核関数の z 方向積分

$$I(\cancel{x}x,y,z) = \int_{-T}^{0}\left[Z(z')\frac{\partial}{\partial x'}\left(\frac{1}{r_1}+\frac{1}{r_2}\right) - \frac{Z(z')}{K_0}\frac{\partial^3}{\partial x'^3}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\right]dz',$$

$$I(x,y,z) = -\int_{-T}^{0}\left[Z'(z')\frac{\partial}{\partial x}\left(\frac{1}{r_1}+\frac{1}{r_2}\right) - \frac{Z(z')}{K_0}\frac{\partial^3}{\partial x^3}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\right]dz',$$

$$= -\int_{-T}^{T}\left[Z'(z')\frac{\partial}{\partial x}\frac{1}{r_1} - \frac{Z(z')}{K_0}\frac{\partial^3}{\partial x^3}\frac{1}{r_1}\right]dz' \quad , (1)$$

但し $Z(z') = -Z(-z')$, $Z'(z') = Z'(-z')$,

$r_1 = \sqrt{x^2+y^2+(z-z')^2}$

$$Z(z') = az' - \frac{(a-1)}{T^2}z'^3$$
$$Z'(z') = a - \frac{3(a-1)}{T^2}z'^2 \qquad \left\{\begin{array}{l} 1 \le a \le \frac{4}{3}, \\ \underline{a=1 \text{ or } \frac{4}{3}} \end{array}\right. \qquad (2)$$

$$-\frac{\partial I}{\partial x} = \int_{-T}^{T}\left[Z'(z')\frac{\partial^2}{\partial x^2}\frac{1}{r_1} - \frac{Z(z')}{K_0}\frac{\partial^4}{\partial x^4}\frac{1}{r_1}\right]dz'$$

$z - z' = \zeta$

$$= \int_{z-T}^{z+T}\left[Z'(z-\zeta)\frac{\partial^2}{\partial x^2}\frac{1}{r} - \frac{Z(z-\zeta)}{K_0}\frac{\partial^4}{\partial x^4}\frac{1}{r}\right]d\zeta \quad , (3)$$

$r = \sqrt{x^2+y^2+\zeta^2}$

$$-\frac{\partial I}{\partial y} = \int_{z-T}^{z+T}\left[Z'(z-\zeta)\frac{\partial^2}{\partial x\partial y}\frac{1}{r} - \frac{Z(z-\zeta)}{K_0}\frac{\partial^4}{\partial x^3\partial y}\frac{1}{r}\right]d\zeta \quad , (4)$$

$$-\frac{\partial I}{\partial z} = -\int_{-T}^{T}\left[Z'(z')\frac{\partial^2}{\partial x\,\partial z'}\frac{1}{r_1} - \frac{Z(z')}{K_0}\frac{\partial^4}{\partial x^3\,\partial z'}\frac{1}{r_1}\right]dz'$$

$$\simeq -\left[Z'(z')\frac{\partial}{\partial x}\frac{1}{r_1} - \frac{Z(z')}{K_0}\frac{\partial^3}{\partial x^3}\frac{1}{r_1}\right]_{z'=-T}^{z'=T}$$

$$+\int_{-T}^{T}\left[Z''(z')\frac{\partial}{\partial x}\frac{1}{r_1} - \frac{Z'(z')}{K_0}\frac{\partial^3}{\partial x^3}\frac{1}{r_1}\right]dz'$$

$$\left(\begin{array}{l} Z'(\cancel{T}) = 3-2a = Z'(-T), \quad \cancel{Z}(T) = aT-(a-1)T = T = -Z(-T) \\ Z''(z') = -\dfrac{6(a-1)}{T^2}z', \end{array}\right)$$

$$-\frac{\partial I}{\partial z} = (3-2a)\left[\frac{\partial}{\partial x}\left(\frac{1}{r}\right)\right]_{z-T}^{z+T} + \frac{T}{K_0}\left[\frac{\partial^3}{\partial x^3}\frac{1}{r}\right]_{z-T}^{z+T}$$

sgn$(\zeta - z)$ //

$$+\int_{z-T}^{z+T}\left[Z''(z-\zeta)\frac{\partial}{\partial x}\frac{1}{r} - \frac{Z'(z-\zeta)}{K_0}\frac{\partial^3}{\partial x^3}\frac{1}{r}\right]d\zeta, \quad (6)$$

$$Z(z-\zeta) = a(z-\zeta) - \frac{(a-1)}{T^2}(z^3 - 3z^2\zeta + 3z\zeta^2 - \zeta^3)$$

$$= az - \frac{(a-1)}{T^2}z^3 + \zeta\left\{-a + \frac{3z^2(a-1)}{T^2}\right\} - \frac{3(a-1)z}{T^2}\zeta^2 + \frac{(a-1)}{T^2}\zeta^3,$$

$$Z'(z-\zeta) = a - \frac{3(a-1)}{T^2}(z^2 - 2z\zeta + \zeta^2) =$$

$$= a - \frac{3(a-1)}{T^2}z^2 + \frac{6z(a-1)}{T^2}\zeta - \frac{3(a-1)}{T^2}\zeta^2 \qquad (7)$$

$$Z''(z-\zeta) = -\frac{6(a-1)}{T^2}(z-\zeta)$$

$a=1$ ならば

$$\left.\begin{aligned} Z(z-\zeta) &= z-\zeta \\ Z'(z-\zeta) &= 1 \\ Z''(z-\zeta) &= 0 \end{aligned}\right\} \qquad (8)$$

以下 先ず $a=1$ の式をあげよう。

$$\int_{z-T}^{z+T} \frac{\partial^2}{\partial x^2}\frac{1}{r}\,d\zeta = \int\left(-\frac{1}{r^3}+\frac{3x^2}{r^5}\right)d\zeta = \left[-\frac{\zeta}{\rho^2 r}+\frac{x^2\zeta}{\rho^4 r}\left(2+\frac{\rho^2}{r^2}\right)\right]_{z-T}^{z+T},$$

$\rho^2 = x^2+y^2$ (9)

$$\int(z-\zeta)\frac{\partial^4}{\partial x^4}\frac{1}{r}\,d\zeta = \int(z-\zeta)\left(\frac{9}{r^5}-\frac{90x^2}{r^7}+\frac{105x^4}{r^9}\right)d\zeta$$

$$= z\left[\frac{3\zeta}{\rho^4 r}\left(2+\frac{\rho^2}{r^2}\right)-\frac{6x^2\zeta}{\rho^6 r}\left(8+\frac{4\rho^2}{r^2}+\frac{3\rho^4}{r^4}\right)+\frac{x^4\zeta}{\rho^8 r}\left(48+24\frac{\rho^2}{r^2}+\frac{18\rho^4}{r^4}+\frac{15\rho^6}{r^6}\right)\right]$$

$$-\left[-\frac{3}{r^3}+\frac{18x^2}{r^5}-\frac{15x^4}{r^7}\right], \quad (10)$$

$$\int_{z-T}^{z+T}\frac{\partial^2}{\partial x\partial y}\frac{1}{r}\,d\zeta = \int\frac{3xy}{r^5}\,d\zeta = \frac{xy\zeta}{\rho^4 r}\left(2+\frac{\rho^2}{r^2}\right), \quad (11)$$

$$\int(z-\zeta)\frac{\partial^4}{\partial x^3\partial y}\frac{1}{r}\,d\zeta = \int(z-\zeta)\left\{-\frac{45xy}{r^7}+105\frac{x^3y}{r^9}\right\}d\zeta$$

$$= z\left[-\frac{3xy\zeta}{\rho^6 r}\left(8+\frac{4\rho^2}{r^2}+\frac{3\rho^4}{r^4}\right)+\frac{x^3y\zeta}{\rho^8 r}\left(48+2\frac{4\rho^2}{r^2}+\frac{18\rho^4}{r^4}+\frac{15\rho^6}{r^6}\right)\right]$$

$$-\left[\frac{9xy}{r^5}-\frac{15x^3y}{r^7}\right], \quad (12)$$

$$\int \frac{\partial^3}{\partial x^3}\frac{1}{r}\,d\zeta = \int\left(\frac{9x}{r^5} - \frac{15x^3}{r^7}\right)d\zeta = \frac{3x\zeta}{\rho^4 r}\left(2+\frac{\rho^2}{r^2}\right) - \frac{x^3\zeta}{\rho^6 r}\left(8+\frac{4\rho^2}{r^2}+\frac{3\rho^4}{\rho^4}\right), \quad (13)$$

q=1 の場合 計算

$$-\frac{\partial I}{\partial x} = \frac{\zeta}{\rho^4 r}\left\{-\rho^2 + x^2\left(2+\frac{\rho^2}{r^2}\right)\right\} - \frac{z\zeta}{K_0 r\rho^8}\left\{3\rho^4\left(2+\frac{\rho^2}{r^2}\right) - 6x^2\rho^2\left(8+\frac{4\rho^2}{r^2}+\frac{3\rho^4}{r^4}\right) + x^4\left(48+24\frac{\rho^2}{r^2}+\frac{18\rho^4}{r^4}+15\frac{\rho^6}{r^6}\right)\right\} + \frac{1}{K_0}\left(-\frac{3}{r^3}+\frac{18x^2}{r^5}-\frac{15x^4}{r^7}\right)\Bigg|_{\zeta=z-T}^{z+T} \quad (14)$$

$$-\frac{\partial I}{\partial y} = \frac{xy\zeta}{\rho^4 r}\left(2+\frac{\rho^2}{r^2}\right) + \frac{xy}{K_0 r^7}\left(9r^2-15x^2\right) - \frac{xyz\zeta}{K_0 r\rho^8}\left\{-3\rho^2\left(8+\frac{4\rho^2}{r^2}+\frac{3\rho^4}{r^4}\right) + x^2\left(48+24\frac{\rho^2}{r^2}+18\frac{\rho^4}{r^4}+15\frac{\rho^6}{r^6}\right)\right\}\Bigg|_{\zeta=z-T}^{z+T} \quad (15)$$

$$-\frac{\partial I}{\partial z} = -\frac{x}{r^3} + \frac{Tx}{K_0}\left(\frac{9}{r^5}-\frac{15x^2}{r^7}\right)\underline{\operatorname{sgn}(\zeta-z)} - \frac{x\zeta}{K_0 r\rho^6}\left\{3\rho^2\left(2+\frac{\rho^2}{r^2}\right) - x^2\left(8+\frac{4\rho^2}{r^2}+\frac{3\rho^4}{r^4}\right)\right\}\Bigg|_{\zeta=z-T}^{z+T}$$

(16)

$$\int_{\zeta_1,\zeta_2} (z-\zeta)^2 \frac{\partial^2}{\partial x^2}\frac{1}{r}\,d\zeta \quad , \quad \int_{\zeta_2,\zeta_3} (z-\zeta)^3 \frac{\partial^4}{\partial x^4}\frac{1}{r}\,d\zeta . \qquad 4$$

$$\int_{\zeta_1,\zeta_2} (z-\zeta)^2 \frac{\partial^2}{\partial x\partial y}\frac{1}{r}\,d\zeta \quad , \quad \int_{\zeta_2,\zeta_3} (z-\zeta)^3 \frac{\partial^5}{\partial x^5\partial y}\frac{1}{r}\,d\zeta . \qquad 21$$

$$\int (z-\zeta)\frac{\partial}{\partial x}\frac{1}{r}\,d\zeta \;\checkmark \quad , \quad \int_{\zeta_1,\zeta_2} (z-\zeta)^2 \frac{\partial^3}{\partial x^3}\frac{1}{r}\,d\zeta \quad \checkmark \qquad x1$$

$$\int (z-\zeta)\frac{\partial}{\partial x}\frac{1}{r}\,d\zeta = z\int \frac{\partial}{\partial x}\frac{1}{r}\,d\zeta - \int \frac{\partial}{\partial x}\frac{\zeta}{r}\,d\zeta = -xz\int\frac{d\zeta}{r^3} - \frac{\partial}{\partial x}r$$

$$= -\frac{xz\zeta}{\rho^2 r} - \frac{x}{r} , \qquad (17) .$$

$$\int \frac{\partial^3}{\partial x^3}\frac{\zeta}{r}\,d\zeta = \int\left(\frac{9x}{r^5} - \frac{15x^3}{r^7}\right)\zeta\,d\zeta = -\frac{3x}{r^3} + \frac{3x^3}{r^5} , \qquad (18)$$

$$\int \frac{\partial^3}{\partial x^3}\frac{\zeta^2}{r}\,d\zeta = \int\left(\frac{9x}{r^5} - \frac{15x^3}{r^7}\right)\zeta^2 d\zeta = -\frac{3x\zeta}{r^3} + \frac{3x\zeta}{\rho^2 r} - 15x^3\zeta\left\{-\frac{1}{5r^5} + \frac{3}{15r\rho^4}\left(2+\frac{\rho^2}{r^2}\right)\right\}$$

$$= -\frac{3x\zeta}{r^3}\left(1-\frac{r^2}{\rho^2}\right) + x^3\zeta\left\{\frac{3}{r^5} - \frac{1}{r\rho^4}\left(2+\frac{\rho^2}{r^2}\right)\right\}$$

$$= -\frac{3x\zeta}{r^3}\left(1-\frac{x^2+\rho^2}{r^2}\right) - \frac{x^3\zeta}{r\rho^4}\left(2+\frac{\rho^2}{r^2}\right) \qquad (19)$$

$$-6? \quad \int \zeta\frac{\partial^2}{\partial x^2}\frac{1}{r}\,d\zeta = \int\left(-\frac{1}{r^3} + \frac{3x^2}{r^5}\right)\zeta\,d\zeta = +\frac{1}{r} - \frac{x^2}{r^3} , \qquad (20)$$

$$+3 \quad \int \zeta^2\frac{\partial^2}{\partial x^2}\frac{1}{r}\,d\zeta = \int\left(-\frac{1}{r^3} + \frac{3x^2}{r^5}\right)\zeta^2 d\zeta = +\frac{\zeta}{r} - \log(r+\zeta) + 3x^2\left(-\frac{\zeta}{3r^3} + \frac{\zeta}{3\rho^2 r}\right)$$

$$= -\ln(r+\zeta) + \frac{\zeta}{r} - \frac{x^2\zeta}{r^3} + \frac{x^2\zeta}{\rho^2 r} , \qquad (21)$$

$$\int \zeta^2 \frac{\partial^4}{\partial x^4}\frac{1}{r}\,d\zeta = \int \zeta^2\left(\frac{9}{r^5} - \frac{90x^2}{r^7} + \frac{105x^4}{r^9}\right)d\zeta$$

$$= 9\zeta\left(-\frac{1}{3r^3} + \frac{1}{3\rho^2 r}\right) - 90x^2\zeta\left\{-\frac{5}{5r^5} + \frac{5}{15r\rho^4}\left(2 + \frac{\rho^2}{r^2}\right)\right\}$$

$$+ 105x^4\zeta\left\{-\frac{1}{7r^7} + \frac{1}{105r\rho^6}\left(8 + \frac{4\rho^2}{r^2} + \frac{3\rho^4}{r^4}\right)\right\}$$

$$= -\frac{3\zeta}{r^3} + \frac{3\zeta}{\rho^2 r} + \frac{18x^2\zeta}{r^5} - \frac{6x^2\zeta}{r\rho^4}\left(2 + \frac{\rho^2}{r^2}\right) - \frac{15x^4\zeta}{r^7}$$

$$+ \frac{x^4\zeta}{r\rho^6}\left(8 + \frac{4\rho^2}{r^2} + \frac{3\rho^4}{r^4}\right), \qquad (22)$$

$$\int \zeta^3 \frac{\partial^4}{\partial x^4}\frac{1}{r}\,d\zeta = \int \zeta^3\left(\frac{9}{r^5} - \frac{90x^2}{r^7} + \frac{105x^4}{r^9}\right)d\zeta$$

$$= -\frac{6}{r} - \frac{3\zeta^2}{r^3} + 90x^2\left(\frac{2}{15r^3} + \frac{\zeta^2}{5r^5}\right) - 105x^4\left(\frac{2}{35r^5} + \frac{\zeta^2}{7r^7}\right),$$

$$= -\frac{6}{r} + \frac{12x^2}{r^3} - \frac{6x^4}{r^5} + \frac{\zeta^2}{r^3}\left(-3 + \frac{18x^2}{r^2} - \frac{15x^4}{r^4}\right), \qquad (23)$$

$$\int \zeta \frac{\partial^2}{\partial x\partial y}\frac{1}{r}\,d\zeta = \int \zeta\left[\frac{3xy}{r^5}\right]d\zeta = -\frac{xy}{r^3}, \qquad (24)$$

$$\int \zeta^2 \frac{\partial^2}{\partial x\partial y}\frac{1}{r}\,d\zeta = \int \zeta^2\frac{3xy}{r^5}\,d\zeta = xy\zeta\left(-\frac{1}{r^3} + \frac{1}{\rho^2 r}\right), \qquad (25)$$

$$\int \zeta^2 \frac{\partial^4}{\partial x^3\partial y}\frac{1}{r}\,d\zeta = \int \zeta^2\left[-\frac{45xy}{r^7} + \frac{105x^3y}{r^9}\right)d\zeta = -45xy\zeta\left(-\frac{1}{5r^5} + \frac{2 + \frac{\rho^2}{r^2}}{15r\rho^4}\right)$$

$$+ 105x^3y\zeta\left\{-\frac{1}{7r^7} + \frac{1}{105r\rho^6}\left(8 + \frac{4\rho^2}{r^2} + \frac{3\rho^4}{r^4}\right)\right\}$$

$$= xy\zeta\left\{\frac{9}{r^5} - \frac{3}{r\rho^4}\left(2 + \frac{\rho^2}{r^2}\right)\right\} + x^3y\zeta\left\{-\frac{15}{r^7} + \frac{1}{r\rho^6}\left(8 + \frac{4\rho^2}{r^2} + \frac{3\rho^4}{r^4}\right)\right\} \qquad (26)$$

$$\int \zeta^3 \frac{\partial^4}{\partial x^3\partial y}\frac{1}{r}\,d\zeta = \int \zeta^3\left(-\frac{45xy}{r^7} + \frac{105x^3y}{r^9}\right)d\zeta = \frac{6xy}{r^3} + \frac{9xy\zeta^2}{r^5} - x^3y\left(\frac{6}{r^5} + \frac{15\zeta^2}{r^7}\right)$$

$$= \frac{6xy}{r^3}\left(1 - \frac{x^2}{r^2}\right) + \frac{3xy\zeta^2}{r^5}\left(3 - \frac{15x^2}{r^2}\right), \qquad (27)$$

$$I = a\left[I\right]_{a=1} - \frac{(a-1)}{T^2} J \quad , \qquad \cdots\cdots (28)$$

とおく。

$$-\frac{\partial J}{\partial x} = \int\left[3(z-\zeta)^2 \frac{\partial^2}{\partial x^2}\frac{1}{r} - \frac{(z-\zeta)^3}{K_0}\frac{\partial^4}{\partial x^4}\frac{1}{r}\right]d\zeta ,$$

$$-\frac{\partial J}{\partial y} = \int\left[3(z-\zeta)^2 \frac{\partial^2}{\partial x \partial y}\frac{1}{r} - \frac{(z-\zeta)^3}{K_0}\frac{\partial^4}{\partial x^3 \partial y}\frac{1}{r}\right]d\zeta ,$$

$$-\frac{\partial J}{\partial z} = +2T^2\left[\frac{\partial}{\partial x}\frac{1}{r}\right] + \int\left[6(z-\zeta)\frac{\partial}{\partial x}\frac{1}{r} - \frac{3(z-\zeta)^2}{K_0}\frac{\partial^3}{\partial x^3}\frac{1}{r}\right]d\zeta , \qquad (29)$$

$$-\frac{\partial J}{\partial x} = \frac{3z^2\zeta}{r\rho^2}\left\{-1 + \frac{x^2}{\rho^2}\left(2 + \frac{\rho^2}{r^2}\right)\right\} - 6z\left(\frac{1}{r} - \frac{x^2}{r^3}\right)$$
$$+ 3\left\{-\ln(r+\zeta) + \frac{\zeta}{r}\left(4 - \frac{x^2}{r^2} + \frac{x^2}{\rho^2}\right)\right\}$$

$$-\frac{z^3\zeta}{K_0 r\rho^4}\left\{3\left(2 + \frac{\rho^2}{r^2}\right) - \frac{6x^2}{\rho^2}\left(8 + \frac{4\rho^2}{r^2} + \frac{3\rho^4}{r^4}\right) + \frac{x^4}{\rho^4}\left(48 + 24\frac{\rho^2}{r^2} + \frac{18\rho^4}{r^4} + \frac{15\rho^6}{r^4}\right)\right\}$$

$$+ \frac{3z^2}{K_0}\left[-\frac{3}{r^3} + \frac{18x^2}{r^5} - \frac{15x^4}{r^7}\right] +$$

$$-\frac{3z\zeta}{K_0}\left[-\frac{3}{r^3} + \frac{3}{\rho^2 r} + \frac{18x^2}{r^5} - \frac{6x^2}{r\rho^4}\left(2 + \frac{\rho^2}{r^2}\right) - \frac{15x^4}{r^4} + \frac{x^4}{r\rho^6}\left(8 + \frac{4\rho^2}{r^2} + \frac{3\rho^4}{r^4}\right)\right]$$

$$+ \frac{1}{K_0}\left[-\frac{6}{r} + \frac{15x^2}{r^3} - \frac{6x^4}{r^5} + \frac{\zeta^2}{r^3}\left(-3 + \frac{18x^2}{r^2} - \frac{15x^4}{r^4}\right)\right] , \qquad (30)$$

$$-\frac{\partial J}{\partial y} = \frac{3xyz^2\zeta}{r\rho^4}\left(2+\frac{\rho^2}{r^2}\right) + \frac{6xyz}{r^3} + 3xy\zeta\left(\frac{1}{\rho^2 r}-\frac{1}{r^3}\right)$$

$$-\frac{xyz^3\zeta}{K_0 r\rho^6}\left[-24-\frac{12\rho^2}{r^2}-\frac{9\rho^4}{r^4}+\frac{x^2}{\rho^2}\left(48+\frac{24\rho^2}{r^2}+\frac{18\rho^4}{r^4}+\frac{15\rho^6}{r^6}\right)\right]$$

$$+\frac{3xyz^2}{K_0 r^5}\left(9-\frac{15x^2}{r^2}\right)$$

$$-\frac{3xyz\zeta}{K_0}\left[\frac{9}{r^5}-\frac{3}{r\rho^4}\left(2+\frac{\rho^2}{r^2}\right)+\frac{x^2}{r\rho^6}\left\{8+\frac{4\rho^2}{r^2}+\frac{3\rho^4}{r^4}-\frac{15\rho^6}{r^6}\right\}\right]$$

$$+\frac{xy}{K_0 r^3}\left[6-\frac{6x^2}{r^2}+\frac{\zeta^2}{r^2}\left(9-\frac{15x^2}{r^2}\right)\right], \quad \cdots\cdots \quad (31)$$

$$-\frac{\partial J}{\partial z} = -2\Gamma^2\cdot\frac{x}{r^3} - \frac{6x}{r}\left(1+\frac{z\zeta}{\rho^2}\right)$$

$$-\frac{3xz^2\zeta}{K_0 r\rho^4}\left[6+\frac{3\rho^2}{r^2}-\frac{x^2}{\rho^2}\left(8+\frac{4\rho^2}{r^2}+\frac{3\rho^4}{r^4}\right)\right]$$

$$+\frac{18x^2}{K_0 r^3}\left(-1+\frac{x^2}{r^2}\right)$$

$$+\frac{3x\zeta}{K_0 r\rho^4}\left[x^2\left(2+\frac{\rho^2}{r^2}\right)+\frac{3\rho^4}{r^2}\left(1-\frac{x^2}{r^2}\right)\right] \quad // \quad (32)$$

x^2

$r^2 \quad \lambda = 0 \quad \rho^2$

$\int$ 極限値 $(z=0)$. $r = \sqrt{x^2+y^2+\zeta^2}$ $\Big|_{\zeta=-T}^{\zeta=T}$

$$\frac{\zeta}{r}\Big]_{\zeta=-T}^{\zeta=T} = \frac{T}{\sqrt{x^2+y^2+T^2}} + \frac{T}{\sqrt{\;}} = \frac{2T}{r_T}, \quad r_T = \sqrt{x^2+y^2+T^2}$$

$$\frac{\zeta}{r^3}\Big] = \frac{2T}{r_T^3}, \quad \frac{1}{r^n}\Big] = 0$$

$$-\frac{\partial I}{\partial x} = \frac{2T}{r_T}\left\{-\frac{1}{\rho^2} + \frac{x^2}{\rho^4}\left(2+\frac{\rho^2}{r_T^2}\right)\right\}$$

$$-\frac{\partial I}{\partial y} = -\frac{2xyT}{r_T\rho^4}\left(2+\frac{\rho^2}{r_T^2}\right),$$

$$-\frac{\partial I}{\partial z} = \left\{\frac{2Tx}{K_0}\left(\frac{9}{r_T^5} - \frac{15x^2}{r_T^7}\right) + \; - \frac{2xT}{K_0 r_T \rho^6}\left[3\rho^2\left(2+\frac{\rho^2}{r_T^2}\right) - x^2\left(8+\frac{4\rho^2}{r_T^2}+\frac{3\rho^4}{r_T^4}\right)\right]\right. \qquad (33)$$

$\int$ 極限値 $(y=0)$, , , $\rho^2 = x^2$

$$-\frac{\partial I}{\partial x} = \frac{\zeta}{x^4 r}\left(x^2+\frac{x^4}{r^2}\right) + \qquad \left(6+\frac{15x^6}{r^6}\right)$$

$$-\frac{2\zeta}{K_0 r x^4}\left[3\left(2+\cancel{\frac{x^2}{r^2}}\right) - 6\left(\cancel{8}+\frac{4x^2}{r^2}+\cancel{\frac{3x^4}{r^4}}\right) + \cancel{18} + 24\frac{x^2}{r^2} + \cancel{\frac{18x^4}{r^4}} + \frac{15x^6}{r^6}\right]$$

$$+\frac{1}{K_0}\left(-\frac{3}{r^3}+\frac{18x^2}{r^5}-\frac{15x^4}{r^7}\right)\Bigg|_{\zeta=z-T}^{\zeta=z+T},$$

$$-\frac{\partial I}{\partial y} = 0 \qquad (34)$$

$$-\frac{\partial I}{\partial z} = -\frac{x}{r^3} + \frac{Tx}{K_0}\left(\frac{9}{r^5}-\frac{15x^2}{r^7}\right)\mathrm{sgn}(\zeta-z)$$

$$+\frac{x\zeta}{K_0 r x^4}\left(+2+\frac{x^2}{r^2}+\frac{3x^4}{r^4}\right)\Bigg|_{\zeta=z-T}^{\zeta=z+T},$$

§ (y = z = 0)

$$-\frac{\partial I}{\partial x} = \frac{2T}{x^2\sqrt{x^2+T^2}}\left(1+\frac{x^2}{x^2+T^2}\right),$$

$$-\frac{\partial I}{\partial y} = 0 \qquad (35)$$

$$-\frac{\partial I}{\partial z} = \frac{2TX}{K_0(X^2+T^2)^{\frac{5}{2}}}\left(9-\frac{15X^2}{X^2+T^2}\right) + \frac{2T}{K_0X^2\sqrt{X^2+T^2}}\left(+2+\frac{X^2}{X^2+T^2}+\frac{3X^4}{(X^2+T^2)^2}\right),$$

§ (x = 0)

$$-\frac{\partial I}{\partial x} = \left[-\frac{\zeta}{\rho^2 r} - \frac{3z\zeta}{K_0 r\rho^4}\left(2+\frac{\rho^2}{r^2}\right) - \frac{3}{K_0 r^3}\right]_{\zeta=z-T}^{\zeta=z+T}, \qquad (36)$$

$$-\frac{\partial I}{\partial y} = 0 = -\frac{\partial I}{\partial z} = 0$$

$$\Big|_{\zeta=z-T}^{z+T} \qquad (37)$$

$$I\Big|_{a=1} = -\int_{-T}^{T} \frac{\partial}{\partial x}\frac{1}{r}\,d\zeta = \int \frac{x}{r^3}\,d\zeta = \frac{x\zeta}{\rho^2 r}\Bigg|_{\zeta=-T}^{T} = \frac{2xT}{\rho^2\sqrt{\rho^2+T^2}}$$

$$J = +3x\int_{-T}^{T}\frac{\zeta^2}{r^3}\,d\zeta = 3x\left[\lg(r+\zeta) - \frac{\zeta}{r}\right]$$

$$= 3x\left[\lg\left(\frac{\sqrt{\rho^2+T^2}+T}{\sqrt{\rho^2+T^2}-T}\right) - \frac{2T}{\sqrt{\rho^2+T^2}}\right]$$

$$r = \sqrt{x^2+y^2+\zeta^2}$$

$$\frac{\partial}{\partial x}\frac{1}{r} = -\frac{x}{r^3}, \quad \frac{\partial^2}{\partial x^2}\frac{1}{r} = -\frac{1}{r^3} + \frac{3x^2}{r^5}$$

$$\frac{\partial^3}{\partial x^3}\frac{1}{r} = \frac{3x}{r^5} + \frac{6x}{r^5} - \frac{15x^3}{r^7} = \frac{9x}{r^5} - \frac{15x^3}{r^7}$$

$$\frac{\partial^4}{\partial x^4}\frac{1}{r} = \frac{9}{r^5} - \frac{45x^2}{r^7} + \frac{105x^4}{r^9} - \frac{45x^2}{r^7}$$

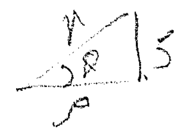
$$= \frac{9}{r^5} - \frac{90x^2}{r^7} + \frac{105x^4}{r^9}$$

$$\int \frac{d\zeta}{r} = \log(r+\zeta), \quad \int \frac{\zeta}{r} d\zeta = r$$

$$d\zeta = \rho \sec^2\theta \, d\theta.$$

$$r = \sqrt{\rho^2+\zeta^2}, \quad \rho^2 = x^2+y^2, \quad \zeta = \rho\tan\theta, \quad r = \rho\sec\theta,$$

$$r\sin\theta = \zeta$$



$$\int \frac{d\zeta}{r^n} = \frac{1}{\rho^{n-1}} \int \cos^{n-2}\theta \, d\theta$$

$n \geq 3$

$$\int \frac{\zeta}{r^n} d\zeta = -\frac{1}{(n-2)r^{n-2}}, \quad \int \frac{\zeta^2}{r^n} d\zeta = -\frac{\zeta}{(n-2)r^{n-2}}\Big| + \frac{1}{n-2}\int \frac{d\zeta}{r^{n-2}}$$

$$\int \frac{\zeta^3}{r^n} d\zeta = -\frac{\zeta^2}{(n-2)r^{n-2}} + \frac{2}{(n-2)}\int \frac{\zeta \, d\zeta}{r^{n-2}}$$

$$\int \frac{d\zeta}{r^3} = \frac{1}{\rho^2}\int \cos\theta \, d\theta = \frac{\sin\theta}{\rho^2} = \frac{\zeta}{\rho^2 r} \;//$$

$$\int \frac{d\zeta}{r^5} = \frac{1}{\rho^4}\int \cos^3\theta \, d\theta = \frac{1}{3\rho^4}\left[\sin\theta(2+\cos^2\theta)\right] = \frac{\zeta}{3\rho^4 r}\left(2 + \frac{\rho^2}{r^2}\right) \;//$$

$$\int \frac{d\zeta}{r^7} = \frac{1}{\rho^6}\int \cos^5\theta \, d\theta = \frac{\zeta}{15\rho^6 r}\left(8 + \frac{4\rho^2}{r^2} + \frac{3\rho^4}{r^4}\right) \;//$$

$$\int \frac{d\zeta}{r^9} = \frac{1}{\rho^8}\int \cos^7\theta \, d\theta = \frac{\zeta}{105\rho^8 r}\left[48 + 24\frac{\rho^2}{r^2} + 18\frac{\rho^4}{r^4} + 15\frac{\rho^6}{r^6}\right] \;//$$

$$A_{2n+1} = \int \cos^{2n+1}\theta\, d\theta = \sin\theta\cos^{2n}\theta + 2n\int \sin^2\theta\cos^{2n-1}\theta\, d\theta.$$

$$= \sin\theta\cos^{2n}\theta + 2n\int (1-\cos^2\theta)\cos^{2n-1}\theta\, d\theta.$$

$$= -2n A_{2n+1} + 2n A_{2n-1} + \sin\theta\cos^{2n}\theta.$$

$\cos^{2n+1}\theta = (1-\sin^2\theta)\cos^{2n-1}\theta$

$$(2n+1)A_{2n+1} = 2n A_{2n-1} + \sin\theta\cos^{2n}\theta.$$

$$A_{2n+1} = \frac{2n}{2n+1}A_{2n-1} + \frac{1}{2n+1}\sin\theta\cos^{2n}\theta$$

$$A_1 = \sin\theta$$

$$A_3 = \frac{2}{3}A_1 + \frac{1}{3}\sin\theta\cos^2\theta = \frac{2}{3}\sin\theta + \frac{1}{3}\sin\theta\cos^2\theta$$
$$= \frac{\sin\theta}{3}(2+\cos^2\theta)$$

$$A_5 = \frac{4}{5}A_3 + \frac{1}{5}\sin\theta\cos^4\theta = \frac{4}{15}\sin\theta(2+\cos^2\theta) + \frac{\sin\theta\cos^4\theta}{5}$$
$$= \frac{\sin\theta}{15}(8+4\cos^2\theta+3\cos^4\theta)$$

$$A_7 = \frac{6}{7}A_5 + \frac{1}{7}\sin\theta\cos^6\theta = \frac{\sin\theta}{105}\Big[\, 6(8+4\cos^2\theta+3\cos^4\theta) + 15\cos^6\theta$$

$$\int \frac{\zeta}{r^3} d\zeta = -\frac{1}{r}, \quad \int \frac{\zeta}{r^5} d\zeta = -\frac{1}{3r^3}, \quad \int \frac{\zeta\, d\zeta}{r^7} = -\frac{1}{5r^5}$$

$$\int \frac{\zeta}{r^9} d\zeta = -\frac{1}{7r^7},$$

$$\int \frac{\zeta^2}{r^3} d\zeta = -\frac{\zeta}{r} + \int \frac{d\zeta}{r} = -\frac{\zeta}{r} + \log(r+\zeta),$$

$$\int \frac{\zeta^2}{r^5} d\zeta = -\frac{\zeta}{3r^3} + \frac{1}{3}\int \frac{d\zeta}{r^3} = -\frac{\zeta}{3r^3} + \frac{\zeta}{3\rho^2 r},$$

$$\int \frac{\zeta^2}{r^7} d\zeta = -\frac{\zeta}{5r^5} + \frac{1}{5}\int \frac{d\zeta}{r^5} = -\frac{\zeta}{5r^5} + \frac{\zeta}{15\rho^4 r}\left(2 + \frac{\rho^2}{r^2}\right)$$

$$\int \frac{\zeta^2}{r^9} d\zeta = -\frac{\zeta}{7r^7} + \frac{1}{7}\int \frac{d\zeta}{r^7} = -\frac{\zeta}{7r^7} + \frac{\zeta}{105\rho^6 r}\left(8 + \frac{4\rho^2}{r^2} + \frac{3\rho^4}{r^4}\right)$$

$$\int \frac{\zeta^3}{r^3} d\zeta = -\frac{\zeta^2}{r} + 2\int \frac{\zeta}{r} d\zeta = 2r - \frac{\zeta^2}{r},$$

$$\int \frac{\zeta^3}{r^5} d\zeta = -\frac{\zeta^2}{3r^3} + \frac{2}{3}\int \frac{\zeta}{r^3} d\zeta = -\frac{2}{3r} - \frac{\zeta^2}{3r^3},$$

$$\int \frac{\zeta^3}{r^7} d\zeta = -\frac{\zeta^2}{5r^5} + \frac{2}{5}\int \frac{\zeta\, d\zeta}{r^5} = -\frac{2}{15r^3} - \frac{\zeta^2}{5r^5},$$

$$\int \frac{\zeta^3}{r^9} d\zeta = -\frac{\zeta^2}{7r^7} + \frac{2}{7}\int \frac{\zeta}{r^7} d\zeta = -\frac{2}{35r^5} - \frac{\zeta^2}{7r^7},$$

check

$$A = \int \frac{d\zeta}{r} = \log(r+\zeta)$$

不同

$$A_x = \frac{\frac{x}{r}}{r+\zeta} = \frac{x}{r(r+\zeta)} \qquad \left(A_{xx} = \frac{1}{r(r+\zeta)} - \frac{x^2}{r^3(r+\zeta)} - \frac{\frac{x^2}{r}}{r(r+\zeta)^2} \right.$$

$$= \frac{x(r-\zeta)}{r\rho^2} = \frac{x}{\rho^2} - \frac{x\zeta}{r\rho^2}$$

$$A_{xx} = -\frac{\zeta}{r\rho^2} + \frac{x^2\zeta}{r^3\rho^2} + \frac{2x^2\zeta}{r\rho^4} = \frac{\zeta}{r\rho^2}\left(-1 + \frac{x^2}{r^2} + \frac{2x^2}{\rho^2}\right) \quad /\!/ \text{o.k. } (9)$$

$$A_{xxx} = -\zeta\left(\frac{-x}{r^3\rho^2} - \frac{2x}{r\rho^4}\right) + \frac{2\zeta x}{r^3\rho^2} + \frac{4x\zeta}{r\rho^4}$$

$$-\zeta x^3\left(\frac{3}{r^5\rho^2} + \frac{2}{r^3\rho^4}\right) - 2\zeta x^3\left(\frac{1}{r^3\rho^4} + \frac{4}{r\rho^6}\right)$$

$$= \frac{6x\zeta}{r\rho^4} + \frac{3\zeta x}{r^3\rho^2} - \frac{8\zeta x^3}{r\rho^6} - \frac{3\zeta x^3}{\rho^2 r^5} - \frac{4\zeta x^3}{r^3\rho^4} \quad /\!/ \text{o.k.}$$

$$\left[-\frac{x^3\zeta}{r\rho^6}\left(8 + \frac{4\rho^2}{r^2} + \frac{3\rho^4}{r^4}\right) \right. \quad /\!/ \text{o.k. } (13)$$

$$\frac{d^4}{dx^4}A = \frac{6\zeta}{r\rho^4} + \frac{3\zeta}{r^3\rho^2} - \frac{24\zeta x^2}{r\rho^6} - \frac{9\zeta x^2}{\rho^2 r^5} - \frac{12\zeta x^2}{r^3\rho^4}$$

-12

6

$$+ 6x\zeta\left(-\frac{x}{r^3\rho^4} - \frac{4x}{r\rho^6}\right) + 3x\zeta\left(\frac{-3x}{r^5\rho^2} - \frac{2x}{r^3\rho^4}\right)$$

$$+ 8\zeta x^3\left(\frac{x}{r^3\rho^6} + \frac{6x}{r\rho^8}\right) + 3\zeta x^4\left(\frac{2}{\rho^4 r^5} + \frac{5}{\rho^2 r^7}\right) + 4\zeta x^4\left(\frac{3}{r^5\rho^4} + \frac{4}{r^3\rho^6}\right)$$

$$= \frac{\zeta}{r\rho^4}\left(6 + \frac{3\rho^2}{r^2}\right) + \frac{\zeta x^2}{r\rho^6}\left(-48 - \cancel{24}\frac{\rho^2}{r^2} - \frac{18\rho^4}{r^4}\right)$$

$$+ \frac{x^4\zeta}{r\rho^8}\left(48 + 24\frac{\rho^2}{r^2} + 18\frac{\rho^4}{r^4} + 15\frac{\rho^6}{r^6}\right) \quad /\!/ \text{o.k. } (10)$$

$$\int \frac{\zeta}{r} d\zeta = r \quad , \quad \frac{\partial}{\partial x} r = \frac{x}{r} \quad , \quad \frac{\partial^2}{\partial x^2} r = \frac{1}{r} - \frac{x^2}{r^3}$$

$$\frac{\partial^3}{\partial x^3} r = -\frac{3x}{r^3} + \frac{3x^3}{r^5} \quad , \quad \frac{\partial^4}{\partial x^4} r = -\frac{3}{r^3} + \frac{9x^2}{r^5} + \frac{9x^2}{r^5} - \frac{15x^4}{r^7} \quad //$$

(10)の大9格式.

$$A_{xy} = + x\zeta\left(\frac{y}{r^3\rho^2} + \frac{2y}{r\rho^4}\right) = \frac{xy\zeta}{r\rho^4}\left(2 + \frac{\rho^2}{r^2}\right) \quad // \quad (11)$$

$$A_{xxxy} = -6xy\zeta\left(\frac{1}{r^3\rho^4} + \frac{4}{r\rho^6}\right) - 3xy\zeta\left(\frac{3}{r^5\rho^2} + \frac{2}{r^3\rho^4}\right)$$

$$+ x^3y\zeta\left[8\left(\frac{1}{r^3\rho^6} + \frac{6}{r\rho^8}\right) + 4\left(\frac{3}{r^5\rho^4} + \frac{4}{r^3\rho^6}\right) + 3\left(\frac{5}{r^7\rho^2} + \frac{2}{r^5\rho^4}\right)\right]$$

$$= \frac{-xy\zeta}{r\rho^6}\left[\underset{8}{24} + \underset{4}{12}\frac{\rho^2}{r^2} + \underset{3}{9}\frac{\rho^4}{r^4}\right]$$

$$+ \frac{x^3y\zeta}{r\rho^8}\left[48 + 24\frac{\rho^2}{r^2} + 18\frac{\rho^4}{r^4} + 15\frac{\rho^6}{r^6}\right] \quad //$$

(12)の前半.

$$\frac{\partial^4}{\partial x^3 \partial y}\int \frac{\zeta}{r} d\zeta = \frac{\partial^4}{\partial x^3 \partial y} r = + \frac{9xy}{r^5} - \frac{15x^3y}{r^7} \quad //$$

OK!! (12)の後半.