[滑走板の造波抵抗] M-117-3

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滑走板(圧力分布)の造波抵抗

波高と造波抵抗

圧力分布 による水面の波高は
$P(x,y)=\frac{1}{2}\rho U^2 p(x,y)$

$\kappa =$

$$\kappa\zeta(x,y) = -p(x,y) + \frac{1}{4\pi\kappa}\iint p_\xi W_\xi \, d\xi d\eta$$

$$= -p(x,y) - \frac{1}{4\pi\kappa}\iint p \, W_{\xi\xi} \, d\xi d\eta \qquad ;\ 1$$

と書ける。

$x\to-\infty$ では $p(x,y)=0$ とすると

$$\kappa\zeta(x,y) = -\frac{1}{4\pi\kappa}\iint p \, W_{\xi\xi} \, d\xi d\eta$$

$x\to\infty$ で

$$W_{\xi\xi} \sim -4i\kappa^3\int_{-\pi/2}^{\pi/2} e^{-i\kappa\sec^2\theta\{(x-\xi)\cos\theta+(y-\eta)\sin\theta\}}\sec^4\theta \, d\theta \qquad ;\ 1/L^3$$

であるから

$$\zeta \sim \frac{i}{\pi}\kappa\iint p\int_{-\pi/2}^{\pi/2} e^{\sim}\sec^4\theta \, d\theta \; d\xi d\eta$$

$$= \int_{-\pi/2}^{\pi/2} H(\theta)\, e^{-i\kappa\sec^2\theta(x\cos\theta+y\sin\theta)}\sec^4\theta \, d\theta \qquad ;\ L$$

where

$$H(\theta) = \frac{i}{\pi}\kappa\iint p\, e^{i\kappa\sec^2\theta(\xi\cos\theta+\eta\sin\theta)} \, d\xi d\eta \qquad ;\ L$$

従って造波抵抗は

$$R_W = \pi\rho U^2\int_0^{\pi/2}|H(\theta)|^2\sec^5\theta \, d\theta$$

振巾関数.

$$H(\theta) = \frac{i\kappa}{\pi} \iint \underbrace{\frac{P(\xi,\eta)}{\frac{1}{2}\rho U^2}}_{= p(\xi,\eta)} \underbrace{e^{i\kappa\sec^2\theta(\xi\cos\theta + \eta\sin\theta)}}_{E(\theta;\xi,\eta)} \, d\xi\, d\eta$$

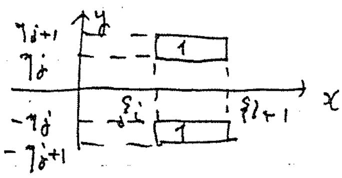
とおく。圧力分布を一定圧力値で細かく分割すると

$$H(\theta) = \frac{1}{\pi} \sum p_{ij} H_{ij}(\theta)$$

$$H_{ij}(\theta) = i\kappa \iint_{S_{ij}} E(\theta;\xi,\eta)\, d\xi\, d\eta \qquad ; L$$

$$= -2i\kappa \frac{\cos^3\theta}{\kappa^2 \sin\theta} \Big[\sin(\kappa\eta\sec^2\theta\sin\theta)\Big]_{\eta_j}^{\eta_{j+1}} \Big[e^{i\kappa\eta\sec^2\theta\sin\theta}\Big]_{\xi_i}^{\xi_{i+1}}$$

S_{ij}



$$-2i \frac{\cos^3\theta}{\kappa \sin\theta}$$

$-2i\cos\theta / SK$

$$R_W = \pi\rho U^2 \int_0^{\pi/2} |H(\theta)|^2 \sec^5\theta\, d\theta$$

揚力係数.

$\cancel{\frac{1}{2}\rho U^2}$

$$P(\xi,\eta) = p_0 \frac{1}{2}\rho U^2 P_X(\xi) P_Y(\eta)$$; $\underline{\underline{p_0, P_X, P_Y は無次元量}}$

としておく. 揚力 L は

$$L = \iint P(\xi,\eta)\, d\xi\, d\eta$$

$$= \frac{1}{2}\rho U^2 p_0 \int_{-L/2}^{L/2} P_X(\xi)\, d\xi \int_{-B/2}^{B/2} P_Y(\eta)\, d\eta$$

P_X, P_Y は各々

$$\underline{\int_{L/2}^{L/2} P_X(\xi)\, d\xi = L} \; ; \; \underline{\int_{-B/2}^{B/2} P_Y(\eta)\, d\eta = B}$$

としておくと

$$L = \frac{1}{2}\rho U^2 p_0 LB$$

揚力係数は

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 LB} = p_0$$

抗重比

造波抵抗の抗重比 C_w

$$\varepsilon_w \leftarrow C_w = \frac{R_w}{\underset{\nwarrow \text{Lift}}{L}} = \frac{\pi\rho U^2}{\frac{1}{2}\rho U^2 p_0 L B}\int_0^{\pi/2}|H(\theta)|^2\sec^5\theta\, d\theta$$

$$= \frac{2\pi}{p_0 L B}\int_0^{\pi/2}|H(\theta)|^2\sec^5\theta\, d\theta$$

$$= \frac{2\pi}{p_0 L^2\cdot\lambda}\int_0^{\pi/2}|H(\theta)|^2\sec^5\theta\, d\theta$$

where λ はアスペクト比で

$\lambda = B/L$

造波抵抗を解析的に求める方法

圧力分布.（$\frac{1}{2}\rho U^2 p_0$ で無次元化してある）

$$p(x,y) = P_X(x) \cdot P_Y(y)$$

X−1)	$P_X = 1 + 2x$	三角形分布
X−2)	$P_X = \frac{4}{\pi}\sqrt{1-4x^2}$	楕円 〃
X−3)	$P_X = \frac{2}{\pi}\frac{1+2x}{\sqrt{1-4x^2}}$	S-line 〃
Y−1)	$P_Y = \frac{2}{\pi}\frac{1}{\sqrt{1-(2y/B)^2}}$	逆楕円 〃
Y−2)	$P_Y = \frac{4}{\pi}\sqrt{1-(2y/B)^2}$	楕円 〃
Y−3)	$P_Y = 1$	一定 〃

$$\int_{-0.5}^{0.5} P_X\,dx = 1$$

$$\int_{-B/2}^{B/2} P_Y\,dy = B$$

振巾関数

$$H(\theta) = \frac{ik}{\pi} \iint P(x, y)\, e^{ik\sec^2\theta(x\cos\theta + y\sin\theta)}\, dxdy$$

$$= \frac{ik}{\pi} \int_{-0.5}^{0.5} P_X(x)\, e^{ikx\sec^2\theta}\, dx \int_{-B/2}^{B/2} P_Y(y)\, e^{iky\sin\theta\sec^2\theta}\, dy$$

$$= \frac{ik}{\pi}\quad H_X(\theta) \cdot H_Y(\theta)$$

X−0) 一様分布

$$P_x = 1$$

$$H_x = \int_{-0.5}^{0.5} e^{i\kappa x \sec\theta}\, d\cancel{\theta}x$$

$$\cancel{= \frac{1}{2}\int_0^{\pi} e^{i\frac{\kappa}{2}\sec\theta\cdot\cos t}\sin t\, dt}$$

$$= \frac{\cos\theta}{i\kappa}\left[e^{i\kappa x \sec\theta}\right]_{-0.5}^{0.5}$$

$$\quad\quad\quad\quad \overset{\shortparallel}{2i\sin\left(\frac{\kappa}{2}\sec\theta\right)}$$

$$= 2\frac{\cos\theta}{\kappa}\sin\left(\frac{\kappa}{2}\sec\theta\right)$$

X−1) 三角形分布

$$P_x = 1 + 2x$$

$$H_x(\theta) = \int_{-0.5}^{0.5} (1+2x)\, e^{i\kappa x \sec\theta}\, dx$$

~~$x = 1/2 \cos t \qquad t = \pi \sim 0$~~
~~$dx = -1/2 \sin t\, dt$~~

$$= \frac{1}{2}\int_0^{\pi} (1+\cos t)\, e^{i\frac{\kappa}{2}\sec\theta \cos t} \sin t\, dt$$

~~$= \frac{1}{2}\int_0^{\pi} \sin t(1+\cos\theta)\{\cos(\frac{\kappa}{2}\sec\theta \cos t) + i\sin(\frac{\kappa}{2}\sec\theta\cos t)\}dt$~~
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~~$= \int_0^{\pi/2} \cos(\frac{\kappa}{2}\sec\theta \cos\theta)\sin t\, dt + i\int_0^{\pi/2} \sin(\frac{\kappa}{2}\sec\theta\cos\theta)\sin t \cos t\, dt$~~

$$= \frac{1}{i\kappa\sec\theta}\Big[(1+2x)\underbrace{\{e^{i\kappa x\sec\theta}}_{=2e^{i\kappa/2\sec\theta}}\Big]$$

$$- \frac{2}{(i\kappa\sec\theta)^2}\Big[e^{i\kappa x\sec\theta}\Big] = 2i\sin(\kappa/2\sec\theta)$$

$$= -2\frac{i\cos\theta}{\kappa}\Big[\cos(\frac{\kappa}{2}\sec\theta) + i\sin(\frac{\kappa}{2}\sec\theta)\Big]$$

$$+ i\frac{4\cos^2\theta}{\kappa^2}\sin(\frac{\kappa}{2}\sec\theta)$$

$$= \frac{2}{\kappa}\sin(\frac{\kappa}{2}\sec\theta)\cdot\cos\theta + i\Big[-\frac{2}{\kappa}\cos(\frac{\kappa}{2}\sec\theta)\cos\theta$$

$$+ \frac{4}{\kappa^2}\sin(\frac{\kappa}{2}\sec\theta)\cos^2\theta\Big]$$

$$2/\kappa \cos\theta\{C_x + 2/\kappa\, S_x \cos\theta\}$$

X-2) 楕円分布.

$$P_x = \frac{4}{\pi}\sqrt{1-4x^2}$$

$$H_x = \frac{4}{\pi}\int_{-0.5}^{0.5}\sqrt{1-4x^2}\, e^{ikx\sec\theta}\, dx$$

$$x = \tfrac{1}{2}\cos t$$

$$= \frac{2}{\pi}\int_0^{\pi}\sin^2 t\; e^{i\frac{k}{2}\sec\theta\cdot\cos t}\, dt$$

$$= \frac{1}{\pi}\int_0^{\pi}(1-\cos 2t)\, e^{i\frac{k}{2}\sec\theta\cdot\cos t}\, dt$$

$$= J_0\left(\frac{k}{2}\sec\theta\right) + J_2\left(\frac{k}{2}\sec\theta\right)$$

X−3) S-Line 分布のとき

$$P_x = \frac{2}{\pi}\frac{1+2x}{\sqrt{1-4x^2}}$$

$$H_x(\theta) = \int_{-0.5}^{0.5} \frac{2}{\pi}\frac{1+2x}{\sqrt{1-4x^2}}\, e^{ikx\sec\theta}\, dx$$

$x = \frac{1}{2}\cos t \qquad t = \pi \sim 0$

$dx = -\frac{1}{2}\sin t\, dt$

$$\frac{1+2x}{\sqrt{1-4x^2}} = \frac{1+\cos t}{\sin t}$$

$$= \frac{1}{2}\cdot\frac{2}{\pi}\int_0^{\pi} e^{i\frac{1}{2}k\sec\theta\cdot\cos t}(1+\cos t)\, dt$$

$$= J_0\left(\frac{1}{2}k\sec\theta\right) + i J_1\left(\frac{1}{2}k\sec\theta\right)$$

$\theta \to \pi/2$

$\sqrt{\cos\theta}$

$z \to 0$

$J_0(z) \to 1$,

$J_1(z) \to z/2$

$z \to \infty$

$$J_0(z) \sim \sqrt{\frac{2}{\pi z}}\cos\left(z - \frac{\pi}{4}\right)$$

$$J_1(z) \sim \sqrt{\frac{2}{\pi z}}\cos\left(z - \frac{3}{4}\pi\right)$$

γ-1) 連続楕円分布

$$P_Y = \frac{2}{\pi}\frac{1}{\sqrt{1-(2y/B)^2}}$$

$$H_Y(\theta) = \int_{-B/2}^{B/2} P_Y(y)\, e^{i\kappa y \sin\theta \sec^2\theta}\, dy$$

$$y = \frac{B}{2}\cos t \qquad t = \pi \sim 0$$

$$dy = -\frac{B}{2}\sin t\, dt$$

$$\sqrt{1-(2y/B)^2} = \sin t$$

$$= \frac{B}{2}\cdot\frac{2}{\pi}\int_0^{\pi} e^{i\kappa \sin\theta \sec^2\theta \frac{B}{2}\cos t}\, dt$$

$$= B\cdot J_0\left(\frac{\kappa B}{2}\sin\theta\sec^2\theta\right)$$

$$\longrightarrow \quad \cos\theta$$

Y(-2) 楕円分布

$$P_Y = \frac{4}{\pi}\sqrt{1-(2y/B)^2}$$

$$H_Y(\theta) = \frac{4}{\pi}\cdot\frac{B}{2}\int_0^{\pi} e^{ik\sin\theta\sec^2\theta\frac{B}{2}\cos t}\sin^2\theta\, d\theta$$

$$= 4\frac{\cos^2\theta}{k\sin\theta}\, J_1\left(\frac{Bk}{2}\sin\theta\sec^2\theta\right)$$

$\theta \to 0$ で

$$H_Y(\theta) \to \cancel{4}\,\frac{1}{\cancel{k\sin\theta}} \times \frac{1}{\cancel{2}}\,\frac{B\cancel{k}}{\cancel{2}}\cancel{\sin\theta}$$

$$= B/$$

r-3) 一定分布

$$P_r = 1$$

$$H_r = \int_{-B/2}^{B/2} e^{i\kappa y \sin\theta \sec^2\theta}\, dy$$

$$= \frac{2\cos^2\theta}{\cancel{2}\kappa \sin\theta} \sin\cancel{\tan}\left(\frac{\kappa B}{2}\sin\theta\sec^2\theta\right)$$

$\theta \to 0$ で

$$\to \frac{2\cos^2\theta}{\cancel{\kappa}\cancel{\sin\theta}} \frac{\cancel{\kappa}B}{2}\cancel{\sin\theta} \to B$$

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三角形滑走面

$$P(x,y) = \frac{8}{\pi^2}\sqrt{\frac{1+2\xi}{1-2\xi}}\sqrt{\frac{1+\eta}{1-\eta}}$$

$$\xi = \frac{2x+\eta}{2(1-\eta)} \Rightarrow x = \xi(1-\eta) - \frac{\eta}{2}$$

$$\eta = {}^{2y}/_{B}$$

$$H(\theta) = \frac{ik}{\pi}\iint p(x,y)\, e^{ik\sec^2\theta(x\cos\theta + y\sin\theta)}\, dxdy$$

$$\frac{\partial x}{\partial \xi} = (1-\eta) \qquad \frac{\partial x}{\partial \eta} = -\xi - \frac{1}{2}$$

$$\frac{\partial y}{\partial \xi} = 0 \qquad \frac{\partial y}{\partial \eta} = \frac{B}{2}$$

$$dxdy = \frac{B}{2}(1-\eta)\, d\xi\, d\eta$$

$$x\cos\theta + y\sin\theta = \left(\xi(1-\eta) - \frac{\eta}{2}\right)\cos\theta + \frac{B}{2}\eta\sin\theta$$

$$\frac{2}{\pi}\int_{-1/2}^{1/2}\sqrt{\frac{1+2\xi}{1-2\xi}}\; e^{ik\sec\theta(1-\eta)\xi}\, d\xi$$

$$= \frac{2}{\pi}\int \frac{1+2\xi}{\sqrt{1-4\xi^2}}\; e^{ik\sec\theta(1-\eta)\xi}\, d\xi$$

$$\xi = \frac{1}{2}\cos t \qquad d\xi = -\frac{1}{2}\sin t\, dt$$

$$= \frac{1}{\pi}\int_0^{\pi} (1+\cos t)\; e^{ik\sec\theta(1-\eta)/2\cdot\cos t}\, dt$$

$$= J_0\left(\frac{k}{2}(1-\eta)\sec\theta\right) + i J_1\left(\frac{k}{2}(1-\eta)\sec\theta\right)$$

$$H(\theta) = \frac{ik}{\pi}\cdot\frac{4}{\pi}\int_{-1}^{1}\left\{J_0\left(\frac{k}{2}(1-\eta)\sec\theta\right) + iJ_1\left(\frac{k}{2}(1-\eta)\sec\theta\right)\right\}$$

$$\times\sqrt{\frac{1+\eta}{1-\eta}}\cdot\frac{B}{2}(1-\eta)\; e^{ik\sec^2\theta\{B\sin\theta-\cos\theta\}\eta}\, d\eta$$