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WAVE-FREE FLOATING-BODY FORMS IN OSCILLATIONS

S. YAMASHITA *

M. BESSHO **

SUMMARY

This paper summarizes the theory and the results of calculations and experiments for the wave-free forms in oscillating problems. The characteristic features of the wave-free forms in heaving and swaying oscillations are described, and the properties of wave exciting forces acting on the wave-free floating bodies are shown.

1. INTRODUCTION

The wave-free frequency or wave-free point means the frequency at which a floating body does not generate radiating waves when it oscillates in calm water. The form of a floating body which has the wave-free point is generally called a wave-free form. The Haskind relation shows the radiating wave is proportional to the wave exciting force on the floating body in waves. Consequently, a floating body having the wave-free point receives no wave exciting force in the incident wave of the frequency corresponding to the wave-free frequency.

It was first pointed out by Ursell (1949) that there was a wave-free form which had a wave-free point in rolling oscillation. Thereafter, the study on the wave-free forms has been followed with the development of fundamental theory for the wave-free form by Bessho (1965) and the experimental investigations by Motora (1965). These studies have shown the characteristic features of the wave-free form in heaving oscillation and contributed greatly to the progress of semi-submersible marine structures.

Through the development of the calculation method of hydrodynamic forces, the wave exciting forces acting on a two-dimensional wave-free body and an axisymmetric wave-free body were calculated and the properties of the wave exciting force were gradually clarified. In recent years wave-free forms in which wave-free points in heaving oscillation appear at two different frequencies were found and a form which generates radiating waves only in one side of the two-dimensional body was also obtained. Moreover, the investigations of wave-free forms have been carried out to find the characteristic features of the wave-free form in swaying oscillation.

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2. WAVE-FREE FORMS IN HEAVING OSCILLATION

Wave-Free Potentials

Let us consider a two-dimensional body oscillating periodically on the free water surface. The circular frequency and amplitude of oscillation are denoted by ω and X respectively and the velocity potential can be represented by $i\omega X\phi(y,z)e^{i\omega t}$. The linearized condition of the free water surface becomes

$$K\phi - \frac{\partial \phi}{\partial z} = 0, \qquad for \quad z=0 \tag{1}$$

where $K = \omega^2/g$ and g is the acceleration of gravity. The free surface condition can be written in the alternative form

$$Re\left[\left(K - i\frac{d}{dt}\right)f(t)\right] = 0, \qquad for \quad z=0 \tag{2}$$

by using the complex variable t and the complex velocity potential f(t) given by

$$t = y + iz$$

$$f(t) = \phi(y,z) + i\,\psi(y,z). \tag{3}$$

Here Re [ ] means taking the real part and ψ represents the stream function.

The wave-free potential is the velocity potential that satisfies the conditions of the free water surface and no radiating waves at infinity. Bessho (1965) has shown that the wave-free potential can be obtained from the adjoint differential operator of the free surface condition (2). If an auxiliary function m(t) is assumed to be a regular function at infinity, the wave-free potential can be expressed in the form

$$f(t) = \left(K + i\frac{d}{dt}\right)m(t) \tag{4}$$

In the case of heaving or swaying oscillation of the two-dimensional body having a cross section symmetric with regard to z axis, m(t) is given by using sources and sinks as follows:

$$m(t) = A\left\{log\frac{t-(\ell+ih)}{t-(\ell-ih)} \pm log\frac{t+(\ell-ih)}{t+(\ell+ih)}\right\} \tag{5}$$

where A is a positive real constant. The sign $\pm$ represents + for heaving and - for swaying oscillation.

Now, let us consider the superposition of the wave-free potential and the potential of the uniform flow toward the positive direction of z axis:

$$F(t) = f(t) + it\,. \tag{6}$$

The streamline through the stagnation point obtained from (6) gives the wave-free form. The method of streamline tracing is applied to the determination of the wave-free form.

The streamlines through the stagnation point for the wave-free singularity (5) where A = 1/2, ℓ = 0 and h = 2 are shown in Fig. 1 (Bessho, 1965). This figure has clarified the wave-free form in heaving oscillation has a bulbous portion below the free water surface.

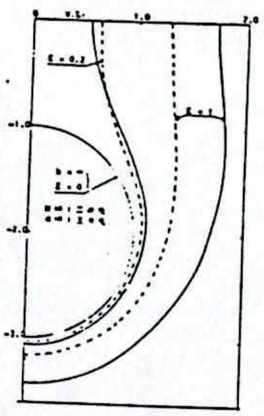

Fig. 1 Streamlines for heaving oscillation

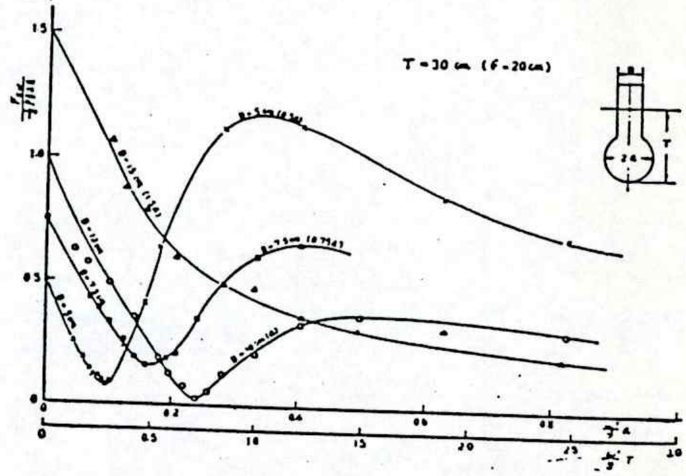

Fig. 2 Measured wave exciting forces

Approximate Approach to Wave-Free Forms

The Kochin function $H^{\pm}(K)$ for the oscillation of the two-dimensional body can be defined by:

$$H^{\pm}(K)=\int_c \left(\frac{\partial\phi}{\partial n}-\phi\frac{\partial}{\partial n}\right)e^{Kz\pm iKy}ds\,, \tag{7}$$

where C is the closed curve expressing the body form and n is the unit normal vector. If the frequency of oscillation is assumed to be small ($K \to 0$), $H^{\pm}(K)$ for heaving oscillation can be approximately expressed by:

$$H^{+}(K)=e^{-Kz_m}\left\{B(1+Kz_m)-K(S+m/\rho)\right\}, \tag{8}$$

where ρ is the fluid density, and B, S and m are respectively the breadth of the body at the water line, the sectional area and the added mass for heaving oscillation of the body. z_m is the representative depth. The Kochin function for the wave-free form becomes zero at a certain frequency. From the Haskind relation

$$Fe/\rho g\zeta_A=-H^{+}(K), \tag{9}$$

it is proved that the wave-free body is free from the wave exciting force Fe acting on the body in waves at a certain wave frequency. Here, ζ_A is the amplitude of the incident wave.

Substitution (8) into (9) shows the wave exciting force for heaving oscillation is composed of the inertia force, the added inertia force and the bouyant force. The bouyant force is usually larger than the sum of the inertia force and the added inertia force, so that the wave exciting force never becomes zero. However, the wave exciting force may become zero at a certain frequency for a body form having a large added inertia force.

Based on this approximate approach, wave excitation tests were conducted with the two-dimensional body having a circular bulbous portion below the free water surface. Fig. 2 shows the measured wave exciting force diminishes at a certain wave frequency (Motora, 1965).

Through the development of the calculation method of hydrodynamic forces, it has become possible to calculate the wave exciting force strictly within the linearized theory. Numerical calculations using the two-dimensional sink-source method show the above form has the wave-free frequency at which the Kochin function will be completely zero as shown in Fig. 3. (Maeda, 1969). The relation between the form and the wave-free frequency was examined for the form having a circular bulbous portion. It has been shown the ratio of the breadth at the waterline to the maximum breadth of the circular bulbous portion is a dominant factor for the wave-free frequency (Maeda, 1969).

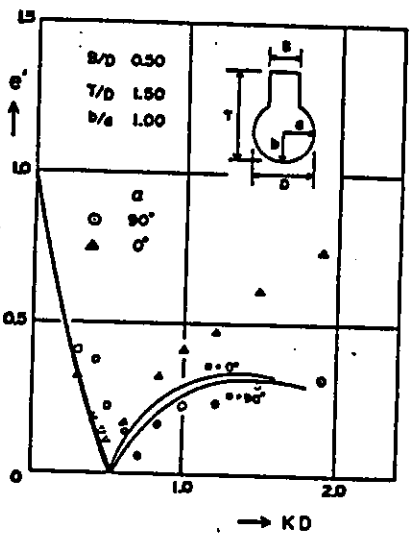

Fig. 3 Wave exciting forces

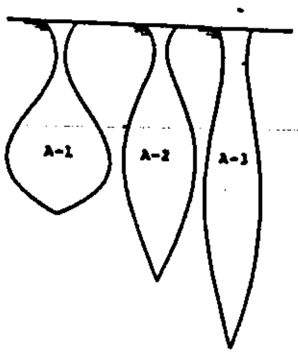

Fig. 4 Calculated doubly wave -free forms

Wave-Free Phenomena in Hydrodynamic Interaction

The twin-hull floating body will be a kind of wave-free body, because the waves that are generated by each hull in oscillation will interact together. When radiating waves are only taken into consideration as the hydrodynamic interaction between two hulls, the Kochin function can be expressed in a simple form for the twin hull floating body. The Kochin function $h^+(K)$ is given by the following expression (Yamashita, 1981):

$$h^+(K) = H^+(K)\left(1 + \frac{C_{T0}}{e^{2iKP} \pm C_{R0}}\right) e^{iKP} \qquad (10)$$

where $H^+(K)$, C_{T0} and C_{R0} are respectively Kochin function for a single body, wave transmission and wave reflection coefficients of the restrained single body. 2P is the distance between the center lines of two hulls. The sign $\pm$ represents - for heaving and + for swaying oscillation. The wave-free phenomena in hydrodynamic interaction occurs when

$$e^{2iKP} = \mp C_{R0} - C_{T0} . \tag{11}$$

If the frequency of oscillation is small ($K \to 0$), the wave-free point will appear near the $KP = \pi/2$. This is equivalent to the case where the half wave length is equal to the distance between the center lines of two hulls.

If each hull of the twin-hull floating body has a wave-free form, two wave-free points exist - the one inherent to the body and the other from hydrodynamic interaction. The theoretical and experimental studies on the hydrodynamic forces of the wave-free twin-hull floating bodies are carried out and the relation between the motions in waves and the wave-free frequencies are investigated (Ohkusu, 1970, 1972).

Doubly Wave-Free and One-Side Wave-Free Forms

There are wave-free forms in which the wave-free point appears at two different frequencies in heaving oscillation. The characteristic features of the form were found by examining the form in which the wave exciting force for heaving approximately becomes zero at two kinds of wave frequencies (Yamashita, 1981). The Kochin function for a slender body in vertical direction is written as follows:

$$H^+(K) = 2\int_{-d}^{0} \frac{db(z)}{dz} e^{Kz} dz , \tag{12}$$

where b(z) is the half-breadth of the form at z and d is draft. The problem minimizing the function

$$J[b] = a\int_{-d}^{0} \left(\frac{db}{dz}\right)^2 dz \tag{13}$$

is considered on the basis of the additional condition where the Kochin function (12) becomes zero at two kinds of wave numbers. The calculated forms are illustrated in Fig. 4. It is clear that the doubly wave-free forms have not only the bulbous portion but also the constricted portion below the free water surface.

The doubly wave-free form in which two different wave-free points overlap each other was derived from the doubly wave-free potential (Kyozuka, 1981). The calculated form has also constricted portion.

The body form generating the radiating waves only in one side of the body, i.e., one-side wave-free form is given (Takagi 1981). This form is good for the wave absorber. It can be derived from the one-side wave-free velocity potential made by the superposition of the symmetric and the anti-symmetric velocity potential with regard to z axis. Figure 5 shows an example of the one-side wave-free form.

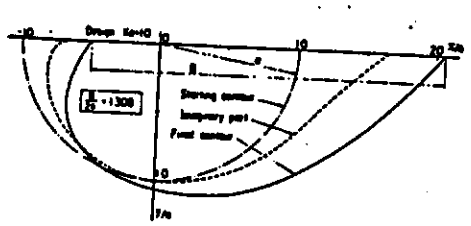

Fig. 5 One-side wave-free forms

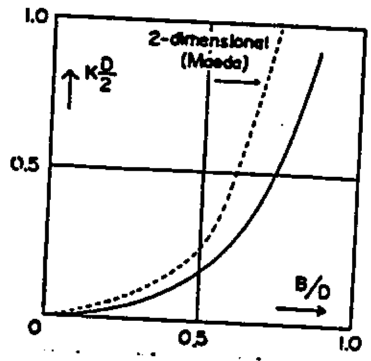

Fig. 6 Chart for the wave-free frequency

3. THREE-DIMENSIONAL WAVE-FREE BODY

Axisymmetric Body

A three-dimensional wave-free body is a floating body which generates no radiating waves in a certain direction when it oscillates. Since the Kochin function H(K, θ) for a three-dimensional floating body can be expressed by

$$H(K,\theta)=\iint_s\left(\frac{\partial\phi}{\partial n}-\phi\frac{\partial}{\partial n}\right)e^{Kz+iK(x\cos\theta+y\sin\theta)}ds\,, \tag{14}$$

this means H(K, θ) = 0 for the specified θ. By the Haskind relation, the floating body does not receive the wave exciting force in the incident waves from the θ direction.

In case of heaving oscillation of an axisymmetric body with z axis,

$$H(K,\theta)=H(K,0) \tag{15}$$

holds. Therefore, if it is wave-free in a certain direction, it will become wave-free in all directions.

As a wave-free form of the axisymmetric body, the form having a spherical bulbous portion below the free water surface was given and it was experimentally shown that the wave exciting force becomes smaller at a certain wave frequency (Motora, 1965). Thereafter, the Kochin function for this form is calculated by the three-dimensional sink-source method, and it is verified the Kochin function diminishes at a certain frequency (Sao, 1971). According to the calculations, the wave-free frequency is approximately determined by the ratio of the diameter of the floating body at the waterline to the maximum diameter of the spherical bulbous portion. This results are given in Fig. 6.

A doubly wave-free form exists with the axisymmetric floating body. The form has the bulbous and constricted portions below the free water surface. The wave-free point appears at two different frequencies as shown in Fig. 7 (Yamashita, 1984). The larger the draft of the doubly wave-free floating body, the closer the two wave-free points will become.

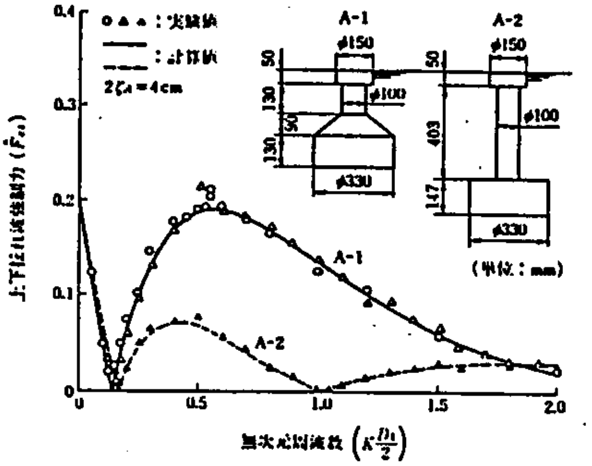

Fig. 7 Wave exciting forces on doubly wave-free bodies

Semi-Submersible

Semi-submersible marine structures which are used for oil drilling rigs have a large hull underwater and a comparatively small water plane area. The characteristics of the semi-submersible are exactly the same as those of the wave-free form and the wave-free frequency appears also in the semi-submersible.

For the lower hull type of semi-submersible in beam waves, the wave exciting force for heaving is given approximately as

$$Fe/\rho g\zeta_A = e^{-Kz_m}\left\{K(\nabla + m/\rho) - A_W(1+Kz_m)\right\}\cos KP\ , \tag{16}$$

where ∇, m and A_W are respectively the volume of displacement, added mass for heaving oscillation and water plane area of the structure. z_m is the depth from the water surface to the center of the lower hull and 2P the distance between the both center lines of the lower hulls. According to this approximate formula, the wave-free point appears at the wave period T_0, where

$$T_0 = \beta_0 T_H \tag{17}$$

$$\beta_0 = \sqrt{1 - K_H z_m} \tag{18}$$

T_H represents the natural period of heaving motion and K_H the wave number corresponding to T_H. The value of $K_H Z_m$ is 0.15 – 0.2 for the conventional semi-submersible. It can be seen from (17) that the wave-free period is always smaller than the natural period. As the natural period is longer and the depth of the lower hull smaller, the wave-free period is closer to the natural period. However, they never coincide.

Since the wave-free form does not receive the wave exciting force at the wave-free point, the motion in waves becomes smaller. For the oil drilling rig, the heaving motion in waves influences the down time of the rig and the wave-free form in heaving oscillation such as the lower hull type of semi-submersible has been adopted. The heaving motion in beam waves z_A is given as follows:

$$\frac{z_A}{\zeta_A} e^{-i\varepsilon_z} = -e^{-Kz_m}\left(1+\frac{Kz_m}{1-K/K_H}\right)\cos KP \qquad (19)$$

by using (16) as the wave exciting force for the lower hull type of semi-submersible.

Ships

There have been very few studies on the application of the wave-free form to ships. One of them is the experimental investigation of the heaving and pitching motions of the ship having a bulbous portion in the lower part of the hull (Maeda, 1969). On the other hand, as a theoretical study the wave-free theory based on the method of representing the ship hull by the singularity distribution in the center plane of the hull has been developed (Bessho, 1967A). It gives an example of the longitudinal distribution of the sectional area of the ship hull which has the wave-free frequency in heaving oscillation.

These studies are made on the assumption that there is no advance speed of the ship. Some investigation of the wave-free form with advance speed is concerned with wave making resistance in calm water (Bessho, 1967B). There is no research on the wave-free form for a body oscillating with advance speed, and this is a main future subject in which a new wave-free theory should be developed.

4. WAVE-FREE FORMS IN SWAYING OSCILLATION

The wave free form in swaying oscillation has been obtained by calculating the streamlines for the wave-free potential for swaying oscillation (Yamashita, 1985). The superposition of the wave-free potential and the potential of the uniform flow toward the negative direction of y axis:

$$F(t)=f(t)+t \qquad (20)$$

is considered and the streamline through the stagnation point is calculated by using the method of streamline tracing. The calculated streamlines shown in Fig. 8 in which the wave-free singularity is given by (5) are classified into

i) Crossing the free water suface,
ii) Entering into the singularity, and
iii) Being closed below the free water surface.

This shows that there are two kinds of wave-free forms for swaying oscillation. The one inferred from i) is a floating body, the surface of which is not vertical and inclined inwardly just below the free water surface. The other inferred from iii) is a fully submerged body. The characteristic features of the former inwardly inclined floating body lies in having the inward slope invariably just below the free water surface.

Figure 9 gives the wave-exciting forces on an inwardly inclined floating body. The wave-free point for swaying oscillation appears in the calculated forces, and the measured forces tend to approach the calculated ones as the wave height becomes smaller. As for the submerged body, the existence of the wave-free point for swaying oscillation is also verified.

The equation of the swaying motion of a two-dimensional body in waves is expressed in the form:

$$\frac{y_A}{\zeta_A} e^{-i\varepsilon_y} = \frac{(D_4 - D_{24}\, l_w) H^+(K)}{D_2 D_4 - D_{24}^2} \tag{21}$$

Therefore the swaying motion diminishes theoretically at the wave-free frequency for swaying oscillation in which $H^+(K) = 0$. The inwardly inclined floating body can be applied to a floating breakwater and the wave transmission for the body was examined theoretically and experimentally (Yamashita, 1986).

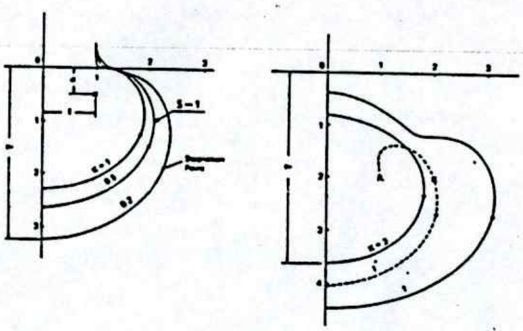

Fig. 8 Streamlines for swaying oscillation

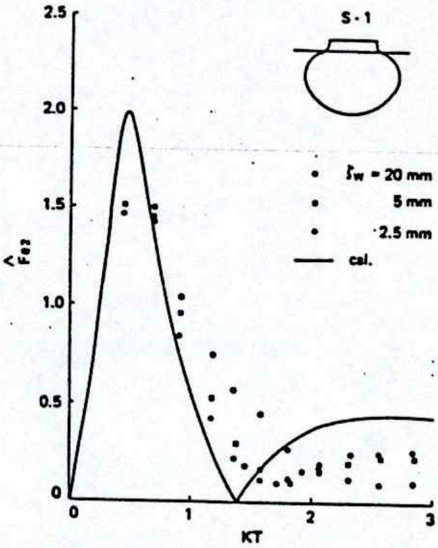

Fig. 9 Wave exciting forces for swaying

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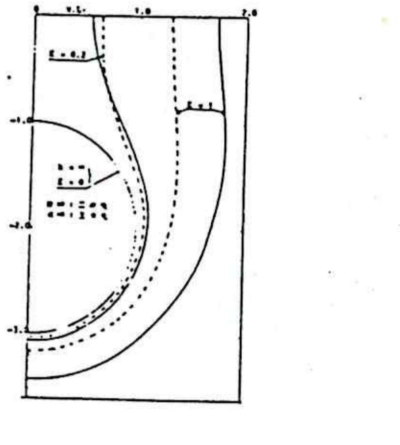
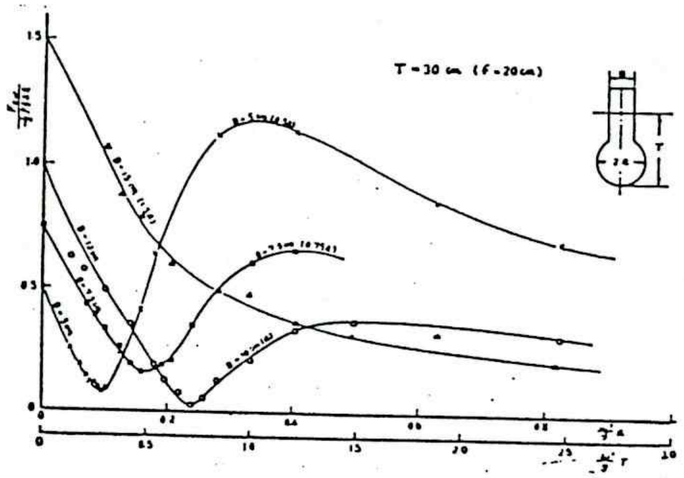


Fig. 1 Streamlines for heaving oscillation



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Substitution (8) into (9) shows the wave exciting force for heaving oscillation is composed of the inertia force, the added inertia force and the bouyant force. The bouyant force is usually larger than the sum of the inertia force and the added inertia force, so that the wave exciting force never becomes zero. However, the wave exciting force may become zero at a certain frequency for a body form having a large added inertia force.

Based on this approximate approach, wave excitation tests were conducted with the two-dimensional body having a circular bulbous portion below the free water surface. Fig. 2 shows the measured wave exciting force diminishes at a certain wave frequency (Motora, 1965).

Through the development of the calculation method of hydrodynamic forces, it has become possible to calculate the wave exciting force strictly within the linearized theory. Numerical calculations using the two-dimensional sink-source method show the above form has the wave-free frequency at which the Kochin function will be completely zero as shown in Fig. 3. (Maeda, 1969). The relation between the form and the wave-free frequency was examined for the form having a circular bulbous portion. It has been shown the ratio of the breadth at the waterline to the maximum breadth of the circular bulbous portion is a dominant factor for the wave-free frequency (Maeda, 1969).

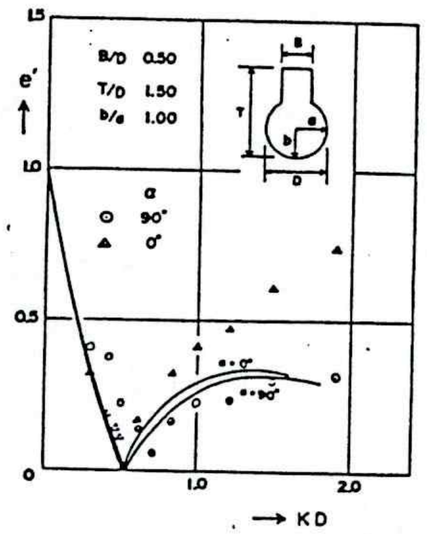

Fig. 3 Wave exciting forces

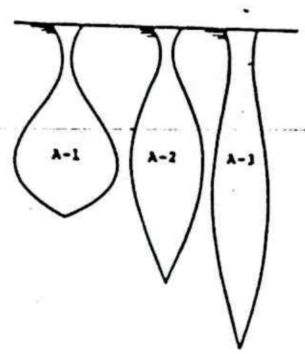

Fig. 4 Calculated doubly wave-free forms

Wave-Free Phenomena in Hydrodynamic Interaction

The twin-hull floating body will be a kind of wave-free body, because the waves that are generated by each hull in oscillation will interact together. When radiating waves are only taken into consideration as the hydrodynamic interaction between two hulls, the Kochin function can be expressed in a simple form for the twin hull floating body. The Kochin function $h^{+}(K)$ is given by the following expression (Yamashita, 1981):

$$h^{+}(K)=H^{+}(K)\left(1+\frac{C_{T0}}{e^{2iKP}\pm C_{R0}}\right)e^{iKP} \qquad (10)$$

where $H^{+}(K)$, C_{T0} and C_{R0} are respectively Kochin function for a single body, wave transmission and wave reflection coefficients of the restrained single body. 2P is the distance between the center lines of two hulls. The sign $\pm$ represents - for heaving and + for swaying oscillation. The wave-free phenomena in hydrodynamic interaction occurs when

$$e^{2iKP}=\mp C_{R0}-C_{T0}\,. \qquad (11)$$

If the frequency of oscillation is small ($K \to 0$), the wave-free point will appear near the $KP = \pi/2$. This is equivalent to the case where the half wave length is equal to the distance between the center lines of two hulls.

If each hull of the twin-hull floating body has a wave-free form, two wave-free points exist - the one inherent to the body and the other from hydrodynamic interaction. The theoretical and experimental studies on the hydrodynamic forces of the wave-free twin-hull floating bodies are carried out and the relation between the motions in waves and the wave-free frequencies are investigated (Ohkusu, 1970, 1972).

Doubly Wave-Free and One-Side Wave-Free Forms

There are wave-free forms in which the wave-free point appears at two different frequencies in heaving oscillation. The characteristic features of the form were found by examining the form in which the wave exciting force for heaving approximately becomes zero at two kinds of wave frequencies (Yamashita, 1981). The Kochin function for a slender body in vertical direction is written as follows:

$$H^{+}(K)=2\int_{-d}^{0}\frac{db(z)}{dz}e^{Kz}dz\,, \qquad (12)$$

where b(z) is the half-breadth of the form at z and d is draft. The problem minimizing the function

$$J[b]=a\int_{-d}^{0}\left(\frac{db}{dz}\right)^{2}dz \qquad (13)$$

is considered on the basis of the additional condition where the Kochin function (12) becomes zero at two kinds of wave numbers. The calculated forms are illustrated in Fig. 4. It is clear that the doubly wave-free forms have not only the bulbous portion but also the constricted portion below the free water surface.

The doubly wave-free form in which two different wave-free points overlap each other was derived from the doubly wave-free potential (Kyozuka, 1981). The calculated form has also constricted portion.

The body form generating the radiating waves only in one side of the body, i.e., one-side wave-free form is given (Takagi 1981). This form is good for the wave absorber. It can be derived from the one-side wave-free velocity potential made by the superposition of the symmetric and the anti-symmetric velocity potential with regard to z axis. Figure 5 shows an example of the one-side wave-free form.

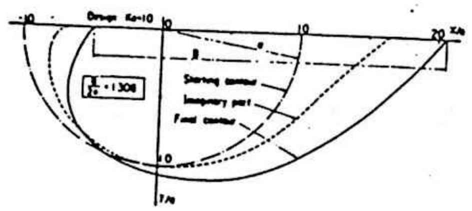

Fig. 5 One-side wave-free forms

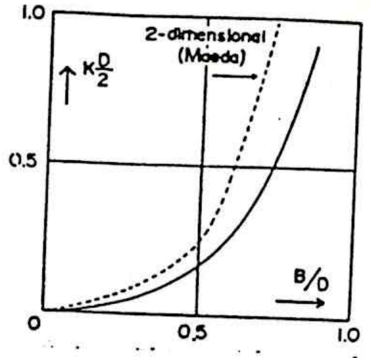

Fig. 6 Chart for the wave-free frequency

3. THREE-DIMENSIONAL WAVE-FREE BODY

Axisymmetric Body

A three-dimensional wave-free body is a floating body which generates no radiating waves in a certain direction when it oscillates. Since the Kochin function H(K, θ) for a three-dimensional floating body can be expressed by

$$H(K,\theta)=\int\int_{s}\left(\frac{\partial\phi}{\partial n}-\phi\frac{\partial}{\partial n}\right)e^{Kz+iK(x\cos\theta+y\sin\theta)}ds\,, \qquad (14)$$

this means H(K, θ) = 0 for the specified θ. By the Haskind relation, the floating body does not receive the wave exciting force in the incident waves from the θ direction.

In case of heaving oscillation of an axisymmetric body with z axis,

$$H(K,\theta)=H(K,0) \qquad (15)$$

holds. Therefore, if it is wave-free in a certain direction, it will become wave-free in all directions.

As a wave-free form of the axisymmetric body, the form having a spherical bulbous portion below the free water surface was given and it was experimentally shown that the wave exciting force becomes smaller at a certain wave frequency (Motora, 1965). Thereafter, the Kochin function for this form is calculated by the three-dimensional sink-source method, and it is verified the Kochin function diminishes at a certain frequency (Sao, 1971). According to the calculations, the wave-free frequency is approximately determined by the ratio of the diameter of the floating body at the waterline to the maximum diameter of the spherical bulbous portion. This results are given in Fig. 6.

A doubly wave-free form exists with the axisymmetric floating body. The form has the bulbous and constricted portions below the free water surface. The wave-free point appears at two different frequencies as shown in Fig. 7 (Yamashita, 1984). The larger the draft of the doubly wave-free floating body, the closer the two wave-free points will become.

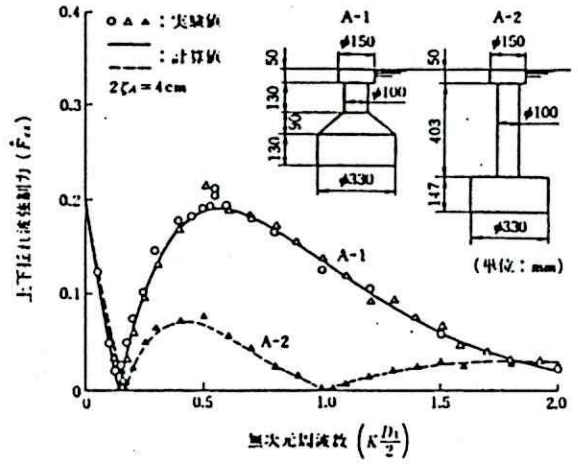

Fig. 7 Wave exciting forces on doubly wave-free bodies

Semi-Submersible

Semi-submersible marine structures which are used for oil drilling rigs have a large hull underwater and a comparatively small water plane area. The characteristics of the semi-submersible are exactly the same as those of the wave-free form and the wave-free frequency appears also in the semi-submersible.

For the lower hull type of semi-submersible in beam waves, the wave exciting force for heaving is given approximately as

$$Fe/\rho g\zeta_A=e^{-Kz_m}\left\{K(\nabla+m/\rho)-A_W(1+Kz_m)\right\}\cos KP\,, \qquad (16)$$

where ∇, m and A_W are respectively the volume of displacement, added mass for heaving oscillation and water plane area of the structure. z_m is the depth from the water surface to the center of the lower hull and 2P the distance between the both center lines of the lower hulls. According to this approximate formula, the wave-free point appears at the wave period T_0, where

$$T_0=\beta_0 T_H \qquad (17)$$

$$\beta_0=\sqrt{1-K_H z_m} \qquad (18)$$

T_H represents the natural period of heaving motion and K_H the wave number corresponding to T_H. The value of $K_H z_m$ is 0.15 - 0.2 for the conventional semi-submersible. It can be seen from (17) that the wave-free period is always smaller than the natural period. As the natural period is longer and the depth of the lower hull smaller, the wave-free period is closer to the natural period. However, they never coincide.

Since the wave-free form does not receive the wave exciting force at the wave-free point, the motion in waves becomes smaller. For the oil drilling rig, the heaving motion in waves influences the down time of the rig and the wave-free form in heaving oscillation such as the lower hull type of semi-submersible has been adopted. The heaving motion in beam waves z_A is given as follows:

$$\frac{z_A}{\zeta_A} e^{-i\epsilon_z} = -e^{-Kz_m}\left(1+\frac{Kz_m}{1-K/K_H}\right)\cos KP \qquad (19)$$

by using (16) as the wave exciting force for the lower hull type of semi-submersible.

Ships

There have been very few studies on the application of the wave-free form to ships. One of them is the experimental investigation of the heaving and pitching motions of the ship having a bulbous portion in the lower part of the hull (Maeda, 1969). On the other hand, as a theoretical study the wave-free theory based on the method of representing the ship hull by the singularity distribution in the center plane of the hull has been developed (Bessho, 1967A). It gives an example of the longitudinal distribution of the sectional area of the ship hull which has the wave-free frequency in heaving oscillation.

These studies are made on the assumption that there is no advance speed of the ship. Some investigation of the wave-free form with advance speed is concerned with wave making resistance in calm water (Bessho, 1967B). There is no research on the wave-free form for a body oscillating with advance speed, and this is a main future subject in which a new wave-free theory should be developed.

4. WAVE-FREE FORMS IN SWAYING OSCILLATION

The wave free form in swaying oscillation has been obtained by calculating the streamlines for the wave-free potential for swaying oscillation (Yamashita, 1985). The superposition of the wave-free potential and the potential of the uniform flow toward the negative direction of y axis:

$$F(t)=f(t)+t \qquad (20)$$

is considered and the streamline through the stagnation point is calculated by using the method of streamline tracing. The calculated streamlines shown in Fig. 8 in which the wave-free singularity is given by (5) are classified into

i) Crossing the free water suface,
ii) Entering into the singularity, and
iii) Being closed below the free water surface.

This shows that there are two kinds of wave-free forms for swayir oscillation. The one inferred from i) is a floating body, the surface or which is not vertical and inclined inwardly just below the free water surface. The other inferred from iii) is a fully submerged body. The characteristic features of the former inwardly inclined floating body lies in having the inward slope invariably just below the free water surface.

Figure 9 gives the wave-exciting forces on an inwardly inclined floating body. The wave-free point for swaying oscillation appears in the calculated forces, and the measured forces tend to approach the calculated ones as the wave height becomes smaller. As for the submerged body, the existence of the wave-free point for swaying oscillation is also verified.

The equation of the swaying motion of a two-dimensional body in waves is expressed in the form:

$$\frac{y_A}{\zeta_A} e^{-i\epsilon_y} = \frac{(D_4-D_{24}\, l_w)H^+(K)}{D_2 D_4-D_{24}^2} \qquad (21)$$

Therefore the swaying motion diminishes theoretically at the wave-free frequency for swaying oscillation in which $H^+(K) = 0$. The inwardly inclined floating body can be applied to a floating breakwater and the wave transmission for the body was examined theoretically and experimentally (Yamashita, 1986).

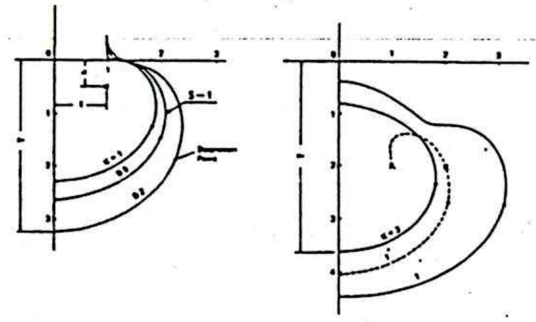

Fig. 8 Streamlines for swaying oscillation

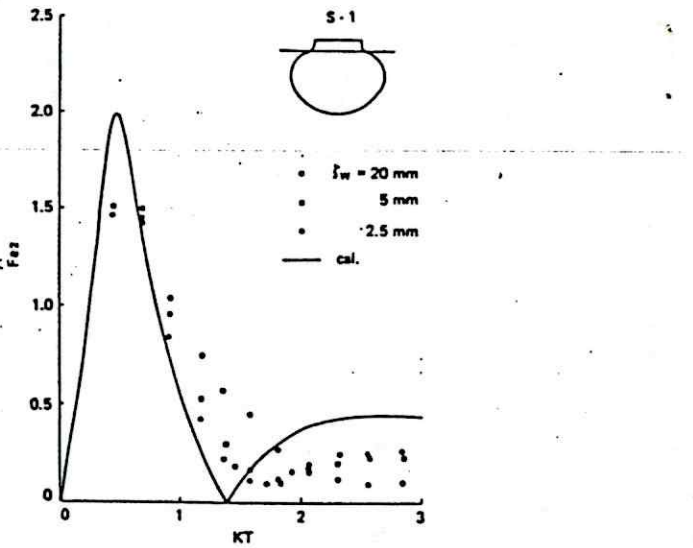

Fig. 9 Wave exciting forces for swaying

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