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大改訂

3次元弾性の場合

1. 応力と歪, 基本特異性
2. Bettiの表現等
3. 軸対称の場合

附録 無限長丸棒中の軸対称特異性

1. 応力と歪，基本特異性

変位のベクトルを $\mathbf{u}(u_1, u_2, u_3)$ とすると歪 e_{ij} は，
$(x \equiv x_1,\ y \equiv x_2,\ z \equiv x_3)$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad \cdots\cdots \quad (1.1)$$

体積増は $\gamma = \sum_{i=1}^{3} e_{ii}$, $\quad \cdots \quad$ (1.2)

フックの法則では応力 f'_{ij} が次の如く表わされる。

$$\frac{f'_{ij}}{G} \equiv f_{ij} = 2e_{ij} + \frac{2\nu}{1-2\nu}\gamma\delta_{ij}, \quad \cdots\cdots \quad (1.3)$$

こゝに $\delta_{ij} = 0$ for $i \neq j$, $\delta_{ii} = 1$,

弾性定数間の関係は

$$G = \frac{E}{2(1+\nu)} = \mu, \quad \lambda = \frac{2\nu}{1-2\nu}G,$$

Bergman, Schiffer のTextの記号との関係

$$\alpha = \frac{1}{1-2\nu}, \quad A = -\frac{\alpha}{2(\alpha+1)} = \frac{-1}{4(1-\nu)}, \quad B = 1$$

$$2A + B = \frac{1-2\nu}{2(1-\nu)},$$

境界力もGで割ってそれを $\boldsymbol{\tau}(\tau_1, \tau_2, \tau_3)$ とすると，

$$\tau_i = \sum_{j=1}^{3} f_{ij}\frac{\partial x_j}{\partial n} = 2\sum_{j=1}^{3} e_{ij}\frac{\partial x_j}{\partial n} + \frac{\nu}{1-2\nu}\gamma\frac{\partial x_i}{\partial n}, \quad (1.4)$$

なお(3)より

$$\sigma = \sum_{i=1}^{3} f_{ii} = \frac{2(1+\nu)}{1-2\nu}\gamma, \quad \cdots\cdots \quad (1.5)$$

適合条件式は

$$\Delta u_i + \frac{1}{1-2\nu}\frac{\partial \gamma}{\partial x_i} = 0, \quad i=1,2,3, \qquad (1.6)$$

今基本特異性として上式を満足する次のものを考える。

$$\left.\begin{aligned} &4\pi U_i^{(k)}(P,Q) = \frac{\delta_{ik}}{R} - \frac{1}{4(1-\nu)}\frac{\partial^2 R}{\partial x_i \partial x_k}, \\ &P \equiv (x_1, x_2, x_3), \quad Q \equiv (x_1', x_2', x_3'), \quad R = \overline{PQ} \\ &U_i^{(k)}(P,Q) = U_i^{(k)}(Q,P), \quad \Delta R = \frac{2}{R} \end{aligned}\right\} \qquad (1.7)$$

これによる歪は (1) より

$$\left.\begin{aligned} 4\pi E_{ij}^{(k)}(P,Q) &= \frac{1}{2}\left(\frac{\partial}{\partial x_i}\frac{\delta_{jk}}{R} + \frac{\partial}{\partial x_j}\frac{\delta_{ik}}{R}\right) \\ &\quad - \frac{1}{4(1-\nu)}\frac{\partial^3 R}{\partial x_i \partial x_j \partial x_k}, \\ E_{ij}^{(k)}(P,Q) &= -E_{ij}^{(k)}(Q,P), \end{aligned}\right\} \qquad (1.8)$$

(2)より

$$4\pi \Gamma^{(k)}(P,Q) = \frac{1-2\nu}{2(1-\nu)}\frac{\partial}{\partial x_k}\frac{1}{R} = -4\pi\Gamma^{(k)}(Q,P), \qquad (1.9)$$

これによる境界力は

$$\begin{aligned} 4\pi T_i^{(k)}(U^{(k)}_{\cdot}; P,Q) &= \delta_{ik}\frac{\partial}{\partial n}\left(\frac{1}{R}\right) + \frac{\partial x_k}{\partial n}\frac{\partial}{\partial x_i}\frac{1}{R} + \frac{\nu}{1-\nu}\frac{\partial x_i}{\partial n}\frac{\partial}{\partial x_k}\frac{1}{R} \\ &\quad - \frac{1}{2(1-\nu)}\frac{\partial^3 R}{\partial n \partial x_i \partial x_k}, \\ &= 8\pi\frac{\partial}{\partial n}U_i^{(k)}(P,Q) + \frac{\partial x_k}{\partial n}\frac{\partial}{\partial x_i}\frac{1}{R} + \frac{\nu}{1-\nu}\frac{\partial x_i}{\partial n}\frac{\partial}{\partial x_k}\frac{1}{R} \\ &\quad - \delta_{ik}\frac{\partial}{\partial n}\frac{1}{R}, \end{aligned} \qquad (1.10)$$

2. Bettiの表現.

歪エネルギーは

$$A = \frac{G}{2}\iiint_D \sum_{i,j=1}^{3} f_{ij}\, e_{ij}\, d\tau, \quad d\tau = dx\,dy\,dz,$$

$$= G\iiint_D \left[\sum_{i,j=1}^{3} e_{ij}^2 + \frac{\nu}{1-2\nu}\left(\sum_{i=1}^{3} e_{ii}\right)^2\right] d\tau, \quad \cdots \quad (2.1)$$

であるから 2次形式

$$E(\mathbf{u},\mathbf{v}) = \iiint_D \left[\sum_{i,j=1}^{3} \frac{\partial u_i}{\partial x_j}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \frac{2\nu}{1-2\nu}\,\mathrm{div}\,\mathbf{u}\,\mathrm{div}\,\mathbf{v}\right] d\tau \quad (2.2)$$

を導入すると

$$e(\mathbf{u},\mathbf{v}) = e(\mathbf{v},\mathbf{u}) \quad (2.3)$$

部分積分して

$$E(\mathbf{u},\mathbf{v}) = \iint_S \mathbf{u}\,\boldsymbol{\tau}(\mathbf{v})\,dS - \iiint_D \mathbf{u}\left\{\Delta\mathbf{v} + \frac{1}{1-2\nu}\,\mathrm{grad}\,\mathrm{div}\,\mathbf{v}\right\} d\tau, \quad (2.4)$$

となり (1.6) が成立つとすると右辺第2項は消えて

(2.3)よりMaxwell-Bettiの相反定理が成立つ

$$\iint_S \left[\mathbf{u}\,\boldsymbol{\tau}(\mathbf{v}) - \mathbf{v}\,\boldsymbol{\tau}(\mathbf{u})\right] dS = 0, \quad (2.5)$$

(Bergman-Schiffの本では法線Dは内向, 今は外向とする)

今 $\boldsymbol{v}$ として (1.7) を考えると 常用手段により

$$\lim_{\varepsilon \to 0} \iint_{\varepsilon} \frac{x_i x_j}{r^4} ds = \frac{4}{3}\pi \delta_{ij}$$

により

$$\boldsymbol{u}(Q) = \iint_S \left[\boldsymbol{\tau}(\boldsymbol{u}(P)) U(P,Q) - \boldsymbol{u}(P) \boldsymbol{T}(U(P,Q)) \right] ds$$

ここに U は マトリックスで 上式を 成分毎に 書くと
前節の記号を使って

$$u_k(Q) = \iint_S \sum_{i=1}^{3} \left[\tau_i(P) U_i^{(k)}(P,Q) - u_i(P) T_i^{(k)}(P,Q) \right] ds(P),$$

$$k = 1, 2, 3 \qquad (2.6)$$

応力等は前節の式を使って計算すればよい。

外部問題（無限遠点を含む領域の）では

吹出し式表現が便利な事があり，それは

$$u_k(Q) = \iint_S \sum_{i=1}^{3} \tau_i(P) U_i^{(k)}(P,Q)\, ds(P), \quad (2.7)$$

$$u_k(Q) = - \iint_S \sum_{i=1}^{3} u_i T_i^{(k)}(P,Q)\, ds(P), \quad (2.8)$$

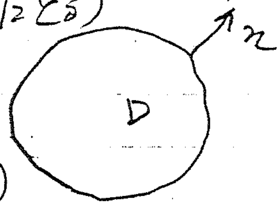
のようになるが 応力集中問題では 2 次元同様

(2.8) が便利であるが，この時 右辺の 核関数

について(1.13)のように応力を計算するのは大変である。

そこで元に戻って今は外部領域なので $\bar{D}$

(2.6)は符号が変って(法線は元のまゝ図のようにとる)



$$u_k(Q) = -\iint_S \sum_{i=1}^{3} \left(t_i U_i^{(k)} - u_i T_i^{(k)} \right) ds(p), \quad (2.9)$$

in $\bar{D}$

内部領域で考えると

$$\iint_S \sum_i^3 \left(t_i' U_i^{k} - u_i' T_i^{(k)} \right) dsp = \begin{cases} u_k'(Q) & \text{in } D \\ 0 & \text{in } \bar{D} \end{cases} \quad (2.10)$$

となるから例えば x一方向の一様引張りを受ける場合の応力集中問題では S 上で

$$f_{ij} = 0 \text{ for } i \neq j, \quad f_{11} = 1, \quad f_{22} = f_{33} = 0$$

であるから外部の境界条件は

$$t_1 = -\frac{\partial x_1}{\partial n}, \quad t_2 = t_3 = 0 \quad \cdots \quad (2.10)$$

一方(2.10)については

$$e_{ij} = \frac{1}{2} f_{ij} - \frac{\nu}{1-2\nu} \gamma \delta_{ij} = \frac{1}{2} f_{ij} - \frac{\nu}{2(1+\nu)} \delta_{ij}$$

$$\frac{\partial u_1'}{\partial x_2} = e_{11} = \frac{1}{2(1+\nu)}, \quad e_{22} = -\frac{\nu}{2(1+\nu)} = \frac{\partial u_2}{\partial x_2} = \frac{\partial u_3}{\partial x_3}$$

$$e_{ij} = 0 \text{ for } i \neq j$$

$$\therefore \quad u_1' = \frac{x_1}{2(1+\nu)}, \quad u_2' = -\frac{\nu x_2}{2(1+\nu)}, \quad u_3' = -\frac{\nu x_3}{2(1+\nu)} \quad (2.11)$$

$$t_1' = \frac{\partial x_1}{\ }, \quad t_2' = t_3' = 0$$

であるから(2.9)から(2.10)を差引いて，

$$u_k(Q) = \iint_S \sum_{i=1}^{3} (u_i + u_i') \, T_i^{(k)} \, ds_P \, , \quad \text{in } \bar{D} \, , \qquad (2.12)$$

を得る。

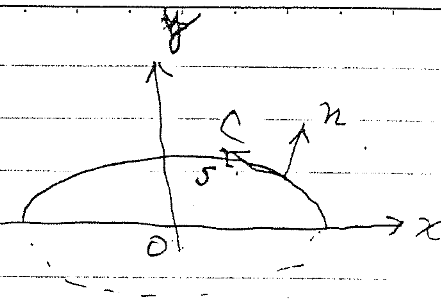
こゝに u_i' は(2.11)によるから $(u_i + u_i')$ は一様引張りも含めた全変位となる。

これを次のように書けばこれは S 上で $(u_k + u_k')$ に関するフレドホルムの第2種方程式となる

$$u_k(Q) + u_k'(Q) - \sum_i \iint_S (u_i + u_i') \, T_i^{(k)} \, dS = u_k'(Q) \, , \qquad (2.13)$$

3. 軸対称の場合

x軸のまわりに対称としよう。
(乱れ応力分布も)



円筒座標を導入し

$$x_1 \equiv x,\quad x_2 \equiv r\cos\theta,\quad x_3 \equiv r\sin\theta,$$

変位は

$$u_1 \equiv u(x,r),\quad u_2 = v(x,r)\cos\theta,\quad u_3 = v(x,r)\sin\theta, \qquad (3.1)$$

$$\left.\begin{aligned}
&e_{11} = \frac{\partial u}{\partial x},\quad e_{22}\Big|_{\theta=0} = \cos^2\theta\frac{\partial v}{\partial r} + \frac{\sin^2\theta}{r}v\Big|_{\theta=0} = \frac{\partial v}{\partial r}\\
&e_{33}\Big|_{\theta=0} = \sin^2\theta\frac{\partial v}{\partial r} + \frac{\cos^2\theta}{r}v\Big|_{\theta=0} = \frac{v}{r},\\
&e_{12}\Big|_{\theta=0} = \frac{1}{2}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right)\cos\theta\Big|_{\theta=0} = \frac{1}{2}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right),\\
&e_{23}\Big|_{\theta=0} = \left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)\sin\theta\cos\theta\Big|_{\theta=0} = 0,
\end{aligned}\right\} \quad (3.2)$$

以下 e_{ij} 等の記号はすべて上のように $\theta=0$ の値を意味するものとす

応力は

$$\left.\begin{aligned}
&f_{ij} = 2e_{ij} + \frac{2\nu}{1-2\nu}\gamma\,\delta_{ij}\\
&f_{11} = 2\frac{\partial u}{\partial x} + \frac{2\nu}{1-2\nu}\gamma,\quad f_{12} = \frac{\partial u}{\partial r} + \frac{\partial v}{\partial x},\\
&f_{22} = 2\frac{\partial v}{\partial r} + \frac{2\nu}{1-2\nu}\gamma,\quad f_{33} = \frac{2v}{r} + \frac{2\nu}{1-2\nu}\gamma\\
&\gamma = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r},\\
&\sum_{i=1}^{3} f_{ii} = 2\gamma + \frac{6\nu\gamma}{1-2\nu} = \frac{2(1+\nu)}{1-2\nu}\gamma
\end{aligned}\right\} \quad (3.3)$$

境界力は

$$\left.\begin{aligned}\tau_1 &= 2\frac{\partial u}{\partial n} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\frac{\partial y}{\partial n} + \frac{2\nu}{1-2\nu}\gamma\frac{\partial x}{\partial n} \\ \tau_2 &= 2\frac{\partial v}{\partial n} - \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\frac{\partial x}{\partial n} + \frac{2\nu}{1-2\nu}\gamma\frac{\partial y}{\partial n}\end{aligned}\right\} \quad (3.4)$$

特に自由境界では

$$\tau_1 = \tau_2 = 0 \quad , \text{---} \quad (3.5)$$

であるから　この時は(3.4)から

$$\left.\begin{aligned}&\frac{\partial u}{\partial n}\frac{\partial x}{\partial n} + \frac{\partial v}{\partial n}\frac{\partial y}{\partial n} + \frac{\nu}{1-2\nu}\gamma = 0 \\ &2\left(\frac{\partial u}{\partial n}\frac{\partial y}{\partial n} - \frac{\partial v}{\partial n}\frac{\partial x}{\partial n}\right) + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0\end{aligned}\right\} \quad (3.6)$$

なる関係となる。

これを代入すると　(3.3)の γ は.

$$\begin{aligned}\gamma &= \frac{\partial u}{\partial s}\frac{\partial x}{\partial s} + \frac{\partial v}{\partial s}\frac{\partial y}{\partial s} + \frac{\partial u}{\partial n}\frac{\partial x}{\partial n} + \frac{\partial v}{\partial n}\frac{\partial y}{\partial n} + \frac{v}{r} \\ &= \frac{\partial u}{\partial s}\frac{\partial x}{\partial s} + \frac{\partial v}{\partial s}\frac{\partial y}{\partial s} + \frac{v}{r} - \frac{\nu}{1-2\nu}\gamma\end{aligned}$$

故

$$\gamma = \frac{1-2\nu}{1-\nu}\left[\frac{\partial u}{\partial s}\frac{\partial x}{\partial s} + \frac{\partial v}{\partial s}\frac{\partial y}{\partial s} + \frac{v}{r}\right], \quad \text{--} \quad (3.7)$$

線素に沿う応力を σ_s とすると

$$\begin{aligned}\sigma_s &= 2\left[\frac{\partial u}{\partial x}\left(\frac{\partial x}{\partial s}\right)^2 + \frac{\partial v}{\partial y}\left(\frac{\partial y}{\partial s}\right)^2 + \frac{\partial x}{\partial s}\frac{\partial y}{\partial s}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] \\ &\qquad + \frac{2\nu}{1-2\nu}\gamma, \\ &= 2\left(\frac{\partial u}{\partial s}\frac{\partial x}{\partial s} + \frac{\partial v}{\partial s}\frac{\partial y}{\partial s}\right) + \frac{2\nu\gamma}{1-2\nu}\end{aligned}$$

となるから (3.7)を代入すると

$$\sigma_s = \frac{2\gamma}{1-2\nu} - \frac{2\nu}{r}, \quad \cdots\cdots \quad (3.8)$$

一方 θ方向の応力 $\sigma_\theta \equiv f_{33}$ であるから

$$\sigma_s + \sigma_\theta = \frac{2(1+\nu)}{1-2\nu}\gamma = \sum_{i=1}^{3} f_{ii}, \quad \cdots\cdots \quad (3.9)$$

これらの式から自由辺の応力は u と v の s 方向の微分によって計算出来る事がわかる。

前同様 suffix をえらぶ事とすると基本特異解は

$$\left.\begin{aligned}
U_1^1 &= \frac{1}{4\pi}\int_0^{2\pi} d\theta \left[\frac{1}{R} - \frac{1}{4(1-\nu)}\frac{\partial^2 R}{\partial x^2}\right], \\
&\quad R^2 = (x-x')^2 + r^2 + r'^2 - 2rr'\cos\theta, \\
U_2^1 &= \frac{1}{4\pi}\int_0^{2\pi} d\theta \left[-\frac{1}{4(1-\nu)}\frac{\partial^2 R}{\partial x \partial r}\right], \\
U_1^2 &= \frac{1}{4\pi}\int_0^{2\pi} d\theta \left[-\frac{1}{4(1-\nu)}\frac{\partial^2 R}{\partial r \partial x}\right], \\
U_2^2 &= \text{〃} \left[\frac{1}{R} - \frac{1}{4(1-\nu)}\frac{\partial^2 R}{\partial r^2}\right],
\end{aligned}\right\} \quad (3.10)$$

$$\left.\begin{aligned}
T_1^1 &= \text{〃} \left[\frac{\partial}{\partial n}\frac{1}{R} + \frac{1}{1-\nu}\frac{\partial x}{\partial n}\frac{\partial}{\partial x}\frac{1}{R} - \frac{1}{2(1-\nu)}\frac{\partial^3 R}{\partial n \partial x^2}\right], \\
T_2^1 &= \text{〃} \left[\frac{\partial x}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} + \frac{\nu}{1-\nu}\frac{\partial r}{\partial n}\frac{\partial}{\partial x}\frac{1}{R} - \frac{1}{2(1-\nu)}\frac{\partial^3 R}{\partial n \partial x \partial r}\right], \\
T_1^2 &= \text{〃} \left[\frac{\partial r}{\partial n}\frac{\partial}{\partial x}\frac{1}{R} + \frac{\nu}{1-\nu}\frac{\partial x}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} - \frac{1}{2(1-\nu)}\frac{\partial^3 R}{\partial n \partial x \partial r}\right], \\
T_2^2 &= \text{〃} \left[\frac{\partial}{\partial n}\frac{1}{R} + \frac{1}{1-\nu}\frac{\partial r}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} - \frac{1}{2(1-\nu)}\frac{\partial^3 R}{\partial n \partial r^2}\right],
\end{aligned}\right\} \quad (3.11)$$

$1 - \frac{1}{2(1-\nu)} = \frac{1-2\nu}{2(1-\nu)}$

許で $\frac{\partial^3 R}{\partial n \partial x^2} = \frac{\partial^2}{\partial n \partial x} \frac{x}{R} = \frac{\partial}{\partial n}\left(\frac{1}{R} + x\frac{\partial}{\partial x}\frac{1}{R}\right)$

$$= 2\frac{\partial x}{\partial n}\frac{\partial}{\partial x}\frac{1}{R} + \frac{\partial r}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} + x\frac{\partial^2}{\partial n \partial x}\frac{1}{R},$$

(但し $x'=0$ としてある。)

$$\frac{\partial^3 R}{\partial n \partial x \partial r} = \frac{\partial^2}{\partial n \partial r}\frac{x}{R} = \frac{\partial x}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} + x\frac{\partial^2}{\partial n \partial r}\frac{1}{R},$$

$$\frac{\partial^3 R}{\partial n \partial r^2} = \frac{\partial}{\partial n}\left[\frac{2}{R} - \frac{1}{r}\frac{\partial R}{\partial r} - \frac{\partial^2 R}{\partial x^2}\right]$$

$\because \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2}\right)R = \frac{2}{R}$ (但し θで積分したとして)

$$\frac{\partial^3 R}{\partial n \partial r^2} = \frac{\partial}{\partial n}\left[\frac{1}{R} - x\frac{\partial}{\partial x}\frac{1}{R} - \frac{1}{r}\frac{\partial R}{\partial r}\right] = \frac{\partial r}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} - x\frac{\partial^2}{\partial n \partial x}\frac{1}{R} + \frac{1}{r^2}\frac{\partial r}{\partial n}\frac{\partial R}{\partial r} - \frac{1}{r}\frac{\partial^2 R}{\partial n \partial r},$$

$$T_1^1 = {}'' \left[\frac{\partial x}{\partial n}\frac{\partial}{\partial x}\frac{1}{R} + \frac{1-2\nu}{2(1-\nu)}\frac{\partial r}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} - \frac{x}{2(1-\nu)}\frac{\partial^2}{\partial n \partial x}\frac{1}{R}\right],$$

$$T_2^1 = {}'' \left[\frac{\nu}{1-\nu}\frac{\partial r}{\partial n}\frac{\partial}{\partial x}\frac{1}{R} + \frac{1-2\nu}{2(1-\nu)}\frac{\partial x}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} - \frac{x}{2(1-\nu)}\frac{\partial^2}{\partial n \partial r}\frac{1}{R}\right]$$

$$T_1^2 = {}'' \left[\frac{\partial r}{\partial n}\frac{\partial}{\partial x}\frac{1}{R} - \frac{1-2\nu}{2(1-\nu)}\frac{\partial x}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} - \frac{x}{2(1-\nu)}\frac{\partial^2}{\partial n \partial r}\frac{1}{R}\right]. \quad (3.12)$$

$$T_2^2 = {}'' \left[\frac{\partial x}{\partial n}\frac{\partial}{\partial x}\frac{1}{R} + \frac{3-2\nu}{2(1-\nu)}\frac{\partial r}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} + \frac{1}{2(1-\nu)}\left\{x\frac{\partial^2}{\partial n \partial x}\frac{1}{R} - \frac{1}{r^2}\frac{\partial r}{\partial n}\frac{\partial R}{\partial r} - \frac{1}{r}\frac{\partial^2 R}{\partial n \partial r}\right\}\right]$$

$$= {}'' \left[\frac{\partial x}{\partial n}\frac{\partial}{\partial x}\frac{1}{R} + \frac{2-\nu}{1-\nu}\frac{\partial r}{\partial n}\frac{\partial}{\partial r}\frac{1}{R} - \frac{1}{2(1-\nu)}\frac{\partial^3 R}{\partial n \partial r^2}\right]$$

$\frac{\partial x}{\partial n} x \frac{\partial}{\partial x}\frac{1}{R} + \frac{\partial r}{\partial n}\frac{\partial^3 R}{\ldots}$

今 $r_1^2 = (x-x')^2 + (r-r')^2, \quad r_2^2 = (x-x')^2 + (r+r')^2.$

$$k^2 = \frac{4rr'}{r_2^2} = 1 - k'^2, \quad k'^2 = (r_1/r_2)^2.$$

とおくと
$$R^2 = (x-x')^2 + r^2 + r'^2 - 2rr'\cos\theta = r_2^2\left(1 - k^2\cos^2\frac{\theta}{2}\right)$$

故に

$$\left.\begin{aligned} \frac{1}{4\pi}\int_0^{2\pi}\frac{d\theta}{R} &= \frac{1}{\pi r_2}\int_0^{\frac{\pi}{2}}\frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}} = \frac{K(k)}{\pi r_2} \\ \frac{1}{4\pi}\int_0^{2\pi}R\,d\theta &= \frac{r_2}{\pi}\int_0^{\frac{\pi}{2}}\sqrt{1-k^2\sin^2\varphi}\,d\varphi = \frac{r_2 E(k)}{\pi} \end{aligned}\right\} \quad (3.13)$$

$$\left.\begin{aligned} \dot{K} &= \frac{dK}{d\mu} = k\frac{dK}{dk} = \frac{E}{k'^2} - K, \\ \dot{E} &= \frac{dE}{d\mu} = k\frac{dE}{dk} = E - K, \end{aligned}\right\} \quad \mu = \log k \quad (3.14)$$

$$\frac{\partial\mu}{\partial x} = \frac{1}{k}\frac{\partial k}{\partial x} = -\frac{x}{r_2^2}, \quad \frac{1}{k}\frac{\partial k}{\partial r} = \frac{1}{2r} - \frac{r+r'}{r_2^2} = \frac{\partial\mu}{\partial r} \quad (x'=0とする)$$

$$\left.\begin{aligned} \frac{\partial K}{\partial x} &= \frac{\partial k}{\partial x}\frac{dK}{dk} = \frac{x}{r_2^2}\left(K - \frac{E}{k'^2}\right), \\ \frac{\partial K}{\partial r} &= \left(\frac{1}{2r} - \frac{r+r'}{r_2^2}\right)\left(\frac{E}{k'^2} - K\right), \\ \frac{\partial E}{\partial x} &= \frac{x}{r_2^2}(K - E), \\ \frac{\partial E}{\partial r} &= \left(\frac{1}{2r} - \frac{r+r'}{r_2^2}\right)(E - K), \end{aligned}\right\} \quad (3.15)$$

$$\frac{\partial}{\partial r}(E-K) = -\frac{k^2}{k'^2}\left(\frac{1}{2r} - \frac{r+r'}{r_2^2}\right)E$$

$$\frac{\partial}{\partial x}\frac{K}{r_2} = -\frac{x}{r_2^3}K + \frac{x}{r_2^3}\left(K - \frac{E}{k'^2}\right) = -\frac{x}{r_2 r_1^2}E,$$

$$\frac{\partial}{\partial r}\frac{K}{r_2} = -\frac{r+r'}{r_2^3}K + \frac{1}{r_2}\left(\frac{1}{2r} - \frac{r+r'}{r_2^2}\right)\left(\frac{E}{k'^2} - K\right)$$

$$= -\frac{K}{2r r_2} + \frac{r_2}{r_1^2}\left(\frac{1}{2r} - \frac{r+r'}{r_2^2}\right)E,$$

$$\frac{\partial}{\partial x} r_2 E = \frac{x}{r_2}E + \frac{x}{r_2}(K - E) = \frac{x}{r_2}K.$$

$$\frac{\partial}{\partial r} r_2 E = \frac{r+r'}{r_2}E + r_2\left(\frac{1}{2r} - \frac{r+r'}{r_2^2}\right)(E - K)$$

$$= \frac{r_2}{2r}E - \left(\frac{r_2}{2r} - \frac{r+r'}{r_2}\right)K$$

$$\frac{\partial^2}{\partial x^2} r_2 E = \frac{K}{r_2} - \frac{x^2}{r_1^2 r_2}E, \quad \text{o.K.}$$

$$\frac{\partial^2}{\partial x \partial r} r_2 E = -\frac{x}{2r r_2}K + \frac{x r_2}{r_1^2}\left(\frac{1}{2r} - \frac{r+r'}{r_2^2}\right)E.$$

$$\frac{\partial^2}{\partial r^2} r_2 E = -\frac{r_2}{2r^2}E + \frac{1}{2r}\left[\frac{r_2}{2r}E - \left(\frac{r_2}{2r} - \frac{r+r'}{r_2}\right)K\right]$$

$$+ \left(-\frac{r_2^2}{2r^2} + \frac{2(r+r')}{2r} - 1\right)\frac{K}{r_2} - \left(\frac{r_2^2}{2r} - (r+r')\right)\left[-\frac{K}{2r r_2} + \frac{r_2}{r_1^2}\left(\frac{1}{2r} - \frac{r+r'}{r_2^2}\right)E\right]$$

$$= \left\{\frac{r_2^3}{r_1^2}\left(\frac{1}{2r} - \frac{r+r'}{r_2^2}\right)^2 - \frac{r_2}{4r^2}\right\}E + \left(\frac{r_2}{2r^2} - \frac{r'}{r r_2}\right)K$$

$$= \left[\frac{x^2}{r_1^2 r_2} - \frac{r_2}{2r^2}\right]E + \left[\frac{r_2^2 - 2rr'}{2r^2 r_2}\right]K \quad // \text{ o.K.}$$

$$-\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial x^2}\right)\cancel{r_2}E = \frac{2K}{r_2},$$

$$\frac{\partial^2}{\partial x^2}\frac{K}{r_2} = -r_2E\left(\frac{1}{r_2^2r_1^2}-\frac{2x^2}{r_1^4r_2^4}(r_1^2+r_2^2)\right) - \frac{x}{r_1^2r_2^2}\frac{x}{r_2}K$$

$$\frac{\partial^2}{\partial x\partial r}\frac{K}{r_2} = \cancel{+2}r_2E\left(\frac{x(r-r')}{r_1^4r_2^2}+\frac{x(r+r')}{r_1^2r_2^4}\right) - \frac{x}{r_1^2r_2^2}\left(\frac{r_2}{2r}E-\left(\frac{r_2}{2r}-\frac{r+r'}{r_2}\right)\right)$$

$$= \frac{x}{r_1^2r_2}\left(\frac{1}{2r}-\frac{r+r'}{r_2^2}\right)K + \left(\frac{2(r-r')}{r_1^4r_2}+\frac{2(r+r')}{r_1^2r_2^3}-\frac{1}{2rr_1^2r_2}\right)xE$$

$$\frac{\partial^2}{\partial r^2}\frac{K}{r_2} = \frac{K}{2r^2r_2} - \frac{1}{2r}\left(-\frac{K}{2rr_2}+\frac{r_2}{r_1^2}\left(\frac{1}{2r}-\frac{r+r'}{r_2^2}\right)E\right)$$

$$+\left\{-\frac{2(r-r')}{r_1^4}\left(\frac{1}{2r}-\frac{r+r'}{r_2^2}\right)+\frac{1}{r_1^2}\left(-\frac{1}{2r^2}-\frac{1}{r_2^2}+\frac{2(r+r')^2}{r_2^4}\right)\right\}r_2E$$

$$+\frac{1}{r_1^2}\left(\frac{1}{2r}-\frac{r+r'}{r_2^2}\right)\left(\frac{r_2E}{2r}-\left(\frac{r_2}{2r}-\frac{r+r'}{r_2}\right)K\right),$$

$$= \left\{\frac{3}{4r^2r_2}-\frac{r_2}{r_1^2}\left(\frac{1}{2r}-\frac{r+r'}{r_2^2}\right)^2\right\}K$$

$$+\left\{\frac{\oplus 2(r^2-r'^2)}{r_1^4r_2^2}-\frac{(r-r')}{rr_1^4}-\frac{1}{r_1^2r_2^2}-\frac{1}{2r^2r_1^2}+\frac{2(r+r')^2}{r_1^2r_2^4}\right\}r_2^2E,$$

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial x^2}\right)\frac{K}{r_2} = \left[-\frac{x^2}{r_1^2r_2^3}-\frac{1}{2r^2r_2}+\frac{3}{4r^2r_2}-\frac{r_2}{r_1^2}\left(\frac{1}{2r}-\frac{r+r'}{r_2^2}\right)^2\right]K$$

o.k.

$$+\left[\frac{-2x^2(r_1^2+r_2^2)}{r_1^4r_2^3}-\frac{1}{r_2r_1^2}+\frac{r_2}{rr_1^2}\left(\frac{1}{2r}-\frac{r+r'}{r_2^2}\right)\right.$$

$$\left.+\frac{2(r^2-r'^2)}{r_1^4r_2}+\frac{2(r+r')^2}{r_1^2r_2^3}-\frac{(r-r')r_2}{rr_1^4}-\frac{1}{r_1^2r_2}-\frac{r_2}{2r^2r_1^2}\right]E$$

$$=0$$ o.k.

以上の諸式によって核関数はすべて楕円積分で表わされる事がわかった。

これらの核関数を使えば(2.12)と全く同形式に

$$u_k(Q)=\sum_i\int_c (u_i+u_i')\,T_i^{(k)}(P,Q)\,r\,ds, \quad \cdots\cdots$$

と書け積分方程式は次のようになる。

$$u_k(Q)+u_k'(Q)-\sum\int_c (u_i+u_i')\,T_i^{(k)}\,r\,ds=u_k'(Q),$$

$T_i^{(k)}$なる核の特異性は$\overbrace{\text{元々}}^{P=Q\text{で}}$係数分だけジャムプするものであったので積分の時はその点注意する必要がある。

また

$$\left.\begin{array}{l} K(k)\xrightarrow[k\to 1]{} 2\log 2-\log(k') \\ E(k)\xrightarrow[k\to 1]{} 1 \end{array}\right\}$$

$$K(k)\xrightarrow[k\to 0]{} E(k)\xrightarrow[k\to 0]{} \frac{\pi}{2}$$

また次頁以下のように$\int_j^i$なる積分を導入すれば直線区間で変位一定ならば上の積分は直ちに実行出来る。

$$S = \frac{1}{4\pi}\int_0^{2\pi}\frac{d\theta}{R} = \frac{1}{2}\int_0^{\infty} e^{-tx} J_0(tr) J_0(tr')\,dt.$$

$$\frac{\partial}{\partial r} J_0(tr) = -t J_1(tr), \quad \frac{d}{dr}\{r J_1(tr)\} = rt J_0(tr)$$

$$S^* = \frac{1}{4\pi}\int_0^{2\pi} R\,d\theta = \frac{r}{2}\int_0^{\infty} e^{-tx} J_1(tr) J_0(tr')\frac{dt}{t},$$

$$\text{or} \quad = \frac{r'}{2}\int_0^{\infty} e^{-tx} J_1(tr') J_0(tr)\frac{dt}{t},$$

$$T = \frac{r}{2}\int_0^{\infty} e^{-tx} J_1(tr) J_0(tr')\,dt,$$

これは第3種楕円積分で表わされる他の方法

$$\frac{\partial S}{\partial x} = -\frac{1}{r}\frac{\partial T}{\partial r}, \quad \frac{\partial S}{\partial r} = \frac{1}{r}\frac{\partial T}{\partial x},$$

$$\frac{\partial S}{\partial n} = -\frac{1}{r}\frac{\partial T}{\partial s}, \quad \frac{\partial S}{\partial s} = \frac{1}{r}\frac{\partial T}{\partial n},$$

$$T^* = -\frac{rr'}{2}\int_0^{\infty} e^{-tx} J_1(tr) J_1(tr')\frac{dt}{t}$$

$$\frac{\partial T^*}{\partial r} = -\frac{rr'}{2}\int_0^{\infty} e^{-tx} J_0(tr) J_1(tr')\,dt = r\frac{\partial S^*}{\partial x}$$

$$\frac{\partial T^*}{\partial x} = \frac{rr'}{2}\int_0^{\infty} e^{-tx} J_1(tr) J_1(tr')\,dt = -r\frac{\partial S^*}{\partial r}$$

$$\frac{\partial}{\partial n} S^* = \frac{1}{r}\frac{\partial T^*}{\partial s}, \quad \frac{\partial S^*}{\partial s} = -\frac{1}{r}\frac{\partial T^*}{\partial n},$$

$$\frac{\partial T^*}{\partial r} = r\frac{\partial S^*}{\partial x} = -rT, \quad \frac{\partial T^*}{\partial x} = -r\frac{\partial S^*}{\partial r} = -r^2 S,$$

$$\frac{\partial^2 T^*}{\partial x^2} = -r^2\frac{\partial S}{\partial x}, \quad \frac{\partial^2 T^*}{\partial r^2} = -T + r^2\frac{\partial S}{\partial x}$$

$$\frac{\partial^2 T^*}{\partial x \partial r} = -r\frac{\partial T}{\partial x} = -r^2\frac{\partial S}{\partial r},$$

$$-\frac{\partial Y}{\partial n}\frac{\partial i}{\partial n}$$

$$T_1^1 = \left\{1+\frac{\left(\frac{\partial X}{\partial n}\right)^2}{1-\nu}\right\}\frac{\partial S}{\partial n} + \frac{\frac{\partial X}{\partial n}\frac{\partial X}{\partial s}}{1-\nu}\frac{\partial S}{\partial s} - \frac{1}{2(1-\nu)}\frac{\partial^2}{\partial X'}\frac{\partial S^*}{\partial n}$$

$$T_2^1 = \frac{1}{1-\nu}\frac{\partial X}{\partial n}\frac{\partial Y}{\partial n}\frac{\partial S}{\partial n} + \left(\frac{\partial X}{\partial n}\frac{\partial Y}{\partial s} + \frac{\nu}{1-\nu}\frac{\partial Y}{\partial n}\frac{\partial X}{\partial s}\right)\frac{\partial S}{\partial s} - \frac{1}{2(1-\nu)}\frac{\partial^2}{\partial X'\partial Y'}\frac{\partial S}{\partial n}$$

$$T_1^2 = \frac{1}{1-\nu}\frac{\partial X}{\partial n}\frac{\partial Y}{\partial n}\frac{\partial S}{\partial n} + \left(\frac{\partial Y}{\partial n}\frac{\partial X}{\partial s} + \frac{\nu}{1-\nu}\frac{\partial X}{\partial n}\frac{\partial Y}{\partial s}\right)\frac{\partial S}{\partial s} - \frac{1}{2(1-\nu)}\frac{\partial^2}{\partial X'\partial Y'}\frac{\partial S}{\partial n}$$

$$T_2^2 = \left\{1+\frac{\left(\frac{\partial Y}{\partial n}\right)^2}{1-\nu}\right\}\frac{\partial S}{\partial n} + \frac{\frac{\partial Y}{\partial n}\frac{\partial Y}{\partial s}}{1-\nu}\frac{\partial S}{\partial s} - \frac{1}{2(1-\nu)}\frac{\partial^2}{\partial Y'^2}\frac{\partial S^*}{\partial n}.$$

$$S_1^1 = \int ds\, r\, T_1^1 = \cancel{-}\left(1+\frac{\left(\frac{\partial X}{\partial n}\right)^2}{1-\nu}\right)T - \frac{X_n Y_n}{1-\nu}S \mp \frac{1}{2(1-\nu)}\frac{\partial^2}{\partial X^2}T^*$$

$$S_2^1 = \int ds\, r\, T_2^1 = -\frac{X_n Y_n}{1-\nu}T + \left(X_n^2 - \frac{\nu}{1-\nu}Y_n^2\right)S - \frac{1}{2(1-\nu)}\frac{\partial^2 T^*}{\partial X\partial Y}$$

$$S_1^2 = \int ds\, r\, T_1^2 = -\frac{X_n Y_n}{1-\nu}T + \left(-Y_n^2 + \frac{\nu}{1-\nu}X_n^2\right)S - \frac{1}{2(1-\nu)}\frac{\partial^2 T^*}{\partial X\partial Y}$$

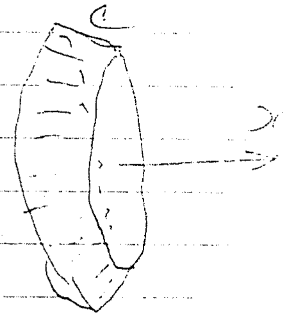
$$S_2^2 = \int ds\, r\, T_2^2 = -\left(1+\frac{Y_n^2}{1-\nu}\right)T + \frac{X_n Y_n}{1-\nu}S - \frac{1}{2(1-\nu)}\frac{\partial^2}{\partial Y^2}T^*.$$

$$\left.\begin{aligned}
S_1^1 &= -\left(1+\frac{Y_n^2}{1-\nu}\right)T - \frac{X_n Y_n}{1-\nu}S + \frac{r^2}{2(1-\nu)}\frac{\partial S}{\partial X}\\
S_2^1 &= -\frac{X_n Y_n}{1-\nu}T + \left(X_n^2 - \frac{\nu Y_n^2}{1-\nu}\right)S + \frac{r^2}{2(1-\nu)}\frac{\partial S}{\partial Y}\\
S_1^2 &= -\frac{X_n Y_n}{1-\nu}T - \left(Y_n^2 - \frac{\nu X_n^2}{1-\nu}\right)S + \frac{r^2}{2(1-\nu)}\frac{\partial S}{\partial Y}\\
S_2^2 &= \left[\frac{1}{2(1-\nu)} - 1 - \frac{Y_n^2}{1-\nu}\right]T + \frac{X_n Y_n}{1-\nu}S - \frac{r^2}{2(1-\nu)}\frac{\partial S}{\partial X}
\end{aligned}\right\}$$

以上の方法を少し一般化して見よう。

微小円環要素上の積分を考えよう。

$$I(Q)=\int_C \left(u T_1^{(k)} + v T_2^{(k)}\right) r\,ds$$



この形でのBettiの相反定理から

Q点が外にあるならば

$$I(Q)=\int_C \left(u T_1^{k} + v T_2^{k}\right) r\,ds = \int_C \left(\tau_1 U_1^{k} + \tau_2 U_2^{k}\right) r\,ds$$

$$+\iint_{A-B}\left[u T_1^{k} + v T_2^{k} - \tau_1 U_1^{k} - \tau_2 U_2^{k}\right] r\,dr$$

Aは前の円板面，Bは後。

今C上で $u = \mathrm{const.}$, $v = v\dfrac{r}{a}$, とすると （$v = \mathrm{const}$ とすると $\tau \propto \dfrac{1}{r}$ で工合が悪い）

$$\gamma = \frac{2v}{a}, \quad \tau_1 = \frac{4\nu}{1-2\nu}\frac{v}{a}\frac{\partial x}{\partial n},$$

$$\tau_2 = 2\frac{v}{a}\frac{\partial r}{\partial n} + \frac{4\nu}{1-2\nu}\frac{v}{a}\frac{\partial r}{\partial n} = \frac{2}{1-2\nu}\frac{v}{a}\frac{\partial r}{\partial n},$$

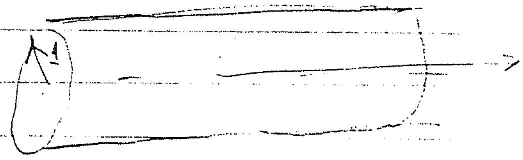
それ故これを上式に代入して

$$I(Q) = \frac{2\left(\frac{v}{a}\right)}{1-2\nu}\int_C\left[2\nu U_1^{k}\frac{\partial x}{\partial n} + U_2^{k}\frac{\partial r}{\partial n}\right] r\,ds$$

$$+\int_{A-B}\left[u T_1^{k} + \frac{v}{a} r T_2^{k} - \frac{4\nu}{1-2\nu}\frac{v}{a} U_1^{k}\right] r\,dr$$

附録A 無限長丸棒の中の軸対称な特異性

丸棒の外周(半径1)は自由辺とする。



そうすると そこでは

$$
\left.\begin{aligned}
&\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\Big|_{r=1}=0\\
&\frac{\partial v}{\partial r}+\frac{\nu}{1-2\nu}\gamma\Big|_{r=1}=0
\end{aligned}\right\}\quad (A.1)
$$

この第2式は $\gamma=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}$ であるから

$$
\frac{1-\nu}{1-2\nu}\gamma\Big|_{r=1}=\frac{\partial u}{\partial x}+\frac{v}{r}\Big|_{r=1}\quad (A.1')
$$

としてもよい。

そして
$$
\left.\begin{aligned}
&\frac{1}{R}=\frac{2}{\pi}\int_0^\infty \cos xt\, K_0(t\varpi)\,dt,\\
&R^2=x^2+\varpi^2,\quad \varpi^2=r^2+r'^2-2rr'\cos\theta
\end{aligned}\right\}\quad (A.2)
$$

であり
$$
K_0(t\varpi)=\sum_{m=-\infty}^{\infty}K_m(tr)\,I_m(tr')\cos m\theta,\quad r>r'
$$

であるから

$$
\left.\begin{aligned}
I&=\frac{1}{4\pi}\int_0^{2\pi}\frac{d\theta}{R}=\frac{1}{\pi}\int_0^\infty \cos xt\,K_0(tr)\,I_0(tr')\,dt,\quad \text{for } r>r'\\
&=\frac{1}{\pi}\int_0^\infty \cos xt\,I_0(tr)\,K_0(tr')\,dt,\quad \text{for } r'>r
\end{aligned}\right\}\quad (A.3)
$$

ここで
$$
\Delta\equiv\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial x^2},\quad \Delta R=\frac{2}{R},
$$

$$
\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-t^2\right)r\begin{Bmatrix}K_1(tr)\\ I_1(tr)\end{Bmatrix}=2t\begin{Bmatrix}-K_0(tr)\\ I_0(tr)\end{Bmatrix},\quad (A.4)
$$

$K_0' = -K_1, \quad I_0' = I_1$

$\frac{\partial}{\partial r}(rK_1) = -rtK_0, \quad \frac{\partial}{\partial r}(rI_1) = rtI_0,$

$\frac{\partial^2}{\partial r^2}(rK_1) = -tK_0 + rt^2K_1, \quad \frac{\partial^2}{\partial r^2}(rI_1) = tI_0 + rt^2I_1,$

No. A-2

Date

$$J = \frac{1}{4\pi}\int_0^{2\pi} R\,d\theta = -\frac{r'}{2\pi}\int_0^\infty \cos xt\,K_1(tr)I_0(tr')\frac{dt}{t} \quad \text{for } r > r'$$

$$= \frac{r'}{\pi}\int_0^\infty \cos xt\,I_1(tr)K_0(tr')\frac{dt}{t}, \quad \text{for } r' > r \qquad (A\cdot5)$$

この積分は t=0 の特異性から積分不可能であるけれど実際はこの微係数のみ必要でそれは積分可能であるから別に不便はないのでこのまゝ書いておく。

これらを使うと (3.10) は

$$U_1^1 = \frac{1}{\pi}\int_0^\infty \cos xt\,I_0(tr')\left[K_0(tr) - \frac{rt}{4(1-\nu)}K_1(tr)\right]dt,$$

$$U_2^1 = \frac{1}{\pi}\int_0^\infty \sin xt\,I_0(tr')\left[-\frac{1}{4(1-\nu)}\frac{\partial}{\partial r}(rK_1(tr))\right]dt, \quad \text{for } r > r' \qquad (A\cdot6)$$

$$U_1^1 = \frac{1}{\pi}\int_0^\infty \cos xt\,K_0(tr')\left[I_0(tr) + \frac{rt}{4(1-\nu)}I_1(tr)\right]dt,$$

$$U_2^1 = \frac{1}{\pi}\int_0^\infty \sin xt\,K_0(tr')\left[+\frac{1}{4(1-\nu)}\frac{d}{dr}(rI_1(tr))\right]dt, \quad \text{for } r < r' \qquad (A\cdot7)$$

$$U_1^2 = U_2^1$$

$$U_2^2 = \frac{1}{\pi}\int_0^\infty \cos xt\,I_0(tr')\left[K_0(tr) + \frac{1}{4(1-\nu)t}\frac{\partial^2}{\partial r^2}(rK_1(tr))\right]dt, \quad \text{for } r > r'$$

$$= \frac{1}{\pi}\int_0^\infty \cos xt\,K_0(tr')\left[I_0(tr) - \frac{1}{4(1-\nu)t}\frac{\partial^2}{\partial r^2}(rI_1)\right]dt, \quad \text{for } r < r' \qquad (A\cdot8)$$

$r=1$で(A·1)の条件を満たすためには外側の領域に特異性のあるものを考えて、まず

$$\left.\begin{aligned} V_1^1 &= \frac{1}{\pi}\int_0^\infty \cos xt\, F_1(t)\left[I_0(tr) + \frac{rt}{4(1-\nu)} I_1(tr)\right] dt, \\ V_2^1 &= \frac{1}{\pi}\int_0^\infty \sin xt\, F_1(t)\left[-\frac{1}{4(1-\nu)}\frac{\partial}{\partial r}(r I_1)\right] dt, \end{aligned}\right\} \quad (A\cdot 9)$$

とさらに調和分を加えて

$$\left.\begin{aligned} W_1^1 &= \frac{1}{\pi}\int_0^\infty \cos xt\, A_1(t) I_0(tr)\, dt, \\ W_2^1 &= \frac{1}{\pi}\int_0^\infty \sin xt\, A_1(t) I_1(tr)\, dt, \end{aligned}\right\} \quad (A\cdot 10)$$

$$\left.\begin{aligned} V_1^2 &= \frac{1}{\pi}\int_0^\infty \sin xt\, F_2(t)\left[\frac{1}{4(1-\nu)}\frac{\partial}{\partial r}(r I_1)\right] dt, \\ V_2^2 &= \frac{1}{\pi}\int_0^\infty \cos xt\, F_2(t)\left[I_0 - \frac{1}{4(1-\nu)t}\frac{\partial^2}{\partial r^2}(r I_1)\right] dt, \end{aligned}\right\} \quad (A\cdot 11)$$

$$\left.\begin{aligned} W_1^2 &= -\frac{1}{\pi}\int_0^\infty \sin xt\, A_2(t) I_0(tr)\, dt, \\ W_2^2 &= \frac{1}{\pi}\int_0^\infty \cos xt\, A_2(t) I_1(tr)\, dt, \end{aligned}\right\} \quad (A\cdot 12)$$

としてこれらを加えて新しくU_j^iを定義しよう。

$$U_j^i + V_j^i + W_j^i \longrightarrow U_j^i$$

その前に

$$\Gamma_U^i = \frac{\partial}{\partial x} U_1^i + \frac{\partial}{\partial r} U_2^i + \frac{1}{r} U_2^i \, , \qquad -1 + \frac{1}{2(1-\nu)} = \frac{-1}{2} \ldots$$

$$-\frac{t}{2}$$

$$\Gamma_U^1 = \frac{1}{\pi}\int_0^\infty \sin xt \, I_0(tr')\Big[-tK_0 + \frac{rt^2}{4(1-\nu)}K_1 - \frac{1}{4(1-\nu)}\frac{\partial^2}{\partial r^2}(rK_1)$$

$$- \frac{1}{4(1-\nu)}\frac{1}{r}\frac{\partial}{\partial r}(rK_1)\Big]dt$$

$$= -\frac{1}{\pi}\frac{1-2\nu}{2(1-\nu)}\int_0^\infty \sin xt \, I_0(tr')K_0(tr)t\,dt \, , \quad \cdots\cdots (A\cdot 13)$$

$$\Gamma_V^1 = -\frac{1}{\pi}\frac{1-2\nu}{2(1-\nu)}\int_0^\infty \sin xt \, F_1(t)\, I_0(tr)t\,dt \, , \qquad (A\cdot 14)$$

$$\Gamma_W^1 = \Gamma_W^2 = 0 \, , \qquad \cdots\cdots (A\cdot 15)$$

$$\Gamma_U^2 = \frac{1}{\pi}\int_0^\infty \cos xt \, I_0(tr')\Big[-\frac{t}{4(1-\nu)}\frac{\partial}{\partial r}(rK_1) - tK_1 + \frac{1}{4(1-\nu)t}\frac{\partial^3}{\partial r^3}(rK_1)$$

$$+ \frac{K_0}{r} + \frac{1}{4(1-\nu)tr}\frac{\partial^2}{\partial r^2}(rK_1)\Big]dt$$

$$= -\frac{1}{\pi}\frac{1-2\nu}{2(1-\nu)}\int_0^\infty \cos xt \, I_0(tr')K_1(tr)t\,dt \, , \quad \cdots (A\cdot 16)$$

$$\Gamma_V^2 = \frac{1}{\pi}\int_0^\infty \cos xt \, F_2\Big[\frac{t}{4(1-\nu)}\frac{\partial}{\partial r}(rI_1) + tI_1 - \frac{1}{4(1-\nu)t}\frac{\partial^3}{\partial r^3}(rI_1)$$

$$+ \frac{I_0}{r} - \frac{1}{4(1-\nu)tr}\frac{\partial^2}{\partial r^2}(rI_1)\Big]dt$$

$$= \frac{1}{\pi}\frac{1-2\nu}{2(1-\nu)}\int_0^\infty \cos xt \, F_2(t)\, I_1(tr)t\,dt \, , \quad \cdots (A\cdot 17)$$

$$\left.\begin{aligned}
\Gamma_U^1 + \Gamma_V^1 = \Gamma^1 &= -\frac{1-2\nu}{\pi\, 2(1-\nu)}\int_0^\infty \sin xt\Big[I_0(tr')K_0(tr) + F_1 I_0(tr)\Big]t\,dt \, , \\
\Gamma_U^2 + \Gamma_V^2 = \Gamma^2 &= -\frac{1-2\nu}{2\pi(1-\nu)}\int_0^\infty \cos xt\Big[I_0(tr')K_1(tr) - F_2 I_1(tr)\Big]t\,dt \, ,
\end{aligned}\right\}$$

(A.18)

$$U_1^1 = \frac{1}{\pi}\int_0^\infty \cos xt\left[I_0(tr')\left\{K_0 - \frac{rt}{4(1-\nu)}K_1\right\} + F_1\left\{I_0 + \frac{rt}{4(1-\nu)}I_1\right\} + A_1 I_0\right]dt,$$

$$U_2^1 = \frac{1}{\pi}\int_0^\infty \sin xt\left[I_0(tr')\left\{+\frac{rt}{4(1-\nu)}K_0\right\} + \frac{F_1}{4(1-\nu)}rtI_0 + A_1 I_1\right]dt,$$

(A·13)

$$U_1^2 = \frac{1}{\pi}\int_0^\infty \sin xt\left[I_0(tr')\left\{+\frac{rt}{4(1-\nu)}K_0\right\} + \frac{F_2}{4(1-\nu)}rtI_0 - A_2 I_0\right]dt$$

$$U_2^2 = \frac{1}{\pi}\int_0^\infty \cos xt\left[\frac{I_0(tr')}{4(1-\nu)}\left\{(3-4\nu)K_0 + rtK_1\right\} + \frac{F_2}{4(1-\nu)}\left\{(3-4\nu)I_0 - rtI_1\right\} + A_2 I_1\right]dt$$

(A·14)

(A·1)の条件は

$$I_0(tr')\left[-tK_1 + \frac{rt^2}{2(1-\nu)}K_0\right] + F_1\left[tI_1 + \frac{rt^2}{2(1-\nu)}I_0\right] + 2A_1 t I_1 = 0$$

$$-\frac{t}{2}\left[I_0(tr')K_0(tr) + F_1 I_0\right] = -t\left[I_0(tr')\left\{K_0 - \frac{rt}{4(1-\nu)}K_1\right\} + F_1\left\{I_0 + \frac{rt}{4(1-\nu)}I_1\right\}\right.$$

$$+ \frac{I_0(tr')}{r}\left\{-\frac{1}{4(1-\nu)}\frac{\partial}{\partial r}(rK_1)\right\} + \frac{F_1}{4(1-\nu)}\frac{\partial}{r\partial r}(rI_1)$$

$$\left. + A_1\left(-I_0 t + \frac{I_1}{r}\right)\right),$$

$$I_0(tr')\left[\left(\frac{1}{2} - \frac{1}{2(1-\nu)}\right)K_0 - \frac{rt}{4(1-\nu)}K_1\right] + F_1\left[\frac{-\nu}{2(1-\nu)}I_0 + \frac{rt}{4(1-\nu)}I_1\right]$$

$$= A_1\left(\frac{I_1}{rt} - I_0\right) = -A_1 I_1'$$

$$I_0(tr')\left[\frac{K_1}{2} - \frac{rtK_0}{4(1-\nu)}\right] + F_1\left[-\frac{I_1}{2} - \frac{rtI_0}{4(1-\nu)}\right] = A_1 I_1,$$

(A·15)

$$I_0(tr')\left[\left(\frac{1}{2rt} + \frac{rt}{4(1-\nu)}\right)K_1 - \frac{1-2\nu}{4(1-\nu)}K_0\right] - F_1\left[\left(\frac{1}{2rt} + \frac{rt}{4(1-\nu)}\right)I_1 + \frac{1-2\nu}{4(1-\nu)}I_0\right]$$

$$= A_1 I_0$$

以下同様 未完