50

No. 4 Paper

Wave-Free Potential and Wave-Making Resistance

By

Masatoshi Bessho

1. Introduction

One of the features difficult to understand on the wave-making resistance is that it seems to be no one-to-one correspondence between it and the ship form. Namely, here is a ship and she has a value of the wave-making resistance at some speed, then many other ships with different shape may have the same one value of the resistance at the same speed. There are many evidences for this phenomena in the experiments and experiences. No one suggests, however, explicitly this principle because it contradicts against the law that the nature does not jump.

One has research and find the most important factors governing the wave-making resistance, but if this principle has once established and it has been clarified how deformation of ship forms does not change the wave-making resistance then, this task will become very much easier than ever. Such potential that does not affect the wave-making character or has no trailing wave is called wave-free and its singularity distribution also by the same word.

In this paper, the theory is explained in thin ship represented by doublet distribution and compared with the principle of D.W.Taylor's experimental research in residual resistance.

2. Wave-making resistance 2),3),4),5),6)

Let us cousider a ship with length L and draft d advancing with uniform velocity V and its half breadth $\eta(x,z)$, where the x-axis is directed to the up-stream, y-axis athwartships, z-axis vertically upwards and the origin is taken at the midship on the mean water level.

Then the wave-making resesistance R of the ship is given by the formula

$$R=\frac{4\rho g^4}{\pi V^6}\int_0^{\pi/2}\left|F(K_o\ \sec^2\theta,\theta)\right|^2 \sec^5\theta d\theta , \qquad (2.1)$$

where ρ means the water density, g the gravity constant and

$$F(k,\ \theta)=\int_{-L/2}^{L/2}dx\int_{-d}^{0}\eta(x,z)e^{kz-ikx\cos\theta}dz \quad ,\ k=K_0\sec^2\ \theta, \quad (2,2)$$

the so-called amplitude function or Kotchin's function.

It is clear from this formula that the wave-making resistance is non-negative, that is, that the resistance vanishes is equivalent to that its Kotchin's function vanishes. Kotchin's function is linear with respect to the half breath so that we may add, for example, two distributions $\eta_1(x,z)$ and $\eta_2(x,z)$ as follows;

$$F=F_1+F_2\ ,\quad F_j=\iint\eta_j(x,z)e^{kz-ikx\cos\theta}dxdz, \qquad j=1,2\ , \qquad (2,3)$$

Then, the resistance will be described as

$$R=R_1+R_2+R_{1,2} \qquad (2.4)$$

where

$$R_j = \frac{4\rho g^4}{\pi V^6}\int_0^{\pi/2} |F_j|^2 \sec^5\theta \, d\theta \,, \qquad j=1,2,$$

and

$$R_{1,2} = \frac{4\rho g^4}{\pi V^6}\int_0^{\pi/2} (F_1 \overline{F_2} + \overline{F_1} F_2)\sec^5 \theta \, d\theta \,, \qquad (2.5)$$

where the bar over the letter means the complex conjugate to be taken.

By the way, if η_2 is very small and confined in very neighbourhood of the point (x,z), its Kotchin's function can be represented as

$$F_2 = \eta_2(x,z) e^{kz - ikx\cos\theta} \Delta x \Delta z \,, \qquad (2.6)$$

and R_2 can be neglected, then

$$R \fallingdotseq R_1 + R_{1,2} \,, \qquad (2.7)$$

and

$$R_{1,2} \fallingdotseq G_1(x,z)\left[2\eta_2(x,z)\Delta x \Delta z\right] \,, \qquad (2.8)$$

where

$$G_1(x,z) = \frac{2\rho g^4}{\pi V^6}\int_0^{\pi/2} (F_1 e^{kz + ikx\cos\theta} + \overline{F_1} e^{kz - ikx\cos\theta})\sec^5 \theta \, d\theta \,, \qquad (2.9)$$

is called the influence function and means the resistance variation when the unit displacement is added at the point (x,z). Making use of this function, the resistance formula can be written as

$$R = 2\iint G(x,z)\,\eta(x,z)\,dx\,dz \,, \qquad (2.10)$$

Thus, vanishing of resistance also means vanishing of the influence function. From these considerations it is neces- and sufficient for vanishing the resistance that its Kotchin's function or influence function vanishes.

Moreover, if there is some distribution which vanishes its Kotchin's function, such a distribution is called wave free and may be added to or subtracted from any distribution without any change of its wave-making resistance as we may see easily from (2.4) and (2.5).

3. Derivation of wave-free distribution[2),5)]

In the representation of Kotchin's function (2.2), its kernel satisfies the partial differential equation,

$$\left(\frac{\partial}{\partial z} + \frac{1}{K_0}\frac{\partial^2}{\partial x^2}\right) e^{kz - ikx\cos\theta} = 0 , \qquad (3.1)$$

where it should be remembered that this operator is the same as of the water surface condition.

Thence, if $\eta(x,z)$ can be represented by a new function $\sigma(x,z)$ as the following equation adjoint to the above,

$$\left(\frac{\partial}{\partial z} - \frac{1}{K_0}\frac{\partial^2}{\partial x^2}\right) \sigma(x,z) = \eta(x,z) , \qquad (3.2)$$

Kotchin's function can be represented by the boundary value of σ as follows;

$$F = \int_{-l/2}^{l/2} \left(\sigma(x,z) e^{kz} \right)_{z=-d}^{0} e^{-ikx} dx$$

$$- \frac{1}{K_0}\int_{-d}^{0} \left(\left\{ \frac{\partial \sigma}{\partial x}(x,z) + ik\cos\theta\, \sigma(x,z) \right\} e^{-ikx} \right)_{x=-l/2}^{l/2} e^{kz} dz , \qquad (3.3)$$

On the other hand, the inhomogeneous differential equation (3.2) is of heat conduction and η stands for heat source, σ temperature, $1/K_0$ coefficient of heat conduction and τ the time. Hence, σ may be determined uniquely except on the boundary and initial conditions, say,

$$\left.\begin{array}{ll} \sigma(x,-d)=0, & \text{for } L/2 > x > -L/2, \\ \sigma(\pm L/2, z)=0, & \text{for } 0 > Z > -d \,. \end{array}\right\} \qquad (3.4)$$

Then, the formula (3.3) may be simplified as [7]

$$F=\int_{-L/2}^{L/2} \sigma(x,0)\, e^{-ikx}\, dx - i \sec\theta \int_{-d}^{0} \left(\sigma(x,z)\, e^{-ikx} \right)_{x=-L/2}^{L/2} e^{kz}\, dz, \qquad (3.5)$$

Thus, only the peripheral value of σ determines Kotchin's function, and if

$$\begin{array}{ll} \sigma(x,0)=0 & \text{for } L/2 > x > -L/2 \,, \\ \sigma(\pm L/2, z)=0 & \text{for } 0 > z > -d \,, \end{array} \qquad (3.6)$$

then F vanishes and we have a wave-free distribution. To solve the equation (3.2) for given η is not a simple calcuation, but to calculate η for given σ is simple.

The formula (3.5) facilitates our research into the wave-making resistance, because it needs only the value on the boundary lines instead of the value over the area. But there remains the wave-free distribution undetermined.

4. Property of wave-free distribution [4),5),6)]

Let us assume that σ may be represented as

$$\sigma(x,z) = \alpha \frac{Bd}{2} X(x')Z(z') , \qquad (4.1)$$

where B means the breadth of ship, α an arbitrary constant,

$x'=2x/L$ and $z'=1+z/d$,

Putting this into (3.2), we have

$$\eta(x,z)= \alpha \frac{B}{2}\left\{X(x')Z'(z') - \frac{4d}{L} F_r^2 X''(x')Z(z')\right\}, \qquad (4.2)$$

where $F_r = V/\sqrt{gL}$.

If X and Z satisfy the conditions

$$X(\pm 1)=X'(\pm 1)=Z(0)=Z(1)=0 , \qquad (4.3)$$

η is wave-free.

Hence, when we add it to or subtract from some distribution, the wave-making resistance does not change. Such deformation of ship forms may be called the invariant deformation with respect to the wave-making resistance.

The invariant deformation has the following properties; Firstly, integrating (4.2) in z, the change of sectional area curve $A_S(x')$ becomes by (4.3)

$$A_s(x')=\int_{-d}^{0}\eta(x,z)dz=-2\alpha\left(\frac{Bd^2}{L} F_r^2\right)X''(x')\int_0^1 Z(z')dz', \qquad (4.4)$$

This change of the sectional area curve is very small in practical applications and only appreciable near extremities. Secondly, integrating (4.4) once more in x, we have by (4.3)

$$\int_{-1}^{1} As(x')dx' = 0, \qquad (4.5)$$

that is, the displacement volume does not change.

Thirdly, integrating (4.4) multiplied by x, we have by (4.3)

$$\int_{-1}^{1} x' As(x')dx' = 0 \quad , \qquad (4.6)$$

that is, the moment of the sectional area curve or the center of buoyancy does not change.

Fourthly, integrating (4.2) in x, the change of water-plane area A_W becomes

$$A_W(z') = \alpha \frac{BL}{4} Z'(z') \int_{-1}^{1} X(x')dx' \ , \qquad (4.7)$$

Lastly, the change of moment of the water plane becomes

$$M_W(z') = \alpha \frac{BL^2}{8} Z'(z') \int_{-1}^{1} x'X(x')dx' \ , \qquad (4.8)$$

Two formulae mean that the displaced volume may be removed vertically without sacrifice of the wave-making resistance.

5. Examples [4),5),6)]

Practical examples are shown in Fig. 2,3 and 4. Fig. 2 and 3 are of broadening of the breadth. The change of the sectional area curve is very slight as we see in these figures and it seems practically to be the same as the experimental principle that the residual resistance is determined almost always by the sectional area curve, we may call it the sectional area curve hypothesis hereafter. Of course, there is a slight change of the sectional area curve, and saying in one word, the volume near bottom of midship removes towards fore and aft ends and swells out there. Fig. 4 shows this tendency exaggerated. The bulbous bow ship form may be equivalent to an extreme U-frame form.

The examples of Fig. 2 and 3 are tested in the model basin and confirmed the theoretical prediction to be appropriate. The example of Fig. 4 is not tested but shows the bulbous bow of such size may not have a merit in the wave-making resistance. (It does not mean that the resistance does not reduce, because the recent research shows the viscous resistance reducing effect of the bulbous bow form.[8])

The similar result as this, that is, the different size and form of bulbous bow has the same residual resistance, is appeared in Taylor's text, which shows his famous researches on the bulbous bow.[1]

6. Conclusion

A general method to obtain wave-free singularity distributions is derived from the formula of the wave-making resistance of the thin ship theory and applied to deform a ship form keeping its wave-making resistance unchanged (invariant deformation).

There is no change of the displacement volume and no removal of the longitudinal center of buoyancy but may be a slight change of the sectional area curve and vertical removal of the displacement volume by such a deformation.

Examining numerical examples, we know the invariant deformation may be understood as a theoretical interpretation of the experimental principle that ships with the same sectional area curve have nearly the same residual resistance.

The present principle, instead of experience, is clear and explains in detail including small change in extremities, so that it may be very usefull for the ship-lines planning and the models deformation planning of the series model testing.

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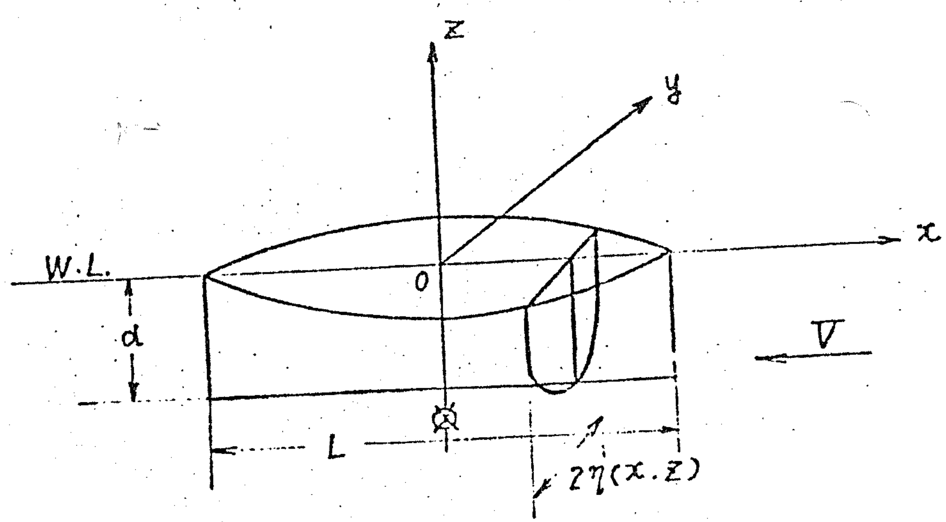

FIG. 1 CO-ORDINATE SYSTEM.

FIG. 3

MODEL	MARK	Δ kg.	S m^2	L_{WL} m	B m	d m	C_b	L/B	L_{WL}/d
PARENT	——	118.75	1.7847	4.000	0.4174	0.1392	0.5111	9.58	28.75
DEF MD	----	118.90	1.8768	4.000	0.4972	0.1392	0.4296	8.05	28.75

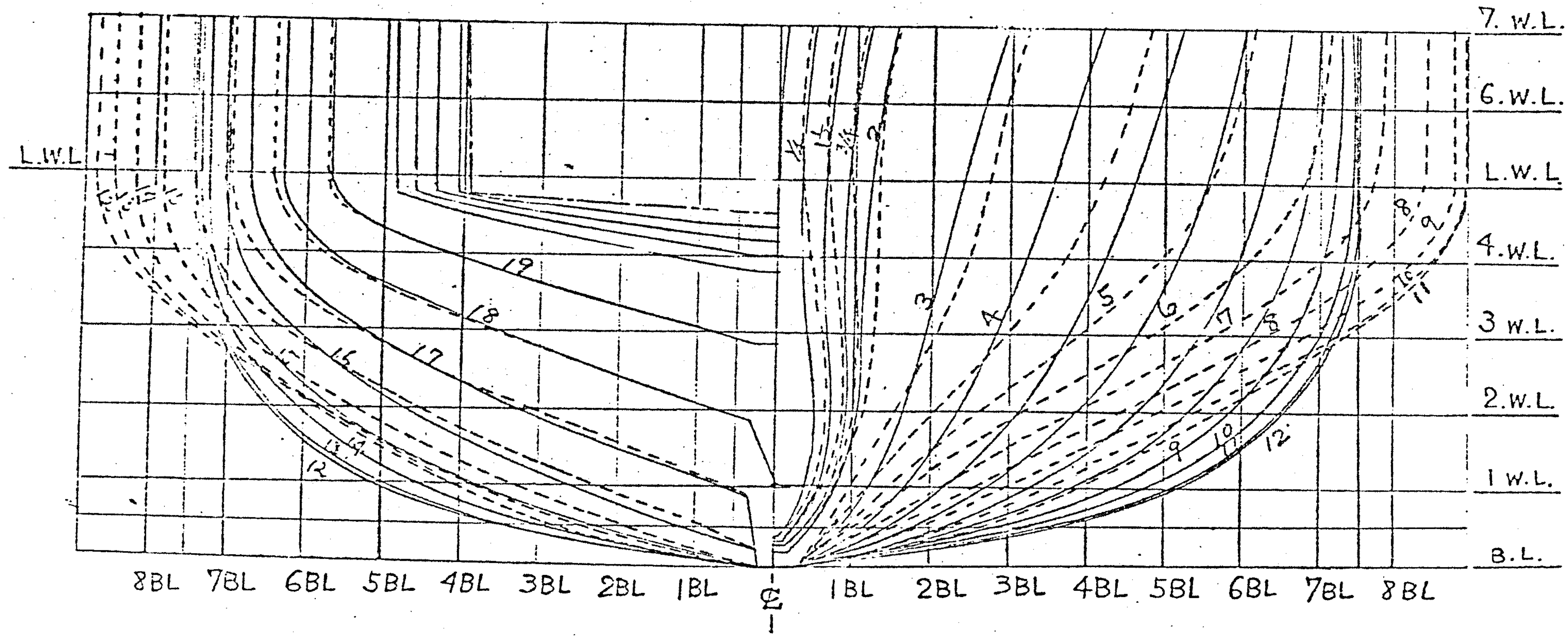

FIG. 2

MODEL	MARK	Δ kg	S m²	$L_{P.P.}$ m	B m	d m	Cb	L/B	L/d	$\nabla/(\frac{L}{10})^3$
PARENT	——	35.15	.6714	1.8	.2452	.0995	.800	7.342	18.08	6.027
DEF MD.	– – –	〃	.6536	1.8	.2941	〃	.533	6.120	〃	〃

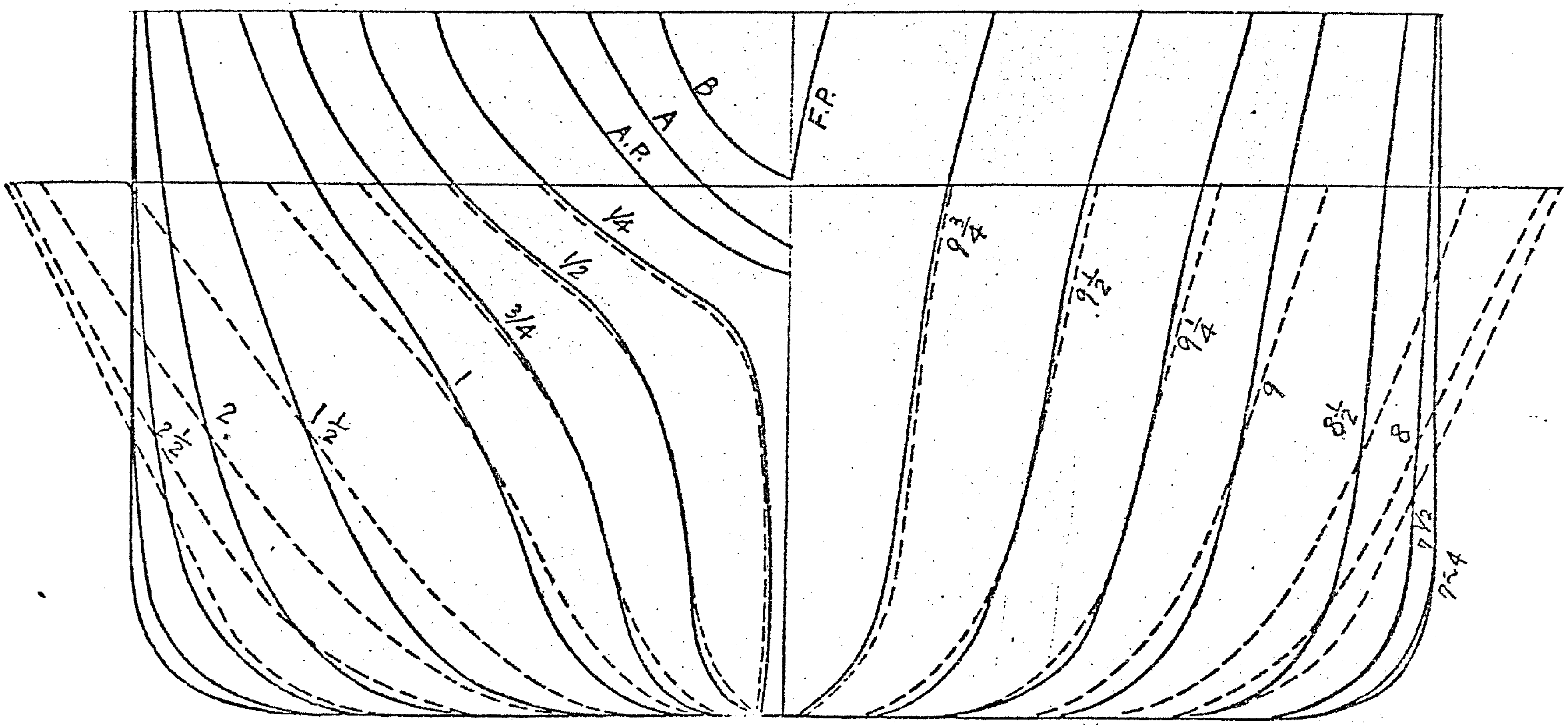

FIG. 4

M.S. No 1342

$C_b = .80$

$L/B = 7.2$

$B/d = 2.46$

$L/d' = 17.7$