学術論文 (98)

The Underwater Sound Scattering Problem from the Floating Elastic Shell

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1 INTRODUCTION

Sound radiation from vibrating structure causes a serious problem in connection with noise pollution and comfortableness to ride in ships, aircrafts, and automobiles. As to this problem, there are many reports which deals with coupled structure-fluid or air vibration system [1,2].
Similarly, sound scattering problems by the structure applying an incident acoustic wave are studied by many researchers [3,4]. However, these sound radiation and scattering problems have been independently treated as different problems. There is few report which describes the relationship between them [5,6].
In this report, we study the reciprocity for the coupled structure-fluid vibrating system. In the time harmonic vibrating fluid domain, we introduce the reciprocal theorem which have an analogy to the antenna theory in the electro-magnetic wave for transmitting and receiving problem. Subsequently, we expand the theorem into the reciprocity of the coupled structure-fluid vibration system for the relationship between sound radiation and scattering problem. By applying the theorem, the property of the sound scattering problem by the structure can be evaluated from the solution of radiation problem.
The method of the calculation is that the finite element analysis of the structure is matched at the structure-fluid interface with the boundary element analysis of the exterior fluid domain.
To verify its usefulness and accuracy, some numerical examples are shown for semi-submerged cylindrical shell. It is numerically demonstrated that the shell deflection amplitude in sound scattering problem can be estimated from the solution in the radiation problem.

2 FUNDAMENTAL EQUATIONS

Let us consider the two dimensional time harmonic vibration with circular frequency ω and express all dynamical quantities in the form

$$F(x,y;t) = \mathrm{Re}\,[\, f(x,y)e^{i\omega t} \,] \tag{1}$$

with the sign Re and $e^{i\omega t}$ usually omitted.
Assume the homogeneous fluid of density ρ and the fluid motion is nonviscouse and irrotational.
Let Dw be a domain of fluid, Cw be a interface boundary between a shell and fluid, Cf be a free water surface, and Cr be a fictitious boundary on infinite large radius r from a shell, as shown in Fig. 1. For a time harmonic vibration, the fundamental differential equation in fluid domain becomes the Helmholtz equation.

$$\nabla^2\phi + k^2\phi = 0 \qquad \text{in } D_w \tag{2}$$

where $k = \omega/c$; wave number, c is the velocity of the sound in a fluid.
On a free water surface, the pressure is zero, that is,

$$\phi = 0 \qquad \text{on } C_f \tag{3}$$

On the interface boundary, there should be the continuity of normal surface velocity between shell and fluid,

$$\frac{\partial \phi}{\partial n} = i\omega w \qquad \text{on } C_w \tag{4}$$

where w denotes the displacement in the normal direction of the shell.
Radiation condition is as following equation.

$$\frac{\partial \phi}{\partial n} = -ik\phi \qquad \text{on } C_r \tag{5}$$

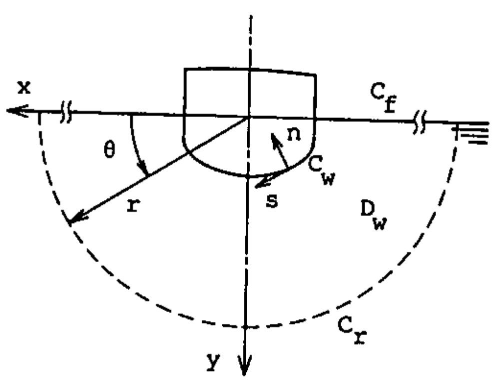

Fig. 1 Co-ordinate syst

3 RECIPROCAL THEOREM

(Part 1) Fluid domain

Let us consider two state of velocity potential Φ_1 and Φ_2 which **satisfies the** governing differential equation (2), free water **boundary** condition (3), and radiation condition (5). The **reciprocal** theorem in fluid domain is expressed as following **equation** [1].

$$\int_{C_w} \phi_1 \frac{\partial \phi_2}{\partial n} ds = \int_{C_w} \phi_2 \frac{\partial \phi_1}{\partial n} ds \tag{6}$$

By using the relationship between normal surface displacement w and velocity potential on interface boundary Cw, equation (6) becomes following equation.

$$\int_{C_w} p_1 \; w_2 \; ds = \int_{C_w} p_2 \; w_1 \; ds \tag{7}$$

where

$$p = -i\rho\omega\phi \qquad ; \text{ pressure on } C_w \tag{8}$$

If we assume p_1 and p_2 to be the resultant pressure when the displacement w_1 and w_2 vibrates at a point s_1 and s_2 as a δ function on the shell, the equation (7) becomes as follows.

$$\int_{C_w} p_1(s) \; \delta(s_2) \; ds = p_1(s_2)$$

$$\int_{C_w} p_2(s) \; \delta(s_1) \; ds = p_2(s_1) \tag{9}$$

$$p_1(s_2) = p_2(s_1)$$

Let us consider the vibration of shell applying an incident plane acoustic wave Φ_0 in the direction α . Let Φ_d be the diffraction potential assuming the shell to be the rigid body and Φ_r be the radiation potential resulting the boundary to deform itself by the displacement w. Total forced vibration energy supplied by the incident wave is as following equation.

$$E = \int_{C_w} \{-(p_0+p_d)\} \; w \; ds$$

$$= \int_{C_w} \{i\rho\omega(\phi_0+\phi_d)\} \frac{1}{i\omega}\frac{\partial \phi_r}{\partial n} ds$$

$$= \rho \int_{C_w} (\phi_0+\phi_d)\frac{\partial \phi_r}{\partial n} ds \tag{10}$$

We can replace the diffraction potential ϕ_d with the radiation potential ϕ_r by using the reciprocal theorem.

$$E = \rho \int_{C_w} (\phi_0 \frac{\partial \phi_r}{\partial n} + \phi_r \frac{\partial \phi_d}{\partial n}) \, ds \qquad (11)$$

The boundary condition of diffraction potential ϕ_d is as following equation.

$$\frac{\partial}{\partial n}(\phi_0 + \phi_d) = 0 \qquad \text{on } C_w \qquad (12)$$

Substituting the equation (12) into (11), Therefore we have.

$$E = \rho \int_{C_w} (\phi_0 \frac{\partial \phi_r}{\partial n} - \phi_r \frac{\partial \phi_0}{\partial n}) \, ds \qquad (13)$$

Incident acoustic plane wave including free water surface effect is expressed following equation.

$$\phi_0 = c\xi(e^{ikz} - e^{ikz'}) \qquad (14)$$

where ξ : amplitude of water particle of incident wave

$$\left.\begin{matrix} z \\ z' \end{matrix}\right\} = x \cos\alpha \pm y \sin\alpha \qquad (15)$$

Substituting equation (14) into (13). We can write the energy as following equation,

$$E = \rho c \xi \int_{C_w} [\frac{\partial \phi_r}{\partial n} - \phi_r \frac{\partial}{\partial n}](e^{ikz} - e^{ikz'}) \, ds \qquad (16)$$

Right hand side of the equation (16) equals to the Kotchin function F(k, α) which means the intensity of far field radiated sound power from vibrating shell in the direction α . Therefore we have

$$E = \rho c \xi \, F(k,\alpha) \qquad (17)$$

If the deflection w is an unit amplitude at point s_1 on the shell, equation (17) can be written as following equation.

$$P(s_1) = \rho c \xi F(k,\alpha) \qquad (18)$$

where $P(s_1) = p_0 + p_d$

This means that the pressure at s_1 applying an incident acoustic wave is proportional to the Kotchin function when the point s_1 is forced to vibrate.

(Part 2) Generalized reciprocal theorem

According to mechanical vibration system, Maxwell-Betti's reciprocal theorem in elastodynamic problem constituted a relation between two different situation $\{ q_1, w_1 \}$ and $\{ q_2, w_2 \}$.

$$\int_{C_w + C_a} q_2 w_1 \, ds = \int_{C_w + C_a} q_1 w_2 \, ds \tag{19}$$

where q is a surface traction.

Surface pressure $p_o + p_d$ applying incident acoustic wave can be considered to be the traction q_2 and we can identify the traction q_1 with external force q acted at point s on C_w.

From equation (19) and (10), we have,

$$\begin{aligned} E &= \int_{C_w} (p_0 + p_d) w_1 \, ds \\ &= \int_{C_w} q_1 w_2 \, ds \quad = q \, w(s) \end{aligned} \tag{20}$$

Therefore, from equation (20) and (18), we can obtain the following equation.

$$\frac{F(k,\alpha)}{q} = \frac{w(s)}{\rho c \xi} \tag{21}$$

The intensity of sound power $F(k,\alpha)$ radiated in the direction acting the external force q at point s is proportional to the deflection w(s) at point s on C_w applying incident acoustic wave with direction α.

This is the general reciprocal theorem including coupled structure-fluid vibration system.

For instance, if we measure a deflection amplitude w at each point on shell surface applying incident acoustic wave in the direction α, we can estimate the intensity of far field radiation acoustic power in the direction α when the external force acts at each points.

Especially, if there is a point whose deflection is equal to zero, the intensity of far field radiation power can not go out in the direction α when the external force acts at the point.

4 FORMULATION

Fluid field

Let us introduce the fundamental solution S(P,Q) as the state in equation (6) which satisfies the free water boundary condition (3) and the radiation condition (5)

$$\Phi_2 = S(P,Q) = \frac{i}{4} [H_0^{(2)}(kR) - H_0^{(2)}(kR')]$$

$$P = (x_P , y_P), \qquad Q = (x_Q , y_Q)$$

$$R = \overline{PQ} = [(x_Q-x_P)^2 + (y_Q-y_P)^2]^{\frac{1}{2}} \tag{22}$$

$$R' = \overline{\bar{P}Q} = [(x_Q-x_P)^2 + (y_Q+y_P)^2]^{\frac{1}{2}}$$

Substituting equation (22) into (6), then we obtain the following equation.

$$\Phi(Q) - \Phi_0(Q) = \int_{C_w} [\Phi(P)\frac{\partial S(P,Q)}{\partial n_P} - \frac{\partial \Phi(P)}{\partial n_P} S(P,Q)] \, ds_P \tag{23}$$

where $\Phi = \Phi_0 + \Phi_r + \Phi_d$

is a total acoustic potential.

To solve the boundary value problem of the equation (23), we divide the boundary C_w into element, Assuming the unknown values Φ and $\partial\Phi/\partial n$ are constant for each element. we can obtain the following simultaneous equations.

$$[H_1 , H_2] \begin{Bmatrix} \frac{\partial \Phi}{\partial n} \\ \Phi \end{Bmatrix} = \{ \Phi_0 \} \tag{24}$$

where $H_{1\ ij} = \int_{C_{w_j}} S(P_j,Q_i) \, ds_P$

$$H_{2\ ij} = \delta_{ij} - \int_{C_{w_j}} \frac{\partial}{\partial n_P} S(P_j,Q_i) \, ds_P$$

δ_{ij} : Kronecker's delta.

Structural equation

The discretization method of the structure by the finite element method is some with former report[1]. The finite equation for the structure with time harmonic vibration are then of the form

$$[-\omega^2 M + K]\{ \delta \} = \{ f_p \} \tag{25}$$

where M and K are mass and stiffness matrix respectively, δ is a displacement vector and f_p is a load vector representing the fluid pressure acting on a structure-fluid interface boundary.

Coupled structure-fluid vibration system

On the interface boundary, the fluid pressure is expressed by the velocity potential as following equation.

$$p = -i\rho\omega\Phi \qquad \text{on } C_w \tag{26}$$

Then the equation (25) becomes as follows.

$$[\ -\omega^2 M + K \quad , \quad -P\ ] \begin{Bmatrix} \delta \\ \Phi \end{Bmatrix} = \{\ 0\ \} \tag{27}$$

where $\{\ f_p\ \} = [\ P\ ]\{\ \Phi\ \}$

On the boundary Cw, the continuity of surface normal velocity between fluid and structure must be designated. The condition of continuity of the normal surface velocity is as following equation.

$$\frac{\partial \Phi}{\partial n} = -i\omega\ w_{mid} \quad = -i\omega[\ B\ ]\{\ \delta\ \} \tag{28}$$

where w_{mid} : surface normal displacement of the middle point of the structure element.

Substituting the equation (28) into (24), we obtain the following coupled structure-fluid equation.

$$\begin{bmatrix} -\omega^2 M + K & -P \\ H_1' & H_2 \end{bmatrix} \begin{Bmatrix} \delta \\ \Phi \end{Bmatrix} = \begin{Bmatrix} 0 \\ \Phi_0 \end{Bmatrix} \tag{29}$$

where

$$H_1' = -i\omega[\ H_1\ ][\ B\ ]$$

Scattering sound power, scattering width

Scattering sound power from vibrating shell is the time averaged value of the product of the real part of the surface pressure and the velocity.

$$P = \int_{C_w} [\ \frac{1}{T}\int_0^T \mathrm{Re}[\ p\ ]\ \mathrm{Re}[\ \frac{\partial \Phi}{\partial n}\ ]\ dt\ ]\ ds \tag{30}$$

From the property of the fundamental solution at the sufficient large distance from the shell, the power can be calculated by the following equation [1].

$$P = \frac{\rho\omega}{16\pi}\int_0^{\pi} |F(k,\theta)|^2\ d\theta \tag{31}$$

As $F(k,\theta)$ is the intensity of far field radiated sound power in direction θ, the directivity of the scattered sound power is the ratio of the intensity of the power in the direction to the all over power.

$$D(\theta) = \frac{|F(k,\theta)|^2}{\frac{1}{\pi}\int_0^{\pi} |F(k,\theta)|^2 \, d\theta} \qquad (32)$$

The scattering width Q is the ratio of the scattering power P to the incident sound power P_o per unit width.

$$Q = P/P_o \qquad (33)$$

where $P_0 = \frac{1}{2}\rho c \omega^2 \xi^2$

5 EXAMPLE OF CALCULATION

In order to verify the property of the present method, computation results by this method will be compared with exact solution for semi-submerged cylindrical shells obtained by Junger's method [3].
Several constants in the computation are assumed such as;

radius of cylindrical shell	$a = 3.0$	(m)
thickness of shell	$t = 0.06$	(m)
Young's modulus	$E = 1.9 \times 10^{11}$	(kg/ms^2)
Poisson's ratio	$\nu = 0.27$	
specific gravity of shell	$\rho_s = 7700$	(kg/m^3)
specific gravity of fluid	$\rho = 1000$	(kg/m^3)
velocity of sound in fluid	$c = 1460$	(m/s)

The direction of incident acoustic wave is $\alpha = \pi/4$.
The numerical result of the vibration mode and scattering sound pattern for 387 Hz (nondimensional wave number $ka = 5.0$) are illustrated in Fig. 2. The present numerical results are marked by circles and the exact solutions by the solid line.
Fig. 3 and 4 shows the relation between the incident wave frequency and the deflecton at bottom and the scattering width. These results show a good agreement over a wide range of frequency.
In order to confirm the reciprocal theorem (21) numerically, we calculate the radiation problem and the scattering problem for submerged rounded corner square shell as shown in Fig. 5 respectively and compare with each phenomenon. The direction of incident acoustic wave is $\alpha = 0, \pi/4, \pi/2, 3\pi/4, \pi$, and the shell deflection amplitude is measured at point A as shown in the figure. In this case, the frequency of incident wave and external force is $ka = 0.10$. Results are shown in Table 1.
The left side column in the table ...

amplitude at point A applying the plane incident acoustic wave in the direction α with amplitude ξ . The right side column shows the intensity of far field radiated sound power in the direction α when the external force acts at point A with amplitude q . Each values agrees precisely.
Applying the reciprocal theorem, we can easily imagine to control the sound radiation pattern. The procedure is that;

(1) To find the point whose deflection amplitude is zero or minimum when the incident wave in the direction applies at shell as shown in Fig. 6.

(2) If the external force acts at the point, the radiation sound does not go out in the direction as shown in Fig. 7.

6 CONCLUSION

We have studied the reciprocity of the coupled structure-fluid time harmonic vibration system for the relationship between the sound radiation and scattering problem.
We have introduced the relationship that the shell surface deflection amplitude applying the incident plane acoustic wave in the direction α is proportional to the intensity of the radiated sound power in the direction α when the external force acts at the point. In order to confirm the reciprocity numerically, we have formulated the sound scattering problem as a coupled structure-fluid interaction system by using F.E.M. for the elastic shell and B.E.M. for fluid domain.

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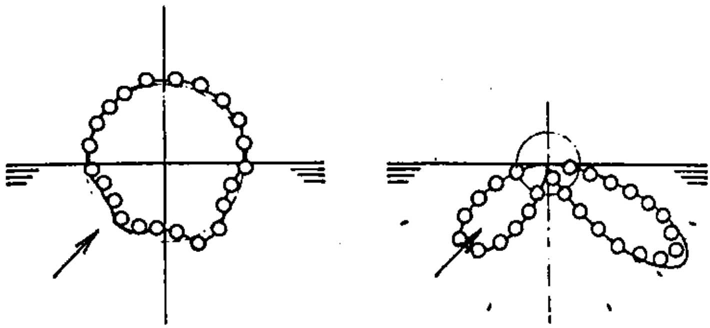

Fig. 2 Vibration mode and scattering sound pattern for ka = 5.0. Circle shows the present method and the solid line by the exact solution.

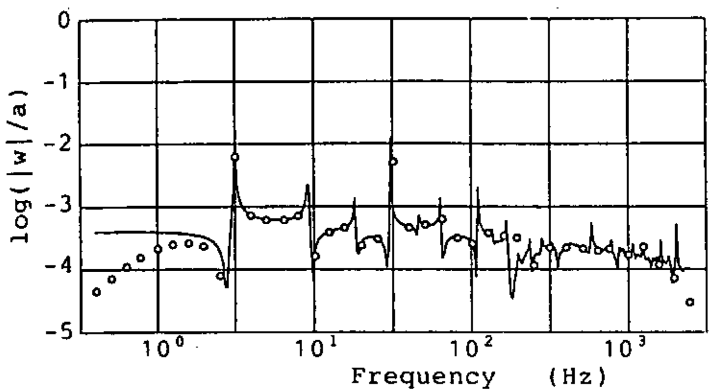

Fig. 3 Relation between incident wave frequency and deflection at bottom.

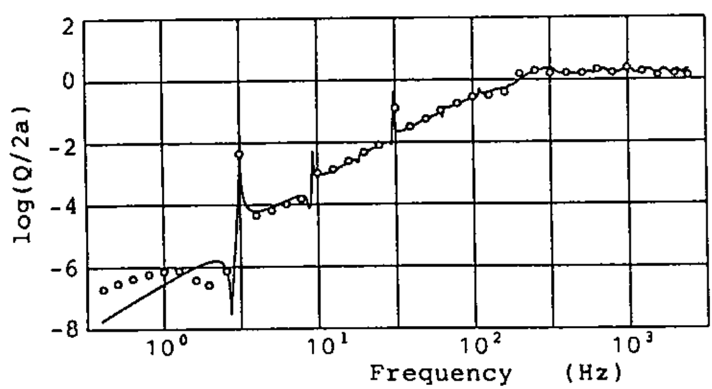

Fig. 4 Relation between incident wave frequency and scattering

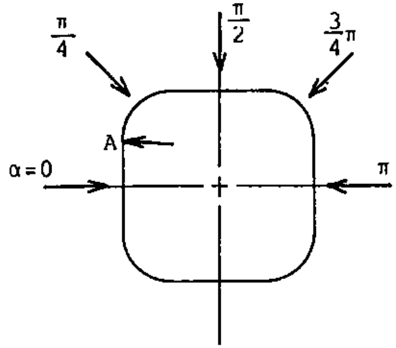

Fig. 5 Submerged rounded corner shell.

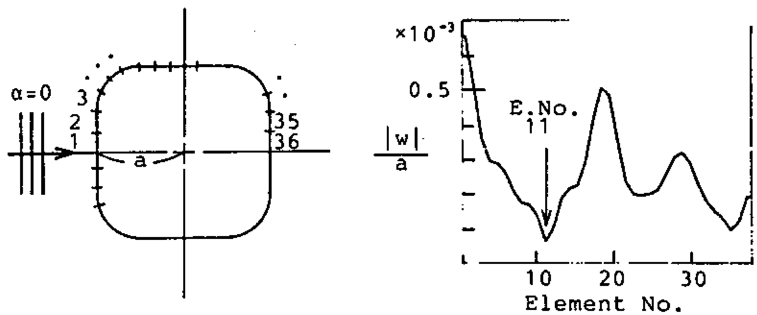

Fig. 6 Solution of sound scattering problem α = 0, ka = 1.0 , right side of figure shows relation between element No. and shell deflection amplitude.

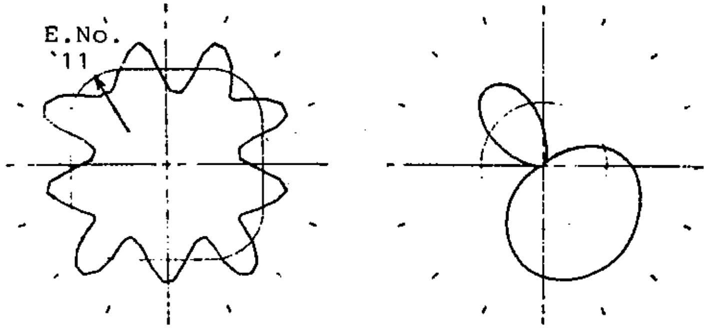

Fig. 7 Vibration mode and sound radiation pattern when the external force acts at element No. 11 .

Table 1 Rceiprocal relationship between acoustic radiation problem and scattering problem

α	$\lvert w(A)\rvert / \rho c \xi$ *	$\lvert F(k,\alpha)\rvert / q$ **	Error (%)
0	0.31490×10^{-5}	0.31565×10^{-5}	0.238
$\pi/4$	0.28599	0.28675	0.266
$\pi/2$	0.23846	0.23927	0.340
$3\pi/4$	0.20401	0.20481	0.392
π	0.19048	0.19124	0.399

* Shell deflection amplitude at point A appling incident plane acoustic wave in the direction α .

** Intensity of radiated sound power in the direction α when the external force q acts at point A.