細長い排水体の極値問題.

$$R_W = \frac{\rho g}{\pi^3} K_0^3 \nabla_0^2 \int_0^{\frac{\pi}{2}} e^{-2K_0 T \sec^2\theta} \left| f^*(K_0 \sec\theta) \right|^2 \sec^5\theta \, d\theta$$

$$f^*(p) = \int_{-l}^{l} A^*(x) e^{-ipx} dx. \qquad (1)$$

$$A^*(x) = \sum_{n=0} \frac{a_n \cos n\theta}{\sin\theta}, \quad a_0 = 1, \quad x = -\cos\theta. \quad (2)$$

$$A^*(\pm 1) = \cancel{\tfrac{d}{dx}} A^{*\prime}|_{x=\pm 1} = 0 \quad , \text{細長条件} \quad \ldots (3)$$

$$f^*(p) = -\frac{1}{ip} A^*(x) e^{-ipx}\Big| + \frac{1}{ip}\int A^{*\prime} e^{-ipx} dx$$

$$= \frac{A^{*\prime}(x)}{p^2} e^{-ipx}\Big| - \frac{1}{p^2}\int A^{*\prime\prime}(x) e^{-ipx} dx$$

$$= -\frac{1}{p^2}\int A^{*\prime\prime}(x) e^{-ipx} dx \quad \ldots (4)$$

$$A^{*\prime\prime}(x) = \frac{1}{\sin\theta}\sum_{n=2} b_n \cos n\theta \quad , b_2 = 8 \, , (5)$$

$$f^*(p) = -\frac{1}{p^2}\sum_{n=1} b_n \int_0^{\pi} \cos n\theta \, e^{ip\cos\theta} d\theta = -\frac{\pi}{p^2}\sum_{n=2} i^n b_n J_n(p), \quad (6)$$

$$\left| f^*(K_0 \sec\theta) \right|^2 = \frac{\pi^2}{(K_0^2 \sec^2\theta)^2} \sum\sum i^{n-m} b_n b_m J_n(K_0\sec\theta) J_m(K_0\sec\theta)$$

$$r = \frac{R_W}{\rho g \frac{\nabla_0^2}{8}} = \frac{8 K_0^3}{\pi^3} \cdot \frac{\pi^2}{K_0^4} \cdot \overset{\frac{8\pi K_0}{\pi}}{\sum_{n,m=2}} i^{n-m} b_n b_m \int_0^{\frac{\pi}{2}} e^{-2K_0 T \sec^2\theta} J_n J_m \sec\theta \, d\theta.$$

$$= \sum_{n,m} \cancel{i^{n-m}} b_n b_m G_{n,m} \quad , \quad (7)$$

$$G_{n,m} = i^{n-m}\frac{8h_0}{\pi K_0}\int_0^{\frac{\pi}{2}} e^{-2k_0 T \sec^2\theta} J_n(k_0 a \sec\theta)\, J_m(k_0 a \sec\theta)\, \sec^{\cancel{3}}\theta\, d\theta, \quad (8')$$

$(L=2)$

$$\frac{\partial^2}{\partial T^2} G_{n,m} = +4 i^{n-m}\frac{8K_0^2}{\pi K_0}\int_0^{\frac{\pi}{2}} e^{-2k_0 T \sec^2\theta} J_n J_m \sec^{\theta} d\theta$$

$$= +\frac{4}{K_0^2} F_{n,m} \quad , \quad (9)$$

$$r = \sum_{n,m=2} b_n b_m G_{n,m} \quad , \quad \underline{b_2 = 8} \quad \cdots (7)$$

故 極値問題は 前と同じ (前後対称分のみ)

$$A^{(i)} = \frac{b_2}{a_0}\cos 2\theta \quad , 1 \longrightarrow \text{MODEL } \underline{S2}$$

$$= \frac{1}{a_0}\left[b_2 \cos 2\theta + b_4 \cos 4\theta \right] \quad \text{Model. } \underline{S\text{-}4}$$

$$= \frac{1}{a_0}\left[b_2 \cos 2\theta + b_4 \cos 4\theta + b_6 \cos 6\theta \right]. \quad \underline{S\text{-}6}$$

$$A(x) = \frac{\pi}{2} \cancel{\,\cdot\,}(x), \quad D_2(x) = \boxed{\sqrt{\frac{4}{\pi}A(x)}} \quad \underline{\text{プロット}}$$

$$A^*(x) = A(x)/a_0' = \sum_n \frac{a_n \cos n\theta}{\sin\theta}, \quad a_0 = 1$$

$$x = -\cos\theta.$$

$$\frac{d^2}{dx^2}A^*(x) = \frac{1}{\sin\theta}\sum_{n=2} b_n \cos n\theta$$

$$\frac{d}{dx}A^* = \int \frac{d^2A^*}{dx^2}dx = \int \sum_1 b_n \cos n\theta \, d\theta = \sum_2 \frac{b_n}{n}\sin n\theta$$

$$A^* = \sum_2 \frac{b_n}{n}\int \sin n\theta \sin\theta \, d\theta = \sum_2 \frac{b_n}{2n}\int (\cos\overline{n-1}\theta - \cos\overline{n+1}\theta)\, d\theta$$

$$= \sum_{n=2} \frac{b_n}{2n}\left\{\frac{\sin(n-1)\theta}{n-1} - \frac{\sin(n+1)\theta}{n+1}\right\}$$

$$= \frac{1}{4\sin\theta}\sum_2 \frac{b_n}{n}\left\{\frac{\cos\overline{n-2}\theta - \cos n\theta}{n-1} - \frac{\cos n\theta - \cos\overline{n+2}\theta}{n+1}\right\}$$

$$= \frac{1}{\sin\theta}\sum_0 a_n \cos n\theta.$$

$$= \frac{1}{4\sin\theta}\Big[\frac{b_2}{2}\left\{1 - \cos 2\theta - \frac{\cos\theta - \cos 4\theta}{3}\right\}$$

$$+ \frac{b_3}{3}\left\{\frac{\cos\theta - \cos 3\theta}{2} - \frac{\cos 3\theta - \cos 5\theta}{4}\right\}$$

$$+ \frac{b_4}{4}\left\{\frac{\cos 2\theta - \cos 4\theta}{3} - \frac{\cos 4\theta - \cos 6\theta}{5}\right\}$$

$$+ \frac{b_5}{5}\left\{\frac{\cos 3\theta - \cos 5\theta}{4} - \frac{\cos 5\theta - \cos 7\theta}{6}\right\}$$

$$+ \frac{b_6}{6}\left\{\frac{\cos 4\theta - \cos 6\theta}{5} - \frac{\cos 6\theta - \cos 8\theta}{7}\right\}$$

$$+ \frac{b_7}{7}\left\{\frac{\cos 5\theta - \cos 7\theta}{6} - \frac{\cos 7\theta - \cos 9\theta}{8}\right\}$$

$$+ \frac{b_8}{8}\left\{\frac{\cos 6\theta - \cos 8\theta}{7} - \frac{\cos 8\theta - \cos 10\theta}{9}\right\} + \cdots\Big]$$

$$\therefore \quad a_0 = 1 = \frac{b_2}{8}, \qquad a_1 = \frac{b_3}{24}$$

$$a_2 = -\frac{b_2}{6} + \frac{b_4}{48}, \qquad a_3 = -\frac{b_3}{16} + \frac{b_5}{80}$$

$$a_4 = \frac{b_2}{24} - \frac{b_4}{30} + \frac{b_6}{120}, \qquad a_5 =$$

$$a_6 = \frac{b_4}{80} - \frac{b_6}{\underset{2}{\cancel{24}} \times 35} + \frac{b_8}{32 \times 7}$$

$$a_8 = \frac{b_6}{24 \times 7} - \frac{16\, b_8}{\underset{2}{\cancel{4}}18 \times 63}$$

$$a_{10} = \frac{b_8}{4 \cdot 8 \cdot 9}$$

Date

$$\gamma = a_0^2 F_{00} + a_0 a_2 F_{02} + a_0 a_4 F_{04} + a_0 a_6 F_{06} + a_0 a_8 F_{08} \cdots$$
$$+ a_2 a_0 F_{20} + a_2^2 F_{22} + a_2 a_4 F_{24} + a_2 a_6 F_{26} + a_2 a_8 F_{28} \cdots$$
$$+ a_4 a_0 F_{40} + a_4 a_2 F_{42} + a_4^2 F_{44} + a_4 a_6 F_{46} + a_4 a_8 F_{48} \cdots$$
$$+ a_6 a_0 F_{60} + a_6 a_2 F_{62} + a_6 a_4 F_{64} + a_6^2 F_{66} + a_6 a_8 F_{68}$$
$$+ a_8 a_0 F_{80} + a_8 a_2 F_{82} + a_8 a_4 F_{84} + a_8 a_6 F_{86} + a_8^2 F_{88}$$

Model

S-2, $a_0 = 1$, $a_2 = -\frac{4}{3}$, $a_4 = \frac{1}{3}$, $a_6 = a_8 = 0$

S-4, $a_0 = 1$, $a_2 = -\frac{4}{3} + \frac{b_4}{48}$, $a_4 = \frac{1}{3} - \frac{b_4}{30}$, $a_6 = \frac{b_4}{80}$, $a_8 = 0$

S-6, $a_0 = 1$, $a_2 = -\frac{4}{3} + \frac{b_4}{48}$, $a_4 = \frac{1}{3} - \frac{b_4}{30} + \frac{b_6}{120}$, $a_6 = \frac{b_4}{80} - \frac{b_6}{70}$,
$a_8 = \frac{b_6}{168}$