E　M-57　No. 1

Date 54.5.16

種々の場合の核関数

1. $S^{(3)}$, $S^{(4)}$
2. 直線自由辺
3. 直線固定辺
4. 円孔自由辺
5. 帯板自由辺
6. 周期性（一方向）
7. 〃　（2方向）
8. 割れ目（直線分）の応力集中

1,

$$\frac{4\pi}{E} S^{(3)}(x,y;x',y') = -(y-y')\lg R = \frac{i}{2}(z-z'-\bar{z}+\bar{z}')\lg R = \frac{Re}{4}(f_3)$$

$$\frac{4\pi}{E} S^{4} = (x-x')\lg R = \frac{z-z'+\overline{z-z'}}{2}\lg R = \frac{1}{4}Re(f_4)$$

$$f = -\bar{z}\varphi + \psi, \quad \frac{\partial f}{\partial x} = i\varphi + \bar{z}\varphi' + \psi',$$

$$\frac{\partial f}{\partial y} = i[\varphi + \bar{z}\varphi' + \psi']$$

$$f_3 = i(z-\bar{z}-z'+\bar{z}')\lg(z-z')$$

$$\varphi_3 = -i\lg(z-z'), \quad \psi_3 = i(z-z'+\bar{z}')\lg(z-z')$$

$$\varphi_3' = \frac{-i}{z-z'}, \quad \psi_3' = i\lg(z-z') + i\frac{\bar{z}'}{z-z'} + i$$

$$f_4 = (z-z'+\bar{z}-\bar{z}')\lg(z-z')$$

$$\varphi_4 = \lg(z-z'), \quad \psi_4 = (z-z'-\bar{z}')\lg(z-z')$$

$$\varphi_4' = \frac{1}{z-z'}, \quad \psi_4' = \lg(z-z') + \frac{-\bar{z}'}{z-z'} + 1$$

$$\psi'' = \frac{1}{z-z'} + \frac{\bar{z}'}{(z-z')^2}$$

$$2\pi U^{(3)} = (H) + \frac{1+\nu}{2}\frac{(x-x')(y-y')}{R^2},$$

$$2\pi V^{(3)} = -\frac{1-\nu}{2}\lg R + \frac{1+\nu}{2}\frac{(y-y')^2}{R^2},$$

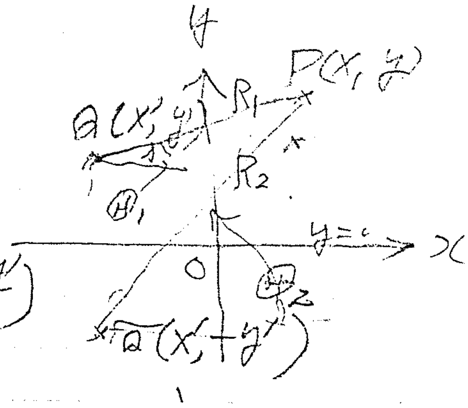
$$2\pi U^{(4)} = \frac{1-\nu}{2}\lg R - \frac{1+\nu}{2}\frac{(x-x')^2}{R^2},$$

$$2\pi V^{(4)} = (H) - \frac{1+\nu}{2}\frac{(x-x')(y-y')}{R^2},$$

z'; pure ima.

$$\varphi' + z\varphi'' = (z\varphi_4')' = \left(\frac{z'}{z-z'}\right)' = -\frac{z'}{(z-z')^2} \xrightarrow{\downarrow} \psi'' - \varphi'$$

2, 直線自由辺 $(y=0)$



先ずQに負の特異点をおくと、

$y=0$ で $f=0$ 故 $\frac{\partial f}{\partial x}=0$.

2次に

$$\frac{\partial}{\partial y}(y-y')\log\frac{R_1}{R_2} = \log\frac{R_1}{R_2} + (y-y')\left(\frac{y-y'}{R_1^2}-\frac{y+y'}{R_2^2}\right)$$

$$\xrightarrow[y\to 0]{} y'\frac{2y'}{R_2^2} = 2y'\frac{\partial}{\partial y}\log R_2$$

これから

$$-(y-y')\log\frac{R_1}{R_2} + 2yy'\frac{\partial}{\partial y}\log R_2 = \frac{4\pi}{E}G^{(3)}$$

$\left[\because\ \Delta^2 y f(x,y) = 0,\ y\Delta f = 0\right]$

同様にして

$$\frac{\partial}{\partial y}(x-x')\log\frac{R_1}{R_2} = (x-x')\left(\frac{y-y'}{R_1^2}-\frac{y+y'}{R_2^2}\right) = \frac{2(x-x')(-y')}{R_2^2}$$

$$= -2y'\frac{\partial}{\partial x}\log R_2$$

$$\frac{4\pi}{E}G^{(4)} = (x-x')\log\frac{R_1}{R_2} + 2yy'\frac{\partial}{\partial x}\log R_2$$

3. 直線固定辺（$y=0$ で $u=v=0$）

$$u-iv=\frac{1}{2G}\left[\mu\overline{\varphi(z)}-\{\bar{z}\varphi'+\psi'\}\right]=0 \quad \text{for } y=0.$$

$$\mu=\frac{(3-\nu)}{(1+\nu)}$$

森口の鏡像原理を使う.

$$\chi_3=z\varphi_3'+\psi_3\approx\frac{-iz}{z-z'}+i\lg(z-z')+\frac{i\bar{z}'}{z-z'}+i$$

$$=i\lg(z-z')+\frac{i(\bar{z}'-z')}{z-z'}$$

$$\bar{\chi}_3(z)=-i\lg(z-\bar{z}')-\frac{i(z'-\bar{z}')}{z-\bar{z}'}$$

$$\mu\varphi(z)=\bar{\chi}_3(z)+\cdots=-i\mu\lg(z-z')-i\lg(z-\bar{z}')-\frac{i(z'-\bar{z}')}{z-\bar{z}'}$$

$$\chi(z)=i\mu\lg(z-\bar{z}')+i\lg(z-z')+\frac{i(\bar{z}'-z')}{z-z'}$$

$$\varphi'=\frac{-i}{z-z'}-\frac{i}{\mu(z-\bar{z}')}+\frac{i(z'-\bar{z}')}{\mu(z-\bar{z}')^2}$$

$$z\varphi'=-i\left(1+\frac{1}{\mu}\right)-\frac{iz'}{z-z'}-\frac{i\bar{z}'}{\mu(z-\bar{z}')}+\frac{i(z'-\bar{z}')}{\mu(z-\bar{z}')}+\frac{i\bar{z}'(z'-\bar{z}')}{\mu(\ldots)}$$

$$=-i\left(1+\frac{1}{\mu}\right)-\frac{iz'}{z-z'}+\frac{i(z'-2\bar{z}')}{\mu(z-\bar{z}')}+\frac{i\bar{z}'(z'-\bar{z}')}{\mu(z-\bar{z}')^2}$$

$$-\psi'=-i\lg(z-z')-i\mu\lg(z-\bar{z}')-\frac{i(\bar{z}'-z')}{z-z'}-i\left(1+\frac{1}{\mu}\right)$$

$$-\frac{iz'}{z-z'}+\frac{i(z'-2\bar{z}')}{\mu(z-\bar{z}')}+\frac{i\bar{z}'(z'-\bar{z}')}{\mu(z-\bar{z}')^2}.$$

$$-\psi=-i(z-z')\left[\lg(z-z')-1\right]-i\mu(z-\bar{z}')\left[\lg(z-\bar{z}')-1\right]$$

$$-i\bar{z}'\lg(z-z')+\frac{i(z'-2\bar{z}')}{\mu}\lg(z-\bar{z}')-\frac{i\bar{z}'(z'-\bar{z}')}{\mu(z-\bar{z}')}$$

$$g_3 = \bar{z}\varphi + \psi = -i\bar{z}\lg(z-z') - \frac{i\bar{z}}{\mu}\lg(z-\bar{z}') - i\frac{(z'-\bar{z}')\bar{z}}{\mu(z-\bar{z}')}$$

$$+ i(z-z')[\lg(z-z') - 1] + i\mu(z-\bar{z}')[\lg(z-\bar{z}') - \cancel{1}]$$

$$+ i\bar{z}'\lg(z-z') + i\frac{(z'-2\bar{z}')}{\mu}\lg(z-\bar{z}') + \frac{i\bar{z}'(z'-\bar{z}')}{\mu(z-\bar{z}')}$$

$$= i(z-\bar{z}-z'+\bar{z}')\lg(z-z') + i\left\{\mu(z-\bar{z}') - \frac{\bar{z}+z'-2\bar{z}'}{\mu}\right\}\lg(z-\bar{z}')$$

$$+ \frac{i(z'-\bar{z}')(\bar{z}'-\bar{z})}{\mu(z-\bar{z}')} + (az + b +)$$

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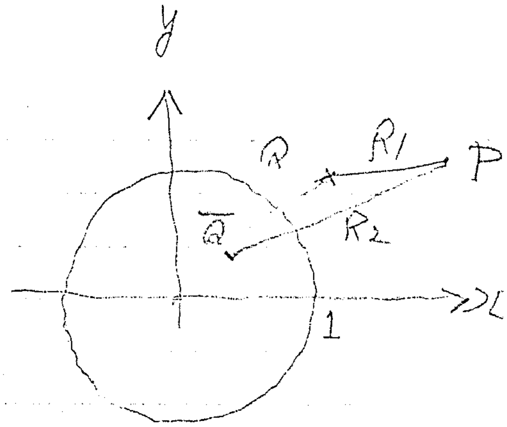
$$z^2 - z\bar{z}' = 1 - z\bar{z}' + z^2 - 1$$
$$= \frac{1}{\bar{z}'^2}\left(z\bar{z}'^2 + 1 - 2z\bar{z}'\right) +$$
$$= \cdots$$

4. 円孔 自由辺

$$\bar{\varphi} + z\varphi' + \psi' = 0.$$
$$\bar{\varphi} + \frac{1}{z}\varphi' + \psi' = 0.$$



$$\chi_z = \frac{\varphi'}{z} + \psi' \approx \frac{-i}{z(z-z')} + i\log(z-z') + i\frac{\bar{z}'}{z-z'}$$

$$\bar{\chi}_z\left(\frac{1}{z}\right) \approx \frac{iz^2}{(1-z\bar{z}')} - i\log\left(\frac{1}{z} - \bar{z}'\right) - \frac{iz\bar{z}'}{1-z\bar{z}'}$$

$$\varphi(z) = -i\log(z-z') + i\log\left(\frac{1}{z} - \bar{z}'\right) - \frac{iz(z-z')}{1-z\bar{z}'} \quad //$$

$$\chi(z) = -i\log\left(\frac{1}{z} - \bar{z}'\right) + i\log(z-z') - \frac{i\left(\frac{1}{z} - \bar{z}'\right)}{(z-z')} \quad //$$

$$z(z-z') = \frac{1}{\bar{z}'^2}\left[z\bar{z}'(z\bar{z}' - z'\bar{z}')\right] = \frac{1}{\bar{z}'^2}\left[(z\bar{z}' - 1 + 1)(z\bar{z}' - 1 + 1 + z'\bar{z}')\right]$$
$$= \frac{1}{\bar{z}'^2}\left[(z\bar{z}'-1)^2 + (2 + \bar{z}'^2)(z\bar{z}'-1) + (1 + z'\bar{z}')\right]$$

$$\varphi(z) = -i\log(z-z') + i\log\left(\frac{1-z\bar{z}'}{z}\right) - \frac{i}{\bar{z}'^2}(1 - z\bar{z}') + i\frac{2 + z'\bar{z}'}{\bar{z}'^2}$$
$$- i\frac{(1+z'\bar{z}')}{(1-z\bar{z}')\bar{z}'^2} \quad //$$

$$\varphi'(z) = -\frac{i}{z-z'} + \frac{-i\bar{z}'}{1-z\bar{z}'} - \frac{i}{z} + \frac{i}{\bar{z}'} + \frac{i\left(z' + \frac{1}{\bar{z}'}\right)}{(1-z\bar{z}')^2}$$

$$\psi' = +i\log(z-z') - i\log\left(\frac{1}{z} - \bar{z}'\right) - \frac{i\left(\frac{1}{z} - \bar{z}'\right)}{z-z'} + \frac{i}{z-z'} + \frac{i}{z}$$
$$- \frac{i}{\bar{z}'} + \frac{i\bar{z}'}{1-z\bar{z}'} + \frac{i\left(z' + \frac{1}{\bar{z}'}\right)}{(1-z\bar{z}')^2} \quad //$$

$$\frac{i(2z - z')}{z(z-z')}$$

$$\frac{1}{z(z-z')} = \left(\frac{1}{z-z'} - \frac{1}{z}\right)\frac{1}{z'}$$

$$\psi' = i \lg\left(\frac{z - z'}{\frac{1}{\bar{z}} - \bar{z}'}\right) + \frac{i\left(1 - \frac{1}{\bar{z}'}\right)}{z - z'} + \frac{i}{z}\left(1 + \frac{1}{z'}\right) - \frac{i}{\bar{z}'}$$

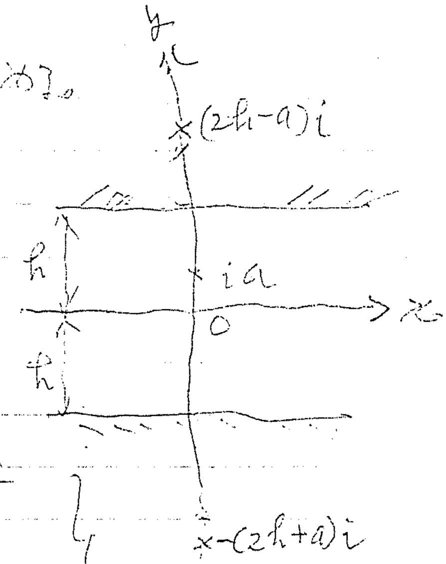
$$+ \frac{i\bar{z}'}{1 - z\bar{z}'} + \frac{i\left(z' + \frac{1}{\bar{z}}\right)}{(1 - z\bar{z}')^2}$$

$\psi =$

5. 帯板 自由辺

核関数を g_3, $\bar{g}$ として次のように f_3 を決める。

$$g_3(x,y) = f_3(x,y) + \bar{F}_3(x,y)$$



$$\left.\begin{array}{l} f_3 = \bar{z}\varphi_3 + \psi_3 \\ \bar{F}_3 = \bar{z}\varphi + \psi \end{array}\right\}$$

$$\frac{\partial^2}{\partial x^2} f_3 = \frac{\partial}{\partial x}\left(\varphi_3 + \bar{z}\varphi_3' + \psi_3'\right) = 2\varphi_3' + \bar{z}\varphi_3'' + \psi_3''$$

$$\frac{\partial^2}{\partial x \partial y} f_3 = i\{\bar{z}\varphi_3'' + \psi_3''\}$$

$$\left\{\begin{array}{l} p = \frac{\partial^2}{\partial x \partial y} f \\ q = -\frac{\partial^2}{\partial x^2} f \end{array}\right\}$$

$$\left(\frac{\partial^2}{\partial x^2} - i\frac{\partial^2}{\partial x \partial y}\right) f_3 = 2\varphi_3' + \bar{z}\varphi_3'' + \psi_3''$$

自由辺上で $p = q = 0$ なる

$$-ip - q = \varphi' + \overline{\varphi'} + \bar{z}\varphi'' + \psi'' \cancel{-} + \{\varphi_3' + \overline{\varphi_3'} + \bar{z}\varphi_3'' + \psi_3''\} = 0$$

$$\left.\begin{array}{l} I \equiv -iP - Q = \varphi' + \overline{\varphi'} + \bar{z}\varphi'' + \psi'' \\ I_3 \equiv -ip_3 - q_3 = \varphi_3' + \overline{\varphi_3'} + \bar{z}\varphi_3'' + \psi'' \end{array}\right\}$$ とおこう。

$(\bar{z} + 2iy - ia + ia)\,i$

$$I_3 = \frac{-a}{(z-ia)^2} + \frac{i}{\bar{z}+ia} + \frac{\bar{z}\,i}{(z-ia)^2}, \quad \bar{z} = z - 2iy$$

$$I_3\big|_{y=h} \equiv I_3(h) = \frac{i}{z+i(a-2h)} + \frac{i}{z-ia} + \frac{2(h-a)}{(z-ia)^2} \quad //$$

$$I_3\big|_{y=-h} \equiv I_3(-h) = \frac{i}{z+i(2h+a)} + \frac{i}{z-ia} - \frac{2(h+a)}{(z-ia)^2} \quad //$$

$$I_3 = \frac{i}{z-ia} + \frac{i}{\bar{z}+ia} + \frac{2(y-a)}{(z-ia)^2} \quad //$$

$z=x+iy$ の正則関数 $f(z)$ は次のように表わされる。

$$f(z)=\int_0^\infty \overline{H}(k)e^{ikz}dk+\int_0^\infty G(k)e^{-ikz}dk \quad \text{for } h>y>-h$$

$$\overline{H}(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(x-ih)e^{-ikx-kh}dx.$$

$$G(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(x+ih)e^{ikx-kh}dx$$

今 $\varphi'=\int_0^\infty(Ae^{ikz}+Be^{-ikz})dk$

$\bar{z}=z-2iy$

$-ik\bar{z}=-ikz-2ky$

$ik\bar{z}=ikz+2ky$

$$\psi'=\int_0^\infty(Ce^{ikz}+De^{-ikz})dk.$$

$$\overline{\varphi'}=\int_0^\infty(\overline{B}e^{2ky+ikz}+\overline{A}e^{-2ky-ikz})dk$$

$$z\varphi''=\int_0^\infty(iA'e^{ikz}-iB'e^{-ikz})dk.$$

$$\varphi'+z\varphi''=-\int_0^\infty(kA'e^{ikz}+kB'e^{-ikz})dk.$$

$$\therefore I(h)=\varphi'+\overline{\varphi'}+(z-2ih)\varphi''+\psi''$$

$$=\int_0^\infty[\overline{B}e^{2kh}-kA'+2khA+C]e^{ikz}dk$$

$$+\int_0^\infty[\overline{A}e^{-2kh}-kB'-2khB+D]e^{-ikz}dk$$

$$I(-h)=\int_0^\infty[\overline{B}e^{-2kh}-kA'-2khA+C]e^{ikz}dk$$

$$+\int_0^\infty[\overline{A}e^{2kh}-kB'+2khB+D]e^{-ikz}dk.$$

一方

$$\frac{i}{z-ia} = -\int_0^\infty e^{ik(z-ia)}\,dk \qquad \text{for } y>a.$$

$$-i \qquad = \int_0^\infty e^{-ik(z-ia)}\,dk \qquad \text{for } y<a.$$

$$\frac{1}{(z-ia)^2} = -(i\cdot i)\int_0^\infty e^{ikz+ka}\,k\,dk \qquad \text{for } y>a.$$

$$i\cdot(-i)\int_0^\infty e^{-ikz-ka}\,k\,dk \qquad \text{for } y<a.$$

$$\therefore\ I_3(h) = \int_0^\infty e^{ikz+ka}(-1+2k\,\overline{h-a})\,dk + \int_0^\infty e^{-ikz-k(2h-a)}\,dk, \quad 2(h-a)>y>a$$

$$I_3(-h) = -\int_0^\infty e^{ikz-k(2h+a)}\,dk + \int_0^\infty e^{-ikz-ka}(1-2k\,\overline{h+a})\,dk.$$

$$-(2h+a)<y<a$$

$$I_3(h) = -\int_0^\infty\left(e^{ikz}P_1 + e^{-ikz}Q_1\right)dk.$$

$$I_3(-h) = -\int_0^\infty\left(e^{ikz}P_2 + e^{-ikz}Q_2\right)dk.$$

$$\left.\begin{aligned}
P_1 &= (1-2k\,\overline{h-a})\,e^{ka}\\
Q_1 &= -e^{-k(2h-a)}\\
P_2 &= +e^{-k(2h+a)}\\
Q_2 &= -e^{-ka}(1-2k\,\overline{h+a})
\end{aligned}\right\}$$

$$C - kA' + 2khA + \bar{B}e^{2kh} = P_1$$
$$D - kB' - 2khB + \bar{A}e^{-2kh} = Q_1$$
$$C - kA' - 2khA + \bar{B}e^{-2kh} = P_2$$
$$D - kB' + 2khB + \bar{A}e^{2kh} = Q_2$$

$$\therefore\ 2khA + \bar{B}\,sh(2kh) = \frac{P_1 - P_2}{2}$$
$$2khB + \bar{A}\,sh\,2kh = \frac{Q_2 - Q_1}{2}$$

P, Q が real なら A, B も real.

$$A = \frac{1}{\Delta_1\Delta_2}\left[\overline{\frac{Q_2 - Q_1}{2}}\,sh\,2kh - (P_1 - P_2)kh\right]$$
$$B = \frac{1}{\Delta_1\Delta_2}\left[\overline{\frac{P_1 - P_2}{2}}\,sh\,2kh - (Q_2 - Q_1)kh\right]$$

$$\left.\begin{matrix}\Delta_1\\ \Delta_2\end{matrix}\right\} = sh\,2kh \pm 2kh, \quad sh\,2kh = \frac{\Delta_1 + \Delta_2}{2},\quad 2kh = \frac{\Delta_1 - \Delta_2}{2}.$$

$$\frac{P_1 - P_2}{2} = e^{-kh}\,sh\,k\overline{h+a} - k\overline{h-a}\,e^{ka}$$
$$\frac{Q_2 - Q_1}{2} = -e^{-kh}\,sh\,k\overline{h-a} + k\overline{h+a}\,e^{-ka}$$

$$M = \frac{1}{2}\left(\frac{P_1 - P_2}{2} + \frac{Q_2 - Q_1}{2}\right) = e^{-kh}\,sh\,ka\,ch\,kh + ka\,ch\,ka - kh\,sh\,ka$$
$$N = \frac{1}{2}\left(\frac{P_1 - P_2}{2} - \frac{Q_2 - Q_1}{2}\right) = e^{-kh}\,sh\,kh\,ch\,ka - kh\,ch\,ka + ka\,sh\,ka$$

$$A = \frac{M}{\Delta_1} - \frac{N}{\Delta_2}$$
$$B = \frac{M}{\Delta_1} + \frac{N}{\Delta_2}$$

$$I(x,y)=\int_0^\infty\left[C-kA'+2kyA+\bar{B}e^{2ky}\right]e^{ikz}\,dk$$

$$+\int_0^\infty\left[D-kB'-2kyB+\bar{A}e^{-2ky}\right]e^{-ikz}\,dk,$$

$$\frac{M}{\Delta_1}\xrightarrow[k\gg1]{}\left[k\,\overline{a-h}\,e^{k|a|-2kh}+\frac{1}{2}e^{k|a|-2kh}\right]$$

$$\frac{N}{\Delta_2}\xrightarrow[k\gg1]{}\left[-k\,\overline{h-a}\,e^{k|a|-2kh}+\frac{1}{2}e^{k|a|-2kh}\right].$$

$$\frac{M}{\Delta_1}\xrightarrow[k\to0]{}\frac{1}{4}\left(\frac{a}{h}-ka\cdots\right)$$

$$\frac{N}{\Delta_2}\xrightarrow[k\to0]{}\frac{3}{4kh}\left\{\frac{a^2}{h^2}-\frac{1}{2}+O(k)\right\}$$

$$\left.\begin{aligned}C-kA'&=P_2+2khA-\bar{B}e^{-2kh}\\ D-kB'&=Q_1+2khB-\bar{A}e^{-2kh}\end{aligned}\right\}.$$

とおくと 積分は $h>y>-h$ で regular

$$I(x,y)=I_0(z)$$

$$+\int_0^\infty\left[2k(y+h)A+\bar{B}\left(e^{2ky}-e^{-2kh}\right)\right]e^{ikz}\,dk$$

$$+\int_0^\infty\left[2k(h-y)B+\bar{A}\left(e^{-2ky}-e^{-2kh}\right)\right]e^{-ikz}\,dk$$

$$I_0(z)=\int_0^\infty\left[P_2e^{ikz}+Q_1e^{-ikz}\right]dk=\frac{i}{z+i(2h+a)}+\frac{i}{z-i(2h-a)}$$

$$I(x,y) = I_0$$

$$+\int_0^\infty dk\,\frac{M}{\Delta_1}\Big[e^{ikz}\{2k(y+h)+e^{2ky}-e^{-2kh}\}+e^{-ikz}\{2k\,\overline{h-y}+e^{-2ky}-e^{-2kh}\}\Big]$$

$$+\int_0^\infty dk\,\frac{N}{\Delta_2}\Big[e^{ikz}\{-2k\,\overline{y+h}+e^{2ky}-e^{-2kh}\}+e^{-ikz}\{2k\,\overline{h-y}+e^{-2kh}-e^{-2ky}\}\Big]$$

$$= I_0$$

$$+2\int_0^\infty\Big[\cos kz\{2kh+\cosh 2ky-e^{-2kh}\}$$
$$+i\sin kz\{2ky+\sinh 2ky\}\Big]\frac{M}{\Delta_1}dk$$

$$+2\int_0^\infty\Big[\cos kz\{-2ky+\sinh 2ky\}$$
$$+i\sin kz\{-2kh+\cosh 2ky-2e^{-2kh}\}\Big]\frac{N\,dk}{\Delta_2}$$

$$= I_0$$

$$+\int_0^\infty\Big[\cos kx\{2k(\overline{y+h}\,e^{-ky}+\overline{y-h}\,e^{ky})+2\cosh ky-2e^{-2kh}\cosh ky\}$$
$$+i\sin kx\{2k(\overline{y+h}\,e^{-ky}-\overline{y-h}\,e^{ky})+2\sinh ky+2e^{-2kh}\sinh ky\}\Big]\frac{M\,dk}{\Delta_1}$$

$$+\int_0^\infty\Big[\cos kx\{-2k(\overline{h-y}\,e^{ky}-\overline{h+y}\,e^{-ky})+2\sinh ky+2e^{-2kh}\sinh ky\}$$
$$+i\sin kx\{-2k(\overline{h-y}\,e^{ky}+\overline{h+y}\,e^{-ky})+2\cosh ky-2e^{-2kh}\cosh ky\}\Big]\frac{N\,dk}{\Delta_2}$$

$$I(x,y) = I_0$$

$$+2\int_0^\infty \cos kx \Big[\{\cosh ky\,(1-e^{-2kh}+2ky) - 2kh \sinh ky\}\frac{M}{\Delta_1}$$

$$+ i\{\sinh ky\,(1+e^{-2kh}-2kh) - 2ky \cosh ky\}\frac{N}{\Delta_2}\Big]dk$$

$$+2i\int_0^\infty \sin kx \Big[\{\sinh ky\,(1+e^{-2kh}-2ky) + 2kh \cosh ky\}\frac{M}{\Delta_1}$$

$$+ \{\cosh ky\,(1-e^{-2kh}-2kh) + 2ky \sinh ky\}\frac{N}{\Delta_2}\Big]dk$$

$$I_4 = \frac{1}{z-ia} + \frac{1}{\bar{z}+ia} - \frac{\bar{z}}{(z-ia)^2} + \frac{1}{z-ia} + \frac{-ia}{(z-ia)^2}$$

$$= \frac{2}{z-ia} + \frac{1}{\bar{z}+ia} - \frac{\bar{z}+ia}{(z-ia)^2}.$$

$$= \frac{2}{z-ia} + \frac{1}{z+i(a+2y)} - \frac{z+i(a-2y)}{(z-ia)^2}$$

$$= \frac{1}{z-ia} + \frac{1}{z+i(2y+a)} - \frac{2i(a-y)}{(z-ia)^2}$$

$$I_4(h) = \int_0^\infty e^{ikz+ka}\left(i - 2i(a-h)k\right)dk + i\int_0^\infty e^{-ikz-k(2h-a)}dk.$$

$$I_4(-h) = \int_0^\infty e^{-ikz-ka}\left(-i-2i(a+h)k\right)dk + i\int_0^\infty e^{ikz-k(2h+a)}dk.$$

$$P_1' = -i\left[1+2k\overline{h-a}\right]e^{ka}.$$

$$Q_1' = ie^{-k(2h-a)}$$

$$Q_2' = i(1+2k\overline{h+a})e^{-ka}.$$

$$P_2' = -ie^{-k(2h+a)}.$$

$$\left.\begin{aligned} \frac{P_1'-P_2'}{2i} &= -e^{-kh}\,\mathrm{sh}\,k\overline{h+a} - k\overline{h-a}\,e^{ka} \\ \frac{Q_2'-Q_1'}{2i} &= +e^{-kh}\,\mathrm{sh}\,k\overline{h-a} + k\overline{h+a}\,e^{-ka} \end{aligned}\right\}$$

$$M' = \frac{1}{4i}(P_1'-P_2'+Q_2'-Q_1') = e^{-kh}\,\mathrm{sh}\,ka\,\mathrm{ch}\,kh - kh\,\mathrm{sh}\,ka + ka\,\mathrm{ch}\,ka$$

$$N' = \frac{1}{4i}(Q_2'-Q_1'-P_1'+P_2') = e^{-kh}\,\mathrm{sh}\,kh\,\mathrm{ch}\,ka + kh\,\mathrm{ch}\,ka - ka\,\mathrm{sh}\,ka$$

$$A = \frac{-1}{4\Delta_1\Delta_2}\left[(P_1'-P_2')(\Delta_1-\Delta_2)+(Q_2'-Q_1')(\Delta_1+\Delta_2)\right]$$

$$= \frac{-Q_2'+Q_1'+(P_1'-P_2')}{\cancel{4}\,\Delta_1} \mp \frac{P_1'-P_2'+Q_2'-Q_1'}{4\,\Delta_2} = -\frac{iN'}{\Delta_1} \mp \frac{iM'}{\Delta_2}$$

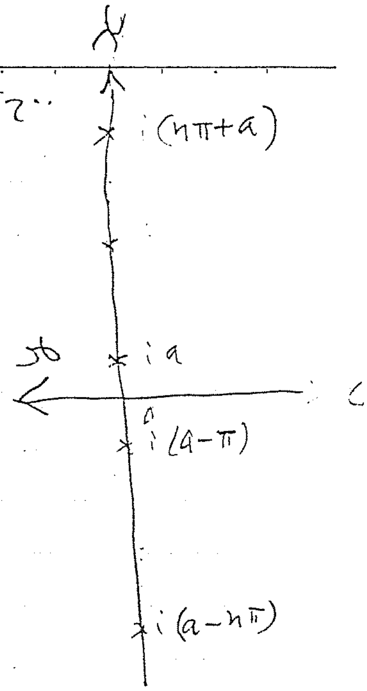
$$B = \frac{-1}{4\Delta_1\Delta_2}\left[(P_1'-P_2')(\Delta_1+\Delta_2)+(Q_2'-Q_1')(\Delta_1-\Delta_2)\right]$$

$$= \frac{Q_2'-Q_1'-(P_1'-P_2')}{4\,\Delta_1} - \frac{P_1'-P_2'+Q_2'-Q_1'}{\Delta_2} = \frac{iN'}{\Delta_1} - \frac{iM'}{\Delta_2}$$

6. 周期性（一方向）

孔が y軸上に 1列 並んでいるような場合で
その間隔を π とすると、f_3 については

$$\varphi_3' = \frac{-i}{z-a}, \quad \psi_3' = i\,lg(z-a) + \frac{ia}{z-a} + i.$$
$$= i\,lg(z-a) + \frac{iz}{z-a}$$



$$\sum_{n=-\infty}^{\infty} lg(z - \overline{i\,a+n\pi}) = lg\{\sin(z-a)\}$$

$$\sum_{n=-\infty}^{\infty} \frac{1}{z-a-n\pi} = \cot(z-a)$$

$$\therefore \bar{z}\varphi' + \psi' = i(z-\bar{z})\cot(z-a) + i\,lg\{\sin(z-a)\} \quad //$$

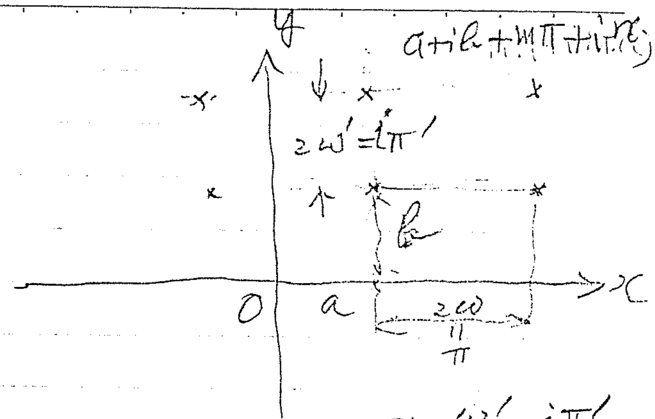
$$\varphi_4' = \frac{1}{z-a}, \quad \psi_4' = i\,lg(z-a) - \frac{z}{z-a}$$

$$\bar{z}\varphi_4' + \psi_4' = lg(z-a) + \frac{\bar{z}-z}{z-a}$$

$$\bar{z}\varphi' + \psi' = (\bar{z}-z)\cot(z-a) + lg\{\sin(z-a)\} \quad //.$$

7. 周期性 (2方向)

$$I = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{\infty} \frac{1}{z-a-ib+n\pi+mi\pi'}$$



$$\tau = \frac{\omega'}{\omega} = \frac{i\pi'}{\pi}$$

$$q = e^{i\pi\tau} = e^{-\frac{\pi'}{\pi}}$$

$$= \sum_{m=-\infty}^{\infty} \cot(z-a-ib+mi\pi')$$

$$= \zeta(z-a-ib) - \frac{2}{\pi}\eta_1 (z-a-ib)$$

$$= \cot(z-a-ib) + 4\sum_{n=1}^{\infty} \frac{q^{2n}}{1-q^{2n}} \sin\{2n(z-a-ib)\}$$

$$q = e^{-\frac{\pi'}{\pi}}$$

$$I = \frac{d}{dz} \log \vartheta_1\left(\frac{z-a-ib}{\pi}\right).$$

$$J = \sum_n \sum_m \log(z-a-ib+n\pi+mi\pi')$$

$$= \log \vartheta_1\left(\frac{z-a-ib}{\pi}\right)$$

$$= \frac{\log C}{4\pi} + \log\{2\sin(z-a-ib)\}$$

$$-2\sum_{n=1}^{\infty} \frac{q^{2n}}{1-q^{2n}} \frac{\cos\{2n(z-a-ib)\}}{n}$$

$$\bar{z}\sum_{n,m}\varphi_3' + \sum_{n,m}\psi_3' = i(z-\bar{z})\sum_{n,m}\frac{1}{z-a-ih} + i\sum_{n,m}\zeta(z-a-ih)$$

$$= -2y\, I + i\, J$$

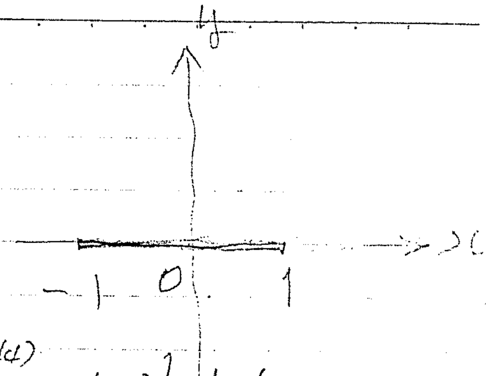
$$\bar{z}\sum\varphi_4' + \sum\psi_3' = -2iy\, I + J$$

$$i\frac{\partial J}{\partial z} = I$$

$$\frac{dI}{dz} = -\frac{2}{\pi}\eta_1 - \wp(z-ia-ih)$$

$$= -\frac{1}{\sin^2(z-ia-ih)} + 8\sum_{n=1}^{\infty}\frac{n q^{2n}}{1-q^{2n}}\cos\{2n(z-ia-ih)\}$$

8. 割れ目の応力集中



$$f(x,y) = \int_c \left(\mu \underline{\underline{S}}^{(3)} + \nu \underline{\underline{S}}^{(4)} \right) ds$$

$$= 2\int_{-1}^{1} \left\{ \mu(x') \underline{\underline{S}}^{(3)}(x-x', y) + \nu(x') \underline{\underline{S}}^{(4)}(x-x', y) \right\} dx'.$$

$$Q = \left.\frac{\partial f}{\partial x}\right|_{y=0} = \frac{1}{2\pi}\int_{-1}^{1} \left[\quad '' \quad + \nu(x')\{ \lg(x-x') + 1 \} \right] dx'$$

$$P = \left.\frac{\partial f}{\partial y}\right|_{y=0} = \frac{1}{2\pi}\int_{-1}^{1} \left[-\mu(x') \lg(x-x') + \quad ; \right] dx'$$

$$Q\Big|_{y\neq 0} = \quad '' \quad + \frac{1}{2\pi}\int_{-1}^{1} \nu(x') \lg(x-x') dx' ;$$

$$P\Big|_{y\neq 0} = \quad - \frac{1}{2\pi}\int_{-1}^{1} \mu(x') \lg(x-x') dx' ,$$

ここに $P=0$, $Q=x$ とすると、上式をもう一度 x で微分して

$$\mu(x) = \frac{A}{\sqrt{1-x^2}} \quad ; \quad A: \text{不定定数}$$

$$\nu(x) = \frac{-2x}{\sqrt{1-x^2}} \quad // = \frac{\partial v}{\partial x}$$

$$v = -2\sqrt{1-x^2} \quad , \quad -\int_{-1}^{1} v\, dx = \mp 2\int_{0}^{\pi} \sin^2\theta\, d\theta = \pi$$

それでは $f = \frac{x^2}{2}$ とすると.

$$x = -\frac{1}{2\pi}\int_{-1}^{1}\frac{\bar{v}\,dx'}{x-x'} \longrightarrow 1 = \frac{1}{2\pi}\int_{-1}^{1}\frac{\bar{v}\,dx'}{(x-x')^2}$$

$$\text{Const.} = \frac{1}{2\pi}\int_{-1}^{1}\frac{\bar{u}\,dx'}{x-x'} \longrightarrow 0 = \frac{1}{2\pi}\int_{-1}^{1}\frac{\bar{u}\,dx'}{(x-x')^2},$$

$\therefore\ \bar{u} = \frac{\alpha x + \beta}{\sqrt{1-x^2}}$, α, β 未定定数

$\cos 2\theta = 2\cos^2\theta - 1$

$$\bar{v} = \frac{\cos 2\theta + \text{Const}}{\sqrt{1-x^2}} = \frac{2(x^2-1)}{\sqrt{1-x^2}} = -2\sqrt{1-x^2}$$

$\because\ v(\pm 1) = 0$.

$$\left.\frac{\partial \bar{u}}{\partial x}\right|_{x=0} = \left. f_{yy}\right|_{\substack{y=0\\x=0}} = \frac{1}{2\pi}\int_{-1}^{1}\frac{\bar{v}\,dx'}{(x-x')^2} = 1.$$

$$\therefore\ \left.\frac{\partial \bar{u}}{\partial x}\right|_{x=0} = \left.\frac{\alpha x}{\sqrt{1-x^2}}\right|_{x=0} = \alpha. \quad \therefore\ \alpha = 1$$