SESSION I - 3

LINE INTEGRAL, UNIQUENESS AND DIFFRACTION OF WAVE IN THE LINEARIZED THEORY

M. *Bessho*

National Defense Academy

Japan

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1. Introduction

There are two opinions with regard to the contribution of the so-called line integral associate with the surface-piercing ship to its wave-making resistance. The one is that it is of higher order and must be neglected in the linearized theory [23, 36]. The other is that it contributes very much to the wave-making resistance and must be put to the account even in the linearized theory [3,16]. The former contains a hypothesis that the change of the wave-making resistance may be little if the change of the velocity potential is little in higher order. But this hypothesis is not always justified because the wave-making resistance has neither lower limit nor upper limit and changes very sensibly with the given ship form. Thus, the true question of this problem may be asked such as in which case the line integral will be neglected or not.

The other difficulty with the existence of the line integral is the presentation of our boundary value problem. That is whether our usual boundary value problem is unique or not. The same question appears also in the theory of a planing boat and in that case the solution satisfying the boundary condition on her wetted surface is not always a correct one, because the water surface elevation does not lie on her surface in general, so that the wetted surface can not be determined a priori [26,30,38].

A way to detour this difficulty is to introduce an eigen solution which represents her little rise or sinkage and has a homogeneous boundary value. In other words, we imagine a submerged thin wing with infinitely little depth and solve the boundary value problem then we can compute the water surface elevation on the top of the wing. If the water surface coincides precisely with the lower surface of the wing by adjusting the immersion and this is a planing-surface [7,3]. The same procedure may be traced in our case to obtain a surface-piercing ship.

The last theme of this paper is the diffraction of wave which is radiated by the ship herself. It is of course a term of higher order but must be important at low speed or the diverging wave where the wave-length is very small. In spite of the importance in the above reason and in the theory of oscillating ship, there are few researches in this problem with respect to the theory of wave-making resistance. Hence, the aim of that paragraph is to show similar formulae in our case as in the case of oscillation problems and obtain deeper understandings into the phenomena of diffraction of wave.

2. On the line integral

Let us imagine a ship floating on still water surface and let her wetted surface be S and the free surface of water be F. And then, let us push the ship as far as she advances at a constant unit speed, where we imagine her sinkage and trim is not permitted, and her wetted surface changes to S* and free water surface to F* and the water motion has a velocity potential $\phi(P)$, $P\equiv(x,y,z)$.

The co-ordinate system will be taken as in Fig.1 and the ship has a symmetry with respect to the right and left.

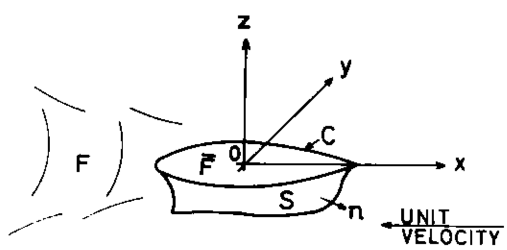

FIG1. CO-ORDINATE SYSTEM

Here, let us introduce a fundamental singularity as

$$S(P,Q)=\frac{1}{4\pi r(P,Q)}+A(P,Q) \quad , \tag{1}$$

where $P\equiv(x,y,z)$, $Q\equiv(x',y',z')$ and $r=\overline{PQ}$, and $A(P,Q)$ be regular in the whole water domain.
Then, under the usual assumption on the properties at infinity, we have the representation

$$\phi(P)=\iint_{S^*+F^*}\{\phi(Q)\frac{\partial}{\partial n}S(P,Q)$$

$$-\frac{\partial\phi}{\partial n}S(P,Q)\}dS(Q) \quad , \tag{2}$$

by Green's theorem.

In the linearized theory, the condition on the water surface F^* is approximated as

$$(\frac{\partial^2}{\partial x^2}+g\frac{\partial}{\partial z})\phi(x,y,o)=o \text{ on } F \quad , \tag{3}$$

where g is the gravity constant in our unit system and the surface elevation ζ given

$$\zeta(x,y)=-\frac{1}{g}\frac{\partial}{\partial x}\phi(x,y,o) \quad . \tag{4}$$

Moreover, the regular part of S(P,Q) is written down, as well known, as

$$A(P,Q)=\frac{-1}{4\pi^2}\lim_{\mu\to+o}\int_{-\pi}^{\pi}\int_{o}^{\infty}\frac{(k\cos^2\theta+g)e^{k(z+z')+ik(\bar{w}-\bar{w}')}}{k\cos^2\theta-g+\mu i\cos\theta}dkd\theta \quad , \tag{5}$$

where $\bar{w}=x\cos\theta+y\sin\theta$ and $\bar{w}'=x'\cos\theta+y'\sin\theta$.
Then, the integral over F^* in the formula (2) becomes by partial integration

$$\iint_{F^*}[\phi\frac{\partial}{\partial n}S-S\frac{\partial\phi}{\partial n}]dS\fallingdotseq-\iint_F[\phi\frac{\partial}{\partial z}S-S\frac{\partial\phi}{\partial z}]dx'dy'$$

$$=\frac{1}{g}\int_C[\phi\frac{\partial}{\partial x'}S-\frac{\partial\phi}{\partial x'}S]dy' \quad , \tag{6}$$

where C is a cross curve of S with F [3,4, 36, 39]. This is the so-called line integral which is considered in this section.

Before entering the discussion, we may remember the case of a flat ship or a planing surface. In that case, there is no difficulty such as line integral, a consistent linearized theory exists and the theory agrees well with the experiment [19].
Then, introducing the linearized pressure as

$$\frac{1}{\rho}p(x,y)=-\frac{\partial}{\partial x}\phi(x,y,o)-g\zeta(x,y) \quad , \tag{7}$$

where ρ is the density of water, and assuming the continuity of the surface elevation, we have by partial integration from the formula(2)

$$\phi(P)=\frac{1}{\rho g}\iint_S p(x',y')\frac{\partial}{\partial x'}S(P;x',y',o)dx'dy' \quad . \tag{8}$$

This process will be suggestive to the following argument.
Now, at first, let us consider the total sum of the line source of the second term in the right hand side of the equation (6), it becomes by (3) and (4)

$$-\frac{1}{g}\int\frac{\partial\phi}{\partial x}dy=\int_C\zeta(x,y)dy=-\iint_F\frac{\partial}{\partial z}\phi dxdy \quad , \tag{9}$$

thence this is the outward flux from F in a unit time. This should not be finite but does not vanish in reality [3,5] and cancells out with the flux from the ship surface as follows.
The flux from S^* is

$$-\iint_{S^*}\frac{\partial\phi}{\partial n}dS=\iint_{S^*}\frac{\partial x}{\partial n}dS \quad , \tag{10}$$

because

$$\frac{\partial\phi}{\partial n}=-\frac{\partial x}{\partial n} \text{ on } S^* \quad . \tag{11}$$

If we integrate this normal velocity over S and the flux becomes zero, but S^* does not coincides with S and then it does not zero. This difference of the flux from between S and S^* will be

$$\frac{\partial x}{\partial n}dS\fallingdotseq\zeta(x,y)dy \quad , \tag{12}$$

if the ship surface cuts the water surface vertically.
Thus the flux from the ship surface (12) cancells out each other with the flux (9) from the water surface [3,36,39, Appendix C]. Therefore, neglecting more higher order terms arising from the change of the wetted surface, we may have a new representation of the velocity potential.

$$\phi(P)=\phi_S(P)+\phi_L(P) \quad , \tag{13}$$

$$\phi_S(P)=\iint_S[\phi(Q)\frac{\partial}{\partial n}S(P,Q)-S(P,Q)\frac{\partial}{\partial n}\phi]dS(Q) \quad , \tag{14}$$

$$\phi_L(P)=\frac{1}{g}\int_C\phi(Q)\frac{\partial}{\partial x'}S(P;x',y',o)dy' \quad , \tag{15}$$

and the latter is a proposed line integral term [3].
It is well known at high speed that

$$\phi(P)\xrightarrow[g\to o]{} o(g) \quad , \tag{16}$$

and for a thin ship

$$\phi_S(P)=O(B/L) \quad , \qquad (17)$$

then we have

$$\phi_L(P) \xrightarrow[g\to o]{} o(g)O(B/L) \quad , \qquad (18)$$

which shows that the line integral does not very much contribute to the water motion at high speed and this conclusion may be true compared with the experimental results.

However, the situation changes drastically at low speed or the practical range of speed of displacement ships and the contribution of the line integral term to the wave-making resistance becomes so large that the resistance curve might change its asymptotic character in low speed [2,3]. The general tendency of the effect of the line integral to the wave-making resistance may be understood as follows:
Let us represent ϕs of a thin ship by doublet distribution and compare it with the one of ϕL as in Fig.2.

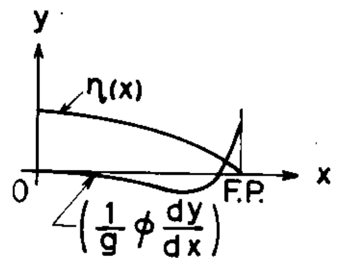

FIG 2. CONTRIBUTION OF LINE INTEGRAL

Then we may understand that the line integral causes a larger change of the resistance at low speed because the correction doublet will be relatively larger than the original near the bow and stern. We may calculate the effect of this correction term by the slender ship approximation but it must be failed in general because the correction term and its derivative do not vanish at both ends so that the resistance may tend to infinity. Hence, to obtain a finite resistance, we must calculate it when the line doublet situates at correct posision. [Appendix E]. Another feature of the effect may be that this effect may be small or large according to the ship form because the contribution of the line doublet is very much sensible to the form of the main hull. Thus, it is possible that the less the wave-making resistance, the less the effect of the line doublet and the better the agreement between the theory and expreiments is [28].

The last feature of the line doublet is that it relates with the vertical force acting on the ship, because its total sum becomes

$$\frac{1}{g}\int_C\phi(x,y,o)dy \doteq -\frac{2}{g}\int_C\eta(x)\frac{\partial}{\partial x}\phi(x,\eta,o)dx$$

$$=2\int_C\zeta(x,\eta)\eta(x)dx \quad , \qquad (19)$$

that is, the virtual increment of the displacement volume and ρg times of it is the increment of the statical buoyancy [5], and this must be added to Lagally's force so that the vertical force may coincide with the one deduced from the thin ship theory [5,37], where $\eta(x)$ is the half breadth of the ship.

By the way, we have also

$$\frac{1}{g}\int_C\phi(x,y,o)dy=\iint_F\zeta(x,y)dxdy \quad , \qquad (20)$$

that is, the increment of the water above the mean water level.
In the right hand side of the equation (20) if we assume that the surface elevation is true one and is not linearized, this gives the total vertical force by the extension of Archimedes' principle [11, Appendix D].

3. Uniqueness of solution

The preceding approach to a surface-piercing ship is so sophisticated and difficult to understanding that we may seek another way through a submerged ship for which the line integral does not appear and the uniqueness of solution is proved [22].
Now let us imagine a submerged ship of which water plane $\bar{F}$ is covered by a solid deck plate as in Fig.3, where is an arbitrarity for the deck [6] but we may not loose a generality by this assumption because we consider its limit when the ship is piercing the water and there is no concern with the state of the water plane.

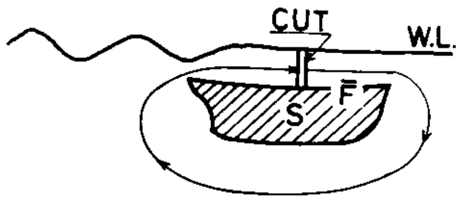

FIG 3. CONNECTIVITY OF THE DOMAIN.

At first, for the simplicity, consider the case of two dimensional motion and the velocity potential is determined uniquely except an arbitrarity associate with a circulation. Usually we do not take the circulation into consideration for a displacement ship, but in our case it is rather natural to introduce a circulation because we try to consider a surface-piercing ship as the limit when a water over top of the submerged ship vanishes and there is no assurance of one-valued nature of the potential. However, there is no Kutta condition in our case, so that that part of solution may be completely arbitrary and it is an arbitrarity associate topologically with the open domain [3], and corresponds to the question of the point singularity at the cross point of ship hull and water surface in the two-dimensional problem, which corresponds to the line integral in the three dimensional case [9,11,12]. This conclusion may be easily extended to the three dimensional case.

Now, let us imagine a wake vortex surface W as in Fig.4 and there be a jump $\Delta\phi$ of the velocity potential between the upper and lower surface.

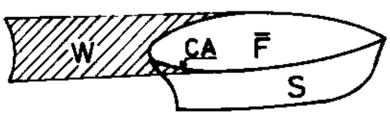

FIG 4. WAKE VORTEX SURFACE

Then, we have by usual method

$$\phi(P)=\iint_{S+\bar{F}}\{\phi(Q)\frac{\partial}{\partial n}S(P,Q)-S\frac{\partial}{\partial n}\phi\}dS$$
$$+\iint_{W}\Delta\phi(Q)\frac{\partial}{\partial z'}S(P,Q)\,dx'dy' \quad , \qquad (21)$$

where

$$\Delta\phi=\phi|_{W+}-\phi|_{W-} \quad . \qquad (22)$$

Between the upper and lower surface of W the normal velocity must be continuous and also the axial one does so that

$$\frac{\partial}{\partial x}\{\Delta\phi(P)\}=o \quad , \text{ on } W \quad , \qquad (23)$$

hence the second term of the right hand side of (21) will become by partial integration

$$\int_W\Delta\phi\frac{\partial}{\partial z'}S\,dx'dy'=-\frac{1}{g}\int_{CA}\Delta\phi\frac{\partial}{\partial x'}S(P;x',y',o)\,dy' \ , \quad (24)$$

where CA means the cross line of W and F. Then, let the ship make approach so nearer the water surface that there may be left very thin sheet of water over the deck of the ship and the vertical velocity may vanish there.
That is, by the water surface condition,

$$\frac{\partial}{\partial z}\phi=-\frac{1}{g}\frac{\partial^2}{\partial x^2}\phi=o \text{ on } F \text{ and } \bar{F} \quad , \quad (25)$$

Therefore, we have by integration [10],

$$-\frac{1}{g}\frac{\partial}{\partial x}\phi=\zeta(x,y)=\text{func. }(y) \text{ on } \bar{F} \quad . \qquad (26)$$

Now, let us consider here the boundary value problem and assume the motion without circulation and so the velocity potential ϕS be one-valued, and we have uniquely the potential

$$\phi_s(P)=\iint_{S+F}\{\phi_s\frac{\partial}{\partial n}S-S\frac{\partial}{\partial n}\phi_s\}dS \quad , \qquad (27)$$

satisfying the condition (11) on both S and F [22]. From this potential we may calculate the water surface elevation $\zeta S(x,y)\equiv\zeta S(y)$ on F which is not zero in general. Hence, if we add to it one more potential ϕ_L which is not already circulation-free and has the boundary conditions

$$\zeta_h(x,y)\equiv\zeta_h(y)=-\zeta_s(y) \quad , \qquad (28)$$

$$\frac{\partial}{\partial n}\phi_h(P)=o \text{ on } S \text{ and } \bar{F} \quad , \qquad (29)$$

there is no water sheet on the deck of a ship and we have a surface-piercing ship. This process of calculation is followed approximately by K. Mori [24] who reports its better result.

The existence of such potential having the homogeneous boundary condition (29) may be curious but it becomes natural when we remember the analogy to the planing boat, then it stands for a slight change of vertical position [7,3]. In fact, we may easily prove in the present case too that there is no change about the ship surface condition except higher order terms when the ship rises or sinks very slightly. This consideration leads us to one more such potential corresponding to sinkage, because the sinkage is not pre-determined. Thus, after all, we have a general representation

$$\phi(P)=\phi_s(P)+\phi_h(P)+\alpha\phi_{ho}(P) \quad , \qquad (30)$$

where α is an arbitrary constant, and ϕ_{ho} satisfies the condition (29) and

$$-\frac{1}{g}\frac{\partial}{\partial x}\phi_{ho}=\zeta_{ho}(x,y)=1 \text{ on } \bar{F} \quad . \tag{31}$$

Following the above derivation α may be equal to the sinkage or rise of ship but there may be another way, for example, to determine it so that the vertical force acting on the ship from the water might balance to the external force [11]. In any way, it must be carefully selected because the immersion of a submerged body induces sensibly the change of the wave-resistance [5,29,34, Appendix B].

The above derivation of the velocity potential will be followed heuristically as follows. In practiacal case, we may represent a ship by singularity over the skeleton surface S [28, Appendix F] on which singularity is swept in from the ship surface S as in Fig.5, and we may have the velocity potential satisfying the ship surface condition;

$$\phi_s(P)=\iint_{\bar{S}}\sigma_s(Q)S(P,Q)dS(Q) \tag{32}$$

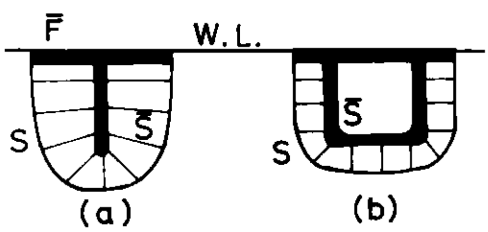

FIG 5. SKELETON SURFACE

There must be non-zero water surface elevation over $\bar{F}$ from this water motion. Then, to suppress the surface elevation, we must introduce a velocity potential satisfying conditions (28) and (29) and, if necessary, (31).
It may be represented as

$$\phi_h(P)=\iint_{\bar{S}}\sigma_h(Q)S(P,Q)dS(Q)$$
$$+\frac{1}{\rho g}\iint_{\bar{F}}p(Q)\frac{\partial}{\partial x}S(P;x',y',o)dx'dy' \ , \tag{33}$$

Thus, we arrive the model of yacht-type ship [31], and we may easily understand by this construction of the velocity potential that this stands in fact for a so-called sheltering or shielding effect [20,21] literally.
The first approximation for a thin ship may be

$$\phi_h(P)\doteqdot\iint_F[\zeta_s(x',y')+\alpha]\frac{\partial}{\partial x}S(P;x',y',o)dx'dy', \tag{34}$$

by a usual flat ship approximation.
Comparing with the line integral (15), we will find that both representation equals each other but higher order terms.

4. Diffraction of wave

The diffraction of wave which is radiated from a ship herself has been considerd of higher order and to be negligible but its importance is being revealed recently in theory and experiments [1,32,33]. On the other hand, its knowledge is indispensable in the theory of ships among waves because otherwise we can not predict the wave-exciting force and that it gives also the wave-damping coefficient [14].
Hence, we may hope usefull formulas derived from the study of diffraction of wave in our case too.
Now, let us introduce the diffraction potential ϕ_d which is a velocity potential of the wave diffracting an elmentary plane wave (A.7) by the ship, its boundary conditions should be

$$\frac{\partial}{\partial n}\phi_d(P;\theta)=-\frac{\partial}{\partial n}\phi_o(P,\theta) \text{ on S and } \bar{F} \quad , \tag{35}$$

$$\zeta_d(x,y;\theta)=-\zeta_o(x,y;\theta) \text{ on } \bar{F} \quad , \tag{36}$$

to assure the uniqueness.
By definition in Appendix A, its reverse flow potential should be

$$\widetilde{\phi_o}(P;\theta)=-\overline{\phi_o(P,\theta)} \quad , \tag{37}$$

$$\frac{\partial}{\partial n}\tilde{\phi}_d(P,\theta)=-\frac{\partial}{\partial n}\phi_d(P,\theta)=-\frac{\partial}{\partial n}\tilde{\phi}_o(P,\theta) \quad , \tag{38}$$

and

$$\tilde{\zeta}_d(x,y;\theta)=-\tilde{\zeta}_o(x,y;\theta)=-\frac{1}{g}\frac{\partial}{\partial x}\widetilde{\phi_c}=\frac{1}{g}\frac{\partial}{\partial x}\phi_o \quad . \tag{39}$$

At first, we show that the amplitude function is given by this diffraction potential as like as in the theory of oscillation [14]. By the reciprocity (A.11), the amplitude function (A.10) becomes

$$H(\theta)=-\iint_{S+\bar{F}+\bar{W}}\widetilde{\Phi_d}(P;\theta)\frac{\partial}{\partial n}\phi(P)dS(P) \quad , \tag{40}$$

where

$$\Phi_d(P;\theta)=\phi_o(P;\theta)+\phi_d(P;\theta) \quad , \tag{41}$$

because the second term of (A.10) can be written by the boundary condition (37) with (36) and applying the reciprocity as

$$-\iint_{S+\bar{F}+W}\phi\frac{\partial}{\partial n}\phi_o dS=\iint_{S+\bar{F}+W}\phi\frac{\partial}{\partial n}\bar{\phi}_o dS$$

$$=-\iint_{S+\bar{F}+W}\phi\frac{\partial}{\partial n}\bar{\bar{\phi}}_d dS=-\iint_{S+\bar{F}+\tilde{W}}\bar{\bar{\phi}}_d\frac{\partial}{\partial n}\phi dS \quad . \quad (42)$$

For a surface-piercing ship as a limit of submerged one, putting the boundary condition (11) and integrating partially the integral over W, we may have

$$H(\theta)=\iint_S \bar{\bar{\phi}}_d(P;\theta)\frac{\partial x}{\partial n}dS-\int_{CF}\Delta\bar{\bar{\phi}}_d(P;\theta)\zeta(x,y)dy \quad . \quad (43)$$

Hence, if we decompose the velocity potential as the formula (3), the above goes to

$$H(\theta)=H_s(\theta)+\alpha H_{ho}(\theta) \quad , \quad (44)$$

where

$$H_s(\theta)=\iint_S\bar{\bar{\phi}}_d(P;\theta)\frac{\partial x}{\partial n}dS \quad , \quad (45)$$

$$H_{ho}(\theta)=-\int_{CF}\Delta\bar{\bar{\phi}}_d(P;\theta)dy \quad , \quad (46)$$

that is, Hs is an amplitude function for ($\phi s+\phi h$) and Hho for ϕho.
Now, if we neglect here the diffraction of wave in the above formula, we have

$$H(\theta)\fallingdotseq-\iint_S\phi_o(P;\theta)\frac{\partial x}{\partial n}dS \quad , \quad (47)$$

and this equals that of the thin or slender ship and that Froude-Kriloff's formula with the wave-exciting force.

Proceeding in this way, we may easily derive the next approximation in the same way as in the oscillation problem [6,12]. The only difference is the existence of the line integral as the second term of the right hand side of (42), but it appears when the ship has the advance speed even in such case [25]. In the first approximation, this line integral equals the one from (33) and so nearly the one derived in §1 when we assume $\phi d=-\phi o$ on the free surface. However, the importance and usefullness lie on the treatment of a short wave, that is, at low speed or diverging wave.
Namely, we may apply to it the theory of geometrical optics [35], but the difficulty arises in our case from the asymmetry of our kernel function S.
Therefore, it must be better to decompose the kernel into even and odd part with respect to x and so the velocity potential too.
In this connection, we have the following important identity between a direct flow and the reversed one.

$$\phi(P)+\tilde{\phi}(P)=\frac{ig}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[\phi_d(P,\theta)\bar{\tilde{H}}(\theta)-\bar{\phi}_d(P,\theta)\tilde{H}(\theta)] \times\sec^2\theta d\theta \quad , \quad (48)$$

because ϕ and ϕd leave Kelvin wave systems as (A.9) in far down stream ($x\to-\infty$) and not in far up stream side but there exists a Kelvin wave system (A.6) from $\tilde{\phi}$ and the normal velocity of each sides of (47) vanishes by the condition (A.2) and (3), so that the both side must be equal to each other by the uniqueness of the boundary value problem. In a special but practical case of a ship which is symmetric in fore and aft and right and left, the above identity shows that the even part of ϕ in x is represented only by the diffraction of her own wave because of the relation (A.13). Therefore, it must be usefull only to solving the boundary value problem but also it supplies various identities about the diffraction.
For example, composing the amplitude function, we have

$$H(\theta)+\tilde{H}(\theta)=\frac{ig}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[H_d(\theta,\theta')\bar{\tilde{H}}(\theta)-\bar{H}_d(\theta,\theta)\tilde{H}(\theta)] \times\sec^2\theta d\theta \quad , \quad (49)$$

where

$$H_d(\theta,\theta')=H_d(\theta',\theta)=-\iint_{S+\bar{F}+W}\phi_d(P;\theta')\frac{\partial}{\partial n}\phi_o(P;\theta)dS. \quad (50)$$

If the ship is symmetric in fore and aft and the diffraction is neglected, that is,

$$H_d(\theta,\theta')\fallingdotseq-\iint_{S+\bar{F}}\Phi_o(P;\theta')\frac{\partial}{\partial n}\phi_o(P,\theta)dS \quad , \quad (51)$$

the above gives an estimation of the out of phase component of the amplitude function [6].

5. Conclusion

Summing up foregoing results,

i) On the line integral of the velocity potential of a surface-piercing ship,
   a) its source-term cancells out with the one by the change of wetted surface of the ship,
   b) the sum of its doublet equals the virtual volume change of the displacement,
   c) it is not important in high speed but in low speed and diverging wave system.

ii) On the uniqueness,
   a) to assure the uniqueness of the boundary value problem, a surface -piercing ship is considered as the limit of a submerged one of which water plane approaches infinitely to the free water surface,
   b) the water surface elevation over that water plane is not determined,
   c) to determine it to a given, it is necessary to introduce a solution having a wake vortex surface as in

a wing theory,
d) it contains a solution corresponding to a slight sinkage,
e) it corresponds to the above line integral,
f) it stands for the so-called sheltering effect by heuristic derivation.

iii) On the diffraction,
a) the diffraction potential of a elementary plane wave is introduced,
b) it is the influence function of the amplitude function, that is, the latter is represented by the former and the normal velocity,
c) the velocity potential of the reversed flow is represented by the one of the direct flow and the diffraction potential,
d) this representation may be usefull.

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Appendix A Reverse flow and Reciprocity [10,12].

Against a direct water flow around a ship as in Fig.1, imagine a reverse flow in which the direction of the unit uniform velocity is reversed and mean quantities of the reverse flow by adding the tilde on letters.
The water surface condition (3) must be unchanged by reversing the flow direction but the surface elevation and the normal velocity on the ship surface change their signs.

$$\tilde{\zeta}(x,y)=\frac{1}{g}\frac{\partial}{\partial x}\tilde{\phi}(x,y,o) \quad , \qquad (A.1)$$

and

$$\frac{\partial}{\partial n}\tilde{\phi}(P)=-\frac{\partial}{\partial n}\phi(P) \text{ on S and } \bar{F}. \qquad (A.2)$$

By the argument of §2 reverse flow velocity potential may be determined uniquely when surface elevation is given over $\bar{F}$.
Therefore, we specify it as

$$\tilde{\zeta}(x,y)=\zeta(x,y) \text{ on } \bar{F} . \qquad (A.3)$$

Now, the velocity potential in reverse flow has of course the same representation as the one in direct flow (21)

$$\tilde{\phi}(P)=\iint_{S+\bar{F}+\tilde{W}}[\tilde{\phi}\frac{\partial}{\partial n}\tilde{S}-\tilde{S}\frac{\partial}{\partial n}\tilde{\phi}]dS \quad , \qquad (A.4)$$

where

$$\tilde{S}(P,Q)=S(Q,P) \quad , \qquad (A.5)$$

and $\tilde{W}$ means the wake vortex surface in the reverse flow.
Then, we have an asymptotic expansion

$$\tilde{\phi}(P) \xrightarrow[x\to+\infty]{} \frac{ig}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[\phi_o(P,\theta)\bar{\tilde{H}}(\theta)$$

$$-\bar{\phi}_o(P,\theta)\tilde{H}(\theta)]\sec^2\theta d\theta \quad , \qquad (A.6)$$

where

$$\phi_o(P,Q)=\exp.[g\{z+i(x\cos\theta+y\sin\theta)\}\sec^2\theta] \quad , \qquad (A.7)$$

and

$$\tilde{H}(\theta)=\iint_{S+\bar{F}+\tilde{W}}[\phi_o(P,\theta)\frac{\partial}{\partial n}\tilde{\phi}-\tilde{\phi}\frac{\partial}{\partial n}\phi_o]dS \quad , \qquad (A.8)$$

may be called amplitude function and the bar on the letter stands for the complex conjugate value to be taken.
Corresponding formula for the direct flow is as follows.

$$\phi(P) \xrightarrow[x\to-\infty]{} \frac{g}{2\pi i}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[\phi_o(P,\theta)\bar{H}(\theta)$$

$$-\bar{\phi}_o(P,\theta)H(\theta)]\sec^2\theta d\theta \quad , \qquad (A.9)$$

$$H(\theta)=\iint_{S+\bar{F}+W}[\phi_o(P,\theta)\frac{\partial}{\partial n}\phi-\phi\frac{\partial}{\partial n}\phi_o]dS \quad , \qquad (A.10)$$

The aim to introduce such reverse flow is to obtain the following reciprocity between arbitrary two potentials:

$$\iint_{S+\bar{F}+W}\phi_i(P)\frac{\partial}{\partial n}\tilde{\phi}_j(P)dS$$

$$=\iint_{S+\bar{F}+\tilde{W}}\tilde{\phi}_j\frac{\partial}{\partial n}\phi_i dS \quad , \qquad (A.11)$$

which is a mere extension of Hanaoka's second theorem [18].
If a ship is thin or flat, we may have the formula from Hanaoka's first theorem but it must be used only when the interchange of order of integration is possible.
When a ship approaches infinitely to the water surface, the above formula goes to, by partial integration like as in (24),

$$\iint_{S+\bar{F}}\phi_i\frac{\partial}{\partial n}\tilde{\phi}_j dS+\int_{CA}(\Delta\phi_i)\zeta_j dy$$

$$=\iint_{S+\bar{F}}\tilde{\phi}_j\frac{\partial}{\partial n}\phi_i dS+\int_{CF}(\Delta\tilde{\phi}_j)\zeta_i dy \quad , \qquad (A.12)$$

where CF means the cross line of W and $\overline{F}$. It is clear that

$$\tilde{\phi}(x,y,z)=\phi(-x,y,z) \quad , \tag{A.13}$$

and

$$\widetilde{H}(\theta)=\overline{H}(\theta) \quad , \tag{A.14}$$

if the ship is symmetric about y-z coordinate plane.

Appendix B

Variational principle [7,8,10,13]

The integral (A.11) or (A.12) may be called the modified Lagrangean because we can write it by Green's theorem and the water surface condition [10] as

$$L(\phi_i,\tilde{\phi}_j)=\frac{-\rho}{2}\iint_{S+\overline{F}+W}\phi_i\frac{\partial}{\partial n}\tilde{\phi}_j dS$$

$$=+\frac{\rho}{2}\iiint_D \nabla\phi_i\nabla\tilde{\phi}_j dxdydz+\frac{\rho g}{2}\iint_F \zeta_i\tilde{\zeta}_j dxdy. \tag{B.1}$$

Making use of this integral we may compose an extremum problem equivalent to the boundary value problem.

Let ϕ be a velocity potential satisfying the water surface condition and consider the functional

$$I=\iint_{S+\overline{F}}[\phi\frac{\partial\tilde{\phi}}{\partial n}-\phi f+\phi f]dS$$

$$+\int_C[\tilde{\zeta}\Delta\phi-w\Delta\tilde{\phi}]dy \quad , \tag{B.2}$$

where

$$\frac{\partial\phi}{\partial n}=-\frac{\partial\tilde{\phi}}{\partial n}=-f \text{ on S and } \overline{F} \quad , \tag{B.3}$$

$$\zeta=\tilde{\zeta}=w \quad \text{on} \quad \overline{F} \quad , \tag{B.4}$$

$$\frac{dw}{dx}=f \quad \text{on} \quad \overline{F} \quad . \tag{B.5}$$

Then, we may easily verify that the extremum condition of I equals the boundary conditions (B.3) and (B.4).

For practical purpose, it is preferable to assume the pressure distribution over the water plane area $\overline{F}$.

Then, the modified Lagrangean becomes by partial integration

$$-\frac{2}{\rho}L(\phi_i,\tilde{\phi}_j)=\iint_{S+\overline{F}L}\phi_i\frac{\partial}{\partial n}\tilde{\phi}_j dS+\iint_{\overline{F}L}p_i(x,y)\tilde{\zeta}_j dxdy, \tag{B.6}$$

where $\overline{F}L$ means the lower side of the pressure distribution, and (B.2) goes to

$$I=\iint_{S+\overline{F}L}[\phi\frac{\partial\tilde{\phi}}{\partial n}-\phi f+\tilde{\phi}f]dS$$

$$+\iint_{\overline{F}L}[p\tilde{\zeta}-\tilde{p}w]dxdy \tag{B.7}$$

For a flat ship, only the second term of the right hand side is to be taken.

In this way of formulation of the boundary condition we must recognize the necessity to specify the wave profile around the ship or fictitious surface elevation over the water plane area.

Appendix C Law of continuity

The outflow from the ship surface and the free water surface must not exist, so that the flux from the ship surface

$$\Delta A=\int_C\zeta(x,y)dy \quad , \tag{C.1}$$

must cancel out with the one from the free surface as in the equation (9). More accurately, including higher order terms, this outflow can be integrated as follows;

$$\iint_{S^*+F^*}\frac{\partial\phi}{\partial n}dS=-\iint_{S^*+F^*}\frac{\partial x}{\partial n}dS$$

$$=\iint_{S^*+F^*}dydz\doteqdot\int_{C-L}\zeta(x,y)dy \quad , \tag{C.2}$$

where L means a line parallel to the y-axis at far down stream.

Hence, to vanish the above, the integral on L must equal (C.1), but if we take the velocity potential as linearized and it has an asymptotic expansion (A.9), the line integral on L vanishes in the mean. This contradiction can be released by introduction of higher order terms, in fact, there exists D.C. part for the surface elevation at far down stream in higher order theory [36].

On the other hand, the flux from the inspection surface I at far down stream must equal the influx at far up stream, that is,

$$\int_L\zeta dy+\iint_I\frac{\partial\phi}{\partial x}\Big]_{X=-\infty}dydz=0\doteqdot\int_L\zeta dy+\int_{-\infty}^{0}dz\int_{-\infty}^{\infty}\frac{\partial\phi}{\partial x}\Big]_{X=-\infty}dy$$

$$+\int_L\frac{\partial\phi}{\partial x}\zeta dy \quad . \tag{C.3}$$

If the velocity potential has an asympotic expansion (A.9), the first and second integrals of this right hand side vanish in the mean, and the left hand side

integral may not vanish.
Thus, although it is necessary to take the higher order term in consideration for the accurate discussion, there may exist a stand point that the flux (C.2) cancels out with the right hand side of (C.3) to keep the law of continuity, then

$$\Delta A=g\int_{-\infty}^{\infty}\zeta^2(-\infty,y)dy \quad , \qquad (C.4)$$

and putting the expansion (A.9) and integrating we have

$$\Delta A=\frac{4g^2}{\pi}\int_0^{\frac{\pi}{2}}|H(\theta)|^2\frac{\sec^3\theta d\theta}{1+\sin^2\theta} \quad . \qquad (C.5)$$

The similar representation will be directly obtained from (C.1) for a thin ship.

This is the projected area of the wave profile on the body plan as in Fig.6, and if the water loose its momentum through this area, its resistance must be

$$\rho U^2(\Delta A) \quad , \qquad (C.6)$$

and this is nearly equal to four times of the wave resistance (E.1) at low speed [4].

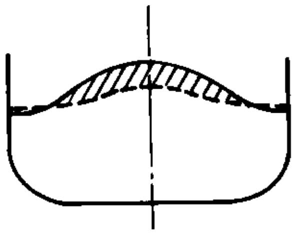

FIG.6 PROJECTION OF WAVE PROFILE

Appendix D

Extension of Archimedes' Principle

"The vertical force acting on a ship in a uniform stream equals the displacement weight including the free water surface displacement from the still water surface".

Let be the vertical force Z, and it is given by the formula

$$Z=-\iint_{S^*}p\frac{\partial z}{\partial n}dS \quad , \qquad (D.1)$$

the pressure p by Bernoulli's theorem

$$\frac{p}{\rho}=-\frac{\partial}{\partial x}\phi-\frac{1}{2}(\nabla\phi)^2-gz \quad , \qquad (D.2)$$

and it vanishes on the free surface. Therefore, the formula (D.1) is written also

$$Z=-\iint_{S^*+F^*}p\frac{\partial z}{\partial n}dS \quad , \qquad (D.3)$$

putting (2) into the above and making use of the boundary condition (11) and

$$\iint_{S^*+F^*}\left[\frac{\partial}{\partial x}\phi\frac{\partial z}{\partial n}-\frac{\partial x}{\partial n}\frac{\partial}{\partial z}\phi\right]dS=o \quad , \qquad (D.4)$$

we have

$$\iint_{S^*+F^*}\left[\frac{\partial}{\partial x}\phi+\frac{1}{2}(\nabla\phi)^2\right]\frac{\partial z}{\partial n}dS$$

$$=\iint_{S^*+F^*}\left[-\frac{\partial\phi}{\partial n}\frac{\partial\phi}{\partial z}+\frac{1}{2}(\nabla\phi)^2\frac{\partial z}{\partial n}\right]dS=o \quad . \qquad (D.5)$$

Hence, we have the theorem

$$Z=\rho g\iint_{S^*+F^*}z\frac{\partial z}{\partial n}dS=\rho g\nabla-\rho g\iint_{F^*}\zeta(x,y)dxdy, \qquad (D.6)$$

where

$$\nabla=\iint_S z\frac{\partial z}{\partial n}dS \quad . \qquad (D.7)$$

If the surface elevation is linearized the formula (D.6) becomes

$$Z'\equiv Z-\rho g\nabla=\rho\iint_F\frac{\partial}{\partial x}\phi dxdy=-\rho\int_C\phi dy$$

$$=2\rho\int\eta(x)\frac{\partial}{\partial x}\phi dx=-2\rho g\int\zeta(x)\eta(x)dx \quad , \qquad (D.8)$$

namely, it equals the virtual change of the displacement weight of the ship. Corresponding formula for a flat ship is

$$Z=\iint_{S+F}pdxdy=-\rho\iint_{S+F}\left[\frac{\partial\phi}{\partial x}+g\zeta\right]dxdy$$

$$=-\rho g\iint_{S+F}\zeta(x,y)dxdy \quad , \qquad (D.9)$$

because ϕ vanishes at infinity.
Therefore, the formula (D.8) may be considered as a first approximation of the vertical force for a displacement type ship.
On the other hand, the vertical force for a thin ship except statical buoyancy is

given by

$$Z' = -\rho\iint_S \frac{\partial\phi}{\partial x}\frac{\partial n(x,y)}{\partial z}dxdz \quad , \quad (D.10)$$

and this formula explanes the experimental sinkage [4,37].
In another way, we may calculate it by Lagally's theorem, but in this case we obtain the difference of (D.8) and (D.10). This may be a paradox originating from disregarding the free surface singularity [4].
Considering the similarity and the corresponding formula for a flat ship, the formula (D.8) and (D.10) may be equally an approximation for a vertical force.

Appendix E

On the resistance integral

The wave-making resistance is given by Mitchel integral

$$R = \frac{\rho g^2}{\pi}\int_0^{\frac{\pi}{2}} |H(\theta)|^2 \sec^3\theta d\theta \quad , \qquad (E.1)$$

where $H(\theta)$ is defined by (A.10).
This integral may not exist in some cases. The typical is of a slender ship and its sectional area curve must have cusps at both ends to exist this integral.
On the other hand, there is no such narrow confinement of the class of ship forms for thin and flat ships.
Therefore, this confinement shows merely that the slender ship approximation is a poor one for the ship having sharp wedged or round ends.
The non-existence of resistance integral results from large amplitude of diverging wave system, but since this wave has small wave-length, the assumption that the beam and draft is small compared with the wave-length does not adequate so that we might treat more cautiously.
For example, consider a slender ship as a limit of the flat ship, which has a rectangular water plane with breadth 2b and a constant draft 2t.
By a slender ship approximation we have

$$H(\theta) = 8igbt\sin(g\sec\theta) \quad , \qquad (E.2)$$

and the wave resistance does not exist

$$R = O(b^2t^2) \longrightarrow \infty \quad . \qquad (E.3)$$

By a flat ship approximation, however, we have the finite wave resistance, that is,

$$H(\theta) = 8it\frac{\cos^2\theta}{\sin\theta}\sin(g\sec\theta)\sin(gb\sec^2\theta\sin\theta), \qquad (E.4)$$

$$R = \frac{64\rho t^2}{\pi}\int_0^{\frac{\pi}{2}}\sin^2(g\sec\theta)\times$$

$$\times\sin^2(gb\sec^2\theta\sin\theta)\frac{\cos\theta d\theta}{\sin^2\theta}. \qquad (E.5)$$

At low speed, this integral can be roughly estimated as

$$R \fallingdotseq 8\rho t^2 gb = O(bt^2) \quad , \qquad (E.6)$$

because we have for large g,

$$R \fallingdotseq \frac{32\rho}{\pi}t^2\int_0^{\frac{\pi}{2}}\sin^2(gb\sec^2\theta\sin\theta)\frac{\cos\theta}{\sin^2\theta}d\theta, \qquad (E.7)$$

and moreover

$$\int_0^{\frac{\pi}{2}}\sin^2(gb\sec^2\theta\sin\theta)\frac{\cos\theta}{\sin^2\theta}d\theta$$

$$\fallingdotseq\int_0^{\frac{\pi}{2}}\sin^2(gb\sec^2\theta)\cos\theta d\theta = \frac{gb}{2}\int_0^{\infty}\sin^2 u\frac{du}{u^2} = \frac{\pi}{4}gb. \quad (E.8)$$

If we start from a thin ship to the same slender one, we may arrive at a different resistance.
Thus the non-existence of the resistance integral of a slender ship means its non-determinancy and more accurately it is of order $(bt)^2$ in general but becomes of different order if we take another approximation.
We may easily find the same property as the thin ship theory too [2,3], so that we must treat the resistance integral more cautiously than ever.
However, these difference between integrals originates from their poor approximation for the diverging wave system, for which our theory must be wrong for lack of the consideration of the diffraction.
Therefore, it is hoped seriously to have the knowledge of short wave diffraction.

Appendix F

Analytical cotinuation

A velocity potential around a ship, that is, a harmonic function may be continuated analytically through ship surface S to a skeleton surface $\overline{S}$ as in Fig.5 by the mirror image principle as far as there lies no cross point of normals of S and $\overline{S}$ in the space between both surfaces. The free water surface is also a singularity surface but its singularity is swept out to infinity and around a water line of ship and it composes a line integral owing to the water surface condition.
Therefore, this curve around the water line must be included in the skeleton surface at least and if we include the water plane area as a part of the skeleton surface this is satisfied.