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ON THE VISCOUS FLOW AROUND A THIN CYLINDER

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SUMMARY

Reciprocity theorems are introduced into the two-dimensional steady Navier-Stokes flow.
They could be useful to estimate approximately the resistace, frictional or pressure and other forces in the same way as Rayleigh-Ritz or Galerkin's method.
They give also the representation of Navier-Stokes flow field which was originally obtained by C.W.Oseen.

The boundary integral equation to solve the boundary value problem are discussed, proposed an iterative method to solve them and shown, for a thin cylinder, the relation to the boundary layer theory.

The approximate solution for a flat plate are given, which gives the frictional resistance very close to the Blasius one.

1. Introduction

In general, Oseen flow is a good approximation of real flow qualitatively in laminar region but wrong quantitatively and, for example, it gives the drag of a flat plate about two times higher than Blasius formula[4]. However, his original scheme was by so-called Oseen kernel to express Navier-Stokes flow field of which equation is assumed as non-homogeous one of Oseen's linearized one [1]. We could hardly find the study in this method actually, but the mathematician used to this method by making use of Stokes kernel instead of Oseen's[2].

In fact, Oseen's equation is not self-adjoint so that we could not define non-negative metric of functions like as in Stokes equation in which we could taken the dissipation or kinetic energy integral as such metric. This difficulty could be detoured by introducing a reverse flow, that is, a flow in which the uniform flow is reversed or of which equation is adjoint to the original flow.

Thus, at first, introducing a bi-linear functional similar to the dissipation integral, we get the reciprocity between Navier-Stokes, Oseen and potential flow.

This reciprocity gives many useful formula to estimate the forces acting on the body and also the general representation of velocity, vorticity and total pressure.

These may be useful to a recent trend to solve, Navier-Stokes flow numerically by the so-called boundary element method [5].

When we consider a very thin body, it will be confirmed that the theory would be consistent with the boundary layer one and a flat plate resistance is calculated approximately and found that it is nearly equal to Blasius one at the second approximation [6].

2. Equation of motion and boundary condition

Let us consider Navier-Stokes flow around a cylinder in a stream of the unit speed taking the co-ordinate system like as Figure. Writing the perturbation velocity as **u**=(u,v), the pressure p, the viscosity μ, the density ρ, the vorticity ζ and the total pressure G. We may write the equation of motion as follows:

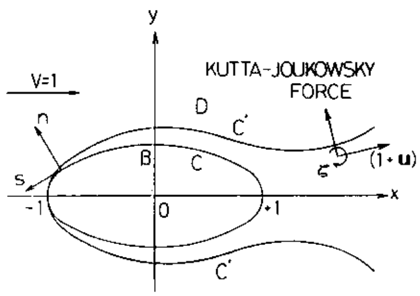

FIGURE Co-ordinate system

$$\frac{1}{\rho} G_x(x,y) - \nu\nabla^2 u = v\zeta$$

$$\nu = \mu/\rho \quad , \qquad (1)$$

$$\frac{1}{\rho} G_y(x,y) - \nu\nabla^2 v = -(1+u)\zeta$$

$$G(x,y) = p + \frac{\rho}{2}\{(1+u)^2 + v^2\} \quad , \qquad (2)$$

$$\zeta(x,y) = v_x - u_y \quad , \tag{3}$$

and the equation of continuity

$$u_x + v_y = 0 \quad , \tag{4}$$

where the suffix means partial differentiation.

Differentiating and adding both equation (1) and using the continuity, we have alternatively two equations, namely,

$$\frac{1}{\rho}\nabla^2 G = (v\zeta)_x - \{(1+u)\zeta\}_y \tag{5}$$

$$\nu\nabla^2\zeta = (v\zeta)_y + \{(1+u)\zeta\}_x = (1+u)\zeta_x + v\zeta_y \tag{6}$$

On the boundary C of a cylinder, the velocity must be

$$u = -1 \quad , \; v = 0 \qquad \text{on } C \tag{7}$$

and the equation of motion (1) becomes as

$$G = p$$
$$p_x = -\mu\zeta_y \quad , \; p_y = \mu\zeta_x \qquad \text{on } C \tag{8}$$

or $\quad p_n = -\mu\zeta_s \quad , \; p_s = \mu\zeta_n$

where n means outward normal and s the distance along C.

The forces acting on the cylinder are given in the following form.

$$X = -px_n + \mu\zeta x_s - 2\mu v_s$$
$$Y = -py_n + \mu\zeta y_s + 2\mu u_s \tag{9}$$

so that the last term of both formula can be omitted by the boundary condition (7).
Now, the equations (1) are not self-adjoint so that we

may need adjoint solutions to discuss their properties. Thence, let us introduce a reverse flow field, that is, a flow in which the uniform flow direction is reversed and quatities related to this flow may be shown by the mark (~).

The equations of motion are

$$\frac{1}{\rho}\tilde{G}_x - \nu\nabla^2\tilde{u} = \tilde{v}\tilde{\zeta}$$

$$\frac{1}{\rho}\tilde{G}_y - \nu\nabla^2\tilde{v} = -(\tilde{u}-1)\tilde{\zeta} \tag{1'}$$

$$\tilde{G} = p + \frac{\rho}{2}\{(\tilde{u}-1)^2 + \tilde{v}^2\} \quad , \tag{2'}$$

that is, in all equations the sign of uniform flow must be changed, and boundary condition

$$\tilde{u} = 1 \quad , \quad \tilde{v} = 0 \qquad \text{on } C \tag{7'}$$

Lastly, we define the stream function as

$$\left.\begin{aligned} \psi_x &= -v \quad , \quad \psi_y = u \\ \nabla^2\psi &= -\zeta \end{aligned}\right\} \tag{10}$$

3. Reciprocity [6,8]

Let as introduce a functional ;

$$\tilde{E}(\mathbf{u},\mathbf{u}) = \mu\iint_D [\ 2(u_x\tilde{u}_x+v_y\tilde{v}_y) + (u_y+v_x)(\tilde{u}_y+\tilde{v}_x)]\ dxdy \tag{11}$$

This is similar to the dissipation integral but has no physical meaning and serve to introduce the reciprocity of flow.

Differentiating partially and using the equation of motion and continuity, we may write (11) as

$$\tilde{E} = \int_C [\ G(\tilde{\mathbf{u}},\mathbf{n}) - \mu\zeta(\tilde{\mathbf{u}},\mathbf{s})\ ]\ ds$$

$$+ \rho\iint_D \zeta[\ v\tilde{u} - (1+u)\tilde{v}\ ]\ dxdy \quad , \qquad (12)$$

and alternatively

$$\tilde{E} = \int_C [\ \tilde{G}(\mathbf{u},\mathbf{n}) - \mu\tilde{\zeta}(\mathbf{u},\mathbf{s})\ ]\ ds$$

$$+ \rho\iint_D \tilde{\zeta}[\ \tilde{v}u - (\tilde{u}-1)\tilde{v}\ ]\ dxdy \quad , \qquad (13)$$

These formulas must be the same, that is,

$$\int_C [\ p(\tilde{\mathbf{u}},\mathbf{n}) - \mu\zeta(\tilde{\mathbf{u}},\mathbf{s}) - \tilde{p}(\mathbf{u},\mathbf{n}) + \mu\tilde{\zeta}(\mathbf{u},\mathbf{s})\ ]\ ds$$

$$= \rho\iint_D [\ \tilde{\zeta}\{\tilde{v}u - (\tilde{u}-1)v\} - \zeta\{\ v\tilde{u} - (u+1)\tilde{v}\}\ ]\ dxdy \qquad (14)$$

This is the first reciprocity.

Secondly, if the reverse flow is a linear Oseen's and we have instead of (13)

$$\tilde{E} = \int_C [\ (\tilde{p}+\rho\tilde{u})(\mathbf{u},\mathbf{n}) - \mu\tilde{\zeta}(\mathbf{u},\mathbf{s})\ ]\ ds$$

$$+ \rho\iint_D \tilde{\zeta}v\ dxdy \quad . \qquad (15)$$

Equating this to the formula (12), we have the second reciprocity ,

$$\int_C [\ \tilde{p}(\mathbf{u},\mathbf{n}) - \mu\tilde{\zeta}(\mathbf{u},\mathbf{s}) - \{\ p + \frac{\rho}{2}(u^2+v^2)\}(\tilde{\mathbf{u}},\mathbf{n})$$

$$+ \mu\zeta(\tilde{\mathbf{u}},\mathbf{s}) + \rho(u\tilde{u}+v\tilde{v})x_n\ ]\ ds$$

$$= \rho\iint_D \zeta(v\tilde{u}-u\tilde{v})\ dxdy \quad , \qquad (16)$$

Thirdly, if the reverse flow has a potential like as

$$\tilde{u} = \tilde{\Phi}_x \quad , \quad \tilde{v} = \tilde{\Phi}_y \qquad , \tag{17}$$

and E vanishes by (13), then we have third reciprocity from (12) as follows ;

$$\int_C (G\tilde{\Phi}_n - \mu\zeta\tilde{\Phi}_s)\, ds = \rho\iint_D \zeta[(1+u)\tilde{v} - v\tilde{u}]\, dxdy \quad , \tag{18}$$

Lastly, if the reverse flow is a degenarate Oseen flow which is defined as its frictional part and the reciprocity becomes, like as (16),

$$\int_C [\ -\mu\tilde{\zeta}(\mathbf{u},\mathbf{s}) - G\tilde{\Psi}_s - \mu\zeta\tilde{\Psi}_n + \rho(u\tilde{u}+v\tilde{v})x_n\ ]\, ds$$

$$= \quad \rho\iint_D \zeta(v\tilde{\Psi}_y + u\tilde{\Psi}_x)\, dxdy \qquad , \tag{19}$$

4. Velocity field [6,8]

In the second reciprocity (16), putting Oseen's kernel as the reverse flow and after some manipulations, we have the following representation of Navier-Stokes flow, namely,

$$\Psi(Q) = \frac{1}{\rho}\int_C [\ p(P)K_s(P,Q) + \mu\zeta(P)K_n(P,Q)\ ]\, ds(P)$$

$$+ \iint_D \zeta(P)[\ v(P)K_y(P,Q) + u(P)K_x(P,Q)\ ]\, dxdy(P) \tag{20}$$

Hence, we may have by direct calculation,

$$\mu\zeta(Q) = \frac{1}{2\pi}\int_C [\ p\Phi_s + \mu\zeta\Phi_n\ ]\ ds$$

$$+\frac{1}{2\pi}\iint_D \zeta[\ v\Phi_y + u\Phi_x\ ]\ dxdy \qquad (21)$$

where

$$K(P,Q) = \frac{1}{2\pi}\int_{-\infty}^{x'} [\ \log R + \phi(P,Q)\ ]\ dx' \qquad (22)$$

$$\Phi(P,Q) = K_o(kR)\ e^{k(x'-x)} \to \begin{cases} 0 \text{ for } x' < x \\ \sqrt{\frac{\pi}{2kR}}\, e^{-\frac{k(y-y')^2}{2(x'-x)}} \end{cases} \qquad (23)$$

Here, it is noticed that ($\zeta dxdy$) is a circulation around an area element so that $\rho(\ \zeta dxdy) \times (u,v)$ is a Kutta-Joukowsky force acting on the fluid element.
Thus, a stream function is represented by forces acting on the fluid multiplied by Oseen kernel.
The velocity field is clearly composed by two components, that is,

$$\psi = \psi^P + \psi^F \ , \quad u = u^P + u^F \ , \quad v = v^P + v^F \ , \ (24)$$

$$u^P(Q) = \frac{1}{2\pi\rho}\int_C \left(p\frac{\partial}{\partial n} - \mu\zeta\frac{\partial}{\partial s} \right) \log R\ ds$$

$$+\frac{1}{2\pi}\iint_D \zeta \left(v\frac{\partial}{\partial x} - u\frac{\partial}{\partial y} \right) \log R\ dxdy \quad , \ (25)$$

$$u^F(Q) = \frac{-1}{2\pi\rho}\int_C [\ X(2k\phi+\phi_x) + Y\phi_y\ ]\ ds$$

$$+\frac{1}{2\pi}\iint_D \zeta [\ v(2k\phi+\phi_x) + u\phi_y\ ]\ dxdy \ , \qquad (26)$$

$$v^P(Q) = \frac{1}{2\pi\rho}\int_C \left(p\frac{\partial}{\partial s} + \mu\zeta\frac{\partial}{\partial n} \right) \log R \, ds + \frac{1}{2\pi}\iint_D \left(v\frac{\partial}{\partial y} + u\frac{\partial}{\partial x} \right) \log R \, dxdy \quad , \tag{27}$$

$$v^F(Q) = \frac{-1}{2\pi\rho}\int_C \left(X\Phi_y - Y\Phi_x \right) ds \quad , \tag{28}$$

At first, comparing (28) with (21), we have

$$\nu\zeta = v^F \quad , \tag{29}$$

Secondly, by making use of the reciprocity (18) and the logarithmic potential as the reverse flow, we have

$$G(Q) - \frac{\rho}{2} = \frac{-1}{2\pi}\int_C \left[G(P)\frac{\partial}{\partial n} - \mu\zeta\frac{\partial}{\partial s} \right] \log R \, ds - \frac{\rho}{2\pi}\iint_D \zeta \left[V\frac{\partial}{\partial x} - (1+u)\frac{\partial}{\partial y} \right] \log R \, dxdy \tag{30}$$

and that, putting the boundary condition (7), we have

$$\frac{1}{2\pi}\iint_D \zeta\frac{\partial}{\partial y}\log R \, dxdy = u(Q) \quad , \tag{31}$$

Then, comparing (30) with (25), we have

$$\frac{G}{\rho} - \frac{1}{2} = u - u^P = u^F \tag{32}$$

and then $P/\rho = \frac{1}{2} + u^F = -\frac{1}{2} - u^P$, on C , (33)

Putting (26) into the right hand side of (32) this expression is an alternative form with (30).

Lastly, in the same way as (30) ; we have

$$\frac{1}{2\pi}\iint_D \zeta\frac{\partial}{\partial x}\log R \, dxdy = -v(Q) \quad , \tag{34}$$

Then, adding this to (27) side by side, we have

$$v^F = \nu\zeta = v - v^P = \frac{-1}{2\pi\rho} \int_C \left(p\frac{\partial}{\partial s} + \mu\zeta\frac{\partial}{\partial n} \right) \log R \, ds$$

$$- \frac{1}{2\pi} \iint_D \zeta \left[v\frac{\partial}{\partial y} + (1+u)\frac{\partial}{\partial x} \right] \log R \, dxdy \quad , \qquad (35)$$

This is an alternative expression with (21).

Now, in far field outside the wake, the velocity has a potential, as is well known, and it becomes as follows from (20).

$$\psi^P(P) \rightarrow \frac{D}{2\pi\rho}\, \theta + \frac{L}{2\pi\rho} \log r + \frac{1}{2\pi r^2} (My - xN) \; , \; (36)$$

where $x = r \cos \theta$, $y = r \sin \theta$.

$$D = \frac{1}{\rho}\int_C X \, ds + \rho\iint_D \zeta v \, dxdy = \int_C X \, ds \; ,$$

$$L = \frac{1}{\rho}\int_C Y \, ds - \rho\iint_D \zeta u \, dxdy = \int_C Y \, ds \; , \qquad (37)$$

The double integral terms vanishes, as easily verfied. The coefficients of doublet terms are

$$M = \frac{1}{\rho} \int_C (Xx - Yy) \, ds + \rho\iint_D \zeta (vx + uy) \, dxdy \; ,$$

$$N = \frac{1}{\rho} \int_C (Xy + Yx) \, ds + \rho\iint_D \zeta (vy - ux) \, dxdy \; , \qquad (38)$$

By applying the theorem (18) to the domain surrounded by C and C' which is just outside boundary curve of frictional wake and on which the total pressure is constant and the vorticity vanishes, we have

$$M = \iint_D \zeta y \, dxdy = \iint_D [\ \psi \nabla^2 y - y \nabla^2 \psi\ ]\, dxdy$$

$$= \int_{C'} (\ \psi y_n - y\, \psi_n\ )\, ds \quad , \tag{39}$$

$$N = -\iint_D \zeta x \, dxdy = \int_{C'} (\ x\, \psi_n - \psi\, x_n\ )\, ds \quad ,$$

These are well-known expressions for the potential flow.

On the other hand, far aftwards in the frictional wake, the dominant term will be u and that

$$u^P = O(\frac{1}{r}) \quad , \tag{40}$$

by (36), so that u may equal the frictional term u^F and it becomes by (26)

$$u \fallingdotseq u^F \fallingdotseq - \frac{kD}{\pi\rho}\phi + \frac{1}{2\pi}\iint_D \zeta(\ 2k\Phi - u\Phi_y\ )\, dxdy \quad , \tag{41}$$

This first approximation is to be

$$u(P) \fallingdotseq - \frac{D}{\rho}\sqrt{\frac{k}{2\pi x}}\ e^{\frac{-ky^2}{2x}} \quad , \tag{42}$$

and then, putting this into the double integral term of (41), it is of the order

$$\frac{k}{x}(\frac{D}{\rho})^2 e^{\frac{-\alpha k y^2}{2x}} \quad , \qquad \alpha \geq 1 \quad , \tag{43}$$

that is, this vanishes as small as the potential part (40) , so that this may be neglected in this stage of approximation.

Therefore, the approximation (42) may be valid in this stage.

The observed wake distribution may be represented in a similar form except that the kinematic viscosity is replaced by the eddy one [3].
This means that the actual wake zone is much broader than the one of steady Navier-Stokes flow.

5. Boundary integral equation and boundary layer theory

The boundary integral equations are give by (20) to (28) and may be written in the following form.

$$\mathbf{u} = A_C(\mathbf{X}) + A_D(\zeta ; \mathbf{u}), \qquad \zeta = B_C(\mathbf{X}) + B_D(\zeta ; \mathbf{u}), \tag{44}$$

where $\mathbf{u} = (u,v)$ and $\mathbf{X} = (X,Y)$.
These equations must satisfy the boundary condition (7) on C and be consistent in the whole fluid.

They are non-linear so that the iterative method may be used practically in any way and the first approximation may be taken as Oseen flow.
That is,

$$\mathbf{u}_o = A_C(\mathbf{X}_o) \qquad \zeta_o = B_C(\mathbf{X}_o) \tag{45}$$

For example, one of methods is to proceed iteratively as follows :

$$\mathbf{u}_N = A_C(\mathbf{X}_N) + A_D(\zeta_{N-1} ; \mathbf{u}_N) , \qquad \zeta_N = B_C(\mathbf{X}_N) + B_D(\zeta_{N-1} ; \mathbf{u}_N) , \tag{46}$$

The first equation of the above is quasi-linear, can be solved, gives $\mathbf{u}_n$ and $\mathbf{X}_n$ and the second one gives the next approximation of vorticity.

This method of numerical solution, so-called boundary element method, has now started and there are few reports but it must become of the important method [5].

Now, if the breadth of a cylinder is very narrow and then these equations must contain or be consistent with the boundary layer theory, therefore let us consider these relations.

At first, for a very thin cylinder, we may write the equation (35) approximately as follows, after some reduction,

$$\nu\zeta = v - v^P = -\frac{1}{\pi}\int_{-1}^{1} \nu\zeta\frac{\partial}{\partial y}\log R\,dx$$

$$-\frac{1}{2\pi}\iint_D [\,(1+u)\,\zeta_x + v\,\zeta_y\,]\,\log R\,dxdy \qquad (47)$$

Just outside thin boundary layer where the velocity vanishes, the stream function must be zero, so that it may become

$$v^P = v = -(\,y + \delta\,)_x \quad , \qquad (48)$$

where y means offset of the boundary and δ the thickness of boundary layer. This is ordinary approximation [3].

On the other hand, the integral of the right hand side may be estimated there by its residue as follows .

$$\int_0^{\delta} [\,(1+u)\,\zeta_x + v\,\zeta_y\,]\,y\,dy = \nu\zeta\Big|_{y=0} \quad , \qquad (49)$$

because the logarithmic kernel varies more slowly compared with the vorticity.

This is the same equation as von Karman's momentum integral as easily seen [3].

Therefore, our boundary integral equation contains the boundary layer theory in the limit.
Moreover, taking up the equation (30), we may have following approximation in the same way,

$$\frac{G}{\rho} - \frac{1}{2} = -1 - u^P \Big|_{y=0}$$

$$= \frac{-1}{\pi\rho} \int_{-1}^{1} [\, p\frac{\partial}{\partial y} \log R + \mu\zeta\frac{\partial}{\partial x} \log R \,]\, dx$$

$$- \frac{1}{2\pi} \iint_D [\, \zeta v - (1+u)\frac{\partial}{\partial x} \,]\, \frac{\partial}{\partial x} \log R \, dxdy \,, \quad (50)$$

Putting the equation (49) into the above, we have the same formula as (33).

$$- u^P \Big|_{y=0} = \frac{p}{\rho} + \frac{1}{2} \quad . \qquad (51)$$

Then, adding this to (26) and taking up dominant terms, we have as the boundary equation,

$$u = -1 = \frac{1}{\pi} \int_{-1}^{1} \zeta\Phi \, dx$$

$$+ \frac{1}{2\pi} \iint_D (\, 2kv\Phi - u\,\Phi_y \,)\zeta \, dxdy \,, \qquad (52)$$

This equation may determine directly the frictional stress by an appropriate method.

For the simplicity sake, we will estimate a flat plate resistance.
Oseen solution of a flat plate is as follows [4];

$$u = -\frac{2}{\sqrt{\pi}} \int_{\eta}^{\infty} e^{-u^2} du \fallingdotseq -(\, 1 - \frac{2}{\sqrt{\pi}}\eta \,) \,,$$

$$v = \frac{1}{\sqrt{2\pi k(1+x)}} \left(1 - e^{-\eta^2} \right) \fallingdotseq \frac{\eta^2}{\sqrt{2\pi k(1+x)}} , \qquad (53)$$

$$\zeta = -\sqrt{\frac{2k}{\pi(1+x)}}\, e^{-\eta^2} , \qquad \eta^2 = \frac{ky^2}{2(1+x)} ,$$

Then, we put these values into (52) except that ζ is multiplied by a constant A and integrating approximately by using the asymptotic expansion (23), we have

$$A = .568 , \qquad (54)$$

The flat plate resistance of Oseen flow is given as follows [4,6];

$$C_F = \frac{D}{\rho L V^2} = \frac{4}{\sqrt{\pi R}} = \frac{2.257}{\sqrt{R}} , \quad R = \frac{VL}{\nu} = 4k , \qquad (55)$$

where L means its length.
Therefore the above value A gives the second approximation;

$$C_F = 1.281/\sqrt{R} , \qquad (56)$$

This is to be compared with Blasius value 1.328 and 3.5% less than that [6].
This result may encourage us and show that the present theory might be reliable.

Lastly, the pressure resistance may be integrated from (51) as in the boundary layer theory.
In the present theory, we have the other formula (18) but in this case the pressure resistance becomes zero under the situation where the equation (49) is valid [3].

6. Conclusion

For two dimensional Navier-Stokes flow, we have introduced various reciprocity theorems and represented the velocity field and total pressure by making use of Oseen kernel and logarithmic potential.
The reciprocity theorem may be used to obtain various approximation like as in Rayleigh-Ritz or Galerkin's method.

The bounbary integral equation may be solved iteratively by converting it into quasi-liear one.
When a cylinder becomes very thin, these equations gives von Karman's momentum integral equation and the other one to determine the frictional stress, namely, the present theory in this limit contains the boundary layer theory.

The similar studies have been carried out in three dimensional flow and two-dimensional periodically oscillating flow and they give us many interesting results [7,8].

For an example, we could discuss oscillating wake behind a cylinder and estimate its Strouhal number approximately [7],
Thus, the present formulation is useful not only numerical computations but approximate evaluations of various quantities of a real flow.

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