97

Underwater Sound Radiated From a Vibrating Ship Hull

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1 INTRODUCTION

A method of numerical analysis for the sound radiation problem are dealt with in the present report in which the vibration of main or auxiliary engines is transmitted to the ship hull and then its hull vibration radiates sound into the water, so that it may be treated as the mutual coupled vibration between the ship hull and the surrounding exterior fluid.

Junger[1] solved at first numerically for the mutual coupled vibration of fluid-structure interaction on infinite circular cylindrical shells and spherical shells in a fluid. And then, he calculated finite circular cylindrical shells and the reinforced cases. His method of approach was analytical and this is appropriate to understand the basic property of this problem, but the shapes that can be computed are limited to simpler ones such as a circular cylinder or sphere.

Then, Chen and Schneiker[2] showed a method of calculation for the sound radiated from a vibrating arbitrary shaped body, but it cannot be regarded as the exact solution of the mutual coupled vibration of fluid-structure interaction, because the vibration of the structure are given without taking account of sound radiation.

On the contrary, Craggs[3], Smith et al.[4], Wilton[5] showed the exact numerical analysis of the mutual coupled vibration of fluid-structure interaction, in which the vibrations of structure are solved by F.E.M. and the sound into fluid by B.E.M. , fitting the condition of the boundary. This method was applied to the interior noise problem like an automobile or the exterior noise problem like Sonar systems.

In this paper, we discuss the problem of ships by the last method. In general, a ship is floating on the free surface, so that we have to consider how the underwater sound due to a ship hull vibration is influenced by the water surface.

Hence, we show the method of numerical analysis for the characteristics of the sound which is radiated into the water from the ship hull floating on the free surface. Namely, we analyze the two dimensional steady forced vibration problem of an infinitely long cylindrical structure floating on the free surface.

Especially, we discuss the property of the sigularity on the numerical integration of the kernel function appearing in the sound radiation, and show an accurate method of numerical integration.

As a numerical example, we calculate the vibration problem of the free surface, compare with another exact solutions[6], and satisfy a good agreement between results by the present method and exact solutions.

2 GENERAL FORMULATION OF THE COUPLED FLUID-STRUCTURE VIBRATION PROBLEM

Let us consider the two dimensional time harmonic vibrating problem when the external force acts on a ship hull floating on a free water surface, Figure 1.

As the velocity potential field Φ and the displacement field are the time harmonics with circular frequency ω, we usually use abbreviation such as

$$\Phi(x,y;t) = \mathrm{Re}[\phi(x,y)e^{i\omega t}], \tag{1}$$

where $\phi(x,y)$ is complex variable which is independent of time, Re[ ] means the real part to be taken, and $i=\sqrt{-1}$.

In the following, we use the function ϕ expressed by the location (x,y) and abbreviate $e^{i\omega t}$.

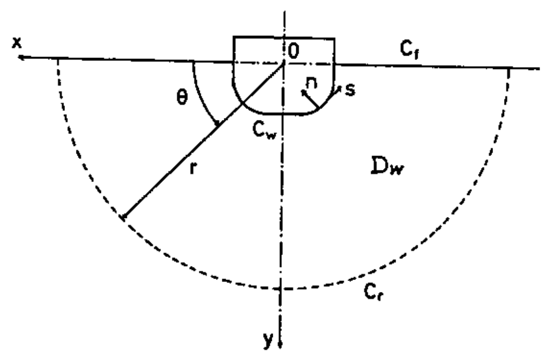

Figure 1. Coordinate system

2.1 Fundamental equation of fluid field

Assume the homogeneous fluid of density ρ and the fluid motion is nonviscous and irrotational.

Let D_w be a domain of fluid, C_w be a interface boundary between a ship hull and fluid, and C_f be a free water surface. For a time harmonic vibration, the equation of motion of a fluid becomes the Helmholtz equation,

$$\nabla^2\phi + k^2\phi = 0 \qquad \text{in } D_w, \tag{2}$$

where $k=\omega/c$, wave number, c is the velocity of the sound in a fluid.

On a free water surface, the pressure is zero, that is,

$$\phi = 0 \qquad \text{on } C_f . \tag{3}$$

On the interface boundary, the normal surface velocity of the ships hull is equal to the fluid velocity,

$$\frac{\partial \phi}{\partial n} = i\omega w \qquad \text{on } C_w , \tag{4}$$

where w denotes the displacement in the normal direction of the ships hull.

When the motion is the time harmonics, the radiation condition that the out-going wave still remains at infinity boundary have to be considered. On the fictitious boundary C_r which is the circle with infinitely large radius r, the radiation condition is as follows,

$$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial r} = -ik\phi \qquad \text{on } C_r . \tag{5}$$

<u>Reciprocal theorem and boundary integral equation</u> Let us consider two regular potentials $\phi^{(1)}$ and $\phi^{(2)}$. They vary sinusoidally in time with the same frequency. Then let us introduce the following functional,

$$L^*(\phi^{(1)},\phi^{(2)}) = \frac{\rho}{2}\iint_D [\nabla\phi^{(1)}\nabla\phi^{(2)} - k^2\phi^{(1)}\phi^{(2)}]d\Omega . \tag{6}$$

If $\phi^{(1)}$ is equal to $\phi^{(2)}$, the functional L^* stands for the modified Lagrangean.

It is clear that there exists the reciprocal relation in equation (6). If the sequence of the states of potential $\phi^{(1)}$ and $\phi^{(2)}$ are changed, the value of the functional L^* does not change. That is,

$$L^*(\phi^{(1)},\phi^{(2)}) = L^*(\phi^{(2)},\phi^{(1)}) . \tag{7}$$

As $\phi^{(1)}$ and $\phi^{(2)}$ are regular and satisfies the fundamental equation (2) in the fluid domain D_w, we can integrate the equation (6) by parts. We have

$$L^*(\phi^{(1)},\phi^{(2)}) = \frac{\rho}{2}\int_C \phi^{(2)}\frac{\partial \phi^{(1)}}{\partial n}ds , \tag{8}$$

and the equation (7) can be written in the following form as the reciprocity.

$$\int_C \phi^{(2)}\frac{\partial \phi^{(1)}}{\partial n}ds = \int_C \phi^{(1)}\frac{\partial \phi^{(2)}}{\partial n}ds , \tag{9}$$

where $C=C_w+C_f+C_r$.

This formula can be called the reciprocal theorem.

Let us introduce the fundamental solution S(P,Q) as the state $\phi^{(2)}$ which satisfies the free water boundary condition (3) and the radiation condition (5),

$$\phi^{(2)} = S(P,Q) = \frac{i}{4}[H_0^{(2)}(kR) - H_0^{(2)}(kR')] , \tag{10}$$

where

$$P = (x_P, y_P) \qquad \text{on } C ,$$

$$Q = (x_Q, y_Q) \qquad \text{in } D_w ,$$

$$R = \overline{PQ} = [(x_Q-x_P)^2 + (y_Q-y_P)^2]^{\frac{1}{2}},$$

$$R' = \overline{\overline{P}Q} = [(x_Q-x_P)^2 + (y_Q+y_P)^2]^{\frac{1}{2}},$$

$H_0^{(2)}(\ )$: the zero-th order Hankel function of the second kind.

Substituting the equation (10) into (9), then we obtain the following representation

$$\phi(Q) = \int_{C_w} [\phi(P)\frac{\partial S(P,Q)}{\partial n_P} - \frac{\partial \phi(P)}{\partial n_P} S(P,Q)]ds_P, \qquad (11)$$

where $\phi^{(1)} = \phi$

This equation permits us to solve the boundary value problem in the fluid by the integral equation.

<u>Discretization</u> Numerical procedure to solve the boundary value problem by making use of the equation (11) is as follows. At first, the boundary C_w may be divided into elements. Assuming that the unknown values ϕ and $\partial\phi/\partial n$ are constant for each element and giving boundary condition at the middle point of each element, we can write the equation (11) as following simultaneous equations,

$$[\ H_1\ ,\ H_2\ ]\begin{Bmatrix}\dfrac{\partial\phi}{\partial n}\\ \phi\end{Bmatrix} = \{0\}, \qquad (12)$$

where

$$H_{1\ ij} = \int_{C_{w\,j}} S(P_j,Q_i)ds_P,$$

$$H_{2\ ij} = \delta_{ij} - \int_{C_{w\,j}} \frac{\partial S(P_j,Q_i)}{\partial n_P} ds_P,$$

δ_{ij} means Kronecker's delta.

<u>Radiated under water sound power</u> Radiated under water sound power P from vibrating hull is the time averaged value of the product of the real part of the surface pressure and the velocity.

$$P = \int_{C_w} [\frac{1}{T}\int_0^T Re[p]\cdot Re[i\omega w]dt\ ]ds$$

$$= -\frac{i\rho\omega}{4}\int_{C_w} [\ \phi\,\overline{\frac{\partial\phi}{\partial n}} - \overline{\phi}\,\frac{\partial\phi}{\partial n}\ ]ds, \qquad (13)$$

where "——" denotes the complex conjugate value to be taken.
All energy of the radiated underwater sound is spread without decreasing because the viscosity of a fluid is neglected. Therefore, the power can be calculated from the integration over the circle located at sufficient large distance from the ship. From the property of the kernel function, the velocity potential at the point $Q(r_Q,\theta_Q)$ on the boundary C_r behaves like as follows

$$\phi(Q) \doteqdot \frac{i}{2\sqrt{2\pi k r_Q}} e^{-ikr_Q+\frac{\pi}{4}i} F(k,\theta_Q) \qquad \text{on } C_r, \tag{14}$$

where

$$F(k,\theta_Q) = \int_{C_w} [\ \phi(P)\frac{\partial}{\partial n_P} - \frac{\partial\phi(P)}{\partial n_P}][e^{ik\tilde{\omega}} - e^{ik\tilde{\omega}^*}]ds_P, \tag{15}$$

$$\left.\begin{matrix}\tilde{\omega}\\ \tilde{\omega}^*\end{matrix}\right\} = x_P\cos\theta_Q \pm y_P\sin\theta_Q \ .$$

Applying this property of the far field potential, the power is calculated by the following equation.

$$P = \frac{\rho\omega}{16\pi}\int_{C_w} F(k,\theta_Q)\cdot\overline{F(k,\theta_Q)}\ d\theta_Q \ . \tag{16}$$

From this equation, we can understand the physical meaning of the function $F(k,\theta_Q)$ that the square of the function is the intensity of the power in the direction θ_Q on the boundary C_r with the radius r_θ . Therefore the ratio of all over power to the intensity of the power which is radiated in the direction θ is given as following equation.

$$D(\theta) = \frac{|F(k,\theta)|^2}{\frac{1}{\pi}\int_0^\pi |F(k,\theta)|^2\ d\theta} \ . \tag{17}$$

2.2 Ship structural equation

The discretization method of the ship hull structure is expressed by the finite element method (with straight beam element). The finite element equations for the structure with time harmonic vibration are then of the form

$$[\ -\omega^2 M + K\ ]\{\ \delta\ \} = \{\ f_p\ \} + \{\ f_q\ \}\ , \tag{18}$$

where M and K are mass and stiffness matrix respectively, δ is displacement vector, f_q is a load vector derived from known applied forces, and f_p is a load vector representing the fluid pressure acting on the fluid-structure interface boundary.
On the interface boundary, the fluid pressure is expressed by the velocity potential as following equation.

$$p = -\ i\rho\omega\phi \qquad \text{on } C_w \ . \tag{19}$$

Explicitly, the vector f_p can be associated with velocity potential

$$\{ f_p \} = [P]\{ \phi \} .$$

Then the equation (18) becomes as follows.

$$[-\omega^2 M + K]\{ \delta \} - [P]\{ \phi \} = \{ f_q \} . \qquad (20)$$

2.3 Coupled fluid-structure equations

The complete solution of the fluid-structure interaction problem may now be described by combining the fluid equation (12) with structure equation (20). On the interface boundary between fluid and structure, the continuity of surface normal velocity between them must be designated.

The continuity of normal surface velocity is achieved by matching at surface middle point of structural element.

$$\frac{\partial \phi}{\partial n} = - i\omega w_{mid}$$
$$= - i\omega[B]\{ \delta \}, \qquad (21)$$

where w_{mid} : surface normal displacement of the middle point of structural element.

Substituting equation (21) into equation (12), we obtain the following coupled fluid-structure equation.

$$\left[\begin{array}{c|c} -\omega^2 M + K & -P \\ \hline H_1' & H_2 \end{array} \right] \left\{ \begin{array}{c} \delta \\ \hline \phi \end{array} \right\} = \left\{ \begin{array}{c} f_q \\ \hline 0 \end{array} \right\}, \qquad (22)$$

where

$$H_1' = - i\omega[H_1][B].$$

3 NUMERICAL PROCEDURE OF INTEGRATION OF KERNEL FUNCTION OF BOUNDARY ELEMENT

The kernel function S(P,Q) and its derivative in the boundary element equation (11) are very complex and the integration must be done by the numerical integration. Generally speaking, the accuracy of the boundary element analysis depends on the accuracy of numerical integration especially, of singular kernel. Then the integration must be done carefully.

Therefore, let us study the numerical property of kernel function and improve the accuracy of its integration.

3.1 Property of the singularity of the kernel function

The kernel function S(P,Q) is the sum of Bessel function and Neumann function. Integrating these functions on each element, the most serious problem is that Neumann function becomes infinite when the control point Q is near to the element. Taking this into consideration, in the vicinity of the origin, Neumann function behaves like as follows,

$$Y_0(kR) \xrightarrow[kR<<1]{} \frac{2}{\pi}(\log(kR/2)+\gamma) \simeq \frac{2}{\pi}\log R . \qquad (23)$$

Therefore, the dominant term of the singularity of Neumann function is the logarithmic term log R . Applying this property to the integral of Neumann function, we propose the accurate integration method as follows. Namely, let us separate the singularity term from Neumann function as follows.

$$Y_0(kR) = [\ Y_0(kR) - \frac{2}{\pi}\log R\ ] + \frac{2}{\pi}\log R\ . \qquad (24)$$

Then the integration of the first term of the right hand side of equation (24) can be done easily by the simple numerical integration such Gaussian quadrature or the trapezoidal rule, because the singularity is eliminated.

The integration of the last term of the right hand side of equation (24) can be done analytically by the following method. By this method, we can evaluate the singularity of Neumann function accurately.

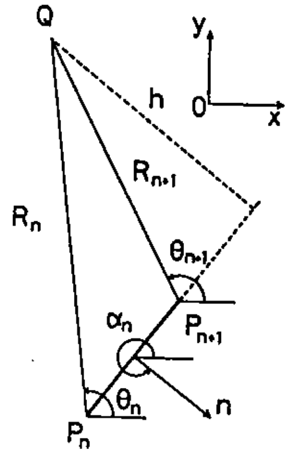

Figure 2. Local coordinate system

3.2 Integration of the logarithmic singularity

Let us study the integration over the arbitrary element $\overline{P_nP_{n+1}}$ on the boundary when the control point Q is random location in the domain, as shown in Figure 2. Notations are shown in the figure and positive direction of angles are counter-clockwise. By transforming the variable suitably, the integration can be done analytically. The integration of log R will be as follows

$$\int_{P_n}^{P_{n+1}} \log R\, ds_p = \int_{\theta_n}^{\theta_{n+1}} \log\{\frac{-h}{\cos(\theta-\alpha_n)}\} \frac{h}{\cos^2(\theta-\alpha_n)}\, d\theta, \qquad (25)$$

where $h = -R\cos(\theta-\alpha_n)$,

$$s_p = h\tan(\theta-\alpha_n).$$

Therefore

$$\int_{P_n}^{P_{n+1}} \log R\, ds_p = h\ [\ \tan t\ \{\log(\frac{-h}{\cos t})-1\} + t\ ]_{\theta_n-\alpha_n}^{\theta_{n+1}-\alpha_n}$$

$$= -R_{n+1}\sin(\theta_{n+1}-\alpha_n)(\log R_{n+1}-1)$$

$$+R_n\sin(\theta_n-\alpha_n)(\log R_n-1) + h(\theta_{n+1}-\theta_n). \qquad (26)$$

The integration of the differential value of logarithmic singularity can be easily done by the following relation

$$\frac{\partial}{\partial n_P}\log R = \frac{\partial \theta}{\partial s_P} \ . \tag{27}$$

Therefore

$$\int_{P_n}^{P_{n+1}} \frac{\partial}{\partial n_P}\log R \, ds_P = \int_{\theta_n}^{\theta_{n+1}} \frac{\partial \theta}{\partial s_P} ds_P = [\ \theta\ ]_{\theta_n}^{\theta_{n+1}}$$

$$= \theta_{n+1} - \theta_n \ . \tag{28}$$

Taking the limit from the interior domain to the boundary, integration reaches as follows.

$$\theta_{n+1} - \theta_n \xrightarrow[Q \text{ on } \overline{P_n P_{n+1}}]{} \pi \ . \tag{29}$$

3.3 Numerical example of integration

For an example, when the range of the integration is taken between the point $P_n(-a/2,0)$ and the point $P_{n+1}(a/2,0)$ on the x-axis (element length is a), we consider the accuracy of the numerical integration of Neumann function.

At first, when the observation point Q is in the middle point of the element, the results of the integration for various non-dimensional wave number ak are shown in Table 1. The values of integration are non-dimensionalized by the element length a . When the wave number ak is smaller than 1.0 , the computation error is less than 4 % . Although the present integration method is

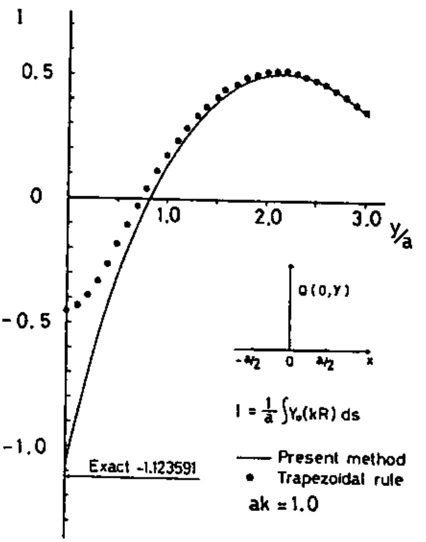

Figure 3. Integration of Neumann function

Table 1. Accuracy of integration of Neumann function

ak	exact[7]	present method	error (%)
0.2	-2.174306	-2.170858	0.159
0.4	-1.728544	-1.717725	0.626
0.6	-1.464277	-1.443893	1.392
0.8	-1.273812	-1.242644	2.447
1.0	-1.123591	-1.081138	3.778
2.0	-0.637069	-0.548363	13.924

very simple, the accuracy is good enough. In the discretization of the fluid model, if the element lengths ak are selected less than 1.0 , there is no problem for the accuracy of the integration of kernel function.

When the observation point Q moves from the origin to arbitrary point of y-axis, the integration of Neumann function is shown in Figure 3 . In this example, the non-dimensional wave number ak is 1.0 . In the figure, the dot mark means the result by the trapezoidal rule. When the control point Q lies near the element, value of y_Q/a is less than 1.0 , there is a difference between the present method and trapezoidal rule. Therefore, the accurate estimation of the singularity of Neumann function can not be done by simple numerical integration such as trapezoidal rule or Gaussian quadrature. But, when y_Q/a is larger than 1.0 , namely, the distance from the observation point Q to the element is larger than one element length, difference between them is negligible.

4 EXAMPLES OF THE CALCULATION

In order to evaluate the propriety of the present method, it is compared with exact solutions[6] of sound problem for semi-submerged cylindrical shells obtained by Junger's method.

The several constants used in the numerical calculation are the same as Junger's[1] as follows.

radius of cylindrical shell	$a = 3.0$	(m)
thickness of cylindrical shell	$t = 0.06$	(m)
Young's modulus	$E = 1.9\times10^{10}$	(Kg/ms^2)
Poisson's ratio	$\nu = 0.27$	
specific gravity of water	$\rho = 1000$	(Kg/m^3)
specific gravity of shell material	$\rho_S = 7700$	(Kg/m^3)
velocity of sound in water	$c = 1460$	(m/s)
exciting force	$q = 1000$	(Kg/ms^2)

The exciting force acts on the bottom perpendicular downward.

The numerical results of the vibration mode and underwater radiation pattern for 1.0 Hz and 300.0 Hz are illustrated in Figure 4,5 . In these figures, the present numerical results are marked by circles and the exact solutions by the solid lines.

Figure 6 shows the relationship between the vibrating frequency and the radiated sound power. It shows a good agreement over a wide range of frequency, so that the present method may be useful for a cylinder of an arbitrary shape.

5 CONCLUSIONS

We have studied the numerical analysis on the problem of underwater radiated sound from a two dimensional circular cylindrical shell floating on the free surface which vibrates periodically. This problem has been formulated generally as a coupled vibration of fluid-structure interaction by using both boundary conditions, making use of the F.E.M. for ship hull vibration and B.E.M. for sound radiation. Especially the property of singularity of the kernel function of sound radiation has been studied and an accurate method of numerical integrations has been proposed.

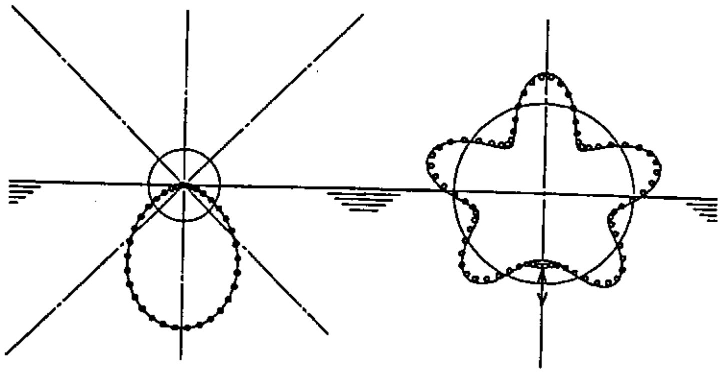

Figure 4. Radiation pattern and deflection mode (f=11.0 Hz)

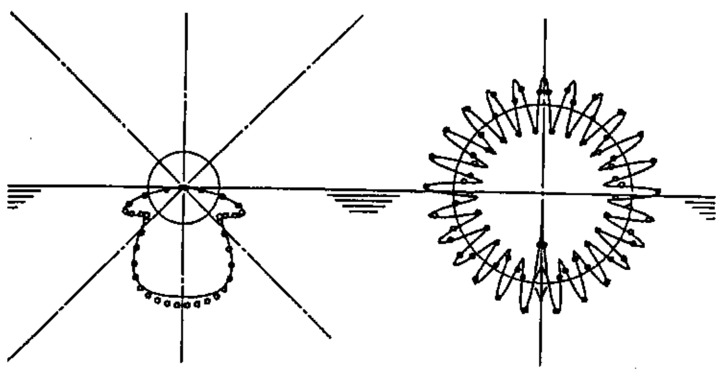

Figure 5. Radiation pattern and deflection mode (f=300.0 Hz)

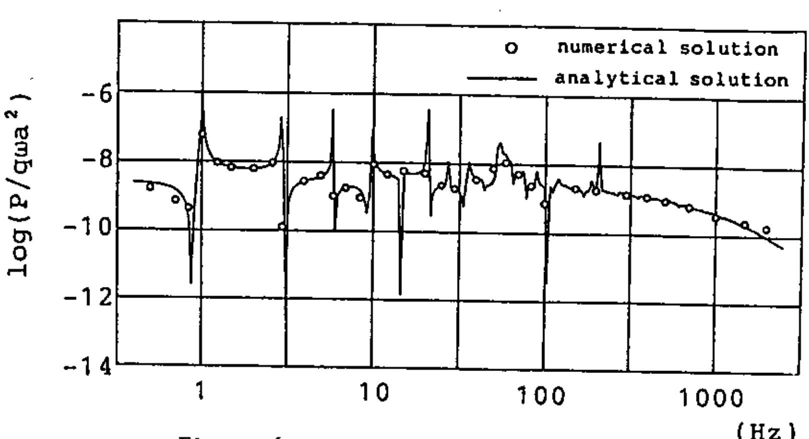

Figure 6. Total radiated sound power

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