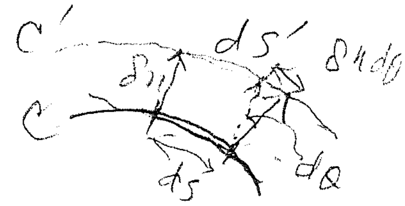
添附別紙の如く変分をとれば曲率の影響は出て来ない。また計算結果でもこの影響は無視出来た。
注) 59.2.19.

変分式の修正（曲率の影響補正）

1. 変分

$$ds' = \left(1+\frac{\delta n}{\rho}\right)ds, \qquad (1)$$



$$\frac{1}{\rho} = \frac{\partial \theta}{\partial s} \quad ;\text{曲率}$$

$$g'\big|_{c'} = g + \delta g + \frac{\partial g}{\partial n}\delta n$$

で $\dfrac{\partial g}{\partial n} = -\dfrac{g}{\rho}$

$$g'\big|_{c'} = g + \delta g - \frac{g}{\rho}\delta n, \qquad (2)$$

$$y'\big|_{c'} = y + \delta y = y + \frac{\partial y}{\partial n}\delta n, \qquad (3)$$

それ故 今

$$I = \int_c g^{\mu} y^{\nu} ds, \qquad (4)$$

の変分は

$$g'^{\mu}\big|_{c'} = g'^{\mu}\big|_{c} + \mu (g')^{\mu-1}\frac{\partial g'}{\partial n}\delta n\Big|_{c}$$

$$= g^{\mu}\big|_{c} + \mu g^{\mu-1}\left[\delta g - \frac{g}{\rho}\delta n\right], \qquad (5)$$

$$y'^{\nu}\big|_{c'} = y^{\nu}\big|_{c} + \nu y^{\nu-1}\frac{\partial y}{\partial n}\delta n, \qquad (6)$$

$$\delta I = \int_C \Big[\Big[\xi^{\mu} + \mu \xi^{\mu-1}\big(\delta\xi - \frac{\xi}{\rho}\delta n\big)\Big]\Big[\eta^{\nu} + \nu\eta^{\nu-1}\frac{\partial \eta}{\partial n}\delta n\Big] \times \Big[1 + \frac{\delta n}{\rho}\Big] - \xi^{\mu}\eta^{\nu}\Big] ds$$

$$= \int_C \Big[\Big\{\xi^{\mu} + \frac{\xi^{\mu}}{\rho}(1-\mu)\delta n + \mu\xi^{\mu-1}\delta\xi\Big\}\Big\{\eta^{\nu} + \nu\eta^{\nu-1}\frac{\partial\eta}{\partial n}\delta n\Big\} - \xi^{\mu}\eta^{\nu}\Big] ds$$

$$= \int_C \Big[\eta^{\nu}\Big\{\mu\xi^{\mu-1}\delta\xi + \frac{(1-\mu)}{\rho}\xi^{\mu}\delta n\Big\} + \nu\xi^{\mu}\eta^{\nu-1}\frac{\partial\eta}{\partial n}\delta n\Big] ds$$

$$= \int_C \Big[\mu\xi^{\mu-1}\eta^{\nu}\delta\xi + \nu\xi^{\mu}\eta^{\nu-1}\frac{\partial\eta}{\partial n}\delta n - \underline{\frac{(\mu-1)}{\rho}\xi^{\mu}\eta^{\nu}\delta n}\Big] ds, \qquad (17)$$

この最後の項が曲率修正で $\mu=1$ の時は0となる。

(附加質量積分の時は $\mu=1$ であるから出て来なかった!)

2. 3次元 回転体

$$J = \int_c q^{\frac{10}{3}} y^{\frac{7}{6}} ds$$

$\mu = \frac{10}{3}$, $\nu = \frac{7}{6}$ とする

$$\delta J = \int_c Q_1^* y \, \delta n \, ds$$

$$Q_1 = \frac{10}{3} q^{\frac{7}{3}} y^{\frac{1}{6}} \left(\frac{\delta q}{\delta n}\right) + \frac{7}{6} q^{\frac{10}{3}} y^{-\frac{5}{6}} \frac{\partial y}{\partial n} = \frac{10}{3} \frac{\partial^2 \varphi}{\partial s} q - 1 \cdot \frac{7}{6} q^{\frac{10}{3}} y^{-\frac{5}{6}} \frac{\partial y}{\partial n}$$

$$Q_1^* = Q_1 - \underbrace{\frac{7}{3\rho} q^{\frac{10}{3}} y^{\frac{1}{6}}}_{\text{追加項}}$$

$$\delta Q_1^* = \delta Q_1 - \frac{7}{3\rho} \left[\frac{10}{3} q^{\frac{7}{3}} y^{\frac{1}{6}} \delta q + \frac{1}{6} q^{\frac{10}{3}} y^{-\frac{5}{6}} \frac{\partial y}{\partial n} \delta n \right]$$

3, 2次元

$$J = \int_C q^4 ds.$$

$\mu = 4$, $\nu = 0$

$$\delta J = \int_C Q^* \delta n \, ds$$

$$Q = 4 q^3 \delta q = 4 q \frac{\partial \phi_3}{\partial s}$$

$$Q^* = Q - \frac{3}{\rho} q^4 \delta n$$

抵抗積分の変分

$$J = \int_C q^{\frac{10}{3}} y^{\frac{7}{6}} ds = \int_C q \psi_2 ds = -\int_C \psi_2 \frac{\partial \Phi}{\partial s} ds$$

$$= \int_C \Phi \frac{\partial \psi_2}{\partial s} ds$$

$$\psi_2 = q^{\frac{7}{3}} y^{\frac{7}{6}} , \quad q = -\frac{\partial \Phi}{\partial s} = \frac{1}{y}\frac{\partial \Psi}{\partial n}$$

$$= \int \Phi \frac{\partial \phi_2}{\partial n} y ds$$

$$J' = \int_{C'} \psi_2' \frac{\partial \Phi'}{\partial s} ds = \int_C \psi_2' \frac{\partial \Phi'}{\partial s} ds + \iint_{\delta D} \underbrace{\nabla \Phi \nabla \phi_2}_{\frac{\partial \Phi}{\partial s}\frac{\partial \phi_2}{\partial s}\delta n} \delta n ds$$

$$\psi_2'\Big|_C = \psi_2'\Big|_{C'} - \delta n \frac{\partial \psi_2}{\partial n} = q^{\frac{7}{3}} y^{\frac{7}{6}} + y \delta n \frac{\partial \phi_2}{\partial s}$$

$$q'^{\frac{7}{3}} y'^{\frac{7}{6}} = \underline{q^{\frac{7}{3}} y^{\frac{7}{6}}} + \frac{7}{3} q^{\frac{4}{3}} y^{\frac{7}{6}} \delta q + \frac{7}{6} q^{\frac{7}{3}} y^{\frac{1}{6}} \delta y$$

$$\frac{\partial \Phi'}{\partial s} = -q' = -q - \delta q$$

$$\psi_2' \frac{\partial \Phi'}{\partial s} = -q^{\frac{10}{3}} y^{\frac{7}{6}} - q^{\frac{7}{3}} y^{\frac{7}{6}} \delta q - q\left(\frac{7}{3} q^{\frac{4}{3}} y^{\frac{7}{6}} \delta q + \frac{7}{6} q^{\frac{7}{3}} y^{\frac{1}{6}} \delta y + y \frac{\partial \phi_2}{\partial s} \delta n\right)$$

$$\delta J = J' - J = -\int_C q\left[\frac{10}{3} q^{\frac{4}{3}} y^{\frac{7}{6}} \delta q + \frac{7}{6} q^{\frac{7}{3}} y^{\frac{1}{6}} \delta y + y \frac{\partial \phi_2}{\partial s} \delta n\right] ds$$

$$- \int_C q \frac{\partial \phi_2}{\partial s} \delta n y \, ds$$

cancel

次頁

$$= \int_C [$$

[illegible]

$$\int_C \bar{\Phi} \frac{\partial \psi_2}{\partial s} ds = \int_C \bar{\Phi} \frac{\partial \phi_2}{\partial n} y \, ds = \int_C \left(\bar{\Phi} \frac{\partial \phi_2}{\partial n} - \phi_2 \frac{\partial \bar{\Phi}}{\partial n} \right) y \, ds .$$

$$\int_{C'} \bar{\Phi}' \frac{\partial \psi_2'}{\partial s} ds = \int_C \left(\bar{\Phi}' \frac{\partial \psi_2}{\partial s} - \phi_2 \frac{\partial \bar{\Phi}'}{\partial s} \right) ds$$

$$= \int_C \left(\Psi' \frac{\partial \phi_2}{\partial s} - \psi_2' \frac{\partial \bar{\Phi}'}{\partial s} \right) ds .$$

$$\Psi' \Big|_C = \Psi' \Big|_{C'} - \delta n \frac{\partial \Psi'}{\partial n} \Big|_C = y \frac{\partial \bar{\Phi}}{\partial s} \delta n$$

$$= -g y \, \delta n$$

$$= \int_C \left[-g \frac{\partial \phi_2}{\partial s} y \, \delta n - \psi_2' \frac{\partial \bar{\Phi}'}{\partial s} \right] ds .$$