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# Characteristics of New Wave-Energy Conversion Devices

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ABSTRACT

A theory is given herein for the absorption of wave energy by means of a symmetrical body with wave energy conversion systems oscillating in a sinusoidal wave. Conditions for achieving a complete wave energy absorption were derived from the expressions for complete wave absorption. Moreover, as a practical application of the theory, two new wave energy absorbing devices were proposed and theoretical calculations and model tests were carried out on the wave and wave energy absorption of regular and irregular waves. Results of the experiments agreed well with the theoretical calculations. Moreover the results of the model tests verified a complete wave energy absorption.

## NOMENCLATURE

- $A_R$ = amplitude of reflected wave
- $A_T$ = amplitude of transmitted wave
- B = breadth of floating body
- $C_R$ = reflection coefficient
- $C_T$ = transmission coefficient
- $C_{ij}$ = coefficient of Eq. (12)
- $C_g$ = group velocity
- $C_2$, $C_3$ = hydrostatic restoring coefficient
- D, $D_1$, $D_3$ = determinant of Eq. (12)
- $E_i$ = wave exciting force in i-th direction
- $F_{ij}$, $f_{ij}$ = hydrodynamic force in i-th direction caused by j-th mode's oscillation, Eq. (10)
- g = gravitational acceleration
- $\overline{GM}$ = metacentric height above C.G.
- $H_i^{\pm}(K)$ = Kochin function of i-th mode's oscillation
- I = mass moment of inertia of the body
- $i$ = unit of imaginary
- K = wave number = $2\pi/\lambda$
- M = mass of the body
- n = normal vector
- $R_i$ = reaction force in i-th direction
- $P_e$, $P_o$ = absorbed power in regular wave
- $P_I$ = power of incident regular wave
- $P_A'$ = absorbed power in irregular wave
- $P_I'$ = power of incident irregular wave
- p = pressure on the surface of the body
- $S(\omega)$ = incident wave energy spectrum
- $s_{ij}$ = restoring coefficient of energy absorbing system
- $s$ = girth length of the body
- t = time
- W = electric power absorbed by generator
- $X_i$ = amplitude of i-th mode oscillation
- $\nabla$ = displaced volume
- $\Delta l$ = distance between two wave height meters (Fig. 4)
- $\delta_{ij}$ = mass or mass moment of inertia of energy conversion system
- $\zeta_a$ = amplitude of incident wave
- $\zeta_i$ = displacement of i-th mode oscillation
- $\eta$ = surface elevation
- $\eta_W$ = efficiency of wave power absorption = $1-C_T^2-C_R^2$
- $\eta_E$ = efficiency of wave power conversion
- $\mu_{ij}$ = damping coefficient of energy conversion system
- $\kappa$ = radius of gyration in rolling oscillation
- $\lambda$ = wave length
- $\rho$ = mass density of fluid
- $\omega$ = circular frequency

## 1. INTRODUCTION

Recently many studies have been made concerning the utilization of wave energy. The Wave-Activated Generator is one practical application of wave energy. Masuda[1] invented this device in 1965 and Ryokusei-sha put it into practical use. McCormick [2], [3] developed theoretical analysis of the pneumatic wave-energy conversion bouy system and the results of his analysis agreed with the experiment results of Masuda. Contrary to these studies, Isaacs [4] invented a wave powered pump which converts wave energy into water pressure that accumulates in an accumulator tank and which activates a hydraulic turbogenerator when the water pressure is released from the tank.

Independent of these practical applications of wave energy, Milgram [5] investigated the problem of absorbing two-dimensional water waves in a channel by means of a moving terminator at the end of the channel. Though the object of his study was to devise and develop a self-actuating wave-absorbing system, his idea, to generate a radiation wave that is exactly opposite to the reflected wave, could be easily extended to the problem of achieving wave energy absorption through the use of an oscillating body with wave energy conversion systems. As just such an extension of Milgram's idea, Bessho developed a theory for achieving wave and wave energy absorption by means of a symmetrical body with wave energy conversion systems oscillating on a free surface and his theory was read at the second subcommittee of Japan Towing Tank Committee in 1973.

In 1976, Evans [6] carried out a theoretical study on predicting the absorption of wave energy by means of a damped, oscillating, and partly or completely submerged body. He derived expressions on the efficiency of power absorption for the case where the body is a two-dimensional cylinder oscillating in either a single mode or in a certain combination of two modes and when the body is a heaving half-immersed sphere. In 1978, Count [7] applied Evans' theory to a two-dimensional asymmetrical wave power device and made a comparison between the calculated and measured efficiencies of a Salter duck. Recently Srokosz and Evans [8] investigated the problem of two arbitrary cylinders oscillating independently and capable of absorbing energy in a single mode from a given incident wave.

Contrary to these studies on the two-dimensional problem of wave energy absorption, the recent theoretical studies were directed on the efficiency of three dimensional wave energy absorption. Budal and Falnes [9] began a study on a resonant point absorber and defined power absorption length in 1975. Budal [10] also theoretically analyzed a wave power absorbing systems that consists of a number of rigid, interspaced, oscillating bodies in 1977. Using slender-body approximation, Newman [11] provided a rough analysis of the maximum rate of wave energy absorption by a flexible or hinged raft.

The present paper is intended to outline Bessho's theory and to show the results of theoretical and experimental studies conducted on the two new wave energy conversion devices which were developed as a part of results of Bessho's theory.

On the contrary to other theories, his theory was developed by using the condition for complete wave absorption, which is substantially similar to the condition for achieving 100% wave energy absorption. Following to his theory, it is shown in §2 that the complete wave and wave energy absorption can be attained in the case when the floating body with energy conversion systems oscillates in its natural period and the damping coefficients of the energy conversion systems are the same as the one of the wave diffraction.

One of the two new wave energy conversion devices is moored at dolphins in shallow waters (abbreviate as "Dolphin Type") and the other is moored by chain in the deep sea (abbreviate as "Chain Type"). In the case of the "Dolphin Type", a half-immersed symmetrical body oscillates in only two modes, heaving and rolling, and the swaying motion is constrained so that 100% wave energy absorption may be attained. In the latter case, since the floating symmetrical body oscillates in three modes, complete wave energy absorption is not yet completed. The results of the theoretical and experimental studies made on these two devices are described in §3 and §4. Wave absorption achieved by the "Dolphin Type" in the case of irregular waves is discussed in §5. The experimental data is also given.

## 2. THEORY OF WAVE-ENERGY ABSORPTION

### 2.1. Formulation

Two-dimensional motions are considered and cartesian co-ordinates (x, y) are chosen as shown in Fig. 1. It is assumed that the fluid motion is inviscid and incompressible, and that the resulting oscillatory motions are linear and harmonic.

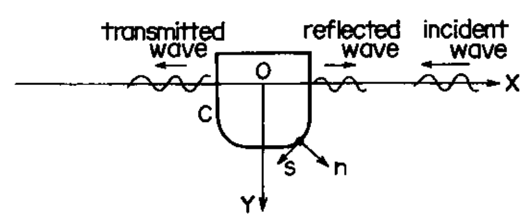


Fig. 1 Co-ordinate system

When a body oscillates in a incident wave of unit amplitude, the amplitudes of the reflected and transmitted wave far from the body can be expressed as:

$$A_R = i K \sum_{j=1}^{4} X_j H_j^{+}(K) \tag{1}$$

$$A_T = 1 + i K \sum_{j=1}^{4} X_j H_j^{+}(K) \tag{2}$$

where K = wave number = $2\pi/\lambda$

$X_j$ = amplitude of j-th mode oscillation

$X_4 = 1/K$

$H_j^{\pm}(K)$ = Kochin function of j-th mode oscillation

Here j = 1, 2, 3 refer to sway, heave, roll, respectively, and j = 4 refers to diffraction of incident wave by the fixed body. When the body is fixed in a incident wave, the expressions for the reflected and transmitted wave (1) and (2) become:

$$A_R = i H_4^+(K) \tag{3}$$

$$A_T = 1 + i H_4^-(K) \tag{4}$$

These waves may be called as the scattered waves and can be considered as the sum of the symmetrical and asymmetrical waves progressing to far away from the body. On the other hand, if the floating body has a symmetrical sectional form, the heaving motion generates a symmetrical radiation wave and the swaying or rolling motion generates an asymmetrical radiation wave.

Using the characteristics of the Kochin functions for a symmetrical body (See Appendix I, (A.14) - (A.17)), the symmetrical and asymmetrical components of the scattered wave can be expressed as following.

$$A_R + A_T = \frac{H_2^+}{\bar{H}_2^+} + 2iKX_2H_2^+ \tag{5}$$

$$A_R - A_T = \frac{H_1^+}{\bar{H}_1^+} + 2iK\left\{X_1H_1^+ + X_3H_3^+\right\} \tag{6}$$

As above mentioned, the conditions for the complete wave absorption are:

$$A_R + A_T = 0 \tag{7}$$

$$A_R - A_T = 0 \tag{8}$$

If we can realize such motions that satisfy the equations (7) and (8) by adding suitable wave-energy conversion systems, the symmetrical and asymmetrical radiation waves cancel out each components of the scattered wave respectively due to interference. In other words, the incident wave is completely absorbed and consequently wave energy absorption can be completely attained.

## 2.2. Equations of Motion

Under the assumptions that the responses are linear and harmonic and that the sectional form of the floating body is symmetry, the equations of motion can be written as:

$$
\begin{aligned}
M\ddot{\zeta}_1 &= F_{11} + F_{13} + E_1 + R_1 \\
M\ddot{\zeta}_2 &= F_{22} - C_2\zeta_2 + E_2 + R_2 \\
I\ddot{\zeta}_3 &= F_{33} + F_{31} - C_3\zeta_3 + E_3 + R_3
\end{aligned} \tag{9}
$$

where   is a displacement of i-th mode's motion, $F_{ij}$ is hydrodynamic force in i-th direction caused by j-th mode of motion, $C_i$ is restoring coefficient, $E_i$ is wave exciting force in i-th direction, and $R_i$ is reaction force in i-th direction acting on the floating body from the energy absorbing system.   , $F_{ij}$ , and $E_i$ can be written as follows:

$$
\begin{aligned}
\zeta_i &= X_i e^{i\omega t} \\
F_{ij} &= \rho\omega^2 X_j f_{ij} e^{i\omega t} \\
&= \rho\omega^2 X_j (f_{ijc} + f_{ijs}) e^{i\omega t} \\
E_i &= -\rho g H_i^+(K)
\end{aligned} \tag{10}
$$

It is assumed that the wave energy absorbing system operates independently with respect to the symmetrical and asymmetrical mode of motion, the reaction forces $R_i$ can be expressed as following:

$$
\begin{aligned}
R_1 &= \rho g\left\{(K\delta_{11} - S_{11} - iK\mu_{11})\zeta_1 + (K\delta_{13} - S_{13} - iK\mu_{13})\zeta_3\right\} \\
R_2 &= \rho g(K\delta_{22} - S_{22} - iK\mu_{22})\zeta_2 \\
R_3 &= \rho g\left\{(K\delta_{33} - S_{33} - iK\mu_{33})\zeta_3 + (K\delta_{31} - S_{31} - iK\mu_{31})\zeta_1\right\}
\end{aligned} \tag{11}
$$

Substituting the expressions (10) and (11) into the equations (9), and abbreviating the term   , the equations of motion become:

$$
\begin{aligned}
C_{11}X_1 + C_{13}X_3 &= H_1^+(K)/K \\
C_{22}X_2 &= H_2^+(K)/K \\
C_{31}X_1 + C_{33}X_3 &= H_3^+(K)/K
\end{aligned} \tag{12}
$$

where

$$
\begin{aligned}
C_{11} &= \nabla + f_{11c} + \delta_{11} - S_{11}/K - i\left\{\mu_{11} + |H_1^+(K)|^2\right\} \\
C_{13} &= f_{13c} + \delta_{13} - S_{13}/K - i\left\{\mu_{13} + H_3^+(K)|H_1^+(K)|\right\} \\
C_{22} &= \nabla + f_{22c} + \delta_{22} - (S_{22} + B)/K - i\left\{\mu_{22} + |H_2^+(K)|^2\right\} \\
C_{31} &= f_{31c} + \delta_{31} - S_{31}/K - i\left\{\mu_{31} + H_1^+(K)|H_3^+(K)|\right\} \\
C_{33} &= \nabla K^2 + f_{33c} + \delta_{33} - (S_{33} + \nabla\overline{GM})/K - i\left\{\mu_{23} + |H_3^+(K)|^2\right\}
\end{aligned} \tag{13}
$$

The solutions of equations (12) can be easily obtained and written as:

$$
\begin{aligned}
KX_1 &= D_1/D \\
KX_2 &= H_2^+(K)/C_{22} \\
KX_3 &= D_3/D
\end{aligned} \tag{14}
$$

where

$$
D = \begin{vmatrix} C_{11} & C_{13} \\ C_{31} & C_{33} \end{vmatrix}
$$

$$
D_1 = \begin{vmatrix} H_1^+(K) & C_{13} \\ H_3^+(K) & C_{33} \end{vmatrix} \qquad
D_3 = \begin{vmatrix} C_{11} & H_1^+(K) \\ C_{31} & H_3^+(K) \end{vmatrix} \tag{15}
$$

### 2.3. Wave Absorption

At first, by substituting the solutions of the equations of motion into (7), the condition that should be satisfied for absorbing the symmetrical component of the scattered wave is obtained as follows:

$$C_{22} = -i\,z\,\left|H_2^+(K)\right|^2 \tag{16}$$

or

$$\left.\begin{aligned} &\nabla + f_{22c} + \delta_{22} - (S_{22} + B)/K = 0 \\ &\mu_{22} = \left|H_2^+(K)\right|^2 \end{aligned}\right\} \tag{17}$$

It is known by (17) that if we have the system resonates with the incident wave in heaving motion and the magnitude of the damping coefficient of the energy conversion system is equalized with that of wave making heave damping of the body, $(A_R + A_T)$ becomes to zero.

On the other hand, the absorbed wave power in the aforementioned condition becomes:

$$P_e = \frac{1}{2}\rho\omega^2\mu_{22}|X_2|^2 = \frac{\rho g^2}{8\omega} = \frac{P_I}{2} \tag{18}$$

where $P_I$ represents the incident wave power

Expression (18) shows that half of the incident wave power is absorbed by the system.

Secondly, let us investigate the condition that satisfies the equation (8). By substituting the expression (14) into (8), the condition that should be satisfied for absorbing the asymmetrical component of the scattered wave is obtained as:

$$\frac{D_1 H_1^+(K) + D_3 H_3^+(K)}{D} = \frac{i\,H_1^+(K)}{2\,H_1^+(K)} \tag{19}$$

It should be noted here that both rolling and swaying motions as known by (6) are not always necessarily employed. If the swaying motion is restricted, Eq. (19) can be simply rewritten as:

$$C_{33} = -i\,2\,\left|\rho H_3^+(K)\right|^2 \tag{20}$$

or

$$\left.\begin{aligned} &\nabla \kappa^2 + f_{33c} + \delta_{33} - (S_{33} + \nabla \overline{GM})/K = 0 \\ &\mu_{33} = \left|H_3^+(K)\right|^2 \end{aligned}\right\} \tag{21}$$

In the same way as the symmetrical part of scattered wave, it is known by (21) that if we have the system resonates with the incident wave in the rolling motion and the magnitude of the damping coefficient of energy conversion system is equalized with that of wave making roll damping of the body, $(A_R - A_T)$ becomes to zero.

In this case, the absorbed wave power becomes:

$$P_e = \frac{1}{2}\rho\omega^2\mu_{33}|X_3|^2 = \frac{\rho g^2}{8\omega} = \frac{P_I}{2} \tag{22}$$

Therefore, if we choose the parameters of wave-energy conversion system so as to satisfy (17) and (21) simultaneously, both the transmitted wave and reflected wave are diminished to zero and the incident wave power is perfectly absorbed by this wave-energy conversion system.

To verify these theoretical results, two kind of wave-energy absorbing systems were made, and the tank test was carried out. One of these is a system using two modes of motion (heaving and rolling) and swaying motion is restricted by dolphins. The other is a system that freed all modes of motion and moored by chain. The former is described in the following section and the latter in §4.

## 3. WAVE ENERGY CONVERSION DEVICE MOORED AT A DOLPHIN

### 3.1. Model Experiments

(1) Model Characteristics

The principal dimensions and characteristics of the model are shown in Table 1.

Table 1 Characteristics of model

| Item | |
|---|---|
| Length | 2.960 m |
| Width | 1.000 m |
| Draft | 0.300 m |
| Displacement | 444.12 Kg |
| Center of bouyancy ($\overline{KB}$) | 0.1957 m |
| Center of gravity ($\overline{KG}$) | 0.300 m |
| Metacenter height ($\overline{GM}$) | 0.451 m |
| Water plane area | 2.960 m |
| Radius of gyration | 0.332 m |
| Natural period of heaving motion | 1.35 sec |
| Natural period of rolling motion | 1.35 sec |

This 1:10 scale model has a Lewis Form section, as shown in Fig. 2.

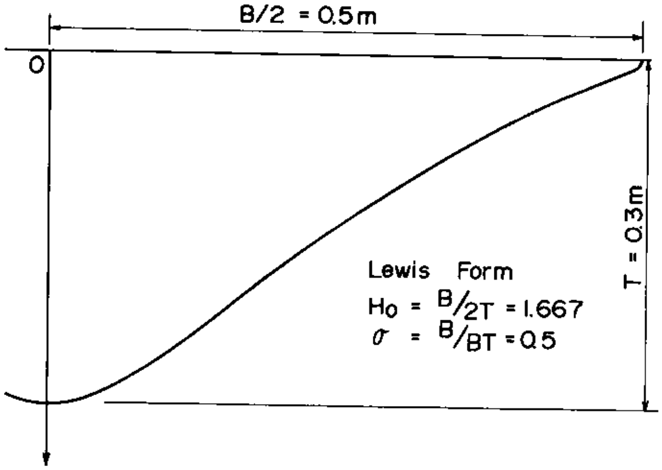


Fig. 2 Section of model.

(2) Experimental Apparatus

The experiments were conducted in 60m (length)x3m(width)x1.5m(water depth) wave basin. The basin was equipped with a piston type wave maker which could generate regular and irregular waves. The maximum wave height is 0.5m and the range of wave period is from 0.5 to 4.0 sec.

The model was set at the center of the basin and was connected to a beam, which had been built over the basin instead of the dolphin by three connecting rods with hinges to permit free oscillation. The arrangement is shown in Fig. 3.

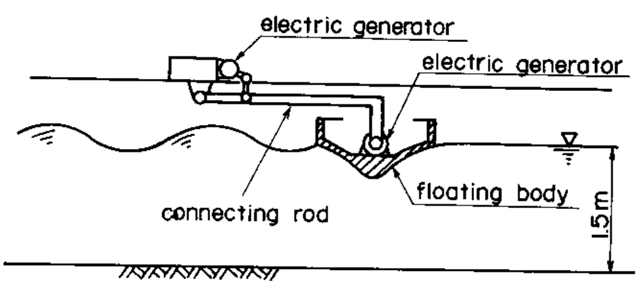


Fig. 3 Experiment apparatus

In this arrangement, the relative motions of the floating body and connecting rods and of the connecting rods and beam (dolphin) drove the electric generators which were located at both ends of the center rod.

In the external circuits of the generators, the variable electric resistors were connected so that the damping factors of the wave energy conversion devices may be adjusted to the wave generation damping factors.

And then one-way clutches and gears were set up at the each electric generators so that the electric generators could revolve in one direction despite the alternative motion of connecting rod. The gears multiplied the rotation 300 times for heaving motion and 200 times for rolling motion.

Since it had been estimated that, in this experiment, the maximum incident wave power would be about 160 watts (wave length 3.0m and wave height 0.2m), two 80 watt direct current generators were selected for heaving motion and rolling motion respectively.

(3) Measurement

The following items were measured:

-Wave height

Incident wave height and reflected wave height were measured by means of capacitance type wave height meters W1 and W2 which had been arranged as shown in Fig. 4. Transmitted wave height was measured by W3 (see Fig. 4). The method used to calculate the incident wave height and reflected wave height is described in Appendix-II.

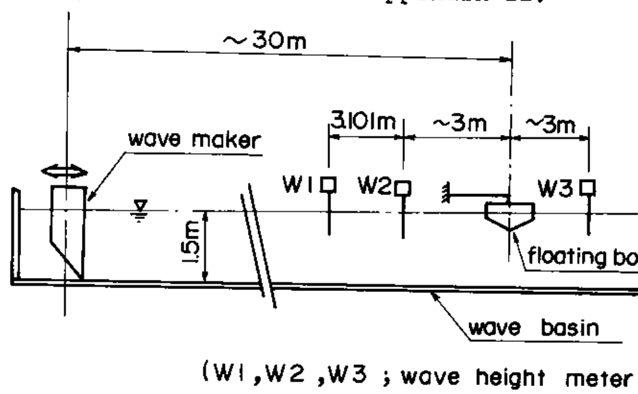


Fig. 4 Measuring of wave height

-Motions of the floating body

The motions of the floating body were measured by means of a displacement meter which was composed of potensio meters, with 6-degrees of freedom. The measuring system is shown in Fig. 5. The measuring arm was attached just at the center of gravity of the floating body.

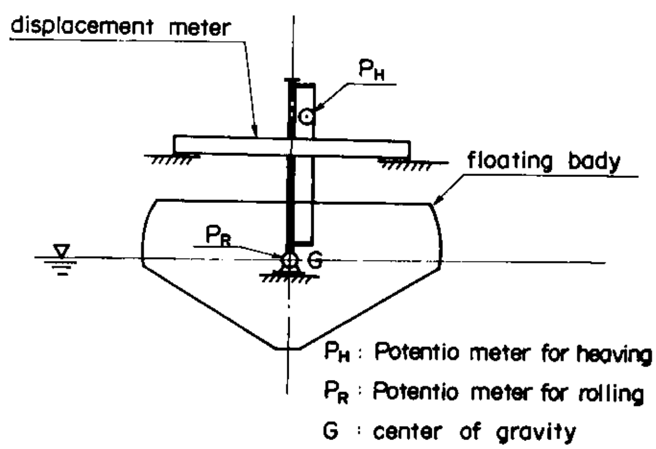


Fig. 5 Measuring of motions

-Generated electric power

The voltages which were generated by the heaving device and rolling device were measured by means of volt meters (see Fig. 6).

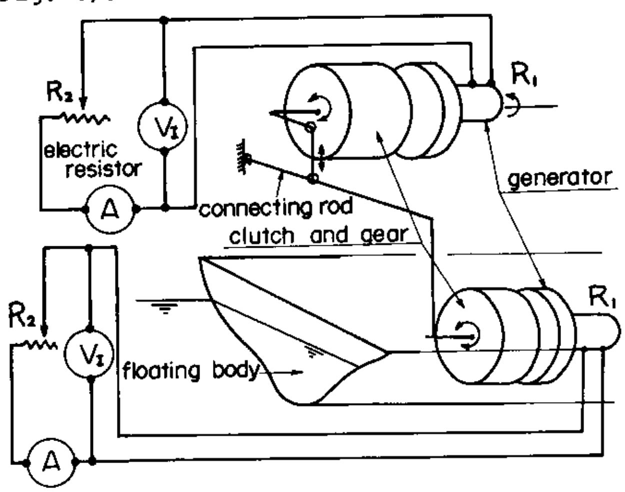


Fig. 6 Damping devices

The circuit current was calculated by dividing the voltage($V_I$) by the resistance of the external circuit($R_2$). Then the electric power was calculated as the product of the circuit current and the square of the total resistance which is the sum of the external and internal resistance($R_1$, see Fig.6).

The measured values were then recorded on an analogue data-recorder and analyzed with the aid of a electric computer.

(4) Wave Conditions

The incident waves were regular ones. The range of wave heights was from 0.1 to 0.2m, and the wave period from 0.9 to 2.82 sec.

## 3.2. Results of the Experiment

(1) Motions of the Floating Body

Figs. 7 and 8 show the heaving motion and rolling motion of the floating body.

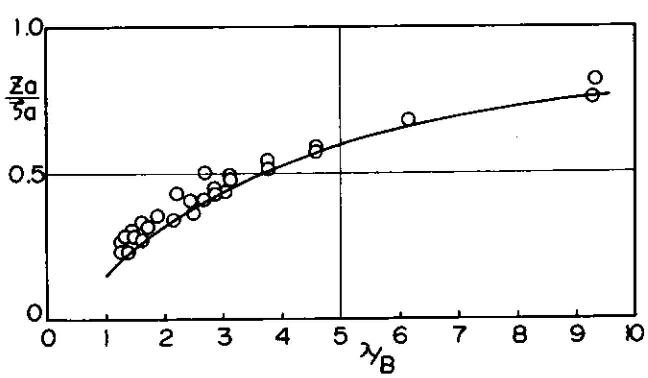


Fig. 7 Heaving motion

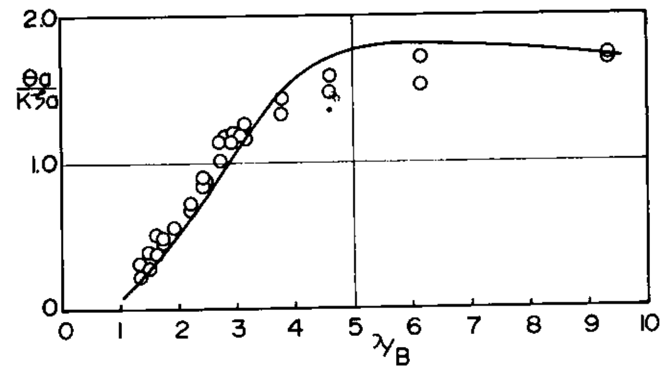


Fig. 8 Rolling motion

The small circles in the figures represent the experimental data, and the full lines represent the theoretical values. These values are divided by the incident wave amplitude (ζa) or wave slope (Kζa). The abscissa represents the ratio of the wave length (λ) to the width of the floating body (B).

It is recognized that the experimental data agree well with the theoretical results.

(2) Characteristics of Wave Transmission and Wave Reflection

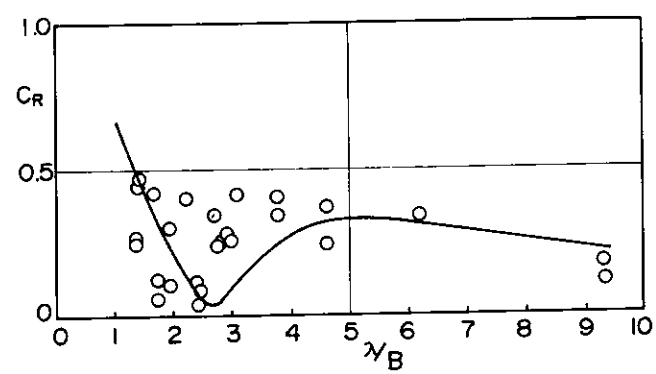


Fig. 9 Wave reflection coefficient

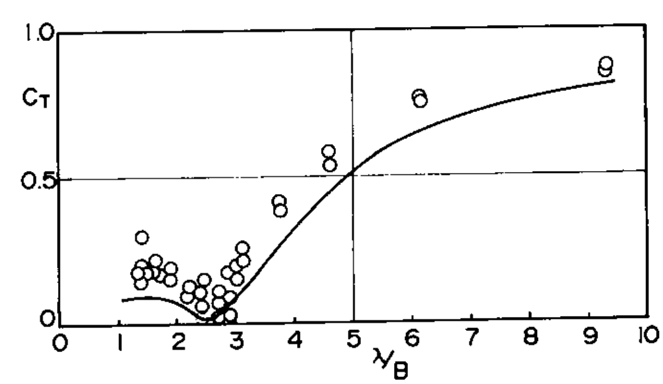


Fig. 10 Wave transmittion coefficient

Figs. 9 and 10 show the reflection coefficient ($C_R$) and the transmission coefficient ($C_T$). $C_R$ means the ratio of the reflected wave height to the incident wave height, and $C_T$, the ratio of the transmitted wave height to the incident wave height. (The method used to calculation $C_R$ and $C_T$ is described in Appendix-II.)

In these figures, $C_R$, together with $C_T$, has the minimum value (nearly equal to zero), when $\lambda/B=2.5$; that is, when wave length reached 2.5 times as long as the width of the floating body, the incident wave energy is almost completely absorbed. Hence, a calm sea appears behind the floating body.

When the incident wave length is less than 4.7m ($\lambda/B\leqq 4.7$), the transmitted wave height is reduced to below 50%.

(3) Efficiency of Wave Power Absorption

The efficiency of wave power absorption is shown in Figs. 11 and 12.

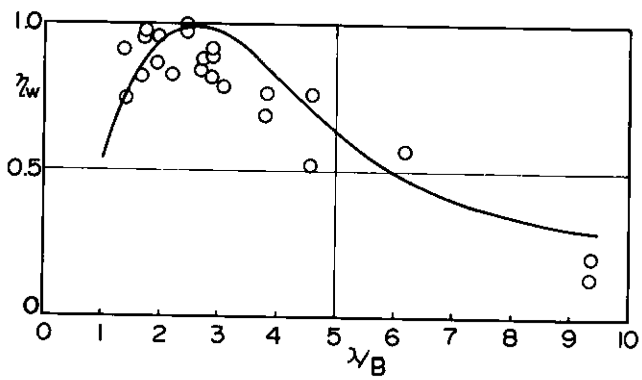


Fig. 11 Efficiency of wave power obsorption

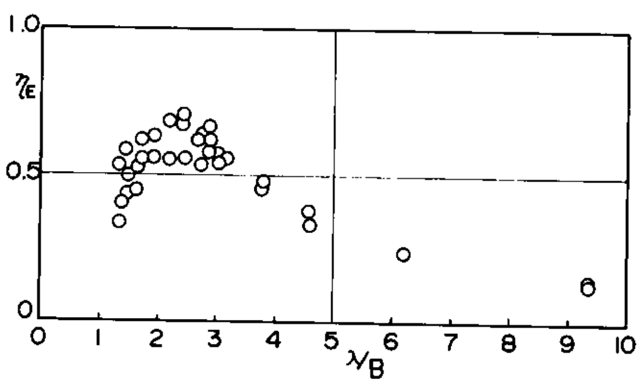


Fig. 12 Efficiency of wave energy conversion

In these figures, wave energy absorption efficiency $\eta_w$ is defined by equation (28).

$$\eta_w = 1 - C_T^2 - C_R^2 \tag{28}$$

Whereas, energy conversion efficiency is expressed by equations (29) and (30).

$$\eta_E = W / P_w \tag{29}$$

$$\begin{cases} W = \frac{1}{t_0}\int_0^{t_0}(R_1+R_2)(V_2(t))^2/R_2^2\,dt \\ P_w = \frac{1}{2}\rho g^2 \zeta_a^2 C_g L \end{cases} \tag{30}$$

where, W means the electric power converted by the electric generator, $P_w$ means the incident wave power and L means the length of the floating body.

In regards to wave absorption efficiency $\eta_w$, it is evident that the wave energy is completely absorbed by the device when the reflected wave and the transmitted wave are diminished simultaneously ($\lambda/B$ = 2.5, see Figs. 11 and 12). On the other hand, the experiment results of energy conversion efficiency $\eta_E$ were short of the theoretical values by 20~30% (see Fig. 12). This loss may be composed of mechanical loss (mainly in the gears and clutches) and of scattering waves of higher order.

To confirm this point, free oscillation tests of the wave energy absorption device were carried out, in which the electric resistors had been removed so that the generated power was not consumed, and about 20~30% absorption of wave energy was verified.

## 4. WAVE ENERGY CONVERSION DEVICE MOORED BY CHAINS

### 4.1. Model Experiments

(1) Model Characteristics

The principal dimensions and characteristics of this model are given in Table 2. This model has a Lewis-Form section, as shown in Fig. 13.

Table 2 Characteristics of model

| Item | |
|---|---|
| Length | 2.900 m |
| Width at water plane | 1.000 m |
| Draft | 0.500 m |
| Displacement | 870.0 Kg |
| Metacenter height ($\overline{GM}$) | 0.200 m |
| Center of gravity ($\overline{KG}$) | 0.418 m |
| Radius of gyration | 0.350 m |
| Natural period of heaving motion | 1.56 sec |
| Natural period of rolling motion | 1.85 sec |

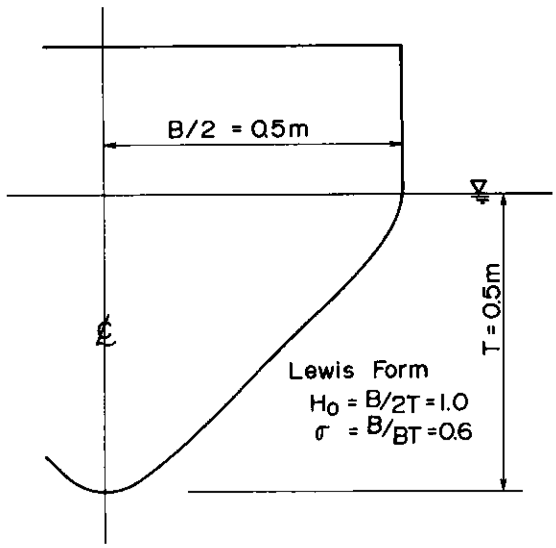


Fig. 13 Section of model.

(2) Experiment Apparatus and Measured Items

Experiments concerning the "Chain Type" device were conducted in the same basin that had been used to test "Dolphin Type" device.

The arrangement of experiment apparatus is shown in Fig. 14.

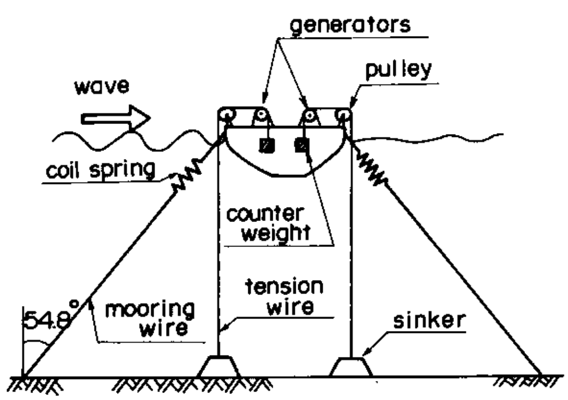


Fig. 14 Expeniment apparatus

The floating body was moored by 4 sets of stainless steel wire and coil spring, instead of by chains. The angle between the mooring wire and vertical line was 55°. The spring constant of the coils was about 1.0 kg/cm.

The damping devices were, as shown in Fig. 14, composed of electric generators, pulleys, sinkers, tension wires and counter weights. One end of the tension wire was connected to the sea bed (this is, to the sinker), and the other end was led into the hull of the floating body via pulleys and a damping device, and was attached to the counter weight (30 kg).

The gears, clutches and electric generators were similar to those used in the "Dolphin Type" device tests.

In order to control the damping factors 70Ω electric resistors were connected to the electric generators in series. The electric circuits were also similar to those used in the "Dolphin Type" device tests.

The measured items were wave heights, motions of the floating body (heaving, rolling and swaying motion) and generated voltage, and these were measured by systems similar to those used in "Dolphin Type" device tests.

(3) Wave Conditions

The experiments were performed in regular waves. The wave height was constant, 0.1m, and the wave length varied from 1.3m to 9.3m.

4.2. Results of the Experiments

(1) Motions of the Floating Body

Figs. 15, 16 and 17 show the heaving, rolling and swaying motions of the floating body.

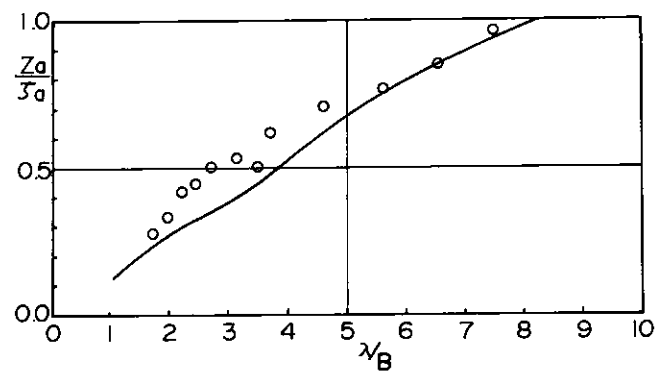


Fig. 15 Heaving motion

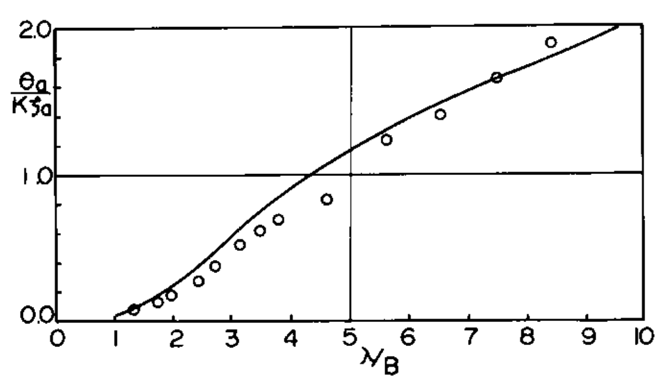


Fig. 16 Rolling motion

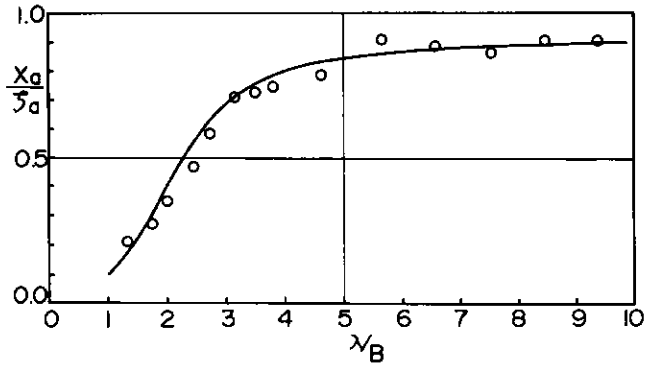


Fig. 17 Swaying motion

In the figures, the small circles represent the results of the experiment and the full lines the theoretical values, which were divided by the incident wave amplitude ($\zeta a$) or wave slope ($K\zeta a$).

Close agreement between the results of the experiment and the theoretical values was observed.

(2) Characteristics of Wave Transmission and Wave Reflection

Figs. 18 and 19 give the wave reflection coefficient $C_R$ and the wave transmission coefficient $C_R$. $C_T$ is about 0.5 for those waves with 3.8m wave length ($\lambda/B$ = 3.8).

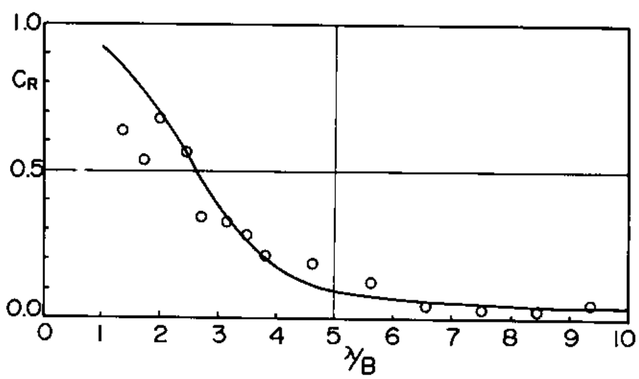


Fig. 18 Wave reflection coefficient

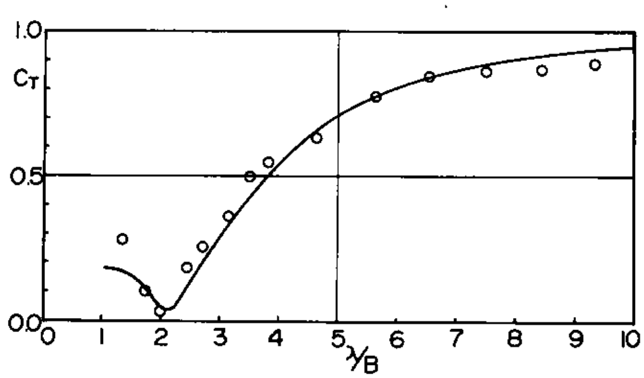


Fig. 19 Wave transmittion coefficient

As for $C_R$ and $C_T$, the results of the experiment agreed well with the theoretical values, and it was thus confirmed that the theory was reasonable.

(3) Efficiency of Wave Power Absorption

Wave energy absorption efficiency $\eta_w$ and energy conversion efficiency $\eta_E$ (defined by equations (28) and (29), respectively) are shown in Figs. 20 and 21.

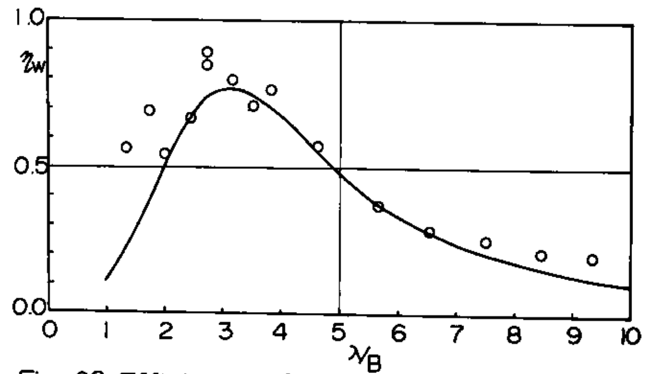


Fig. 20 Efficiency of wave power absorption

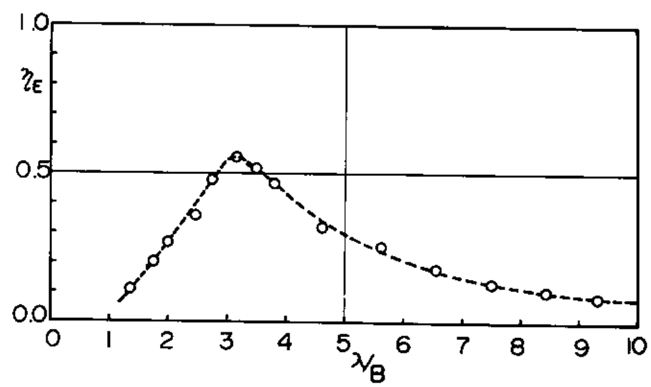


Fig. 21 Efficiency of wave energy conversion

$\eta_w$ has the maximum value for $\lambda/B$=3.0, the theoretical value being 76% and experiment value 80%. The theoretical results and the experiment results agreed well. (Scattering of $\eta_w$ when $\lambda/B$=3 were caused by errors made in the calculation of the reflected wave heights.)

$\eta_E$, as shown in Fig. 21, also has a maximum value for $\lambda/B$=3.0; however, the value is about 55%. It is believed that the difference between $\eta_w$ and $\eta_E$ was due to the mechanical loss in the gears.

Compared with that of the "Dolphin Type" device, $\eta_w$ of the "Chain Type" device is somewhat inferior. The reason may be considered as that the swaying motion of the floating body for the "Chain Type" device is not controlled to absorb wave energy, though the swaying motion of the "Dolphin Type" device is restrained. Therefore, the reflecting energy caused by the non-controlled swaying motion makes absorption efficiency low.

## 5. WAVE POWER ABSORPTION OF IRREGULAR WAVES

It is common to adopt the linear superposition theorem for predicting some phenomena that occur in irregular waves; for example, ship motions in a random seaway. In this section we shall consider the applicability of this theorem for predicting wave power absorption of irregular waves in the two-dimensional case.

The power absorbed by a wave energy conversion device in irregular waves can be represented as the difference between incident wave power and the total power of reflected and transmitted waves. If we adopt the linear superposition theorem to predict reflected and transmitted waves, then we can denote incident wave power $P_I'$, reflected wave power $P_R'$ and transmitted wave power $P_T'$ as the following:

$$\begin{aligned} P_I' &= \frac{1}{2}\rho g^2 \int_0^\infty \frac{S(\omega)}{\omega}\, d\omega \\ P_R' &= \frac{1}{2}\rho g^2 \int_0^\infty \frac{C_R^2(\omega) S(\omega)}{\omega}\, d\omega \\ P_T' &= \frac{1}{2}\rho g^2 \int_0^\infty \frac{C_T^2(\omega) S(\omega)}{\omega}\, d\omega \end{aligned} \tag{31}$$

where $S(\omega)$ is an energy spectrum of an incident wave. Absorbed power $P_A'$ is described in the following:

$$\begin{aligned} P_A' &= P_I' - (P_R' + P_T') \\ &= \frac{1}{2}\rho g^2 \int_0^\infty \frac{\{1 - C_R^2(\omega) - C_T^2(\omega)\} S(\omega)}{\omega}\, d\omega \end{aligned} \tag{32}$$

Consequently, efficiency of wave power absorption is

$$\begin{aligned} \eta_W &= \frac{P_A'}{P_I'} \\ &= \int_0^\infty \frac{\{1 - C_R^2(\omega) - C_T^2(\omega)\} S(\omega)}{\omega}\, d\omega \Big/ \int_0^\infty \frac{S(\omega)}{\omega}\, d\omega \end{aligned} \tag{33}$$

If reflected coefficient $C_R(\omega)$ and transmittion coefficient $C_T(\omega)$ in regular waves are known, absorbed power $P_A'$ and efficiency of wave power absorption $\eta_w$ can be easily calculated by using equation (32) and (33).

In these equations energy spectrums of reflected and transmitted waves are expressed by the product of the incident wave energy spectrum and the square of the reflection or transmission coefficient in regular waves respectively. In order to verify these relations, we conducted an experiment using a model of the "Dolphin Type" at the wave basin.

Characteristics of the model, experiment apparatus and measuring systems are described in §3. A typical example of an incident wave energy spectrum, reflected wave energy spectrum and transmitted wave energy spectrum measured during the model test is shown in Fig. 22 and Fig. 23.

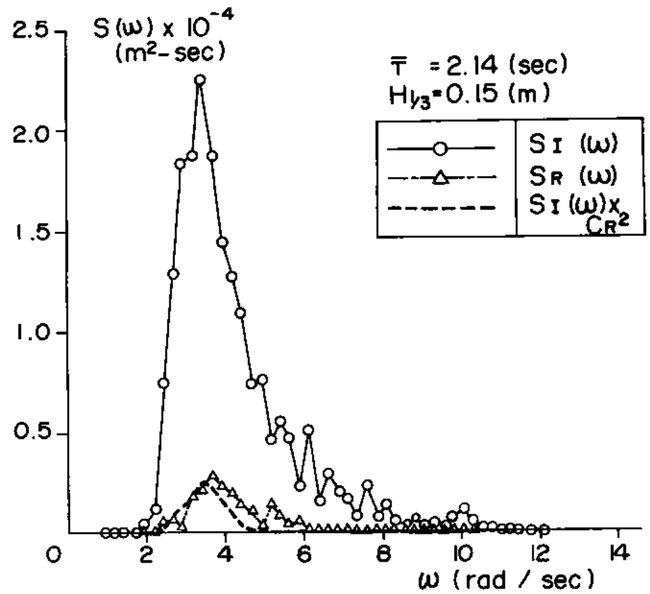


Fig. 22 Energy spectrums of incident wave and reflected wave

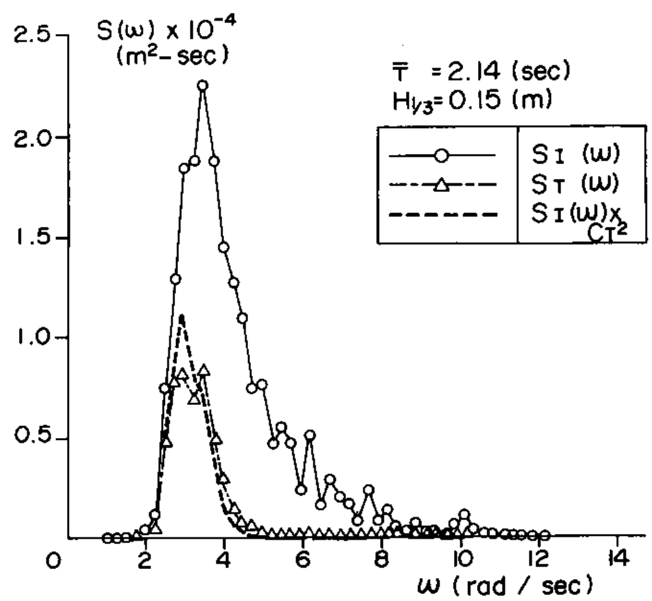


Fig. 23 Energy spectrums of incident wave and transmitted wave

The full lines in these figure represent the results of the model experiment and the broken lines represent the results of calculation in which theoretical values were used for the reflection and transmission coefficients and experimental values for the incident wave energy spectrum. These figures show that the results of calculation agree well with the experimental results on the reflection and transmission coefficients in irregular waves. From this result, we may conclude that equation (31) is useful for predicting the reflected and transmitted wave powers in irregular waves. Therefore we can reasonably predict absorbed power and efficiency of wave power absorption by using equation (32) and (33).

## 6. CONCLUSION

A theory for predicting wave energy absorption by using a two-dimensional, half-immersed, symmetrical body with wave energy conversion systems is outlined in§2. The theory was developed by the use of the condition for achieving complete wave absorption and led to the conclusion, that the complete wave absorption can be attained if the natural frequencies of the symmetrical and asymmetrical oscillation modes are tuned to the desired frequency and the damping coefficients of the wave energy conversion systems are the same as the wave generation damping coefficients for the symmetrical and asymmetrical oscillation modes.
Furthermore it was showed that, if the above conditions are satisfied, a complete wave energy absorption can be also attained and each half of an incident wave power is absorbed by each energy conversion systems for the symmetrical or asymmetrical oscillation modes.

Applying the conclusions of the theory, we proposed two new wave energy conversion devices and carried out theoretical calculations and model experiments. The theoretical and experimental studies concerning motion and wave absorption showed good agreement with each other. The results of model experiments conducted on the "Dolphin Type" were especially good as they verified a complete wave energy absorption which had been predicted by the theoretical calculations.

On the other hand it was difficult to attain a complete wave energy absorption in the case of the "Chain Type". Because the floating body oscillates in three degrees of freedom, two asymmetrical radiation waves are generated by the swaying and rolling motions and, although the natural period of rolling can be easily attuned to a given period, it is very difficult to attune the natural period of swaying in the same manner because the restoring forces of the mooring lines are too small. For these reason, in the present study we could not attain complete wave energy absorption. Thus, the maximum efficiency of wave energy absorption for this type device was only 80% both in the model experiment and in theoretical calculations.

In §5, the prediction of wave energy absorption in irregular waves was considered and it was confirmed that the linear superposition theorem is useful for this purpose. We also showed that the energy spectrums of reflected and transmitted waves can be predicted as the product of the incident wave energy spectrum and the square of the reflection or the transmission coefficient respectively.

## AKNOWLEDGEMENT

We wish to acknowledge kind advices that we received from Professor S. Motora and his constant encouragement.

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## APPENDIX I

We denote the velocity potential $\Phi$(x, y, t) as:

$$\Phi(x,y,t)=R_e\left\{\varphi(x,y)e^{i\omega t}\right\} \tag{A.1}$$

where (x, y) satisfied the following linearized free surface condition.

$$K\varphi(x,0)+\frac{\partial}{\partial y}\varphi(x,0)=0 \tag{A.2}$$

where K = wave number = $2\pi/\lambda$ while the surface elevation is expressed by using as:

$$H(x,t)=R_e\left\{\eta(x)\,e^{i\omega t}\right\} \tag{A.3}$$

where

$$\eta(x)=\frac{\omega}{ig}\varphi(x,0) \tag{A.4}$$

The incident wave and the diffraction wave velocity potentials are written as:

$$\varphi_i(x,y)=i\frac{g}{\omega}\phi_i(x,y) \tag{A.5}$$

where i = 0 : incident wave
i = 4 : diffraction wave

and

$$\phi_o(x,y)=e^{-Ky+iKx} \tag{A.6}$$

On the other hand, the radiation wave velocity potential is written as:

$$\varphi_i(x,y)=i\omega X_i\phi_i(x,y) \tag{A.7}$$

where i = 1 : swaying
= 2 : heaving
= 3 : rolling

and $X_i$ : amplitude of i-th mode oscillation

Assuming that the motions are linear and harmonic, the total velocity potential $\varphi$(x, y) can be written as:

$$\varphi(x,y)=i\omega\sum_{i=0}^{4}X_i\phi_i(x,y) \tag{A.8}$$

where $X_o = X_4 = 1/K$

Here the Kochin function $H_i^{\pm}(K)$ can be introduced as:

$$H_i^{\pm}(K)=\int_c\left(\frac{\partial\phi_i}{\partial n}-\phi_i\frac{\partial}{\partial n}\right)e^{-Ky\pm iKx}\,ds \tag{A.9}$$

Hence, the progressing wave at $x \gg 1$ is expressed asymptotically as:

$$\phi_i(x,y)\longrightarrow iH_i^{\pm}(K)e^{-Ky\mp iKx} \tag{A.10}$$

Let us devide the velocity potential into real part with subscript c, and imaginary part with subscript s, and is described as:

$$\phi=\phi_c+i\phi_s \tag{A.11}$$

The corresponding Kochin functions are written as:

$$H_{c,s}^{\pm}(K)=\int_c\left(\frac{\partial\phi_{c,s}}{\partial n}-\phi_{c,s}\frac{\partial}{\partial n}\right)e^{-Ky\pm iKx}\,ds \tag{A.12}$$

and

$$H_i^{\pm}(K)=H_{ic}^{\pm}(K)+iH_{is}^{\pm}(K) \tag{A.13}$$

If the floating body has a symmetrical sectional form, the following relations are obtained.

$$H_1^{+}(K)=-H_1^{-}(K)\quad(\text{swaying}) \tag{A.14}$$

$$H_2^{+}(K)=H_2^{-}(K)\quad(\text{heaving}) \tag{A.15}$$

$$H_3^{+}(K)=-H_3^{-}(K)\quad(\text{rolling}) \tag{A.16}$$

Furthermore, the Kochin function corresponding to the diffraction wave is described by that of radiation wave as:

$$H_4^{\pm}(K)=\frac{H_{2s}^{+}(K)}{\overline{H_2^{+}(K)}}\pm\frac{H_{1s}^{+}(K)}{\overline{H_1^{+}(K)}} \tag{A.17}$$

where $\overline{Hi^{+}(K)}$ means the conjugate of $H_i^{+}(K)$

Substituting (A.10), (A.8) into expression (A.4), the progressing waves propagate to outward are described at $|x|\to\infty$ as:

$$\eta^{\pm}(x)=iK\sum_{i=1}^{4}X_iH_i^{\pm}(K)e^{\mp iKx} \tag{A.18}$$

From expression (A.18), the amplitude of the wave propagates in the +x direction (reflected wave) is

$$A_R=iK\sum_{i=1}^{4}X_iH_i^{+}(K) \tag{A.19}$$

Similarly that of the transmitted wave, including the incident wave, is

$$A_T=1+iK\sum_{i=1}^{4}X_iH_i^{-}(K) \tag{A.20}$$

## APPENDIX II

This appendix describes the calculation method of reflected wave height[16].

Generally, using the Fourier Analysis, time domain data $J(t)$ expands in a series of wave components with circular frequency $\omega_n$(rad/sec).

The wave amplitude $J_a$, of which wave frequency is $\omega_o$(rad/sec), can be described by the following equations:

$$J_a=\sqrt{A^2+B^2} \tag{A.21}$$

$$A=\frac{1}{t_o}\int_0^{t_o}J(t)\cos\omega_o t\,dt \tag{A.22}$$

$$B=\frac{1}{t_o}\int_0^{t_o}J(t)\sin\omega_o t\,dt \tag{A.23}$$

where $t_o$ is the time duration of $J(t)$.

When the Fourier coefficients A and B are known, $J_{\omega_o}(t)$, wave component having circular frequency $\omega_o$, can be expressed as the following equation:

$$\zeta_{\omega_0}(t) = A\cos\omega_0 t + B\sin\omega_0 t \qquad (A.24)$$

It should be noted that if the Fourier coefficients of the wave data measured by means of one set of height meters located in front of the floating body (see Fig. 4), incident wave amplitude $\zeta_{aI}$ and reflected wave amplitude $\zeta_{aR}$ are calculated by using the following equations:

$$\zeta_{aI} = \frac{1}{2|\sin k\Delta l|}\left[(A_2 - A_1\cos k\Delta l - B_1\sin k\Delta l)^2 + (B_2 + A_1\sin k\Delta l - B_1\cos k\Delta l)^2\right]^{1/2} \qquad (A.25)$$

$$\zeta_{aR} = \frac{1}{2|\sin k\Delta l|}\left[(A_2 - A_1\cos k\Delta l + B_1\sin k\Delta l)^2 + (B_2 - A_1\sin k\Delta l - B_1\cos k\Delta l)^2\right]^{1/2} \qquad (A.26)$$

where $A_1$ and $B_1$ are the Fourier coefficients derived from the measured value of wave height meter W1, and $A_2$ and $B_2$ are the Fourier coefficients of W2, respectively.

Wave reflection coefficient $C_R$ can be described as follow:

$$C_R = \zeta_{aR} / \zeta_{aI} \qquad (A.27)$$