A-12

1. On the Ship Motion Reduction by Anti-Pitching Fins in Head Seas

Masatoshi BESSHO, *Member**, Yusaku KYOZUKA, *Member***
Masayuki HAYASHIDA, *Member****, Shigeru YAMAUCHI***
Michio MIYAZAKI***

(From J.S.N.A. Japan, Vol.153, June 1983 and Vol.155, June 1984)

Summary

This is a study on a heaving–free and/or pitching–free ship with pairs of fins, that is, a ship which is free from the heaving and/or pitching oscillation in head sea at a given wave–length. For this purpose, it is sufficient to cancel the wave–exciting force and/or moment by using pairs of fins to make a ship with fin wave–free for heave and/or pitch motion. Thus, there are two equations in complex variables for the heave and pitch–free.

On the other hand, a pair of fins has three variables to be determined which are the area, the aspect–ratio and the location of fins to be attached. Two pairs of fins are sufficient to make a ship heave–free or pitch–free.

Taking a container–cargo ship for an example, the calculations based on strip–theory are carried out for two cases. And it is confirmed by experiments that the theory agrees well with experimental results.

Nextly, a series of calculations is carried out for ships which have the same displacement but have different water–plane area in order to obtain an optimum fins small fin–area.

Nomenclature

$A_f = C_f S_f$	: area of fin (of one side)
A. R.	: aspect–ratio of fin (of one side)
A_w	: amplitude of in–coming wave
b_{fj}	: moment lever of spring of j–th fin (Fig. 2)
C_{fj}	: chord length of j–th fin
C_L	: lift coefficient of fin
C_p	: prismatic curve of a ship
C_T	: total resistance coefficient
C_w	: water–plane area curve of a ship
$c = \omega_0/K$	: phase velocity of wave
$F_n = V/\sqrt{gL}$	: Froude number
g	: gravity constant
h	: amplitude of heaving oscillation at center of gravity of ship
I_1, I_2	: velocities of heaving and pitching oscillations at center of gravity of ship
I_j, I'_j	: velocity of flapping oscillation of fin
$K = 2\pi/\lambda = \omega_0^2/g$	: wave number of in–coming wave
L	: length of ship or lift of fin
l_j	: position of j–th fin
$l_w = V_2/V_1$	: lever of wave–exciting moment
m_j	: added mass of j–th fin
N	: two dimensional damping coefficient
R	: resistance increase in waves
s_{fj}	: span length of j–th fin
T_f	: thrust by fin
V	: ship velocity
V_1, V_2	: wave–exciting force and moment

* National Defense Academy
** Research Institute for Applied Mechanics, Kyushu University
*** Maritime Defense Agency

v_{wj}	: vertical velocity of water at j–th fin
v_{rj}	: vertical velocity of j–th fin relative to water
W	: dissipated power of radiating wave
Z_{ij}	: i–th mode hydro– and mechanical impedance due to j–th mode oscillation
z_j	: impedance due to j–th pair of fin
ω_0	: circular frequency of in–coming wave
$\omega=\omega_0+KV$	: encounter frequency
ψ	: pitching amplitude
$\alpha_j=\arg\{V_j\}$	: phase of wave–exciting force or moment
δ_j	: frapping angle of j–th fin
θ_j	: attack angle of j–th fin

1. Introduction

The study of ship motion reduction with fins has a long history and many works have been carried out. But there are comparatively few works which deal with anti–pitching fins.

At first, M. A. Abkowitz [3] reported that the pitching oscillation can be reduced to a half by a pair of fins with the area of 3 to 7% of the ship water plane area. M. Matsui [4] reported that the active control anti–pitching fin for a small passenger ship can reduce pitching completely at some wave–length. The added mass effect of fins is neglected in the above two studies. Ir. J. H. Vugts [5] studied the passive and active fins including the added mass effect and showed a good agreement with experiments and theoretical calculations by the strip method.

M. A. Abkowitz and M. Matsui et al. suggested that the motion reduction effect of fins result from large eddy damping. This is true and may be the largest role of fins. On the other hand, the reduction of ship motions by fins can be also explained by the reduction of wave–exciting moment.

It is well known that the semi–submerged ship has a good sea–keeping quality. This is result of its wave–free property, that is, the wave–exciting force vanishes at a certain wave–length. However, the motions of a ship are not always reduced when the wave–exciting force vanishes because the damping coefficient also vanishes consequently. [1, 2]

If it is possible to make a ship wave–free only in one direction, for example, head seas, it must allow us to have heave–free and/or pitch–free ship. The preceding work tried to reduce only pitching motion. On the contrary, we have tried to obtain the fins which make a ship heave–free and/or pitch–free at a given wave–length. The solution have been obtained by the wave–free condition; that is, cancellation of the wave–exciting force ane/or moment by the appropriate combinations of fins.

The calculation is based on the reliable ordinary method which gives correct prediction of sea–keeping properties. Taking a container cargo ship as an example, we have carried out the calculation and studied the practical feasibility and found that there exists actually such a fin in a fairly broad range of wave–length. We have verified the prediction to be reliable by experiments.

However, the area of such fins becomes fairly large so that the practical application would be difficult because of the increase of resistance in still water for one reason. Therefore, we study on the relation between the fin–area and particulars of ship form, and the possibility to reduce fin–area. We carried out a series of calculations for ships which have the same displacement but have different water area in order to obtain optimum fins of smaller area.

2. Ship Model and Fins

Let us calculate fin particulars to make a ship heave and/or pitch–free in head seas. For this purpose, we make use of ordinary strip method with the coordinate systems as shown in Fig. 1. [1, 2]

We calculate the motions of a container ship of which body plan and pricipal dimensions are shown in Fig. 2 and Table 1. Her service speed is Fn=0.26, but the calculation was carried out for Fn=0.18 because of the short length of

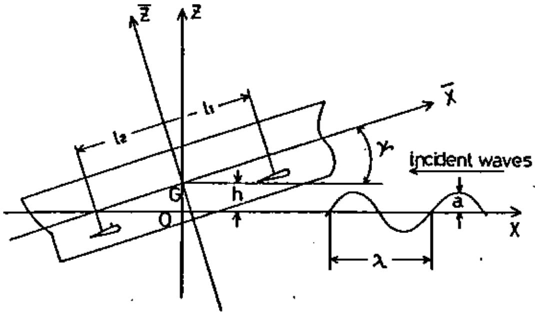

Fig. 1 Coordinate system

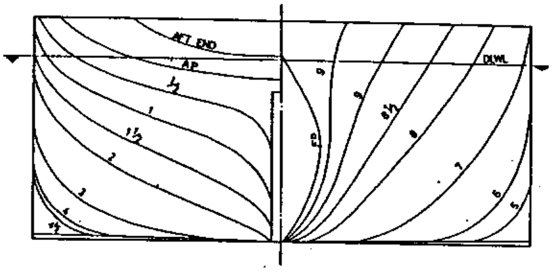

Fig. 2 Body plan of the ship

Table 1 Principal dimensions of ship and model

	Ship	Model
Length between perpendiculars (m)	250	3
Breadth moulded (m)	32	0.384
Draft loaded (m)	11.5	0.138
Block coefficient	0.632	
Midship coefficient	0.964	
Volume of displacement (m^3)	58126	0.1004
Waterplane area (m^2)	6870	0.9893
Wetted surface area (m^2)	10073	1.4505
Radius of gyration / Lpp	0.25	
⊠G / Lpp *	-0.01824	
⊠F / Lpp **	-0.0578	

* ⊠G : distance of C.G. from

** ⊠F : distance of center of flotation from

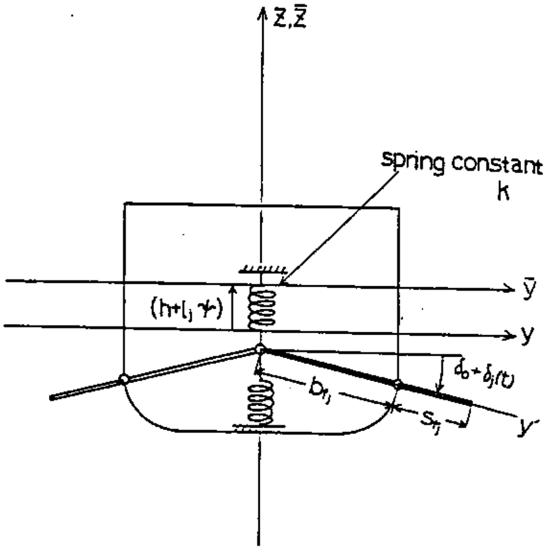

Fig. 3 Flapping fin.

the model testing basin. But the difference of the Froude number would not change the calculated results greatly. The calculation shows that the fin area decreases slightly as the ship speed increases.

Now, we think about a pair of fins which is assumed to be able to flap around a point and supported by two springs at center–line of the ship as shown in Fig. 3. We estimate the force on fins under assumptions that the force acts on one point of ship and its moment about mid–chord of the fin is negligible. Then, the lift L and added mass m of the fin area are estimated as follows, because the reduced frequency must be vary small in practical case.

$$\left.\begin{aligned} L &= \frac{\rho}{2} V v\, S_f\, C_L' \\ C_L' &= \frac{2\pi}{1+\frac{1}{A.R.}}, \quad A.R. = S_f/C_f \\ m &= \frac{\pi}{4} \rho\, C_f^2\, S_f \end{aligned}\right\} \quad (1)$$

where v is relative vertical velocity of fin to the water. The effect of aspect–ratio is assumed as same as the one of elliptic wing taking into account mirror image effect to the ship hull and the lift coefficient is due to theory of wing so that it must be corrected at the stage of practical application.

In the followings, dividing these quantities by the velocity v and we make use of an impedance form. For a pair of fins the impedance may be written as:

$$z = z_r + iz_i$$

$$= \frac{2L}{V} + 2i\omega m = \frac{\pi \rho V A_f}{1 + \frac{1}{A.\ R.}} + \frac{i\pi}{2}\rho\,\omega C_f A_f \quad (2)$$

3. Equation of Motion of a Ship with Fins

In general, in treating mechanical oscillation problem, it is simple, convenient and even fertile to make use of analogy to the electric circuit theory. [12] Hence, we express the equation of motion as follows:

$$\left.\begin{aligned} &Z_{11} + Z_{12} I_2 = V_1 \\ &Z_{21} I_1 + Z_{22} I_2 = V_2 \end{aligned}\right\} \quad (3)$$

where I_1 and I_2 are the complex velocities of heaving and pitching oscillation respectively. Z_{ij} denotes the hydrodynamical and mechanical impedance. V_1 and V_2 are the wave exciting force and moment, which are calculated from the strip method by ordinary process. [1, 2]

Here, we use an analogy in which the velocity and force or moment correspond to electric current and voltage, so that mass, spring constant and damping coefficient correspond to inductance, capacitance and resistance respectively.

Now, let us consider to suppress completely heaving and pitching oscillations. As easily seen, it is sufficient when the following conditions exist.

$$V_1 = V_2 = O \quad (4)$$

These conditions mean the ship form is wave-free, and the ship does not radiate waves to the forward direction when heaving and pitching. [11] Although such ship form is not known yet, but we may fit some appendage to cancel out the wave exciting force and moment.

For such appendage, let us choose two pairs of flapping fins, which are supported by two springs of the constant equal to k as shown in Fig. 3. The chord and span length are denoted by c_{fj} and s_{fj} for one side of a pair of fins. The fore and aft fins are attached to the bow and stern of a ship and their distances from the center of gravity are l_1 and l_2 respectively. Let us denote the impedances of fore- and aft-fins as z_1 and z_2, and the velocities at a point of center-line of ship as I_3 and I_4 respectively.

Then, the equations (3) become:

$$\left.\begin{aligned} &Z_{11}' I_1 + Z_{12}' I_2 + Z_{13}\ I_3 + Z_{14}\ I_4 = V_1 \\ &Z_{21}' I_1 + Z_{22}' I_2 + Z_{23} I_3 + Z_{24} I_4 = V_2 \\ &Z_{31}\ I_1 + Z_{32}\ I_2 + Z_{33}\ I_3 = V_3 \\ &Z_{41}\ I_1 + Z_{42}\ I_2 + Z_{44}\ I_4 = V_4 \end{aligned}\right\} \quad (5)$$

where

$$\left.\begin{aligned} &Z_{13} = Z_{31} = \frac{i}{\omega} k_1 \left(\frac{2b_{f1}}{S_{f1}}\right), \\ &Z_{14} = Z_{41} = \frac{i}{\omega} k_2 \left(\frac{2b_{f2}}{S_{f2}}\right) \end{aligned}\right\} \quad (6)$$

$$\left.\begin{aligned} &Z_{23} = l_1 Z_{13},\ Z_{24} = -l_2 Z_{14}, \\ &Z_{32} = Z_{23} + \frac{iV}{\omega} z_1, \\ &Z_{42} = Z_{24} + \frac{iV}{\omega} z_2 \\ &Z_{11}' = Z_{11} - Z_{13} - Z_{14} \\ &Z_{12}' = Z_{12} - Z_{23} - Z_{24} \\ &Z_{21}' = Z_{21} - Z_{23} - Z_{24} \\ &Z_{22}' = Z_{22} - l_1\ Z_{23} + l_2\ Z_{24} \\ &Z_{33} = z_1 + i\omega m_1 - Z_{13} \\ &Z_{44} = z_2 + i\omega m_2 - Z_{14} \end{aligned}\right\} \quad (7)$$

and

$$\left.\begin{aligned} &V_3 = z_1\, v_{w1},\ V_4 = z_2\, v_{w2} \\ &v_{w1} = i\omega_0\ a e^{Kz1 + iKl1} \\ &v_{w2} = i\omega_0\, a e^{Kz2 - iKl2}, \\ &\omega_0 = \sqrt{K/g} = \omega - KV \end{aligned}\right\} \quad (8)$$

Eqs. (5) can be written by the flapping velocities of the fore- and aft-fins.

$$\left.\begin{aligned} I_3&=I_1+l_1\ I_2+I_3',\ I_3'=-\frac{S_{f1}}{2}\dot{\delta}_1 \\ I_4&=I_1-l_2 I_2+I_4', I_4'=-\frac{S_{f2}}{2}\dot{\delta}_2 \end{aligned}\right\} \quad (9)$$

Substituting above equatins into eq. (5), we obtain

$$\left.\begin{aligned} &Z_{11}''I_1+Z_{12}''I_2+z_1\ I_3'+z_2\ I_4' \\ &\quad =V_1+V_3+V_4, \\ &Z_{21}''I_1+Z_{22}''I_2+l_1\ z_1\ I_3'-l_2\ z_2\ I_4' \\ &\quad =V_2+l_1\ V_3-l_2\ V_4, \\ &Z_{31}''I_1+Z_{32}''I_2+Z_{33}\ I_3'=V_3, \\ &Z_{41}''I_1+Z_{42}''I_2+Z_{44}\ I_4'=V_4 \end{aligned}\right| \quad (10)$$

where

$$\left.\begin{aligned} &Z_{11}''=Z_{11}+z_1+z_2, \\ &Z_{12}''=Z_{12}+\left(l_1+\frac{iV}{\omega}\right)z_1-\left(l_2-\frac{iV}{\omega}\right)z_2, \\ &Z_{21}''=Z_{21}+l_1\ z_1-l_2 z_2, \\ &Z_{22}''=Z_{22}+l_1\left(l_1+\frac{iV}{\omega}\right)z_1 \\ &\qquad +l_2\left(l_2-\frac{iV}{\omega}\right)z_2, \\ &Z_{31}''=z_1,\ Z_{41}''=z_2, \\ &Z_{32}''=\left(l_1+\frac{iV}{\omega}\right)z_1,\ Z_{42}''=\left(-l_2+\frac{iV}{\omega}\right)z_2 \end{aligned}\right| \quad (11)$$

If the fins are fixed, the top two equations in eqs. (10) are sufficient to be considered by putting $I_3=I_4=0$.

4. Two Pairs of Fins for Heave and Pitch Free

At first, we study on a ship with two pairs of fixed fin. Then, the oscillation–free conditions for a ship with two pairs of fixed fins are obvious as in eq. (4)

$$\left.\begin{aligned} V_1+V_3+V_4&=0 \\ V_2+l_1\ V_3-l_2\ V_4&=0 \end{aligned}\right\} \quad (12)$$

Solving eq. (12) with respect to V_3 and V_4, we obtain

$$\left.\begin{aligned} V_3&=z_1\ V_{w1}=-\frac{l_2 V_1+V_2}{l_1+l_2} \\ V_4&=z_2\ V_{w2}=-\frac{l_1\ V_1-V_2}{l_1+l_2} \end{aligned}\right\} \quad (13)$$

Putting following equations into eqs. (13),

$$\left.\begin{aligned} V_1&=|\ V_1\ |\ e^{i\alpha_1} \\ V_2&=l_W\ |\ V_1\ |\ e^{i\alpha_2} \end{aligned}\right\} \quad (14)$$

$$\left.\begin{aligned} l_1+l_2&=2\ l_m \\ l_1-l_2&=2\ l_C \end{aligned}\right\} \quad (15)$$

we obtain

$$\left.\begin{aligned} &\frac{l_m-l_C+l_W\ \cos(\alpha_2-\alpha_1)+il_W}{l_m+l_C-l_W\ \cos(\alpha_2-\alpha_1)-il_W} \\ &\frac{\sin(\alpha_2-\alpha_1)}{\sin(\alpha_2-\alpha_1)}=\frac{Z_1}{Z_2}e^{2iKl_m} \end{aligned}\right\} \quad (16)$$

At first, for the convenient sake, let us assume two fins are attached. Then, we may verify the condition (16) becomes

$$\left.\begin{aligned} &l_C=l_W\ \cos(\alpha_2-\alpha_1) \\ &l_W \sin(\alpha_2-\alpha_1)=l_m \tan Kl_m \end{aligned}\right\} \quad (17)$$

Solving eqs. (17) with respect to l_c and l_m, we obtain fin position l_1 and l_2. Then, putting these conditions in the one of eqs. (13), we may calculate the impedance:

$$z_1=z_2=\frac{i\sqrt{l_m^2+l_W^2}}{2\,\omega_0 a l_m}V_1 e^{-KZ_1-iKl_C} \quad (18)$$

For an actual calculation, we must search at first one of the root l_m of the second equation of eqs. (17) for a given wave number. Then, we may calculate l_c and z_1 from eqs. (17) and (18). Here, z_1 must lie in the first quadrant of the complex plane by the eq. (2). Hence, if it does not lie in the first quadrant, we must search another root which gives appropriate impedance.

From the impedance thus obtained, we may calculate its aspect–ratio and area as follows. Namely, cancelling the fin area from the real and imaginary part of the impendance of eq. (2), we have

$$\frac{z_{\xi}^{2}}{z_{\zeta}^{2}}=\frac{32\pi\rho V^{3}(A.R.)^{4}}{\omega^{2}(1+A.R.)^{3}} \qquad (19)$$

Solving this equation, we may determine the aspect–ratio and then calculate the fin area from the real or imaginary part of the impedance.

Nextly, we consider a ship with flapping fins. We go back to eq. (10) again and putting following coditions,

$$I_1'=I_2'=0 \qquad (20)$$

we obtain

$$\left.\begin{aligned} &z_1 I_3'+z_2 I_4'=V_1+V_3+V_4\\ &l_1 z_1 I_3'-l_2 z_2 I_4'=V_2+l_1 V_3-l_2 V_4\\ &Z_{33} I_3'=V_3\\ &Z_{44} I_4'=V_4 \end{aligned}\right\} \qquad (21)$$

Substituting eqs. (11) into eqs. (21) and subtracting I_3' and I_4', we obtain

$$\left.\begin{aligned} &\frac{Z_{13}}{Z_{33}}V_3+\frac{Z_{14}}{Z_{44}}V_4=V_1\\ &l_1\frac{Z_{13}}{Z_{33}}V_3-l_2\frac{Z_{14}}{Z_{44}}V_4=V_2 \end{aligned}\right\} \qquad (22)$$

Solving above equations with respect to V_3 and V_4, we obtain

$$\left.\begin{aligned} &(l_1+l_2)\frac{Z_{13}}{Z_{33}}V_3=l_2 V_1+V_2\\ &(l_1+l_2)\frac{Z_{14}}{Z_{44}}V_4=l_1 V_1-V_2 \end{aligned}\right\} \qquad (23)$$

These equations are similar to eqs. (13) and we obtain eq. (16) again assuming the same particulars for fore– and aft–fins. Therefore, we obtain the position l_1 and l_2 at first, and the impedance of fins could be calculated by the one of eqs. (23) as

Table 2 Calculated particulars of two–pairs of flapping fins for model ship (F_n=0.18)

λ/L	Fins Position		Fin's Particular			Remarks
	l_1(m)	l_2(m)	A_p(m²)	C_p(m)	k'(Kg)	
0.9	-3.097	4.653	0.113	0.336	23.39	(1)
0.95	-5.986	9.314	0.057	0.240	-6.23	(2) Fixed Fin A_p=0.0146, AR=0.154
1.0	-1.671	5.751	0.058	0.240	11.16	(2)
1.05	0.486	3.707	0.158	0.398	-48.48	(2) Fixed Fin A_p=0.0452, AR=0.239
1.1	1.249	3.024	0.068	0.261	25.35	(2)
1.2	1.805	2.643	0.124	0.351	26.27	(2)
1.4	2.236	2.643	0.268	0.518	51.85	(2)
1.6	2.539	2.835	0.397	0.630	76.44	(2)
1.8	2.823	3.076	0.520	0.724	98.98	(2)
2.0	3.106	3.339	0.616	0.785	127.43	(2)

Notes

(1) : The first root of equation (16).

(2) : The second root of the above equation.

A.R. = 1.0, k' = k x b_f.

$$\frac{Z_{13}}{Z_{33}}z_1=\frac{V_1 e^{-KZ1-iKlc}}{2i\omega_0 a\cos Kl_m} \qquad (24)$$

Otherwise, we can take no account of mass of fin because the added mass is greater than it in this problem. Then, we assume

$$Z_{33}\fallingdotseq z_1-Z_{13},\ Z_{44}\fallingdotseq z_2-Z_{14} \qquad (25)$$

Putting above equations into eq. (24), we obtain

$$\frac{1}{Z_{13}}-\frac{1}{z_1}=\frac{2i\omega_0 a}{V_1}\cos kl_m\ e^{KZ1+iklc}$$

Therefore, fin area could be calculated from the real part of eq. (26) for a given aspect–ratio because Z_{13} is pure imaginary number. Spring constant, k_f, could be obtained from the imaginary part of eq. (26) after that. When spring constant takes negative value, imaginary part of the right hand side of eq. (26) is positive, it means that fixed fins will satisfy the condition given by eq. (18). So that, we can determine the aspect–ratio and the fin area by eq. (19).

Carrying out this calculation for the present model, we have results as shown in Table 2 and we can verify their satisfactry capability by calculation of heaving and pitching responses of the ship. Thus, it seems that the apporopriate combination of two pairs of fins to suppress completely heaving and pitching oscillations at any wave length exists.

However, we see two difficulties in these results. The one is that the aft–fin lies far aft of ship's stern so that the actual installation might be impossible. The other is that there are cases to need fins of the very small aspect–ratio.

5. One Pair of Fins for Pitch Free

In a similar way as the preceding case, we may design one pair of fins. At first, we consider the possibility to make a ship heave–and pitch–free by a pair of fins.

Now, we put the equation of motion reffering to eqs. (5) as:

$$\left.\begin{aligned} Z_{11}' I_1 + Z_{12}' I_2 + Z_{13} I_3 &= V_1 \\ Z_{21}' I_1 + Z_{22}' I_2 + Z_{12} I_3 &= V_2 \\ Z_{33} I_3 &= V_3 \end{aligned}\right\} \qquad (27)$$

Putting $I_1 = I_2 = 0$ into eqs. (27) and using eqs. (7), we obtain

$$\left.\begin{aligned} &\frac{Z_{13}}{Z_{33}} V_3 = V_1 \\ &l_1 V_1 = V_2 \end{aligned}\right\} \qquad (28)$$

However, l must be real value, so that V_1 and V_2 defined by eq. (14) must satisfy the following equation

$$\alpha_2 - \alpha_1 = 0 \text{ or } \pi \qquad (29)$$

Although this condition is satisfied at $\lambda/L =$ 0.654, 0.942 in our model ship, the calculated fin area results in negative value. So that, it seems that a pair of fins would not make a ship wave–free in both oscillations.

Nextly, we may design one pair of fins for pitch–free. Putting $I_2 = 0$ into eq. (27), we obtain

$$\left.\begin{aligned} &Z_{11} I_1 + Z_{13} (I_3 - I_1) = V_1 \\ &Z_{21} I_1 + Z_{23} (I_3 - I_1) = V_2 \\ &(Z_{31} + Z_{33}) I_1 + Z_{33} (I_3 - I_1) = V_3 \end{aligned}\right\} \qquad (30)$$

From the top two equations in eqs. (30), we obtain

Table 3 Calculated particulars of one pair of fixed fins for model ship ($F_n = 0.18$)

Fins Posi. (m)	Fin's Dim.	λ/L 1.05	1.1	1.2	1.4	1.6	1.8	2.0
1.1	$A_F (m^2)$					0.042	0.066	0.081
	$C_F (m)$	*	*	*	*	1.366	1.000	0.937
	A.R.					0.022	0.066	0.092
1.2	A_F				0.046	0.065	0.077	0.087
	C_F	*	*	*	1.000	0.820	0.794	0.795
	A.R.				0.046	0.096	0.122	0.138
1.3	A_F			0.040	0.062	0.072	0.081	0.090
	C_F	*	*	1.015	0.701	0.679	0.688	0.706
	A.R.			0.039	0.126	0.157	0.171	0.180
1.4	A_F		0.050	0.061	0.069	0.076	0.083	0.090
	C_F	*	0.884	0.655	0.593	0.598	0.618	0.641
	A.R.		0.064	0.143	0.197	0.213	0.218	0.219
1.5	A_F	0.077	0.075	0.072	0.074	0.076	0.084	0.089
	C_F	0.746	0.627	0.550	0.526	0.540	0.564	0.590
	A.R.	0.138	0.189	0.238	0.267	0.268	0.263	0.256

Note) * means no solution

$$\left.\begin{aligned} I_1 &= \frac{lV_1 - V_2}{lZ_{11} - Z_{21}}, \\ I_3 - I_1 &= \frac{V_2 Z_{11} - V_1 Z_{21}}{(lZ_{11} - Z_{21}) Z_{13}} \end{aligned}\right\} \qquad (31)$$

Substituting these equations into the third equation of eqs. (30), we obtain

$$\frac{1}{lZ_{11} - Z_{21}} \left[(lV_1 - V_2)(Z_{33} + Z_{13}) + \frac{Z_{33}}{Z_{13}} (V_2 Z_{11} - V_1 Z_{21}) \right] = V_3$$

By using eqs. (8) and (25), we obtain the following equation which is similar to eq. (26)

$$\frac{1}{Z_{13}} - \frac{1}{z} = \frac{v_* (lZ_{11} - Z_{21}) - (lV_1 - V_2)}{V_2 Z_{11} - V_1 Z_{21}} \qquad (32)$$

Therefore, particulars of fin and the spring constant could be determined as the same process as the case for two pairs of fins. However, the fin position is not determined explicitly in this case. So that, we calculate the fin–area and the spring constant for given fin position and aspect–ratio. When the spring

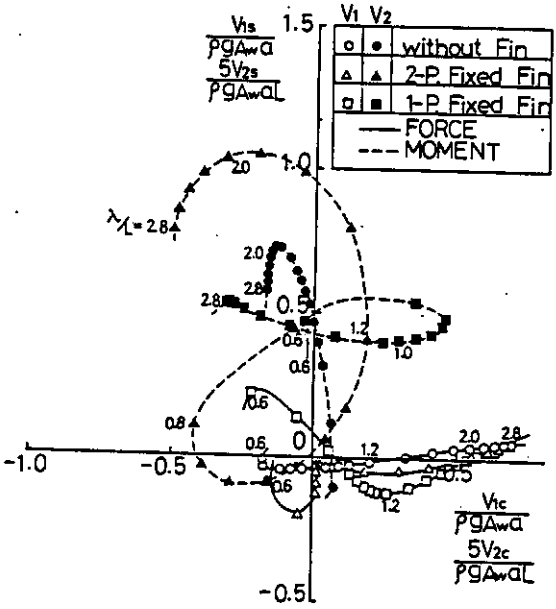

Fig. 4 Vector representation of wave–exciting force and moment

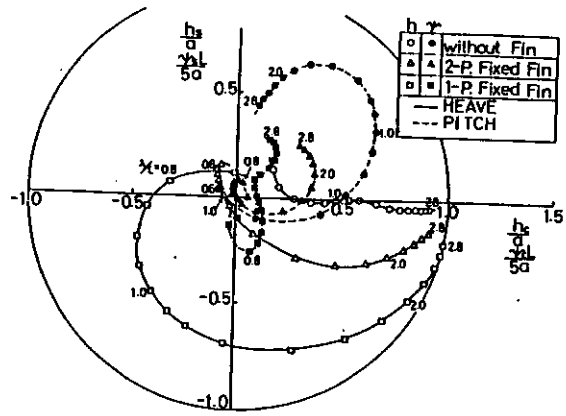

Fig. 5 Vector representation of heave and pitch motions

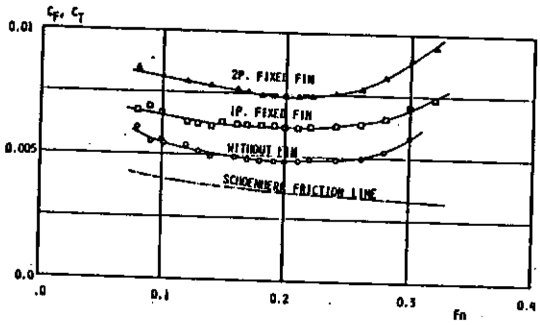

Fig. 7 Resistance in calm sea

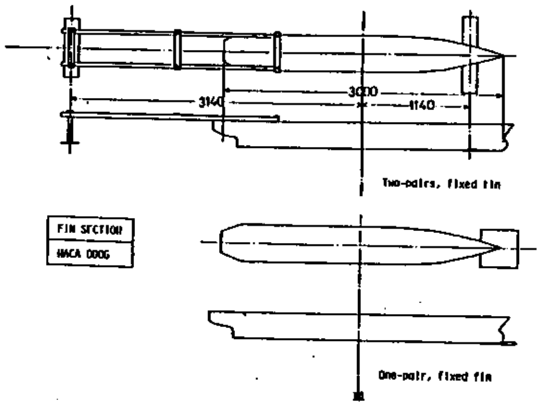

Fig. 6 Arrangement of fins

constant becomes negative, we can find a solution for fixed fins as the same reason as the case for two pairs of fins. Table 3 is a result of such calculation for one pair of fixed fins. As seen in the table, when the fin position approaches near mid–ship, the fin–area becomes smaller and at the same time its aspect–ratio becomes extraordinarily small, so that the fin could not have anti–pitching effect as remarked before. Thus, the only preferable position of the fins may be near the F. P.

As the similar manner, we can calculate the particulars of fins for heave–free by putting the following equation

$$I_1 = 0 \tag{33}$$

and for the heave–free at l_p foward the C. G.

$$I_1 + l_p I_2 = 0 \tag{34}$$

Lastly, both the magnitude and phase of quantities appeared is important in these calculation, so that it is convenient and helpful for our understanding to plot them in the complex plane as shown in Fig. 4 and 5. These figures give us a clear geometrical image with respect to the effect of fin.

6. Experiments

To confirm the reliability of the foregoing theory, we carried out erperiments using

	$A_f(m^2)$	$C_f(m)$	A.R.	$l_1(m)$	$l_2(m)$	$k'(Kg)$	Cal.	Exp.
Without Fin							———	●
2P. Fixed	0.050	0.160	1.938	1.14	3.14	∞	– – –	○
2P. Fixed	0.040	0.144	do.	do.	do.	do.	—·—	

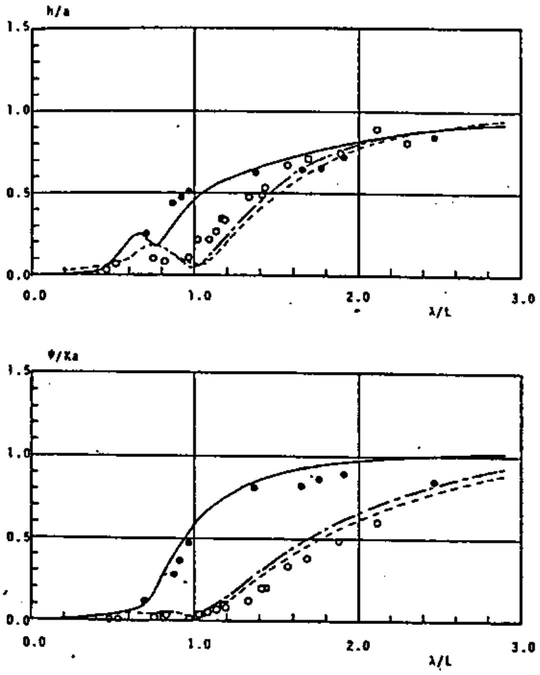

Fig. 8 Experimental results of amplitude responses of a ship with or without two pairs of fins

	$A_f(m^2)$	$C_r(m)$	A.R.	$l(m)$	$k'(Kg)$	Cal.	Exp.
Without Fin						———	●
1P. Fixed	0.080	0.40	0.5	1.50	∞	– – –	○
1P. Fixed	0.065	0.36	do.	do.	do.	—·—	

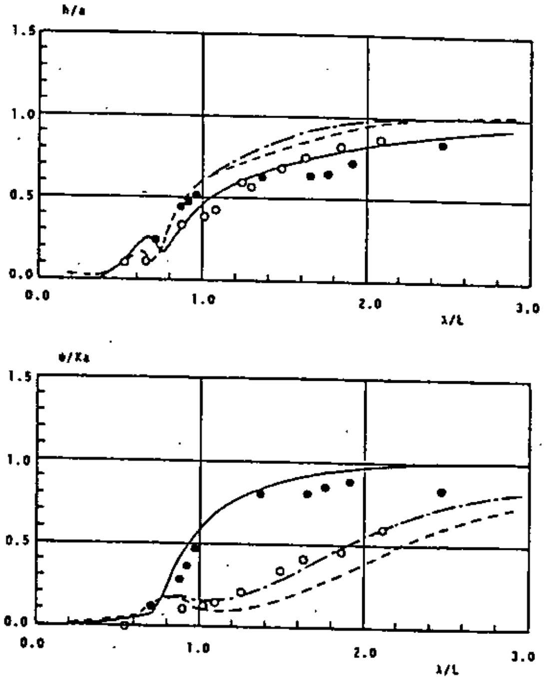

Fig. 9 Experimental results of amplitude responses of a ship with or without one pair of fins

three–meter models shown in Table 1 at Meguro Ship Model Basin of Defense Agency. The first test was carried out for a naked hull without fin, the second with two pairs of fixed fins and the third with one pair of fixed fins arranged as shown in Fig. 6.

Before the test in waves, resistance tests were carried out in calm water. Fig. 7 shows the results. A large added resistance by fin is remarkable and the added resistance coefficient divided by the fin area is about 0.012 for one pair fins and 0.019 for two pairs respectively. These values are extraordinarity higher than the value of 0.006 obtained by wind–tunnel test results at $Re=10^6$. [8] These differences would be explained as that the relative angle of attack of fins to the water might have a certain value. In any way, this is no doubt one of the greatest practical difficulty.

Then, we carried out test in waves. The responses are shown in Figs. 8 and 9 with fin particulars. These fins are selected merely for simplicity sake and do not give strictly heave and/or pitch–free but it might give a sufficient sea–keeping quality as shown in the figures.The agreement with theory and experiments seems fairly well but there are a little differences between them. Then, multiplying factor 0.8 to the fin's impedance, we have responses of chain lines in these figures. This correction seems not always right but do for pitching response for one pair of fins in Fig. 9.

The resistance increase in waves are also measured by a gravity dynamometer and shown in Figs. 10 and 11. The solid and dotted lines are theoretical values calculated by Ger-

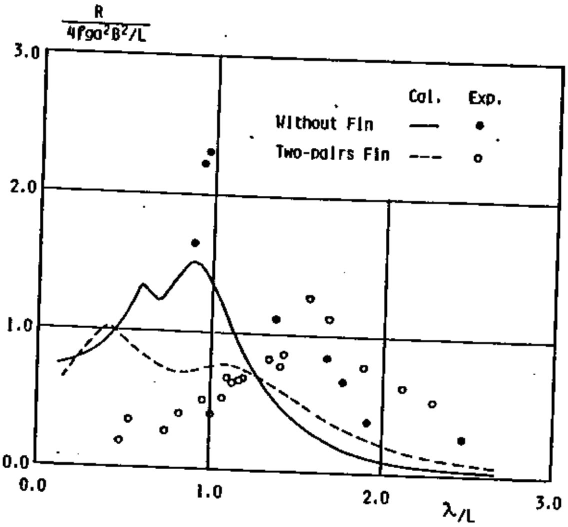

Fig. 10 Resistance increase of a ship with or without two pairs of fins

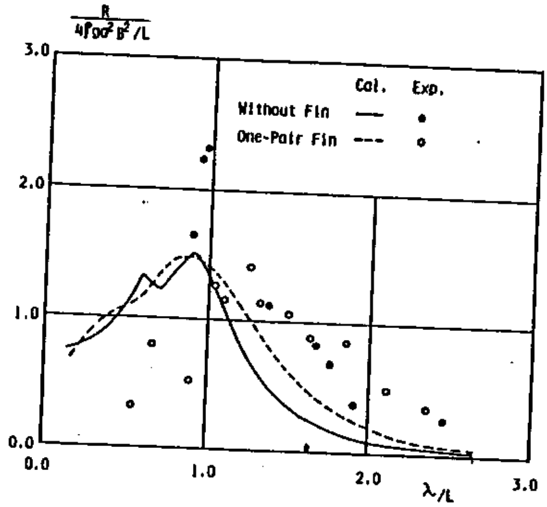

Fig. 11 Resistance increase of a ship with or without one pair of fins

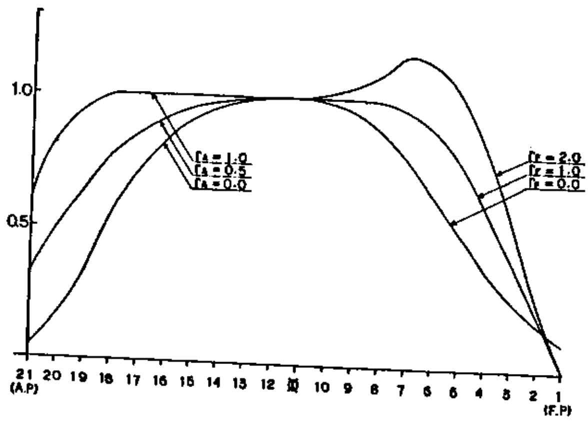

Fig. 12 C_w—curve of a container ship

Table 4 Water plane area and vertical prismatic coefficient of models

Model	$(r_f r_a)$	A_w (m²)	Volume (m³)	Cvpf	Cvpa	Cvp
Container	(0000)*	0.679		1.1	1.1	1.1
	(0000)	0.775			0.964	0.964
	(0005)	0.856		0.964	0.803	0.883
	(0010)	0.937			0.688	0.826
	(1000)	0.848	0.103		0.964	0.885
	(1005)	0.929		0.806	0.803	0.805
	(1010)	1.010	(Proto-type)		0.688	0.747
	(2000)	0.920			0.964	0.828
	(2005)	1.001		0.693	0.803	0.748
	(2010)	1.082			0.688	0.691
Cargo		0.963	0.147	0.928	0.820	0.874
Tanker		1.340	0.205	0.967	0.863	0.915
DC		0.749	0.051	0.764	0.591	0.677

Note) Model length = 3.0 m

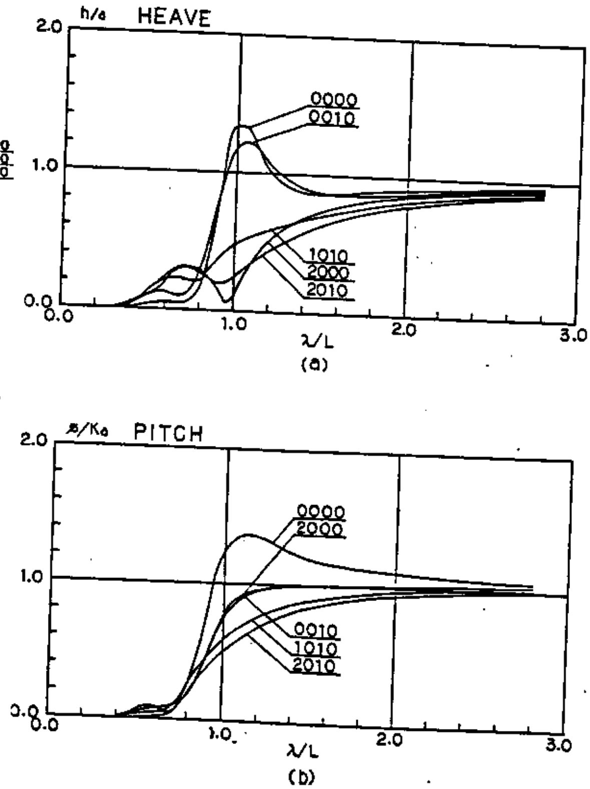

Fig. 13 Amplitude responses of heave and pitch of a ship without fins

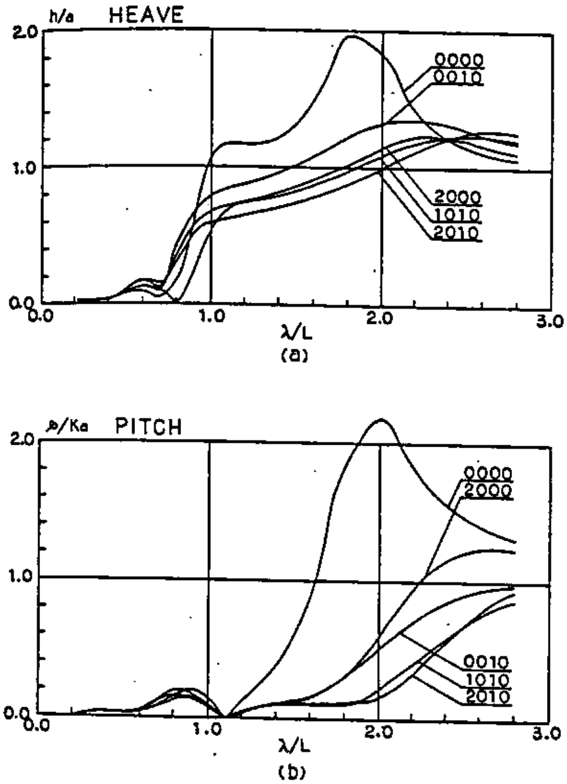

Fig. 14 Amplitude responses of heave and pitch of a ship with one pair of fixed fins

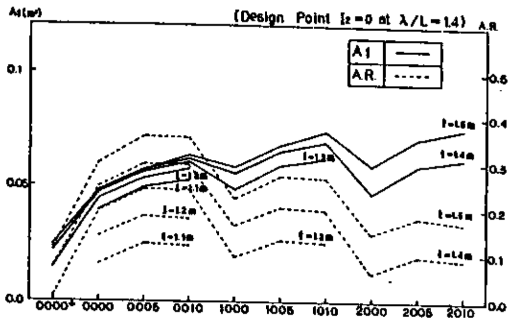

Fig. 15 Optimum fin area and aspect–ratio

ritsma's formula [2] for the only main hull. The test results show that the resistance increase reduces fairly the amount near the wave length equal to the ship length and this is observed also by G. P. Stefun. [7] The one of the reason may be a motion reduction by fin because the resistance increase is proportional to the relative vertical velocity of ship to water and this is seen from theoretical values in Fig. 10. The other may be a thrust produced by oscillating fin [10] but we have little knowledge about this side of phenomena, so that we leave this in the future and show only a rough approximation in Appendix.

7. Deformation of Ship Form and the Necessary Fin Area

Nextly, we study on the relation between the fin area and particulars of ship form. The sea–keeping quality of a ship is thought to depend mainly on the prismatic curve and the water–plane area. However, the prismatic curve of a ship is determined by the performances of resistance and propulsion in still water, so that the water–plane area is only the one we could change to improve sea–keeping quality. This correspond to the existing results that the sea–keeping quality could be classified by the fore– and aft–part of the vertical prismatic curve. We carry out a series of calculations for ships which have the same prismatic curve but have different water–plane area in order to obtain optimum fins of smaller area.

Let us denote the prismatic curve, water–plane area curve and breadth of a section of a ship as $C_p(x)$, $C_w(x)$ and $B(x)$ respectively.A section of a ship is deformed as followings:

(1) $C_p(x)$ and the depth of the section is kept constant

(2)

$$\frac{B(x)}{Bm.s.} = C_p(x) + r \cdot f(x) \qquad (35)$$

where

$f(x) = C_w(x) - C_p(x)$

$Bm.s.$: breadth of the midship section

r : a constant

(3) sections are expressed by Lewis–form

We distinguish the constant, r, between fore– and aft–part, and express them as r_F and

r_A respectively. Here, we choose the values for r_F as 0.0, 1.0 and 2.0, and for r_A as 0.0, 0.5 and 1.0, so that we consider 9 models as their combinations. If r_F and r_A equal to 1.0 which is symbolized by (1010), the model corresponds to the original ship. The water–plane area curve of a container ship are shown in Fig. 12. The water–plane area and vertical prismatic coefficients of the ship are shown in Table 4, where (0000)* denotes a model whose breadth is expressed by

$$\frac{B(x)}{Bm.s.}=\frac{C_p(x)}{1.1} \tag{36}$$

Amplitude responses of heave and pitch of the models are shown in Figs. 13 for the case without fin. In general, both responses of heave and pitch of a ship of small water–plane area such as (0000) seem to become large. However, responses would be suppressed by increase of the water–plane area of fore part such as (2010). On the other hand, the responses of motion of models with one pair of fixed fins at the bow which is designed pitch–free at $\lambda/L=1.1$ are shown in Figs. 14. The optimun fin area and the aspect–ratio for those models are shown in Fig. 15. Pitching of a model with fins is greatly suppressed in wide range of wave length except (0000). On the contrary, heaving becomes larger than that of a model without fins and the response would not much depend on the water–plane area.

When the water–plane area is small, the necessary fin area becomes smaller as shown in Fig. 15. However, aspect ratio of such fins is also small, so that such fins seem not to act like calculations. We should modify our theory to the fins of small aspect ratio.

8. Conclusion

We have discussed the fins necessary to suppress completely the heaving and/or pitching oscillation at a given wave length in head seas and have the following conclusions.

(1) Two pairs of fins may be sufficient almost always to make a ship heave– and pitch–free. Although there are left two arbitrarinesses, we can calculate the position, area and aspect–ratio of fin assuming the same fin fitted. However, the position of fin lies far from the A.P. in almost all cases.

(2) A pair of fins would make a ship either heave–free or pitch–free. In this paper, we have dealt with the pitch–free case and calculated the particulars of fins for a given position near F.P.. However, such fins do not exist in shorter wave range but do in the range longer than resonance. This range shifts slightly if the hull forms of ships are different.

(3) In both cases, the fins thus obtained are much larger than the one studied in the past. The aspect–ratio sometimes becomes very much smaller but these slutions are false because such fins do not act effectively.

(4) The experiments were carried out and the experimental results agree well with the theory.

(5) The greatest difficulty may be, of course, such a large area of fin. Therefore, we have carried out the same calculation with respect to various types of ship and found that the fin area decrease when the vertical prismatic coefficient increases. Thus, a semi–submerged ship needs only a small area of fin but the responses in waves are much larger than the ordinary ship except the designed wave length. These observations suggest a dominant role of the damping force for reduction of oscillations.

9. Acknowledgement

We thank Meguro Ship Model Basin of Defense Agency and Hiratsuka Technical Institute Laboratory of Sumitomo Heavy Industry Co. Ltd. for thier experimental support.

References

1) J. T. T. C. (Jul. 1969): Text of Symp. on Sea–Keeping Performance.
2) J. T. T. C. (Dec. 1977): Text of the second Symp. on Sea–Keeping Performance.
3) Abkowitz, M. A. (1959): The Effect of Anti–Pitching Fins on Ship Motion, S. N. A. M. E.
4) Matsui, M. et al. (1966): On the Controlled Anti–Pitching Fins, J. S. N. A. Japan, Vol.

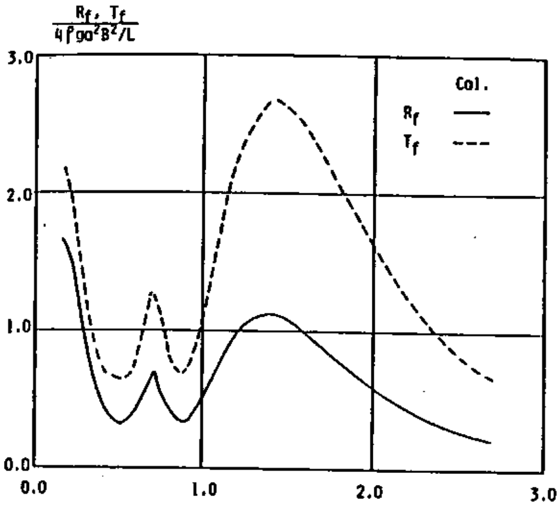

Fig. A-1 Resistance increase and thrust of two pairs of fixed fins

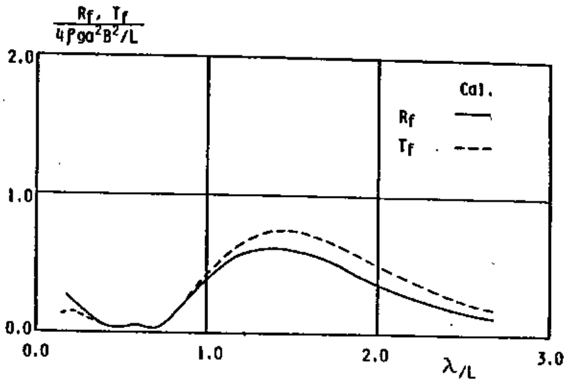

Fig. A-2 Resistance increase and thrust of one pair of fixed fins

119.

5) Vugts, Ir. J. H. (1967): Pitching and Heaving with Fixed and Controlled Bow Fins, I. S. P., No. 153.
6) Fein, J. A. et al. (1980): The Sea-Keeping Characteristics of a SWATH Ship, 13th O. N. R. Symp. Tokyo.
7) Stefun, G. P. (Oct. 1959):Model Experiments with Fixed Bow Anti-Pitching Fins, J. S. R., Vol. 3, No. 2.
8) Abott, I. H. and von Doenhoff, A. E. (1959): Theory of Wing Section, Dover Pub.
9) Lee, C. M. (1983): Preliminary Studies Leading to Seakeeping Hull Design, PRADS'83, Tokyo-Seoul.
10) Jacobsen, E. (Sep. 1981): The Foil Propeller, Wave Power for a Propulsion, 2nd Symp. on Wave and Tidal Energy, Cambridge, England.
11) Bessho, M. (1965): On Wave-Free Distribution in the Oscillating Problems, J. S. N. A. Japan, Vol. 117.
12) Bessho, M. (Apr. 1982): Introduction to Water Wave Engineering, Bull. S. N. A. Japan, No. 634.

Appendix
Resistance Increase in Waves and Thrust of Fin

The resistance increase R in waves may be estimated by Gerritsuma's formula [2]:

$$R = W/(c+V) \qquad (A.1)$$

where

$$W = \frac{1}{2}\int_L N\,|\,v_r\,|^2\,dx \qquad (A.2)$$

and c is the phase velocity of waves. N and v_r denote the wave damping of the ship section and the vertical relative respectively. W is the radiated wave power by a ship which is absorbed from the in-coming waves, so that this resistance results from the mometum loss of the in-coming wave.

Therefore, in the case of fin we could make use of this formula in this sense, that is,

$$R_f = \frac{\pi\rho V A_f\,|\,v_r\,|^2}{(C+V)\,(1+\frac{1}{A.R.})} \qquad (A.3)$$

for a pair of fins, making use of eq. (2).

On the other hand, the oscillating fin may have a thrust. [10] Assuming quasi-steady process because of small reduced frequency for practical case, we may estimate it as follows. Namely, since the thrust of a pair of fins is the time mean of the x-component of the lift by eq.

(1), we may have approximately

$$T=\frac{1}{2}(2L)\left|\frac{v}{V}\right|^2$$

$$=\frac{\pi\rho A_f|v_0|^2}{(1+\frac{1}{A.R.})^2} \quad (A.4)$$

Of course, this must be multiplied some reduction factor, but represents its property qualitatively. Figs. A–1 and A–2 are the calculated results corresponding to the experiments. The order of magnitude agrees well with the experiments but differs in phase.

In any way, if we expect a thrust of fin it is clear from these condition that the aspect–ratio and the relative velocity to water must be large.