楕円積分による揚力ポテンシャルの表示

$$\phi(x,y,0) = \int_{-1}^{1} \mu(\xi) \frac{\partial}{\partial \xi} \frac{1}{R} d\xi \quad , \quad (1)$$

$$\mu(\xi) = \frac{\alpha}{\sin\theta} F(\theta), \quad \xi = -\cos\theta, \quad (2)$$

$$F(\theta) = \sum_{n=0}^{\infty} a_n \cos n\theta \quad , \quad (3)$$

$$\phi(x,y,0) = \alpha \int_0^{\pi} f(\theta) \frac{\partial}{\partial \xi} \frac{1}{R} d\theta = -\alpha \frac{\partial}{\partial x} \int_0^{\pi} f(\theta) \frac{d\theta}{R}, \quad (4)$$

$$\left(\because \frac{\partial}{\partial \xi}\frac{1}{R} = -\frac{\partial}{\partial x}\frac{1}{R}\right)$$

今

$$I(\theta; x,y) = \int_0^{\theta} \frac{d\theta}{R} = \int_0^{\theta} \frac{d\theta}{\sqrt{(x+\cos\theta)^2 + y^2}} \quad , \quad (5)$$

とおくと

$$\phi(x,y,0) = -\alpha \frac{\partial}{\partial x} \left[f(\theta) I(\theta) \Big|_{\theta=0}^{\pi} - \int_0^{\pi} f'(\theta) I(\theta) d\theta \right]$$

$$= -\alpha \frac{\partial}{\partial x} \left[f(\pi) I(\pi; x,y) - \sum_{\theta} f'\left(\theta + \frac{\Delta\theta}{2}\right) \Big[I(\theta) \Big]_{\theta}^{\theta+\Delta\theta} \right], \quad (6)$$

$$f'(\theta) = \sum_{n=1}^{\infty} n a_n \sin n\theta \quad , \quad (7).$$

$$\therefore = -\alpha \frac{\partial}{\partial x} \left[\sum f\left(\theta + \frac{\Delta\theta}{2}\right) \Big[I \Big]_{\theta}^{\theta+\Delta\theta} \right] \quad (6') \text{ 追加}$$

まず $I(\pi; x, 0)$ について積分しよう。

$$I(\pi; x, 0) = \int_0^{\pi} \frac{d\theta}{x + \cos\theta} \quad , \quad \text{---} \quad (8)$$

$x = \cosh t > 1$ とおくと $(\sinh t = \sqrt{x^2 - 1})$

$$\frac{\sinh t}{\cosh t + \cos\theta} = 1 + 2\sum_{n=1}^{\infty} (-1)^n e^{-nt} \cos n\theta \quad , \quad (9)$$

なる展開があるから

$$I(\pi; x, 0) = \frac{\pi}{\sqrt{x^2 - 1}} \quad , \quad (10)$$

$$\left.\begin{aligned} \frac{\partial I}{\partial x} &= -\frac{\pi x}{(x^2-1)^{\frac{3}{2}}} , \\ \frac{\partial^2 I}{\partial x^2} &= -\pi\left[\frac{1}{(x^2-1)^{\frac{3}{2}}} - \frac{3x^2}{(x^2-1)^{\frac{5}{2}}}\right] = \frac{\pi(2x^2+1)}{(x^2-1)^{\frac{5}{2}}} , \end{aligned}\right\} (11)$$

特に $\underline{MO}$ では

$$\mu(\theta) = \frac{\alpha}{\sin\theta} \quad , \quad (12)$$

だから

$$\phi(x, y, 0) = -\alpha \frac{\partial}{\partial x} I(\pi; x, y) \quad , \quad (13)$$

$$\phi(x, 0, 0) = +\frac{\alpha\pi x}{(x^2-1)^{\frac{3}{2}}} \quad , \quad (14)$$

$$\frac{\partial\phi}{\partial x} = -\frac{\pi\alpha(2x^2+1)}{(x^2-1)^{\frac{5}{2}}} \quad , \quad (15)$$

さて一般に

$$I(\theta; x, y) = \int_{-1}^{\xi} \frac{d\xi}{\sqrt{1-\xi^2}\left[(x-\xi)^2+y^2\right]} \qquad (15)$$

公式によると

$$Y(\xi) = (\xi^2-1)\left[(x-\xi)^2+y^2\right], \qquad (16)$$

$$\left.\begin{aligned} & a^2=(x-1)^2+y^2, \quad b^2=(x+1)^2+y^2, \\ & k^2 = \frac{4-(a-b)^2}{4ab}, \quad \xi = \frac{b-a-(b+a)\sqrt{1-u^2}}{b+a-(b-a)\sqrt{1-u^2}} \end{aligned}\right\} \qquad (17)$$

$$\sqrt{1-u^2} = \frac{(b+a)\xi-(b-a)}{(b-a)\xi-(b+a)},$$

とおくと

$$I = \int_{-1}^{\xi} \frac{d\xi}{\sqrt{-Y(\xi)}} = \frac{1}{\sqrt{ab}} \int_0^u \frac{du}{\sqrt{(1-u^2)(1-k^2u^2)}}, \qquad (18)$$

(17) より

$$\left.\begin{aligned} u^2 &= \frac{4ab(1-\xi^2)}{\left[a+b-(a-b)\xi\right]^2} \\ u &= \frac{2\sqrt{ab}\sqrt{1-\xi^2}}{a+b-(a-b)\xi}, \end{aligned}\right\} \qquad (19)$$

(18)は楕円積分により次のように表わされる。

$$\left.\begin{aligned} I &= \frac{1}{\sqrt{ab}} F(k, \varphi), \\ \sin\varphi &= \frac{2\sqrt{ab}\sqrt{1-\xi^2}}{a+b-(a-b)\xi}, \end{aligned}\right\} \qquad (20)$$

まず $q=0$ ならば $k=0$ でまた $\varphi=\pi$ とおくと

確かに上式は(10)に一致する。

$$I(\theta;x,0)=\frac{1}{\sqrt{x^2-1}}\sin^{-1}\left[\frac{\sqrt{x^2-1}\sin\theta}{x+\cos\theta}\right],\qquad(21)$$

[$\sin^{-1}$ は $[0,\pi]$ の区間で 2つ値があるので注意, θ が [illegible] ?]

今 $\sin\varphi=\dfrac{\sqrt{x^2-1}\sin\theta}{x+\cos\theta}$ とおくと $\cos\varphi=\dfrac{1+x\cos\theta}{x+\cos\theta}$,

$$-\sin\varphi\frac{d\varphi}{dx}=\frac{\cos\theta}{x+\cos\theta}-\frac{1+x\cos\theta}{(x+\cos\theta)^2}=\frac{-\sin^2\theta}{(x+\cos\theta)^2}$$

$$\therefore\ \frac{d\varphi}{dx}=\frac{\sin\theta}{\sqrt{x^2-1}\,(x+\cos\theta)},\qquad(22)$$

$$\therefore\ \frac{\partial}{\partial x}I=-\frac{x}{(x^2-1)^{\frac{3}{2}}}(\varphi)+\frac{\sin\theta}{(x^2-1)(x+\cos\theta)},\qquad(23)$$

$$\frac{\partial^2}{\partial x^2}I=\frac{(2x^2+1)}{(x^2-1)^{\frac{5}{2}}}\varphi-\frac{x\sin\theta}{(x^2-1)^2(x+\cos\theta)}-\frac{\sin\theta\left[2x(x+\cos\theta)+x^2-1\right]}{(x^2-1)^2(x+\cos\theta)^2},$$

(24)

― 一般の場合 (17), (20)

$$2a\frac{\partial a}{\partial x} = 2(x-1), \quad b\frac{\partial b}{\partial x} = x+1,$$
$$a\frac{\partial a}{\partial y} = y, \quad b\frac{\partial b}{\partial y} = y$$

$$2\log k = \log[4-(a-b)^2] - \log(4ab)$$

$$\frac{2}{k}\frac{\partial k}{\partial x} = \frac{-2(a-b)(\cancel{a_x}-b_x)}{4-(a-b)^2} - \frac{ba_x - ab_x}{ab}$$
$$= \frac{2(b-a)\{b(x-1)-a(x+1)\}}{[4-(a-b)^2]ab} - \frac{b^2(x-1)-a^2(x+1)}{a^2b^2}$$
$$= \frac{2(b-a)\{(b-a)x-a-b\}}{[4-(a-b)^2]ab} - \frac{2x^2-2-2y^2}{a^2b^2}$$
$$= \frac{-2x}{ab} - \frac{2(x^2+y^2-1)}{a^2b^2}$$

$$\frac{\partial F(k,\varphi)}{\partial k} = \frac{1}{k'^2}\left(\frac{E-k'^2F}{k} - \frac{\sin\varphi\cos\varphi}{\sqrt{1-k^2\sin^2\varphi}}\right)$$

$$\frac{\partial E}{\partial k} = \frac{E-F}{k}$$

$$E(k,\varphi) = \int^{\varphi}\sqrt{1-k^2\sin^2\varphi}\,d\varphi,$$

$$k'^2 = 1-k^2 = \frac{(b+a)^2-4}{4ab},$$

$$\frac{\partial}{\partial x}\frac{1}{\sqrt{ab}} = -\frac{(a b_x + b a_x)}{2(ab)^{\frac{3}{2}}} = -\frac{(a^2+b^2)x + a^2 b^2}{(ab)^{\frac{5}{2}}} = -\frac{x\left[x^2+y^2-1\right]}{(ab)^{\frac{5}{2}}}$$

$$a b_x + b a_x = \frac{1}{ab} x$$

$$k^2 = \frac{4-(a-b)^2}{4ab} = \frac{4-a^2-b^2+2ab}{4ab} = \frac{1}{2} + \frac{1-x^2-y^2}{2ab},$$

$$2k\frac{\partial k}{\partial x} = \frac{-x}{ab} - \frac{2x(1-x^2-y^2)(x^2+y^2-1)}{2(ab)^3} = -\frac{x}{ab}\left[1 - \frac{(x^2+y^2-1)^2}{(ab)^2}\right]$$

$$= -\frac{4xy^2}{(ab)^3},$$

$(x^2-1)^2 + y^4 + 2y^2(x^2+1)$

$$\frac{\partial k}{\partial x} = -\frac{2xy^2}{k(ab)^3}, \quad //$$

$$\frac{\partial}{\partial y}\frac{1}{\sqrt{ab}} = -\frac{a b_y + b a_y}{2(ab)^{\frac{3}{2}}} = -\frac{y(1+x^2+y^2)}{(ab)^{\frac{5}{2}}},$$

$$2k\frac{\partial k}{\partial y} = -\frac{y}{ab} - \frac{y(1-x^2-y^2)}{(ab)^3}(1+x^2+y^2) = -\frac{y\left[(ab)^2 + (1-(x^2+y^2)^2)\right]}{(ab)^3}$$

$$= -\frac{2y}{(ab)^3}(1+y^2-x^2)$$

$$\frac{\partial k}{\partial y} = \frac{y(x^2-1-y^2)}{k(ab)^3} \quad //$$

$$k'^2 = 1-k^2, \quad 2k'\frac{\partial k'}{\partial x} = -2k\frac{\partial k}{\partial x}$$

$$\therefore \frac{\partial k'}{\partial x} = -\frac{k}{k'}\frac{\partial k}{\partial x}, \quad \frac{\partial k'}{\partial y} = -\frac{k}{k'}\frac{\partial k}{\partial y},$$

$$I(\pi;\, x, y) = \frac{1}{\sqrt{ab}} F(k, \pi) = \frac{2}{\sqrt{ab}} F(k) \quad K(k)$$

$$\frac{\partial}{\partial x} I = 2K \frac{\partial}{\partial x}\frac{1}{\sqrt{ab}} + \frac{2}{\sqrt{ab}} \frac{\partial K}{\partial k}\frac{\partial k}{\partial x}$$

$$\frac{\partial^2}{\partial x^2} I = 2\left(\frac{\partial^2}{\partial x^2}\frac{1}{\sqrt{ab}}\right) K + 4 \frac{\partial K}{\partial x}\frac{\partial k}{\partial x}\left(\frac{\partial}{\partial x}\frac{1}{\sqrt{ab}}\right)$$
$$+ \frac{2}{\sqrt{ab}}\left\{\frac{\partial^2 k}{\partial x^2}\frac{\partial K}{\partial k} + \left(\frac{\partial k}{\partial x}\right)^2 \frac{\partial^2 K}{\partial k^2}\right\}$$

$$\frac{\partial^2}{\partial x \partial y} I = 2 \frac{\partial^2}{\partial x \partial y}\frac{1}{\sqrt{ab}} + 2 \frac{\partial K}{\partial y}\frac{\partial}{\partial x}\frac{1}{\sqrt{ab}} + 2 \frac{\partial K}{\partial k}\frac{\partial}{\partial y}\frac{1}{\sqrt{ab}}$$
$$+ 2 \frac{2}{\sqrt{ab}} \frac{\partial^2 K}{\partial x \partial y}$$

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$$\frac{\partial K}{\partial k} = \frac{E}{k'^2} - \frac{K}{k},$$

$$\frac{\partial E}{\partial k} = \frac{E-K}{k},$$

$$\frac{\partial^2 K}{\partial k^2} = \frac{E_k}{1-k^2} + \frac{2kE}{(1-k^2)^2} - \frac{K_k}{k} + \frac{K}{k^2}$$

$$= \frac{E-K}{k k'^2} + \frac{2kE}{k'^4} - \frac{1}{k}\left(\frac{E}{k'^2} - \frac{K}{k}\right) + \frac{K}{k^2}$$

$$= \frac{2kE}{k'^4} + \left(\frac{2}{k^2} - \frac{1}{k k'^2}\right)K,$$

$$\frac{\partial}{\partial x}K = \frac{\partial k}{\partial x}\frac{\partial K}{\partial k}, \quad \frac{\partial}{\partial y}K = \frac{\partial k}{\partial y}\frac{\partial K}{\partial k}$$

$$\frac{\partial^2}{\partial x^2}K = \frac{\partial^2 k}{\partial x^2}\frac{\partial K}{\partial k} + \left(\frac{\partial k}{\partial x}\right)^2 \frac{\partial^2 K}{\partial k^2}, \quad \frac{\partial^2 K}{\partial x \partial y} = \frac{\partial^2 k}{\partial x \partial y}\frac{\partial K}{\partial k} + \frac{\partial k}{\partial x}\frac{\partial k}{\partial y}\frac{\partial^2 K}{\partial k^2}$$

$$\frac{\partial^2 K}{\partial x^2} = k_{xx}\left(\frac{E}{k'^2} - \frac{K}{k}\right) + \left(\frac{\partial k}{\partial x}\right)^2 \left\{\frac{2kE}{k'^4} - \left(\frac{2}{k^2} - \frac{1}{k k'^2}\right)K\right\}$$

$$= \left(\frac{k_{xx}}{k'^2} + \frac{2k}{k'^4}k_x^2\right)E - \left\{\frac{k_{xx}}{k} + \left(\frac{2}{k^2} - \frac{1}{k k'^2}\right)k_x^2\right\}K$$

$$a^2 = (x-1)^2 + y^2, \quad b^2 = (x+1)^2 + y^2$$

$$k^2 = \frac{4-(a-b)^2}{4ab}, \quad \zeta = \frac{B(u)}{A(u)} = \frac{b-a-(b+a)\sqrt{1-u^2}}{b+a-(b-a)\sqrt{1-u^2}}, \quad \frac{|b-a|}{(b+a)}$$

$$\sqrt{1-u^2} = \frac{D(\zeta)}{C(\zeta)} = \frac{(b+a)\zeta-(b-a)}{(b-a)\zeta-(b+a)},$$

$$1-u^2 = \frac{D^2}{C^2}, \quad u^2 = 1-\frac{D^2}{C^2} = \frac{C^2-D^2}{C^2} = \frac{(b-a)^2\zeta}{}$$

$$= \frac{\underbrace{(b-a)^2\zeta^2+(b+a)^2}_{-2(b^2-a^2)\zeta} - \{(b+a)^2\zeta^2+(b-a)^2-2(b^2-a^2)\zeta\}}{C^2}$$

$$u^2 = \frac{(1-\zeta^2)\{(b+a)^2-(b-a)^2\}}{C^2 \qquad 4ab} = \frac{4ab(1-\zeta^2)}{C^2}.$$

$$u = \frac{2\sqrt{ab}\sqrt{1-\zeta^2}}{C(\zeta)} \quad (\text{[illegible]}), \quad 2u\frac{du}{d\zeta} = \frac{-8ab\zeta}{C} - \text{[illegible]}$$

$$\frac{d\zeta}{du} = \frac{+(b+a)\frac{u}{\sqrt{1-u^2}}}{A(u)} - \frac{B\left((b-a)\frac{u}{\sqrt{1-u^2}}\right)}{A^2} = \frac{\frac{u}{\sqrt{1-u^2}}}{A^2}\{(b+a)A-(b-a)B\}$$

$$= \frac{u}{\sqrt{1-u^2}A^2}\left[(b+a)^2-(b^2-a^2)\sqrt{1-u^2}-\{(b-a)^2-(b^2-a^2)\sqrt{1-u^2}\}\right]$$

$$= \frac{4abu}{\sqrt{1-u^2}A^2(u)}$$

$$-\cancel{4ab}(1-\zeta^2)\{4-(a-b)^2\}$$

$$1-k^2u^2 = 1-\frac{4ab(1-\zeta^2)}{C^2}k^2 = \frac{(b-a)^2\zeta^2+(b+a)^2-2(b^2-a^2)\zeta}{C^2}$$

$$= \frac{\overset{2(b^2+a^2)}{(b+a)^2+(b-a)^2} - 2\overset{4x}{(b^2-a^2)}\zeta - 4(1-\zeta^2)}{C^2} = \overset{-2}{4}(1+x^2+y^2) - 8x\zeta - 4(1-\zeta^2)$$

$$= \frac{4(x^2+y^2)-8x\zeta+4\zeta^2 \cancel{2}}{\cancel{4}\ C^2} = \frac{4\{(x-\zeta)^2+y^2\}}{C^2} = \frac{A^2R^2}{\cancel{4}a^2b^2}$$

$$C(\zeta) = \frac{(b-a)B}{A} - (b+a) = \frac{(b-a)^2-(b^2-a^2)\sqrt{1-u^2}-\{(b+a)^2-(b^2-a^2)\sqrt{1-u^2}\}}{A}$$

$$= -4ab$$

$1-k^2u^2 = +\frac{R^2A^2(u)}{4a^2k^2}$, $\frac{d\xi}{du} = \frac{4abu}{\sqrt{1-u^2}\,A^2}$, $\boxed{\frac{\frac{d\xi}{du}}{\sqrt{1-\xi^2}} = \frac{2\sqrt{ab}}{A\sqrt{1-u^2}}}$

$\sqrt{1-k^2u^2} = \frac{RA(u)}{2ak}$, $1-\xi^2 = \frac{4abu^2}{A^2}$, $\sqrt{1-\xi^2} = \frac{2\sqrt{ab}\,u}{A(u)}$

$$\frac{\frac{d\xi}{du}}{R\sqrt{1-\xi^2}} = \frac{A^2}{2ak\sqrt{1-k^2u^2}\;2\sqrt{ab}\,u}\cdot\frac{4abu}{\sqrt{1-u^2}\,A^2} = \frac{1}{\sqrt{1-k^2u^2}\sqrt{1-u^2}\sqrt{ab}}$$

$(1+x^2+y^2)^2 - 16x^2 = 4(1+x^4+y^4+2x^2+2y^2+2x^2y^2) - 16x^2$

$= 4(1+x^4+y^4-2x^2+2y^2+2x^2y^2)$

$= 4a^2b^2$

$a^2b^2 = (x^2-1)^2 + y^4 + \underbrace{y^2[(x+1)^2+(x-1)^2]}_{2y^2(x^2+1)}$

$= x^4+1+y^4-2x^2+2y^2(x^2+1)$

$(x^2+y^2)(\cancel{b+a})^2 + (b+a)^2 - 8x^2$

$= (x^2+y^2+1)(b^2+a^2-2ab) + 4ab - 8x^2$

$= 2(x^2+y^2+1)(x^2+y^2+1) - 8x^2 + 2ab(2-x^2-y^2-1)$

$= 2[x^4+y^4+1+2x^2+2y^2+2x^2y^2]$

$= 2[x^4+y^4+1-2x^2+2y^2+2x^2y^2] + 2ab(1-x^2-y^2)$

$= 2a^2b^2 + 2ab(1-x^2-y^2) = 2a^2b^2\left[\cancel{ab}\,1 - \frac{1-x^2-y^2}{ab}\right]$

$= 2a^2b^2[ab-1+x^2+y^2]$, $(a-b)^2 = a^2+b^2$

A-1 回転楕円体の速度ポテンシャルの積分

$$\phi(x,y,0) = \int_{-1}^{1} \mu(\xi) \frac{\partial}{\partial \xi} \frac{1}{R}\, d\xi, \qquad (1)$$

$$\mu(\xi) = \alpha(1-\xi^2), \qquad (2)$$

$$\frac{\partial}{\partial \xi} \log(\xi - x + R) = \frac{1}{R}, \qquad (3)$$

$$\frac{\partial}{\partial \xi}\left[(\xi - x)\log(\xi - x + R) - R\right] = \log(\xi - x + R), \qquad (4)$$

$$\phi(x,y,0) = \alpha\int_{-1}^{1}(1-\xi^2)\frac{\partial}{\partial \xi}\frac{1}{R}\, d\xi = +2\alpha\int_{-1}^{1}\xi\frac{d\xi}{R}$$

$$= 2\alpha\left[\xi \log(\xi - x + R)\right]_{\xi=-1}^{1} - 2\alpha\int_{-1}^{1}\log(\xi - x + R)\, d\xi$$

$$= 2\alpha\left[\xi\log(\xi - x + R) - (\xi - x)\log(\xi - x + R) + R\right]_{\xi=-1}^{1}$$

$$= 2\alpha\left[x\log(\xi - x + R) + R\right]_{\xi=-1}^{1}, \quad \cdots (5)$$

$$\frac{\partial\phi}{\partial x} = 2\alpha\left[\log(\xi - x + R) - \frac{x}{R} + \frac{x-\xi}{R}\right]$$

$$= 2\alpha\left[\log(\xi - x + R) - \frac{\xi}{R}\right]_{\xi=-1}^{1}, \qquad (6)$$

$$\frac{\partial\phi}{\partial y} = 2\alpha\left[\frac{xy}{R(\xi - x + R)} + \frac{y}{R}\right]_{\xi=-1}^{1} = 2\alpha\left[\frac{y}{R}\left\{\frac{\xi + R}{\xi - x + R}\right\}\right] \qquad (7)$$

$$\frac{\partial\phi}{\partial x} = 2\alpha\left[\log\left(\frac{y^2}{R + x - \xi}\right) - \frac{\xi}{R}\right]_{\xi=-1}^{1}, \qquad (6')$$

$\log(\xi - x + R) = \log($ ...

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$$\frac{\partial\phi}{\partial x}(x,0,0) = 2\alpha\left[\log\left(\frac{x+1}{x-1}\right) - \frac{1}{x-1} + \frac{1}{x+1}\right], \quad x>1$$

$$= 2\alpha\left[\log\left(\frac{x+1}{x-1}\right) - \frac{2}{x^2-1}\right], \quad (8),$$

$x \to 1$, $x = 1+\varepsilon$

$$-\frac{\partial\phi}{\partial x} \doteqdot 2\alpha\left(-\log\frac{2}{\varepsilon} + \frac{1}{\varepsilon}\right)$$

流れ関数は

$$\psi(x,y) = 2\alpha\int_{-1}^{1}\frac{(x-\xi)\xi}{R}d\xi,$$

$$= -2\alpha\,\xi R\Big|_{-1}^{1} + 2\alpha\int_{-1}^{1}R\,d\xi = +2\alpha\left\{\sqrt{(x+1)^2+y^2} - \sqrt{(1-x)^2+y^2}\right\} + 2\alpha\int R\,d\xi$$

$$= \alpha\left[y^2\log(\xi-x+R) - (x+\xi)R\right]_{\xi=-1}^{\xi=1}, \quad (9)$$

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速度ポテンシャル

$$\phi = \alpha \int_{-1}^{1} \frac{\partial}{\partial \xi} \frac{1}{R}\, d\xi = \alpha \left[\frac{1}{R} \right]_{-1}^{1} , \qquad (1)$$

流れの関数

$$\psi = \alpha \left[\frac{\xi - x}{R} \right]_{-1}^{1} , \qquad (2)$$

故 $\alpha \left[\dfrac{1-x}{\sqrt{(1-x)^2+y^2}} + \dfrac{1+x}{\sqrt{(1+x)^2+y^2}} \right] = \dfrac{\delta^2}{2} \qquad (3)$

C. Cocoonoid

$$\phi(x,y,0)=-\alpha\frac{\partial}{\partial x}\int_{-1}^{1}\frac{d\xi}{R\sqrt{1-\xi^2}}, \qquad (1)$$

$$\psi(x,\cancel{y,0})=\alpha y\frac{\partial}{\partial y}\int_{-1}^{1}\frac{d\xi}{R\sqrt{1-\xi^2}}, \qquad (2)$$

$$I(x,y)=\int_{-1}^{1}\frac{d\xi}{R\sqrt{1-\xi^2}}, \qquad (3)$$

$$a^2=(x-1)^2+y^2,\quad b^2=(x+1)^2+y^2$$

$$k^2=\frac{4-(a-b)^2}{4ab},\quad \xi=\frac{B(u)}{A(u)}=\frac{b-a-(b+a)\sqrt{1-u^2}}{b+a-(b-a)\sqrt{1-u^2}},$$

$$k'^2=1-k^2=\frac{(a+b)^2-4}{4ab}$$

$$1-\xi^2=\frac{A^2-B^2}{A^2}=\frac{-(b-a)^2+(b+a)^2-(1-u^2)\{(b+a)^2-(b-a)^2\}}{A^2}=\frac{+4abu^2}{A^2}$$

$$R^2=(x-\xi)^2+y^2=x^2+y^2+\xi^2-2x\xi=\frac{(x^2+y^2)A^2+B^2-2xAB}{A^2}$$

$$A^2R^2=(x^2+y^2)\{(b+a)^2+(b-a)^2(1-u^2)-2(b^2-a^2)\sqrt{1-u^2}\}+(b-a)^2+(b+a)^2(1-u^2)$$
$$-2(b^2-a^2)\sqrt{1-u^2}\quad-2x\Big[\underbrace{b^2-a^2}_{4x}+(b^2-a^2)(1-u^2)-\{\underbrace{(b+a)^2+(b-a)^2}_{2(b^2+a^2)}\}\sqrt{1-u^2}\Big]$$

$$=(x^2+y^2)\{2(b^2+a^2)\}+2(b^2+a^2)-16x^2$$

$$+u^2\{-(x^2+y^2)(b-a)^2-(b+a)^2+8x^2\}$$

$$(1+x^2+y^2)(x^2+y^2-1)$$

$$(a^2+b^2)^2-(b^2-a^2)$$

$$\sqrt{1-u^2}\Big[-8x(x^2+y^2)-\cancel{(b-a)^2}\,8x+8x(1+x^2+y^2)\Big]$$

$$8x(ab-1-x^2-y^2)$$

$$=4a^2b^2(1-k^2u^2)$$

$$\frac{1}{R}=\frac{A}{2ab\sqrt{1-k^2u^2}}$$

$$\frac{d\xi}{du} = \frac{AB'(u) - BA'}{A^2} = \frac{u}{A^2(u)\sqrt{1-u^2}}\Big[(b+a)\{(b+a)-(b-a)\sqrt{1-u^2}\} - (b-a)\{(b-a)-(b+a)\sqrt{1-u^2}\}\Big]$$

$$= \frac{4ab\,u}{\sqrt{1-u^2}\,A^2(u)}$$

$$\frac{d\xi}{\sqrt{1-\xi^2}} = \frac{2\sqrt{ab}\,du}{A(u)\sqrt{1-u^2}}$$

$$\therefore \frac{d\xi}{R\sqrt{1-\xi^2}} = \frac{du}{\sqrt{ab}\sqrt{1-k^2u^2}\sqrt{1-u^2}} \quad ///$$

$$\therefore I(x,y) = \frac{1}{\sqrt{ab}}\int_0^{\pi}\frac{du}{\sqrt{1-k^2u^2}\sqrt{1-u^2}} = \frac{2}{\sqrt{ab}}\int_0^{\frac{\pi}{2}}\frac{du}{\sqrt{1-k^2u^2}\sqrt{1-u^2}},$$

$$= \frac{2}{\sqrt{ab}}K(k), \quad (4)$$ 第1種完全楕円積分

$$\frac{\partial}{\partial x}\frac{1}{\sqrt{ab}} = -\frac{ab_x + a_x b}{2(ab)^{\frac{3}{2}}} = -\frac{a^2(x+1)+b^2(x-1)}{2(ab)^{\frac{3}{2}}} = -\frac{x(x^2+y^2-1)}{(ab)^{\frac{5}{2}}} \; //$$

$$\frac{\partial}{\partial y}\frac{1}{\sqrt{ab}} = -\frac{ab_y + a_y b}{2(ab)^{\frac{3}{2}}} = -\frac{y(1+x^2+y^2)}{(ab)^{\frac{5}{2}}} \; //$$

$$k^2 = \frac{4-\{a^2+b^2-2ab\}}{4ab} = \frac{1}{2} + \frac{1-x^2-y^2}{2ab} \; //$$

$2x(x^2+\cdots) - 4x$

$a^2(x-1)+b^2(x+1)$

$$2k\frac{\partial k}{\partial x} = -\frac{x}{ab} - \frac{(1-x^2-y^2)(ab_x+ba_x)}{2(ab)^2} = -\frac{x}{ab} - \frac{(1-x^2-y^2)(x^2+y^2-1)}{(ab)^3}$$

$$= -\frac{x}{(ab)^3}\left[a^2b^2-(x^2+y^2-1)^2\right] = -\frac{4xy^2}{(ab)^3}, \; ///$$

$$2k\frac{\partial k}{\partial y} = -\frac{y}{ab} - \frac{y(1-x^2-y^2)}{(ab)^3}(1+x^2+y^2) = -\frac{2y(1+y^2-x^2)}{(ab)^3}, \; //$$

x^4+1-2x^2 $\quad 1-\{x^4+y^4+2x^2y^2\}$

$$E(k) = \int_0^1 \sqrt{\frac{1-k^2u^2}{1-u^2}}\,du$$ ，第2種完全楕円積分

$$\frac{\partial K(k)}{\partial k} = \frac{E(k)}{k k'^2} - \frac{K(k)}{k} \quad // \text{ ok.}$$

$$\therefore \frac{\partial I}{\partial x} = 2K \frac{\partial}{\partial x}\frac{1}{\sqrt{ab}} + \frac{2}{k\sqrt{ab}} \frac{\partial k}{\partial x}\left(\frac{E}{k'^2} - K\right)$$

$$= \frac{2}{\sqrt{ab}} \frac{E(k)}{k k'^2} \frac{\partial k}{\partial x} + 2K\left\{\frac{\partial}{\partial x}\frac{1}{\sqrt{ab}} - \frac{1}{k\sqrt{ab}}\frac{\partial k}{\partial x}\right\}$$

$$= -\frac{4E(k)}{(k k'^2)^2}\frac{xy^2}{(ab)^{\frac{5}{2}}} + 2K\left\{-\frac{x(x^2-y^2-1)}{(ab)^{\frac{3}{2}}} + \frac{2xy^2}{k^2(ab)^{\frac{5}{2}}}\right\} \quad //$$

$$\frac{\partial I}{\partial y} = -\frac{2E}{(k k'^2)}\frac{y(1+y^2-x^2)}{(ab)^{\frac{3}{2}}} + 2yK\left\{-\frac{1+x^2+y^2}{(ab)^{\frac{3}{2}}} + \frac{1+y^2-x^2}{k^2(ab)^{\frac{3}{2}}}\right\} \quad //$$

$y \to 0$, $x > 1$ のとき $k \to 0$

$$k^2 \doteqdot k^2(y=0) + 2ky\frac{\partial k}{\partial y} = -\frac{2y(1+y^2-x^2)}{(ab)^{\frac{3}{2}}} + \frac{2y}{(x^2-1)}$$

$$K(0) = E(0) = \frac{\pi}{2}$$

$$\therefore \frac{\partial I}{\partial x} \xrightarrow[y\to 0]{|x|>1} -\frac{\pi x}{(x^2-1)^{\frac{3}{2}}} \quad //$$

$$\frac{\partial I}{\partial y} \longrightarrow \frac{\pi y}{k^2}\left(-\frac{2}{(x^2-1)^{\frac{3}{2}}}\right) \doteqdot -\frac{\pi}{(x^2-1)^{\frac{1}{2}}} \quad //$$

No. C-4

Date

積分方程式は 前文(4)式で

$$\frac{1}{2}y^2 = -\alpha y \frac{\partial I}{\partial y} = \cdots$$

$$1 = 4\alpha\left[\left\{\frac{1+y^2-x^2}{(kk')^2(ab)^{3}}\right\}E(k) + \left\{\frac{1+x^2+y^2}{(ab)^{3}} + \frac{x^2-1-y^2}{k^2(ab)^{3}}\right\}K(k)\right]$$

なお stagnation pt. を求めるのは 二ホで(無次元)で

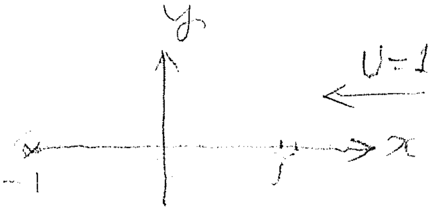
$$-\left.\frac{\partial\phi}{\partial x}\right|_{y=0} = 1, \quad \frac{\pi\alpha(2x^2-1)}{(x^2-1)^{\frac{3}{2}}} = 1$$

D. 流線追跡（没水回転体）

速度ポテンシャルは次式で表わされるとする。

$$\phi(x,y,z)=x+\int_{-1}^{1}\mu(\xi)\left[\frac{\partial}{\partial\xi}\frac{1}{R}\right]d\xi, \qquad (1)$$

$$R=\sqrt{(x-\xi)^2+y^2+z^2},$$



回転体の $z=0$ の面で流線（船体表面）を1本求めればよい。

船長は 2、一様流速は 1 とする。

$$u(x,y,z)=-\frac{\partial\phi}{\partial x}, \quad v(x,y,z)=-\frac{\partial\phi}{\partial y}$$

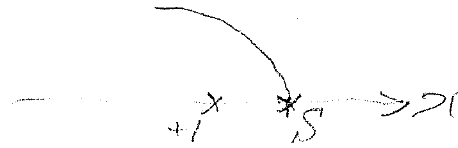
$$\left.\begin{aligned}&\frac{\partial}{\partial\xi}\frac{1}{R}=+\frac{x-\xi}{R^3}, \quad \frac{\partial^2}{\partial x\partial\xi}\frac{1}{R}=-\frac{2(x-\xi)^2-y^2-z^2}{R^5}\\&\frac{\partial^2}{\partial y\partial\xi}\frac{1}{R}=-\frac{3y(x-\xi)}{R^5},\end{aligned}\right\} \quad (2)$$

$$\left.\begin{aligned}&\xi=\cos\theta, \quad \mu(\xi)=\frac{1}{\sin\theta}\sum_n a_n\cos n\theta\\&\mu(\xi)d\xi=-M(\theta)d\theta, \quad M(\theta)=\sum a_n\cos n\theta\end{aligned}\right\}$$

$$\phi(x,y,z)=x+\int_0^{\pi}M(\theta)\left(\frac{\partial}{\partial\xi}\frac{1}{R}\right)d\theta, \qquad (1)'$$

$$\left.\begin{aligned}&u(x,y,0)=-1+\int_0^{\pi}M(\theta)\frac{2(x-\xi)^2-y^2}{[(x-\xi)^2+y^2]^{\frac{5}{2}}}d\theta,\\&v(x,y,0)=\int_0^{\pi}M(\theta)\frac{3(x-\xi)y}{[(x-\xi)^2+y^2]^{\frac{5}{2}}}d\theta,\end{aligned}\right\} \quad (3)$$

まず最初に淀み点 S を求める。



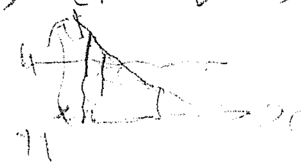
即ち

$$\left[u(x,0,0) = \int_0^l M(\xi) \frac{2}{(x-\xi)^3} d\xi - 1 \quad \cdots\cdots (4) \right]$$

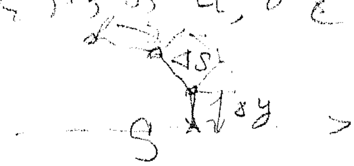
$$u(x,0,0) = 0 \quad \cdots\cdots\cdots (5)$$

となる点を求める。

xが少し0より大きい点の u(x,0,0)を求め それが0より大きければ, xを少し大きくして再び uを求め, それが0より小さくなれば少し戻す。

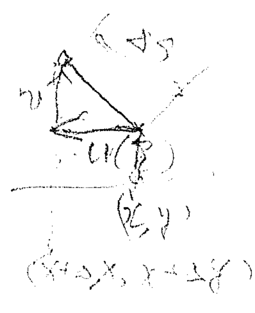


それが求まると S点で Δy だけ離れた点の u, v を計算して ΔS だけ進める。



この操作を繰り返し, x < -l 迄追跡する。

$$\left(\begin{array}{l} \Delta x = u \Delta S \\ \Delta y = v \Delta S \end{array} \right)$$



$$\mu(x) = \frac{1}{4\pi} A(x).$$

$$A(x) = \frac{a_0'}{\sin\theta} \sum a_n \cos n\theta$$

$$V = \int_{-l}^{l} A(x)\,dx = \pi l a_0'$$

$V = 2l \times \frac{\pi}{4} D_E^2$, D_E：等価直径（$2l$の円筒と同体積の直径）

$$a_0' = \frac{1}{\pi l} \times \frac{\pi l}{2} D_E^2 = \frac{D_E^2}{2} = \left(\frac{D_E}{L}\right)^2 (2l^2).$$

従って $l = 1$ とすると　$a_0' = 2\left(\frac{D_E}{L}\right)^2$

$$\mu(x) = \frac{\left(\frac{D_E}{L}\right)^2}{2\pi \sin\theta} \sum a_n \cos n\theta$$

計算は　$\frac{D_E}{L} = 0.15,\ 0.1,\ 0.05$　の3通り

この時の　$\delta = \frac{V}{L^3} = \frac{2l}{2l} \times \frac{\pi}{4}\left(\frac{D_E}{L}\right)^2 = \frac{\pi}{4}\left(\frac{D_E}{L}\right)^2.$

[.01767, .007854, .001963]

積分

$$\int_{\theta+\Delta\theta}^{\theta} f(\theta)\frac{\partial}{\partial\xi}\frac{1}{R}\,d\xi \fallingdotseq f\left(\theta+\frac{\Delta\theta}{2}\right)\left[\frac{1}{R}\right]_{\theta+\Delta\theta}^{\theta}$$

と近似する。　$\xi = \cos\theta$

それ以外

$$\int_{\theta+\Delta\theta}^{\theta} f(\theta)\frac{\partial^2}{\partial x\partial\xi}\frac{1}{R}\,d\xi \fallingdotseq f\left(\theta+\frac{\Delta\theta}{2}\right)\left[\frac{\partial}{\partial x}\frac{1}{R}\right]_{\theta+\Delta\theta}^{\theta}$$

$$\int f(\theta)\frac{\partial^2}{\partial y\partial\xi}\frac{1}{R}\,d\xi \fallingdotseq f\left(\theta+\frac{\Delta\theta}{2}\right)\left[\frac{\partial}{\partial y}\frac{1}{R}\right].$$

$$\left.\frac{\partial}{\partial x}\frac{1}{R}\right|_{z=0} = -\frac{x-\xi}{R^3} = -\frac{x-\xi}{\left[(x-\xi)^2+y^2\right]^{\frac{3}{2}}}$$

$$\frac{\partial}{\partial y}\frac{1}{R} = -\frac{y}{R^3} = \frac{-y}{\left[\quad\right]^{\frac{3}{2}}}$$