\partial}{\partial z}\frac{1}{r}, \qquad r=\sqrt{x^2+y^2+z^2}, \tag{2.5}$$

$$m=\frac{1}{r_1}-\frac{1}{r_2}, \qquad \begin{aligned} r_1&=\sqrt{x^2+y^2+(z+h)^2}, \\ r_2&=\sqrt{x^2+y^2+(z-h)^2}, \end{aligned} \tag{2.6}$$

and consider a heaving oscillation.
Since the motion has an axial symmetry, the body profile may be calculated by making use of Stokes' stream function.
Let it be ψ, then in the present case

$$\psi=\frac{\sin^2\theta}{r}+\frac{3}{Kr^2}\sin^2\theta\cos\theta, \qquad \text{for} \quad (2.5), \tag{2.7}$$

$$\psi=r^2\sin^2\theta\left(\frac{1}{r_{13}}-\frac{1}{r_{23}}\right)+K\left(\frac{z+h}{r_1}-\frac{z-h}{r_2}\right), \quad \text{for} \quad (2.6), \tag{2.8}$$

where $z=-r\cos\theta$,
and at the body surface

$$\psi=\frac{r^2}{2}\sin^2\theta, \tag{2.9}$$

The body profiles can easily be calculated and are shown in Fig. 1 and 2 with dotted lines.

The same procedure can be applied more easily for the two-dimensional case and that the complex velocity potential can be used in this wave-free case because all the motion has the same phase.
Now, let $f(t)$, $t=y+iz$, be the complex velocity potential as

$$f(t)=\phi(y,z)+i\psi(y,z), \tag{2.10}$$

then the surface condition becomes

$$Re\left\{Kf-i\frac{d}{dt}f\right\}=0, \qquad \text{for} \quad z=0, \tag{2.11}$$

and if there is a regular function $m(t)$ such that

$$Re\{m(t)\}=0, \qquad \text{for} \quad z=0, \tag{2.12}$$

the function f computed as

$$f(t)=Km(t)+i\frac{d}{dt}m(t), \tag{2.13}$$

is wave-free.

Some of the simplest examples are

$$m(t)=i/t^n, \ f(t)=\frac{iK}{t^n}-\frac{n}{t^{n+1}}, \tag{2.14}$$

$$m(t)=\log\left(\frac{t+ih}{t-ih}\right), \ f(t)=\frac{1}{t+ih}-\frac{1}{t-ih}+K\log\left(\frac{t+ih}{t-ih}\right), \tag{2.15}$$

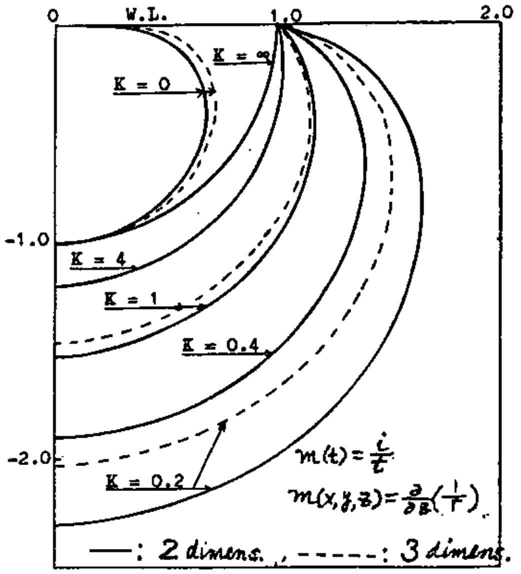

Fig. 1. Heaving oscillation

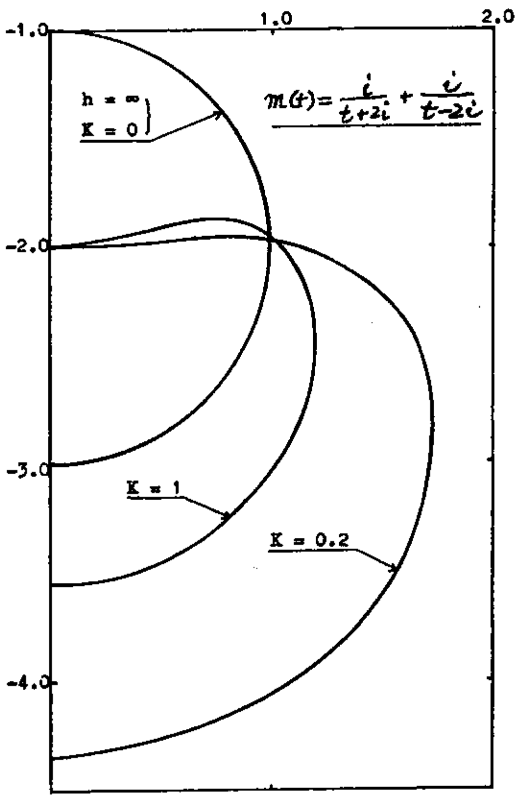

Fig. 2. Heaving oscillation

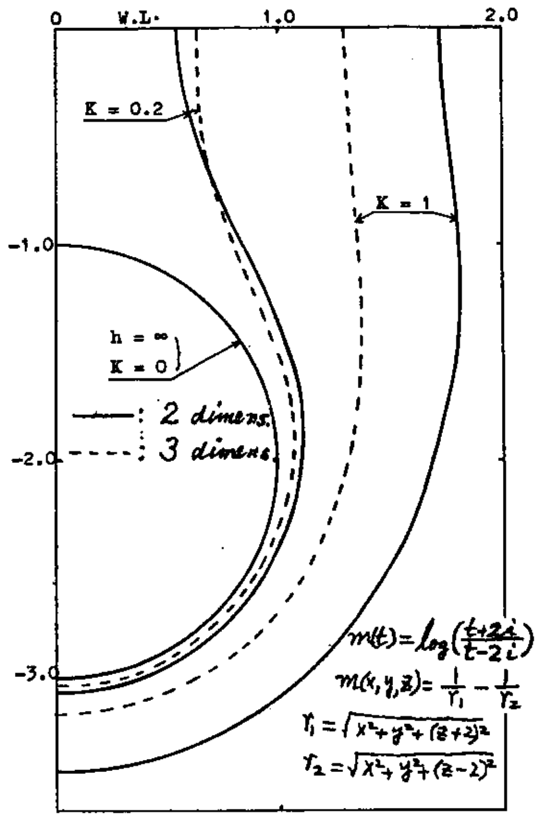

Fig. 3. Heaving oscillation

$$\left.\begin{aligned} m(t)&=\frac{A}{(t+ih)^n}-\frac{\bar{A}}{(t-ih)^n}, \\ f(t)&=\frac{KA}{(t+ih)^n}-\frac{K\bar{A}}{(t-ih)^n}-\frac{niA}{(t+ih)^{n+1}}+\frac{ni\bar{A}}{(t-ih)^{n+1}}, \end{aligned}\right\} \tag{2.16}$$

Since the boundary condition for the heaving motion is

$$\psi=-y, \tag{2.17}$$

solving the equation, the wave-free body profile can be obtained as shown in Fig. 1~3.

In general, let us imagine a half-immersed cylinder and N-points on it except on the water surface and take up N-wave-free functions such as (2.14). Then, (2.18) gives the equations to determine the potential and they may not always be singular or may have a unique solution. This means that the motion of any cylinder may be represented by wave-free potentials except near the water surface. By the same procedure, we may have wave-free ship-like form in the case of the rolling motion, but not of the swaying motion. However, their cases may be more interesting to study by another treatment succeeding.

3. Pressure Distribution

Let us consider the swaying oscillation of an infinitely long and shallow cylindrical vessel floating on the water surface[9)12)].
The boundary condition may be written as

$$\frac{\partial}{\partial z}\phi(y,0)=-\frac{\partial}{\partial y}\psi(y,0)=-\frac{\partial}{\partial y}\zeta(y), \qquad \text{for} \quad |y|\leqq 1, \tag{3.1}$$

where $\zeta(y)$ means the offset of the cylinder, or integrating it,

$$\psi(y,0)=\zeta(y), \qquad \text{for} \quad |y|\leqq 1, \tag{3.2}$$

In this case, the function satisfying the condition (2.12) can be represented as

$$m(t)=\frac{1}{\pi i}\int_{-1}^{1}\frac{\sigma(y')}{t-y'}dy', \tag{3.3}$$

where σ is a real function.

Then, the wave-free potential f becomes

$$f(t)=\frac{K}{\pi i}\int_{-1}^{1}\frac{\sigma(y')}{t-y'}dy'-\frac{1}{\pi}\int_{-1}^{1}\frac{\sigma(y')dy'}{(t-y')^2}. \tag{3.4}$$

Moreover, the linearized pressure $p(y,z)$ defined as

$$\frac{i\omega}{\rho g}p(y,z)=-K\phi+\frac{\partial}{\partial z}\phi\equiv P(y,z), \tag{3.5}$$

is given as

$$P(y, z)=\mathrm{Re}\left\{K^2 m(t)+\frac{d^2}{dt^2}m(t)\right\}, \tag{3.6}$$

and especially

$$P(y, 0)=K^2\sigma(y)+\frac{d^2}{dy^2}\sigma(y), \tag{3.7}$$

where the boundary values are assumed as

$$\sigma(\pm 1)=\frac{d}{dy}\sigma(\pm 1)=0, \tag{3.8}$$

The offset of the cylinder is given by (3.2) as

$$\zeta(y)=\frac{K}{\pi}\int_{-1}^{1}\frac{\sigma(y')dy'}{y'-y}+\frac{d}{dy}\sigma(y). \tag{3.9}$$

Integrating (3.9) and putting the boundary condition (3.8), we have

$$\int_{-1}^{1}\zeta(y)dy=\frac{K}{\pi}\int_{-1}^{1}\sigma(y)\log\left|\frac{1+y}{1-y}\right|dy, \tag{3.10}$$

Then, if σ is an even function, this integral vanishes, and even if it does not, since K-value may be very small in the practical case, the displacement volume of the cylinder is very small.

For example, putting $y=-\cos\theta$ and

$$\sigma(y)=\frac{\sin(n-1)\theta}{n-1}-\frac{\sin(n+1)\theta}{n+1}, \qquad n\geqq 2 \tag{3.11}$$

the offset becomes

$$\zeta(y)=\sin n\theta+K\left\{\frac{\cos(n-1)\theta}{n-1}-\frac{\cos(n+1)\theta}{n+1}\right\}, \tag{3.12}$$

These curves are not so like as any section of ships clearly.

Lastly, it is very interesting to remember the wave-free potential of the uniform motion with constant unit speed.
In that case, the water surface condition is

$$Im\left\{Kf-i\frac{d}{dt}f\right\}=0, \qquad \text{for} \quad z=0 \tag{3.13}$$

and the wave-free potential is given, by the same function $m(t)$ as (2.12), in a manner that

$$f(t)=i\left\{Km(t)+i\frac{d}{dt}m(t)\right\}, \tag{3.14}$$

Then, the surface form becomes

$$\phi(y,0)=Re\left\{Km+i\frac{d}{dt}m\right\}, \tag{3.15}$$

and the pressure

$$p(y)/\rho=Re\left\{K^2m+\frac{d^2}{dt^2}m\right\}, \tag{3.16}$$

Comparing (3.13), (3.14), (3.15), (3.16) with (2.11), (2.13), (3.9), (3.6) respectively, if the same function $m(t)$ is taken up in both cases, it is easily found that the pressure becomes the same and the complex potential is conjugate with each other.[13][14]

4. Long Wave Approximation[15][16][17][18][19][20][21]

In two dimensional problem, the wave-length very much longer than the body dimension usually, that is, K is very small. When K is very small, the velocity potential near the body surface can be approximated by the one such that

$$\frac{\partial}{\partial z}\phi(y,0)=0, \tag{4.1}$$

that is the potential assuming the water surface as a rigid surface.

On the other hand, since the wave height is given as (A.25), expanding its exponential term, $H^+(K)$ will be nearly

$$H^+(K)\doteqdot\alpha+K\beta+O(K^2), \tag{4.2}$$

where

$$\alpha=\int_c\frac{\partial}{\partial n}\phi ds, \tag{4.3}$$

$$\beta=\int_c\left(\frac{\partial}{\partial n}\phi-\phi\frac{\partial}{\partial n}\right)(z+iy)ds. \tag{4.4}$$

Firstly, let us consider the heaving motion. The boundary condition is

$$\frac{\partial\phi}{\partial n}=-\frac{\partial z}{\partial n}, \tag{4.5}$$

Then, α and β is easily found to be

$$\alpha=-\int_c\frac{\partial z}{\partial n}ds=B, \tag{4.6}$$

where B means the breadth of the cylinder at the water line.

$$\beta=-\int_c (z+\phi)\frac{\partial z}{\partial n}ds=-(1+k_z)A\,, \tag{4.7}$$

where A means the displaced area and k_z the added mass coefficient of the cylinder, and also the symmetry of the curve C is assumed throughout this paper.
Collecting these results, H^+ becomes

$$H^+(K)\doteqdot B-K(1+k_z)A\,, \tag{4.8}$$

Hence, the wave-free condition is

$$B\doteqdot K(1+k_z)A\,, \tag{4.9}$$

Here, it must be remembered that k_z is logarithmically infinite for $K\to 0$,[15] so that its value may be taken for a given K-value.
Motora[22] obtained nearly the same conclusion as this, considering the integral (A.25) more physically,[21] and one of his wave-free cylinder is very much like the one shown in Fig. 2.

Secondly, let us consider the swaying motion. The boundary condition is

$$\frac{\partial\phi}{\partial n}=-\frac{\partial y}{\partial n}\,, \tag{4.10}$$

and

$$\alpha=-\int_c \frac{\partial y}{\partial n}ds=0\,, \tag{4.11}$$

$$\beta=-i\int_c (y+\phi)\frac{\partial y}{\partial n}ds=-i(1+k_y)A\,, \tag{4.12}$$

where k_y means the added mass coefficient for the swaying.

Namely,

$$H^+(K)\doteqdot -iK(1+k_y)A\,, \tag{4.13}$$

and the swaying motion can not be wave-free to this order of the approximation.

Lastly, for the rolling motion, the boundary condition is to be

$$\frac{\partial}{\partial n}\phi=y\frac{\partial z}{\partial n}-z\frac{\partial y}{\partial n}\,, \tag{4.14}$$

Then,

$$\alpha=0, \tag{4.15}$$

$$\beta=-iA(\overrightarrow{OM}+k_y\overrightarrow{OC})\,, \tag{4.16}$$

where M means the metacenter and C the center of the added mass, that is,

$$\overrightarrow{OM}A=(\overrightarrow{OB}-\overrightarrow{MB})A=\int_c y\left(z\frac{\partial y}{\partial n}-y\frac{\partial z}{\partial n}\right)ds\ , \tag{4.17}$$

$$k_y\overrightarrow{OC}A=\int_c \phi_1\left(z\frac{\partial y}{\partial n}-y\frac{\partial z}{\partial n}\right)ds\ , \tag{4.18}$$

where ϕ_1 is the potential of the swaying.[18)20)]
Namely, for the rolling motion,

$$H^+(K)\doteqdot -iKA(\overrightarrow{OM}+k_y\overrightarrow{OC})\ , \tag{4.19}$$

so that the wave-free condition may be

$$\overrightarrow{OM}/\overrightarrow{OC}=-k_y\ , \tag{4.20}$$

which means that each of the metacenter and the center of the added mass lies at the other side of the origin with the ratio k_y.

Ursell[15)] found one of such cylinder, but Hishida[17)] found that there is always a point around which the rolling motion has no wave.
In fact, since H^+ function of such motion about a point P can be given as

$$H^+(K)\doteqdot -iKA[(\overrightarrow{OM}+k_y\overrightarrow{OC})+\overrightarrow{OP}(1+k_y)] \tag{4 21}$$

the wave-free condition is

$$\overrightarrow{OP}=-\frac{\overrightarrow{OM}+k_y\overrightarrow{OC}}{(1+k_y)}\ , \tag{4.22}$$

then such point will be always found.[23)]

5. Slender Thin Ship

In the three dimensional problem, the analysis becomes complicated and also it is not always the case that the wave length is very large compared with the ship dimension. Thus, we confine ourselves with the so-called slender thin ship. In this type, we mean the ship which is repersented by the singularity distribution over the $x-z$ plane. Let us consider any singularity distribution $\mu(x, z)$ over this plane. Then, the wave produced will be given by a constant multiple of the function:

$$H(K,\theta)=\int_{-1}^{1}\int_{-d}^{0}\mu(x,z)e^{Kz+iKx\cos\theta}dxdz\ , \tag{5.1}$$

Hence, introducing an auxiliary function $\sigma(x, z)$ as

$$\mu(x, z)=\frac{d}{dz}\sigma(x, z)+K\sigma(x, z)\ , \tag{5.2}$$

with the boundary condition

$$\sigma(x, -d)=0\ , \tag{5.3}$$

and putting this into the above and integrating by parts, we have

$$H(K, \theta)=\int_{-1}^{1}\sigma(x, 0)e^{iKx\cos\theta}dx\ , \tag{5.4}$$

This is the same form as of the slender ship theory,[24] the difference is that $\sigma=0$ does not mean the zero-displacement, although the wave-free condition is

$$\sigma(x, 0)=0\ . \tag{5.5}$$

For example, let

$$\sigma(x, z)=-m(x)z(z+d)\ , \tag{5.6}$$

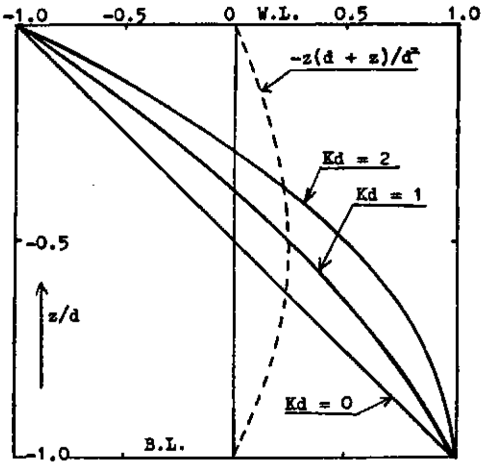

Fig. 4. Draftwise distribution of wave-free singularity

with an arbitrary function $m(x)$, then $\mu(x, z)$ may be calculated as shown in Fig. 4. From these figures, we may easily imagine that the section forms of the ship may be very similar as the forms of Fig. 1 and 2.

Now, the wave-free ship form has no wave-damping obviously, for there is no energy dissipation by the wave. Namely, there is neither wave-exciting force nor the wave-damping, so that the free motion among the wave with no viscosity is indefinite and in fact divergent because the motion is proportional to the exciting force over the damping, but the damping is the square of that force.

Here is a question whether it may be possible to make zero the exciting force but not the damping. This is easily answered by putting.

$$\sigma(x, 0)=\frac{d^2}{dx^2}a(x)+K^2\cos^2\alpha\ a(x)\ , \tag{5.7}$$

with

$$a(\pm 1)=\frac{d}{dx}a(\pm 1)=0\ , \tag{5.8}$$

because, putting these into (5.4) and integrating by parts, we have

$$H(K, \alpha)=0, \quad \text{but} \quad H(K, \theta)\neq 0 \quad \text{for} \quad \theta\neq\alpha \quad \text{or} \quad \alpha+\pi\,, \tag{5.9}$$

and this means that the exiciting force by the wave coming from the direction α is zero but the damping is not.

For example, taking the slender ship approximation,[24] let us consider

$$\sigma(x, 0) \fallingdotseq A(x)\,, \tag{5.10}$$

for the heaving motion, where $A(x)$ means the sectional area curve, and

$$\sigma(x, 0) \fallingdotseq xA(x)\,, \tag{5.11}$$

for the pitching motion.

Let us consider as $\alpha=0$ and also for the simplicity put

$$a(x)=(1-x^2)^2\,, \quad \text{for the heaving,} \tag{5.12}$$

$$a(x)=x(1-x^2)^2\,, \quad \text{for the pitching.} \tag{5.13}$$

After the operation of (5.7), we have the sectional area curve as shown in Fig. 5.

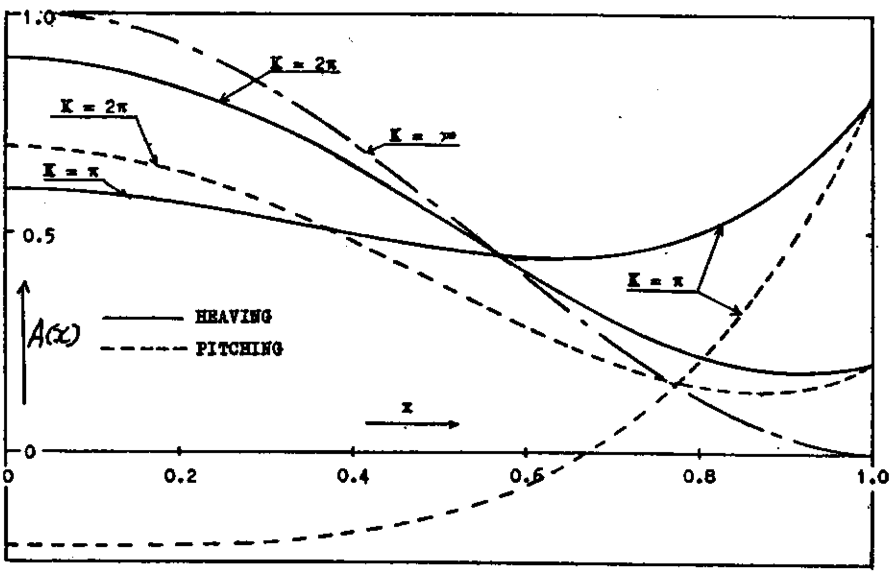

Fig. 5. Sectional area curve

As seen from the figures and the formula (5.7), these sectional area curve are the same as those of the uniform motion with the constant speed for which the transverse wave vanishes, if both wave-length is the same.[4)13)]

6. Summary

There are ship forms having no radiating wave when they oscillate without advance speed. Then they receive no exciting force from the incident wave by Haskind-Hanaoka's formula.

To obtain such ship forms, there are following four ways:

i) To deduce from the wave-free potential introduced by the use of the adjoint differential operator of the water surface condition.

ii) To obtain the wave-free condition by the approximate wave-height formula when the wave-length is very large.

iii) To consider the wave-free singularity distribution, for example, about the slender thin ship.

iv) To cancell out the waves by their interference. This is not explained in the paper, but it will be obvious.

Thence, the two-dimensional wave-free ship form

i) exists for the heaving motion and has large bulges,

ii) practically does not exist for the swaying motion,

iii) exists for the rolling motion (rotatory oscillation about an axis) but we can say more precisely that there exists always a point the motion about which produces no wave. Such ship forms does not experience the wave-exciting force but also has not the wave-damping, so that the motion among the wave may become very large.

In the three dimensional problem, the result may be like the above, which is confirmed principally by the slender thin ship theory. In this case, however, there is another type of the wave-free ship form which produces no wave in one direction but does not other direction so that the wave-damping may not vanish.

The ship motion among waves may be estimated fairly precisely by the present knowledge of the theory, but the dependence of the motion upon the ship form is not always sufficient because of the various difficulties. In this circumstances, such study as developed here may be a method to attack these difficulties.

Acknowlegement:

The author wishes to thank Prof. Motora who advised him to study this problem and gave him all kinds of help. Prof. Motora had a like idea independently and verified it experimentally.[22]

Appendix: Green Function and Haskind-Hanaoka's Formula.

Unit source potentials, call it fundamental singularities, are

$$4\pi S^{(3)}(P, Q)=\frac{1}{r_1}-\frac{1}{r_2}+\lim_{\mu\to+0}\frac{1}{\pi}\int_{-\pi}^{\pi}\int_0^{\infty}\frac{ke^{-k(z+z')+ik(x-x')\cos\theta+ik(y-y')\sin\theta}}{k-K+\mu i}dk\,, \tag{A.1}$$

$$2\pi S^{(2)}(P, Q)=\log\left(\frac{r_2}{r_1}\right)+2\lim_{\mu\to+0}\int_0^\infty \frac{e^{-k(z+z')}\cos(y-y')}{k-K+\mu i}dk\,, \tag{A.2}$$

where

$$P\equiv(x, y, z) \text{ or } (y, z),\ Q\equiv(x', y', z') \text{ or } (y', z')\,,$$
$$r_1=\overline{PQ},\ r_2=\overline{P\bar{Q}},\ \bar{Q}\equiv(x', y', -z') \text{ or } (y', -z')\,.$$

and the subsciript (2) or (3) on the shoulder of S means that it stands for the two-or three-dimensional one respectively.

Necessary Green functions to solve the present problems, Neumann functions, are such that

$$N(P, Q)=S(P, Q)+A(P, Q)\,, \tag{A.3}$$

where $A(P, Q)$ is regular in the water, and

$$\frac{\partial}{\partial n}N(P, Q)=0 \text{ or } \frac{\partial}{\partial n}(S+A)=0, \text{ on } C\,, \tag{A.4}$$

where n means the outward normal of the wetted boundary C of the body. Moreover, since

$$S(P, Q)=S(Q, P)\,, \tag{A.5}$$

it will be also assumed that

$$N(P, Q)=N(Q, P)\,, \tag{A.6}$$

By this Neumann function the potential $\phi(P)$ regular in the water can be represented as follows:

$$\phi(P)=-\int_c \frac{\partial\phi}{\partial n_Q}N(P, Q)ds_Q\,, \tag{A.7}$$

Especially, $A(P, Q)$ of (A.3) may be

$$A(P, R)=\int_c\left\{\frac{\partial}{\partial n_Q}S(P, Q)\right\}N(Q, R)ds_Q\,, \tag{A.8}$$

If the point P lies infinitely far from the body but near the water surface, since $S(P, Q)$ becomes approximately

$$S^{(2)}(P, Q)\to -ie^{-K(z+z')-iK|y-y'|}\,, \tag{A.9}$$

$$S^{(3)}(P, Q)\to\sqrt{\frac{K}{2\pi i\tilde{\omega}}}\exp\{-K(z+z')-iK\tilde{\omega}+iK(x'\cos\theta+y'\sin\theta)\} \tag{A.10}$$

where

$$\tilde{\omega}^2=x^2+y^2,\ x=\tilde{\omega}\cos\theta \quad\text{and}\quad y=\tilde{\omega}\sin\theta\,,$$

$A(P, R)$ will become

$$A^{(2)}(P, R) \to -ie^{-Kz \pm iKy}\phi_d^{\pm}(R)\,, \qquad \text{for} \quad y \gtrless 0\,, \tag{A.11}$$

where

$$\phi_d^{\pm}(y, z) = \int_c \left\{\frac{\partial}{\partial n_Q} e^{-Kz' \pm iKy'}\right\} N(P, Q) ds_Q\,, \tag{A.12}$$

and

$$A^{(3)}(P, R) \to \sqrt{\frac{K}{2\pi i \tilde{\omega}}} \exp\{-Kz - iK\tilde{\omega}\}\phi_d(R, \theta)\,, \tag{A.13}$$

where

$$\phi_d(x, y, z; \theta) = \int_c \left\{\frac{\partial}{\partial n_Q} e^{-Kz' + iK(x'\cos\theta + y'\sin\theta)}\right\} N(P, Q) ds_Q\,, \tag{A.14}$$

From the property of Neumann function, it is clear that

$$\frac{\partial}{\partial n}\phi_d = -\frac{\partial}{\partial n}\phi_0\,, \tag{A.15}$$

where

$$\phi_0^{\pm}(P) = e^{-Kz \pm iKy}\,, \tag{A.16}$$

$$\phi_0(P; \theta) = e^{-Kz + iK(x\cos\theta + y\sin\theta)} \tag{A.17}$$

Since exp. $(-Kz \pm iKy)$ is a regular wave advancing to the positive (under sign) or negative (upper sign) direction and exp. $\{-kz + ik(x\cos\theta + y\sin\theta)\}$ is also a regular plane wave coming from the direction θ, these are all diffraction potentials.

Adding (A.11), (A.13) to (A.9), (A.10), the asymptotic formula of Neumann function will be

$$N^{(2)}(P, Q) \to -i\phi_0^{\pm}(P)\{\phi_0^{\mp}(Q) + \phi_d^{\mp}(Q)\}\,, \qquad \text{for} \quad y \gtrless 0\,, \tag{A.18}$$

$$N^{(3)}(P, Q) \to \sqrt{\frac{K}{2\pi i \tilde{\omega}}} \exp\{-Kz - iK\tilde{\omega}\}\{\phi_0(Q; \theta) + \phi_d(Q; \theta)\}\,, \tag{A.19}$$

Hence, putting these into (A.7), any potential tends to

$$\phi^{(2)} \to ie^{-Kz \mp iKy}H^{\pm}(K) \qquad \text{for} \quad y \gtrless 0\,, \tag{A.20}$$

$$H^{\pm}(K) = \int_c (\phi_0^{\mp} + \phi_d^{\mp})\frac{\partial \phi}{\partial n} ds\,, \tag{A.21}$$

$$\phi^{(3)} \to -\sqrt{\frac{K}{2\pi i \tilde{\omega}}} \exp\{-Kz - iK\tilde{\omega}\} H(K, \pi + \theta)\,, \tag{A.22}$$

$$H(K,\theta)=\int_c \{\phi_0(\theta-\pi)+\phi_d(\theta-\pi)\}\frac{\partial\phi}{\partial n}ds\ , \tag{A.23}$$

Clearly, the function H is a constant multiple of the diverging wave-height. On the other hand, since

$$\int_c\left(\phi_d\frac{\partial\phi}{\partial n}-\phi\frac{\partial}{\partial n}\phi_d\right)ds=0\ , \tag{A.24}$$

the H-function can be written also as

$$H(K,\theta)=\int_c\left(\phi_0\frac{\partial\phi}{\partial n}-\phi\frac{\partial}{\partial n}\phi_0\right)ds\ , \tag{A.25}$$

and this is a usual representation. Now, (A.21) and (A.23) are Haskind-Hanaoka's formulas, that is, the wave-height produced by the body-motion is proportional to the exciting force or moment of the incident wave because the pressure is proportional to the potential.

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