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安定理論について

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1. 2物体の干渉, 可逆定理, ハスキットの定理

非常線型理論の可逆定理 (3.2.3) は物体 C_1

C_1, C_2 の2つになっても成立つ。

C_2

今

$$\left. \begin{aligned} (u_1, v_1)_{C_1} &= \text{given} \\ (u_1, v_1)_{C_2} &= 0 \end{aligned} \right\} (1.1)$$



$$\left. \begin{aligned} (\tilde{u}_2, \tilde{v}_2)_{C_2} &= \text{given} \\ (\tilde{u}_2, \tilde{v}_2)_{C_1} &= 0 \end{aligned} \right\} (1.2)$$

ある速度場について (3.2.3) を書くと

$$\int_{C_1+C_2} (\tilde{u}_2 X_1 + \tilde{v}_2 Y_1) ds = \int_{C_1+C_2} (u_1 \tilde{X}_2 + v_1 \tilde{Y}_2) ds + \rho \int_{C_1+C_2} (u_1 \tilde{u}_2 + v_1 \tilde{v}_2) \frac{\partial X}{\partial n} ds, \quad (1.3)$$

となるので上境界条件を代入すると

$$\int_{C_2} (\tilde{u}_2 X_1 + \tilde{v}_2 Y_1) ds = \int_{C_1} (u_1 \tilde{X}_2 + v_1 \tilde{Y}_2) ds, \quad (1.4)$$

最初の逆流れを考えるとこれは又次のようにも書ける。

$$\int_{C_2} (u_2 \tilde{X}_1 + v_2 \tilde{Y}_1) ds = \int_{C_1} (\tilde{u}_1 X_2 + \tilde{v}_1 Y_2) ds, \quad (1.4')$$

特に $\tilde{u}_1 = 1, \tilde{v}_1 = 0$ on C_1 , $u_2 = -1, v_2 = 0$, on C_2

とすれば

$$-\int_{C_2} \tilde{X}_1 ds = \int_{C_1} X_2 ds, \quad (1.5)$$

これは相互作用に割る可逆定理である。

この定理で特に $C_2 \rightarrow 0$ になった極限を考よう。

C_2 における特異性点の位置 $(\tilde{U}''', \tilde{V}''')$ とするこよう。

$$\text{また } (u_1, v_1) \rightarrow (u, v)$$

$$(u, v)|_{C_1} = \text{given}, \quad (1.6)$$

$(\tilde{u}_2, \tilde{v}_2)$ とには C_2 における特異性 $(\tilde{U}''', \tilde{V}''')$ とそれから

C_1 に干渉して出来る $(\tilde{u}_d, \tilde{v}_d)$ の和としよう。

$$\tilde{u}_2 = \tilde{U}''' + \tilde{u}_d, \quad \tilde{v}_2 = \tilde{V}''' + \tilde{v}_d, \quad \{ \quad (1.7)$$

$$\tilde{u}_2|_{C_1} = \tilde{v}_2|_{C_1} = 0.$$

$$\int_{C_2} \tilde{X}_2 ds = 1, \quad \int_{C_2} \tilde{Y}_2 ds = 0, \quad \int_{C_2} \tilde{u}_2 ds = \int_{C_2} \tilde{v}_2 ds = 0, \quad (1.8)$$

を考慮して (1.3) より

$$\int_{C_1} (u \tilde{X}_2 + v \tilde{Y}_2) ds = -u|_{C_2}, \quad (1.9)$$

特に $u = -1, v = 0$ on C_1 とすれば

$$\int_{C_1} \tilde{X}_2 ds = u|_{C_2}, \quad (1.10)$$

即ち C_2 における特異点 C_2 に及ぼす力は

C_1 の周りの一様流れの速度場の C_2 点における速度に等しい。

これは水波理論におけるハスキントの定理(の拡張)に相当する。

順流^{あき}で考えれば (1.10)は

$$\int_{C_1} x_2 ds = \tilde{u}|_{C_2}, \quad (1.10')$$

となるが、これは C_2 点から逆流^{さか}の伴流の外側にある (3.4.4) 條より

$\tilde{u} \div \tilde{u}_p$ で表わされ、内側にある^{うしろ}粘性の影響を

うける事になり当然の事ではあるが遠場では $\tilde{u}$ の漸近展開が $(\int x_2 ds)$ に比例する事がわかっているので

近似的な見極りが出来るのは便利である。

^{よく知られた}
(ハスキントの定理) 係は遠場の極限である)

2. 境界値問題の変分原理

ストークス流れでは定常, 非定常に拘わらず同じ可逆定理

(1.2.7), (3.2.3) が成立つのでこれを使って 次のように

境界値問題の変分原理を導く事が出来る。

即ち領域 D の中で ストークスの方程式を満足する比較

函数のとき (u, v) , 流れ $(\tilde{u}, \tilde{v})$ を考えよう。

この境界力は $(X, Y), (\tilde{X}, \tilde{Y})$ とする。

C 上の与えられた境界条件は

$$u = u_0, v = v_0; \quad \tilde{u} = \tilde{u}_0, \quad \tilde{v} = \tilde{v}_0 \quad \text{on } C, \quad (2.1)$$

さて汎関数を次のようにとり,

$$\begin{aligned} I = & \int_C [\tilde{u}X + \tilde{v}Y + u\tilde{X} + v\tilde{Y} - \rho(u\tilde{u} + v\tilde{v}) \frac{\partial X}{\partial n}] ds \\ & - 2 \int_C (u_0\tilde{X} + v_0\tilde{Y} + \tilde{u}_0X + \tilde{v}_0Y) ds, \quad (2.2) \end{aligned}$$

変分をとると

$$\delta \int_C [\tilde{u}X + \tilde{v}Y] ds = \int_C (\delta \tilde{u}X + \delta \tilde{v}Y + \tilde{u}\delta X + \tilde{v}\delta Y) ds$$

であるからこの右辺の内 前の二項は可逆定理により

$$\int_C (\delta \tilde{u}X + \delta \tilde{v}Y) ds = \int_C [u\delta \tilde{X} + v\delta \tilde{Y} - \rho(u\delta \tilde{u} + v\delta \tilde{v}) \frac{\partial X}{\partial n}] ds$$

したがって

$$\delta \int_C [\tilde{u}X + \tilde{v}Y] ds = \int_C [u\delta \tilde{X} + v\delta \tilde{Y} + \tilde{u}\delta X + \tilde{v}\delta Y - \rho(u\delta \tilde{u} + v\delta \tilde{v}) \frac{\partial X}{\partial n}] ds,$$

すなわち (2.2) の変分は 結局

$$\delta I = 2 \int_C [(u-u_0)\delta\tilde{X} + (v-v_0)\delta\tilde{Y} + (\tilde{u}-u_0)\delta X + (\tilde{v}-v_0)\delta Y] ds, \quad (2.4)$$

となるから, $(\delta X, \delta Y), (\delta\tilde{X}, \delta\tilde{Y})$ に拘わらず

$\delta I = 0$ となるならば

$$u = u_0, v = v_0, \tilde{u} = \tilde{u}_0, \tilde{v} = \tilde{v}_0, \quad (2.5)$$

つまり (2.2) の汎関数 I の極値を与えるものが
(2.1) の境界条件の解である。

附録A オセーニ流れの別の表現

表現 (1.3.10) は 物理的に 明白な オセーニ レット によるものであって 数値計算 上 も 好 線 的 で 便利 である。

これをさらに (1.3.13) ~ (1.3.15) のように ポテンシャル成分と粘性成分に分けると見通しがよくなるが、さらに X, Y は、
(1.1.12) の

(1.3.18), (1.3.19) の関係式を代入すると、(F=1 とする)

$$\left. \begin{aligned} X &= u_p \frac{\partial X}{\partial n} - v_p \frac{\partial Y}{\partial n} \\ Y &= -u_p \frac{\partial X}{\partial s} + v_p \frac{\partial Y}{\partial s} \end{aligned} \right\} \dots (A.1)$$

となるが 境界条件を

$$\left. \begin{aligned} u_p + v_p &= -1 \\ v_p + u_p &= 0 \end{aligned} \right\} \dots (A.2)$$

とし、速度ポテンシャル、流れ関数を

$$u_p = \frac{\partial \phi_p}{\partial x} = \frac{\partial \psi_p}{\partial y}, \quad v_p = \frac{\partial \phi_p}{\partial y} = -\frac{\partial \psi_p}{\partial x}, \quad (A.3)$$

と導くと (A.1) は

$$\left. \begin{aligned} X &= \frac{\partial \phi_p}{\partial n} = \frac{\partial \psi_p}{\partial s} \\ Y &= -\frac{\partial \phi_p}{\partial s} = \frac{\partial \psi_p}{\partial n} \end{aligned} \right\} \dots (A.4)$$

したがって

$$\left. \begin{aligned} D &= \int_C X ds = \Delta \psi_p \\ \Gamma &= \int_C Y ds = \Delta \phi_p \end{aligned} \right\} \dots (A.5)$$

つまり、 ϕ_p, ψ_p は 一般には 多価関数で 一回まわると

D, P が 15 ジヤム 70 になる。

これは ソース と サークル-イメージ であり、

$$\phi_p(Q) + \frac{D}{2\pi} \log r' - \frac{P}{2\pi} \theta' = \phi_p'(Q), \quad \dots (A.6)$$

とすると ϕ_p' は 1 個 であり、グリーン の 定理 より

$$\phi_p'(Q) = \frac{-1}{2\pi i} \int_C \left[\phi_p' \frac{\partial}{\partial n} \log R - \frac{\partial \phi_p'}{\partial n} \log R \right] ds, \quad (A.7)$$

同じ 表現 が えられる。

流れ関数 には

$$\psi_p(Q) + \frac{D}{2\pi} \theta' + \frac{P}{2\pi} \log r' = \psi_p'(Q), \quad \dots (A.8)$$

$$\psi_p'(Q) = - \frac{1}{2\pi} \int_C \left[\psi_p' \frac{\partial}{\partial n} \log R - \frac{\partial \psi_p'}{\partial n} \log R \right] ds, \quad (A.9)$$

ここで さらに 複素平面 において 流れ関数 を 導入し

$$u_H = \frac{\partial \psi}{\partial y} \quad , \quad v_H = - \frac{\partial \psi}{\partial x} \quad , \quad \dots (A.10)$$

$$\psi = \psi_p + \psi_F \quad , \quad \dots (A.11)$$

ここで 複素平面 (A.12) へ

$$\frac{\partial \psi}{\partial z} = - \frac{\partial \psi}{\partial \bar{z}} = \frac{\partial \psi}{\partial s} \quad , \quad \frac{\partial \psi}{\partial \bar{s}} = - \frac{\partial \psi}{\partial s} = - \frac{\partial \psi}{\partial s} \quad , \quad (A.12)$$

あるいは

$$\frac{\partial \psi_p}{\partial z} = \frac{\partial \psi}{\partial s} - \frac{\partial \psi_F}{\partial z} \quad , \quad \frac{\partial \psi_p}{\partial \bar{s}} = - \frac{\partial \psi}{\partial s} - \frac{\partial \psi_F}{\partial \bar{s}} \quad , \quad (A.13)$$

$$\psi_0 \delta \psi = \frac{1}{2} \delta \psi^2$$

$$\int_C \left[\psi_H \frac{\partial \bar{\Phi}}{\partial x} - \bar{\Phi} \frac{\partial \psi_H}{\partial x} \right] ds = 2\ell \int_C \left(\frac{\bar{\Phi}}{?} \psi_H \right) \frac{\partial x}{\partial \eta} d\eta + 2\pi \psi_F$$

つネリ

$$\mathcal{F}_H(\alpha) = \frac{1}{2\pi} \int_C \left[\mathcal{V}_H \left(\frac{\partial \Phi}{\partial \eta} - 2k\Phi \frac{\partial \chi}{\partial \eta} \right) - \frac{2}{\Phi} \frac{\partial \mathcal{V}_H}{\partial \eta} \right] dS, \quad (A.14)^H$$

$$\left[\int_{-\infty}^{\infty} \frac{e^{is}}{2\pi} ds = -2\pi \right]$$

解けるよ、わかる、あるが、D.P.の果敢発露等よくわかる。

なお 昨らに 二つの関係がある

$$\frac{1}{2\pi} \int_C \left(y \frac{\partial}{\partial n} \log R - \frac{\partial y}{\partial n} \log R \right) dS_p = 0, \quad \begin{cases} Q \in D, \\ \end{cases} \quad (A.15)$$

$$\frac{1}{2\pi} \int_0^{2\pi} \left[y \left(\frac{\partial \phi}{\partial x} - \frac{1}{\phi} \frac{\partial x}{\partial u} \right) - \frac{1}{2\pi} \frac{\partial^2}{\partial x^2} \right] d\zeta_p = 0$$

* ψ_p ist - $\int_{\mathbb{R}^n} \psi_p(x) dx = 1$ ψ_p ist - $\int_{\mathbb{R}^n} \psi_p(x) dx = 1$

ψ_F は -100V 7s, ψ_H の値が 20V ψ_P の値が 12

1. 付号の「~~国~~」を「~~国~~」に訂正

この積分範囲は $(c + \text{cut})^2$ とする。

$\cos \theta = \frac{v}{c}$ $\Rightarrow$ $\sin \theta = \sqrt{1 - \frac{v^2}{c^2}}$ $D = \frac{\lambda}{\sin \theta} = \frac{\lambda}{\sqrt{1 - \frac{v^2}{c^2}}}$

さて (A.11) において ψ_H が ψ_P に対して 充分小さいとすれば

$$\psi \doteq \psi_P \doteq -\psi, \quad \psi_H \doteq 0, \quad (A.16)$$

同じ事であるから D を無視すると (1.8), (A.9) は

$$\psi_P = -\frac{P}{2\pi} \log r' + \frac{1}{2\pi} \int_C \left[\psi \frac{\partial \log R}{\partial n} + \frac{\partial \psi}{\partial n} \log R \right] ds, \quad (A.17)$$

$$\frac{\partial \psi_P}{\partial n} = \frac{\partial \psi_P}{\partial n} + \frac{P}{2\pi} \frac{\partial \log r}{\partial n} \quad (A.18)$$

流出

P は クッタの条件 によつて iR であるものとするならば これは ポテンシャル 流れの問題としてとけて $\frac{\partial \psi_P}{\partial n}$ は 一意に定まる。

境界層理論では 境界層の外側はこのポテンシャルで近似しているから この場合 (A.16) は そんなに無理な仮定ではないだろう。

さうすると (1.3.19) と 境界条件より

$$\oint \gamma = 2\pi V_P \quad (A.19)$$

(1.2.16) より 抵抗力の近似値は

$$\frac{D}{\rho} = \int_C V_P \tilde{g}_P ds, \quad (A.20)$$

$$\left(\tilde{g}_P = -g_P \text{ である } \right)$$

これは レイナルズ 数に 1 対 1 関係しないから のようにして 0 になる事がわかる。

$$\frac{D}{P} = - \int_C v_p q_p ds = \int_C v_p \left(\frac{\partial \psi}{\partial n} + \frac{\partial \psi_p}{\partial n} \right) ds$$

~~(A.21)~~

$$\int_C v_p \frac{\partial \psi_p}{\partial n} ds = \int_C \psi_p \frac{\partial v_p}{\partial n} ds = - \int_C \psi \frac{\partial v_p}{\partial n} ds$$

$$\therefore \psi_p|_C = -\psi$$

$$\therefore \frac{D}{P} = \int_C \left(v_p \frac{\partial \psi}{\partial n} - \psi \frac{\partial v_p}{\partial n} \right) ds$$

(A.21)

この右辺は積分路 \$C\$ の関係し方から無限遠方の
積分面によってゆき、ここで \$v_p \propto M \frac{\partial^2 f}{\partial x \partial y} r \propto M \frac{1}{r^2}\$

を考慮して積分すると

$$D = 0$$

(A.22)

となる。

そういう事で何等かの手段で (1.3.14) (1.3.15) の積分
方程式をとがなると近似値は与えられる。

3. 非定常線型理論 (セーン近似)

3.1 運動方程式

円周波数 ω の周期的運動をしている場合を考えよう。

この時任意の関数 $f(t)$ をフーリエ変換して、

$$\frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-i\omega t} dt = F(\omega) \quad , \quad \dots (3.1.1)$$

とすると 逆に

$$f(t) = \frac{T}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega \quad , \quad \dots (3.1.2)$$

以下はこの意味で $F(\omega)$ を扱うものとする。

こうすると運動方程式は高次の項を省略して

$$\left. \begin{aligned} i\omega u + u_x &= -\frac{1}{\rho} p_x + \nu \nabla^2 u \\ i\omega v + v_x &= -\frac{1}{\rho} p_y + \nu \nabla^2 v \end{aligned} \right\} \dots (3.1.3)$$

また

$$\nabla^2 p = 0$$

$$\left. \begin{aligned} (i\omega + \frac{\partial}{\partial x}) \zeta &= \nu \nabla^2 \zeta, \quad \zeta = u_x - v_y \end{aligned} \right\} (3.1.4)$$

逆流れでは

$$\left. \begin{aligned} i\omega \tilde{u} - \tilde{u}_x &= -\frac{1}{\rho} \tilde{p}_x + \nu \nabla^2 \tilde{u} \\ i\omega \tilde{v} - \tilde{v}_x &= -\frac{1}{\rho} \tilde{p}_y + \nu \nabla^2 \tilde{v} \end{aligned} \right\} \dots (3.1.5)$$

$$\nabla^2 \tilde{p} = 0$$

$$\left. \begin{aligned} (i\omega - \frac{\partial}{\partial x}) \tilde{\zeta} &= \nu \nabla^2 \tilde{\zeta}, \quad \tilde{\zeta} = \tilde{u}_x - \tilde{v}_y \end{aligned} \right\} \dots (3.1.6)$$

そこで流れ関数を導入しよう。

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \zeta = v_x - u_y = -\Delta \psi, \quad (3.1.7)$$

これを2つに分けて

$$\psi = \psi_p + \psi_F, \quad (3.1.8)$$

ψ_p は渦度分とし、そのポテンシャルを ϕ_p とすれば

$$u_p = \frac{\partial \psi_p}{\partial y} = \frac{\partial \phi_p}{\partial x}, \quad v_p = -\frac{\partial \psi_p}{\partial x} = \frac{\partial \phi_p}{\partial y}, \quad (3.1.9)$$

圧力 p は渦度分と数値故、その速度関数を $\tilde{f}$ とすれば

$$\frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{\partial \tilde{f}}{\partial y}, \quad \frac{1}{\rho} \frac{\partial p}{\partial y} = -\frac{\partial \tilde{f}}{\partial x}, \quad (3.1.10)$$

と書く。

(3.1.5) =

すると (3.1.3) は次のように書ける。

$$\left. \begin{aligned} (i\omega + \frac{\partial}{\partial x}) \psi_p &= -\tilde{f} \\ (i\omega + \frac{\partial}{\partial x}) \psi_F &= \nu \nabla^2 \psi_F \end{aligned} \right\} \quad (3.1.11)$$

$$\left. \begin{aligned} (i\omega - \frac{\partial}{\partial x}) \tilde{\psi}_p &= -\tilde{\tilde{f}} \\ (i\omega - \frac{\partial}{\partial x}) \tilde{\psi}_F &= \nu \nabla^2 \tilde{\psi}_F \end{aligned} \right\} \quad (3.1.12')$$

また

$$\left. \begin{aligned} \frac{p}{\rho} &= -(i\omega + \frac{\partial}{\partial x}) \phi_p \\ \nu \zeta &= -(i\omega + \frac{\partial}{\partial x}) \psi_F \end{aligned} \right\} \quad (3.1.12)$$

$$\left. \begin{aligned} \tilde{p}/\rho &= -(i\omega - \frac{\partial}{\partial x}) \tilde{\phi}_p \\ \nu \tilde{\zeta} &= -(i\omega - \frac{\partial}{\partial x}) \tilde{\psi}_F \end{aligned} \right\} \quad (3.1.12')$$

境界条件は、C 上で u, v に対して $\frac{\partial \psi}{\partial n}, \frac{\partial \psi}{\partial s}$ が与えられる。

今 $\psi = f(v, u), \quad \frac{\partial \psi}{\partial n} = \frac{\partial f}{\partial n} \quad (3.1.13)$

とすれば、次に境界 C を 1 と 17

前送りれ口送り

$$f = i\omega y$$

左送りれ

$$f = -i\omega x$$

回転(垂直)

$$f = \frac{i\omega}{2} (x^2 + y^2)$$

(3.1.14)

また境界上では、これらの関係を使う。

$$\frac{1}{P} \frac{\partial P}{\partial n} = \frac{\partial \psi}{\partial s} = -\nu \frac{\partial \psi}{\partial s} - \frac{\partial}{\partial s} \left[\left(i\omega + \frac{\partial}{\partial x} \right) f \right], \quad (3.1.15)$$

$$\frac{1}{P} \frac{\partial P}{\partial s} - \nu \frac{\partial \psi}{\partial n} = - \frac{\partial}{\partial n} (\psi + \nu \psi) = + \frac{\partial}{\partial n} \left[\left(i\omega + \frac{\partial}{\partial x} \right) \psi \right]$$

$$= + i\omega \frac{\partial \psi}{\partial n} - \frac{\partial \psi}{\partial n}$$

$$\frac{\partial \psi}{\partial n} = \frac{\partial \psi}{\partial x} \frac{\partial x}{\partial n} + \frac{\partial \psi}{\partial y} \frac{\partial y}{\partial n} = \left\{ \frac{\partial x}{\partial n} + \frac{\partial y}{\partial s} \right.$$

$$\therefore \frac{1}{P} \frac{\partial P}{\partial s} = \nu \frac{\partial \psi}{\partial n} - \left\{ \frac{\partial x}{\partial n} + i\omega \frac{\partial \psi}{\partial n} + \frac{\partial \psi}{\partial s} \right\}, \quad (3.1.16)$$

境界力については、(1.1.10)の定義より

$$\begin{aligned} X &= -P \frac{\partial x}{\partial n} - \mu \psi \frac{\partial \psi}{\partial n} - 2\mu \frac{\partial \psi}{\partial s} \\ Y &= -P \frac{\partial y}{\partial n} + \mu \psi \frac{\partial x}{\partial n} + 2\mu \frac{\partial \psi}{\partial s} \end{aligned} \quad \left. \begin{aligned} & \\ & \end{aligned} \right\} \quad (3.1.17)$$

となる。

$$\frac{1}{T} = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{\omega^2 - \omega'^2} d\omega'$$

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3.2 可逆定理

(1.2.1) の積分を導入すると 明らかに (1.2.2) なる可逆性方程式成立つ。

(1.2.1) は (1.2.3) のように変形出来るが (3.1.3) を代入すると

$$E = -\rho \iint_D [i\omega(u\tilde{u} + v\tilde{v}) + \tilde{u}u_x + \tilde{v}v_x] dx dy + \int_C (\tilde{u}X + \tilde{v}Y) ds, \quad (3.2.1)$$

(1.2.5) に (3.1.5) を代入すると

$$E = -\rho \iint_D [i\omega(u\tilde{u} + v\tilde{v}) - u\tilde{u}_x - v\tilde{v}_x] dx dy + \int_C (u\tilde{X} + v\tilde{Y}) ds, \quad (3.2.2)$$

可逆性 (2.4), 2式を等置する

$$\begin{aligned} \int_C (\tilde{u}X + \tilde{v}Y) ds - \rho \iint_D (\tilde{u}u_x + \tilde{v}v_x) dx dy \\ = \int_C (u\tilde{X} + v\tilde{Y}) ds + \rho \iint_D (u\tilde{u}_x + v\tilde{v}_x) dx dy, \end{aligned}$$

となり, 部分積分して

$$\int_C (\tilde{u}X + \tilde{v}Y) ds = \int_C (u\tilde{X} + v\tilde{Y}) ds + \rho \int_C (u\tilde{u} + v\tilde{v}) \frac{\partial X}{\partial n} ds, \quad (3.2.3)$$

となり, (1.2.7) と同型となる。

$$u, v = e^{i\omega t}$$

Kodaira / 1970

今 $\tilde{a}, \tilde{v}$ としたポテンシャル, 流束を $\frac{1}{\pi}$ としてみよう.

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad \dots \quad (3, 2, 4)$$

また 力については

$$(i\omega - \frac{\partial}{\partial x}) \hat{\phi} + \frac{\hat{p}}{\rho} = \nu \Delta \hat{\phi} = 0$$

これより式(1)と(2.2.2)から得られ、かつ

$$\left. \begin{aligned} \dot{x}^2 &= -\dot{p} \frac{\partial x}{\partial u} - 2\mu \frac{\partial \tilde{v}}{\partial s} \\ \dot{y}^2 &= -\dot{p} \frac{\partial y}{\partial u} - 2\mu \frac{\partial \tilde{u}}{\partial s} \end{aligned} \right\} \quad (3.2.5'')$$

∴ 2 の値を代入して $(3, 2, 3)$ が成立つ。

(3, 2, 5) 7

$(3, 2, 5)$ へ
 $(3, 1, 7)$ と $(3, 2, 3)$ 123456789

$$-\int_{\mathcal{S}} \left[p \frac{\partial \tilde{\Phi}}{\partial n} - \mu \frac{\partial \tilde{\Phi}}{\partial s} \right] ds$$

$$= - \int_C \vec{B} \cdot \left(u \frac{\partial \vec{x}}{\partial n} + v \frac{\partial \vec{y}}{\partial n} \right) ds + P \int_C \left(u \vec{a} + v \vec{b} \right) \cdot \frac{\partial \vec{x}}{\partial n} ds, \quad (3.2.6)$$

あるいは (3.2.4) に より

$$\int_C \left(\rho \frac{\partial \tilde{\phi}}{\partial n} - \mu S \frac{\partial \tilde{\phi}}{\partial s} \right) ds = \rho \int_C \left(i\omega - \frac{\partial}{\partial n} \right) \tilde{\phi} \left(u \frac{\partial x}{\partial n} + v \frac{\partial y}{\partial n} \right) ds$$

$$+ \rho \int_C \left(u \tilde{u} + v \tilde{v} \right) \frac{\partial x}{\partial n} ds, \quad (0.2.b)$$

さて (3.2.3) に (3.1.17) を代入し、流れの速度を求めると

$$\begin{aligned} uX + vY &= -\rho \left(u \frac{\partial X}{\partial u} + v \frac{\partial Y}{\partial u} \right) + \mu S \left(u \frac{\partial X}{\partial S} + v \frac{\partial Y}{\partial S} \right) \\ &\quad - 2\mu \left(u \frac{\partial v}{\partial S} - v \frac{\partial u}{\partial S} \right) \\ &= -\rho \frac{\partial \psi}{\partial S} - \mu S \frac{\partial \psi}{\partial u} - 2\mu \left(u \frac{\partial v}{\partial S} - v \frac{\partial u}{\partial S} \right) \end{aligned}$$

$$uX + vY = -\rho \frac{\partial \psi}{\partial S} - \mu S \frac{\partial \psi}{\partial u} - 2\mu \left(u \frac{\partial v}{\partial S} - v \frac{\partial u}{\partial S} \right)$$

これから、両辺の微分をとり、(3.2.7) に

$$\int_C \left(\rho \frac{\partial \psi}{\partial S} + \mu S \frac{\partial \psi}{\partial u} \right) dS = \int_C \left(\rho \frac{\partial \psi}{\partial S} + \mu S \frac{\partial \psi}{\partial u} \right) dS - \rho \int_C \nabla \psi \nabla \psi \frac{\partial X}{\partial u} dS, \quad (3.2.7)$$

あるいは

$$\begin{aligned} \int_C \left(\mu S \frac{\partial \psi}{\partial u} - \psi \frac{\partial \rho}{\partial S} \right) dS &= \int_C \left(\mu S \frac{\partial \psi}{\partial u} - \psi \frac{\partial \rho}{\partial S} \right) dS \\ &\quad - \rho \int_C \nabla \psi \nabla \psi \frac{\partial X}{\partial u} dS. \quad (3.2.8) \end{aligned}$$

3.3 基本特異性

を代入すると

$$\bar{\Phi} = K_0(k'R) e^{k'(x-x)}, \quad k' = \sqrt{k^2 + 2i\omega k}, \quad k = \frac{1}{2V}, \quad (3.3.1)$$

を代入すると

$$(i\omega + \frac{\partial}{\partial x'} - \nu \Delta) \bar{\Phi} = 0, \quad (3.3.1')$$

この関数からさらに次の関数を定義しよう。

$$\Psi = e^{-i\omega x'} \int_{-\infty}^{x'} \bar{\Phi} e^{i\omega x'} dx', \quad (3.3.2)$$

$$\text{つまり } (i\omega + \frac{\partial}{\partial x'}) \Psi = \bar{\Phi}, \quad (3.3.2')$$

同様にして

$$(i\omega + \frac{\partial}{\partial x'}) S = \log R, \quad (3.3.3)$$

と次の関数

$$S = e^{-i\omega x'} \int_{-\infty}^{x'} \log R e^{i\omega x'} dx', \quad (3.3.3')$$

を定義しよう。尚この関数は improper だが微分

は proper である。

これらの関数に (3.1.1), (3.1.1') と対応をせよと

$$\psi_P = -\frac{S}{2\pi}, \quad \psi_A = -\frac{\Psi}{2\pi}, \quad \psi = -\frac{\bar{\Phi}}{2\pi}, \quad \psi = -\frac{\log R}{2\pi} \text{ とおけば}$$

核関数が出る,

$$\oint_{\Sigma} \frac{\partial \psi}{\partial n} ds = -1 = - \oint_{\Sigma} \frac{\partial \psi}{\partial n} ds, \quad (3.3.4)$$

となる。

2.2.12

$$\left. \frac{\partial \bar{\psi}}{\partial y} \right|_{\substack{x=0 \\ y=0}} = \frac{1}{2} + e^{-i\omega x'} \int_0^{x'} e^{-\frac{y'^2}{2\bar{z}}} \frac{dy'}{\bar{z}} \quad , \quad \frac{dy'}{\bar{z}} = \eta^2$$

$$-\frac{\sqrt{2}}{2\sqrt{z}} \frac{y' d\bar{z}}{\bar{z}^2} = d\eta \quad , \quad \frac{\sqrt{2}}{\sqrt{2\bar{z}}} y' = \eta$$

$$\left. \frac{\partial \bar{\psi}}{\partial y} \right| = 2\sqrt{\pi} e^{-i\omega x'} \int_{\eta_0}^{\infty} e^{-\eta^2} d\eta \quad , \quad \eta_0 = \sqrt{\frac{2}{2x'}} y' \quad (3.3.10)$$

$$\xrightarrow{\eta_0 \rightarrow 0} \pi e^{-i\omega x'} \operatorname{sgn}(y') \quad , \quad -$$

2) S の性質

$$\frac{\partial S}{\partial x} = e^{-i\omega x'} \int_{-\infty}^{x'} \frac{x-x'}{R^2} e^{i\omega x'} dx' = e^{-i\omega(x-x')} \int_{x-x'}^{\infty} \frac{e^{-i\omega \bar{z}}}{\bar{z}^2 + (y-y')^2} d\bar{z} \quad (3.3.11)$$

$$\frac{\partial S}{\partial y} = e^{-i\omega x'} \int_{-\infty}^{x'} \frac{y-y'}{R^2} e^{i\omega x'} dx' = e^{-i\omega(x-x')} \int_{x-x'}^{\infty} \frac{(y-y') e^{-i\omega \bar{z}}}{\bar{z}^2 + (y-y')^2} d\bar{z}$$

よってこの以下、 $x=y=0$ として考えても一般性を失わないので

この式集に於て、次の積分を導入すると

$$I_1 = e^{-i\omega x'} \int_{-x'}^{\infty} \frac{e^{-i\omega \bar{z}}}{\bar{z} + iy'} d\bar{z} = e^{-\omega y' - i\omega x'} \int_{-i\omega(x'-iy')}^{\infty - i\omega y'} \frac{e^{-t}}{t} dt$$

$$= e^{-\omega(y'+ix')} E_1 \{-i\omega(x'-iy')\} \quad ,$$

$$I_2 = e^{-i\omega x'} \int_{-x'}^{\infty} \frac{e^{-i\omega \bar{z}}}{\bar{z} - iy'} d\bar{z} = e^{\omega y' - i\omega x'} \int_{-i\omega(x'+iy')}^{\infty - i\omega y'} \frac{e^{-t}}{t} dt \quad (3.3.12)$$

$$= e^{\omega(y'-ix')} E_1 \{-i\omega(x'+iy')\} \quad ,$$

とすると

$$\begin{aligned} S_x &\approx \frac{1}{2} (I_1 + I_2) \\ S_y &= \frac{1}{2i} (I_1 - I_2) \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad (2.3.13)$$

また

$$F_1(z) \doteq \lim_{z \rightarrow 0} -\gamma - \lg z,$$

またさらに

$$\begin{aligned} S_x &\xrightarrow{R \rightarrow 0} -\gamma - \lg(\omega R) - \frac{1}{2} \frac{\pi i}{2} \\ S_y &\xrightarrow{R \rightarrow 0} 0, \quad \tan \theta = \frac{y'}{x'} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad (3.3.14)$$

次に $R \rightarrow \infty$ では

i) $x - x' \gg 1$, $x'' \ll -1 + x$ では (3.3.11) を直接部分積分

して

$$S_x \doteq \frac{x - x'}{i\omega R^2}, \quad S_y = \frac{y - y'}{i\omega R^2}, \quad (3.3.15)$$

ii) $x - x' \ll -1$, $x' \gg 1 + x$ では、上式の右の項の他に

項が出てくる。

$$\begin{aligned} S_x &\doteq \frac{-x'}{i\omega R^2} - \pi i e^{-\omega(y') - i\omega x'} \\ S_y &\doteq \frac{-y'}{i\omega R^2} - \pi e^{-\omega(y') - i\omega x'} \operatorname{sgn}(y'), \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad (3.3.16)$$

とすると

$$-y' = \sqrt{x'^2 + y'^2} \sin \theta$$

$$\begin{aligned} -\omega y' - i\omega x' &= -\exp(i\theta + i\omega x') \\ &= \pi \omega y' e^{-i\theta} \\ &= \omega y' e^{-(\frac{\pi}{2} + \theta)i} \\ \omega y' - ix' &= \omega y' e^{i(\theta - \frac{\pi}{2})} \end{aligned}$$

$$\frac{\omega y'}{\sqrt{x'^2 + y'^2}} = \left(\frac{1}{3 + iy'} - \frac{1}{3 - iy'} \right) \frac{1}{i}$$

3.4 速度場の表現

(3.2.8) において ψ に前節の核を導入すると

$$\begin{aligned} \tilde{\psi} &= \frac{-1}{2\pi} (\rho + \bar{\psi}), \quad \tilde{\rho} = -\frac{\bar{\psi}}{2\pi}, \quad \tilde{\rho} = -\frac{1}{2\pi} \bar{\psi} R, \\ \frac{\partial \tilde{\rho}}{\partial s} &= -\frac{\partial \tilde{\psi}}{\partial n} = \frac{1}{2\pi} \frac{\partial}{\partial n} \bar{\psi} R, \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} (3.4.1)$$

と書く

$$\psi(Q) = \int_C \left[\mu(s) \frac{\partial \tilde{\psi}(P, Q)}{\partial n} - \tilde{\psi} \frac{\partial \rho}{\partial s} + \psi \frac{\partial \tilde{\rho}}{\partial s} - \mu \tilde{\rho} \frac{\partial \psi}{\partial n} + \rho \nabla \psi \nabla \tilde{\psi} \frac{\partial x}{\partial n} \right] ds, \quad (3.4.2)$$

をうる。

そこで C の内部領域 D で正則な ψ_i を考え

$$\psi = \psi_i, \quad \frac{\partial \psi_i}{\partial n} = \frac{\partial \psi}{\partial n} \quad \text{on } C, \quad (3.4.3)$$

とすれば (3.2.8) より Q を D 内の点とすれば上と同じ核

$$0 = \int_C \left[\mu(s) \frac{\partial \tilde{\psi}}{\partial n} - \tilde{\psi} \frac{\partial \rho_i}{\partial s} + \psi_i \frac{\partial \tilde{\rho}}{\partial s} - \mu \frac{\partial \psi_i}{\partial n} \tilde{\rho} + \rho \nabla \psi_i \nabla \tilde{\psi} \frac{\partial x}{\partial n} \right] ds, \quad (3.4.3)$$

をうきから、辺々相引くと

$$\psi(Q) = \int_C \left[\mu(s-s_i) \frac{\partial \tilde{\psi}(P, Q)}{\partial n} - \tilde{\psi} \frac{\partial}{\partial s} (\rho - \rho_i) \right] ds, \quad (3.4.4)$$

をうる。

 ψ_i については (3.1.14) より, 前節, 左をわねにすれば

$$\psi_i = i\omega y \text{ ならば } s_i = 0, \quad \frac{1}{\rho} \rho_i = -i\omega x,$$

$$\psi_i = -i\omega x \text{ ならば } s_i = 0, \quad \frac{1}{\rho} \rho_i = -i\omega y, \quad \left. \begin{array}{l} \\ \end{array} \right\} (3.4.5)$$

のようになるから同解問題に対しては内部問題を解いて

 ρ_i, s_i を決めぬばならない。

3.5 遠場の表現

§3.3 により 遠場では

$$\psi(p, a) \xrightarrow{a \rightarrow \infty} \frac{-1}{2\pi} S \doteq \frac{1}{2\pi i \omega} \oint R, \quad \left\{ \begin{array}{l} \text{伴流の場} \\ \text{波場} \end{array} \right. \quad (3.5.1)$$

$$\int^2 \rightarrow 0$$

(3.4.2) に代入すると

$$\psi(a) \doteq \psi_p = \frac{-1}{2\pi} \int_C \left[\mu S \frac{\partial S}{\partial n} + p \frac{\partial S}{\partial s} - \psi \frac{\partial \oint R}{\partial n} + p \nabla \psi \nabla S \frac{\partial X}{\partial n} \right] d$$

$$\frac{1}{i} = A \frac{\partial \oint R}{\partial x} + B \frac{\partial \oint R}{\partial y}, \quad (3.5.2)$$

$$A = \frac{1}{2\pi i \omega} \int_C \left[\mu S \frac{\partial X}{\partial n} + p \frac{\partial X}{\partial s} - i \omega \psi \frac{\partial X}{\partial n} + p \frac{\partial \psi}{\partial x} \frac{\partial X}{\partial n} \right] dS,$$

$$B = \frac{1}{2\pi i \omega} \int_C \left[\mu S \frac{\partial y}{\partial n} + p \frac{\partial y}{\partial s} - i \omega \psi \frac{\partial y}{\partial n} + p \frac{\partial \psi}{\partial y} \frac{\partial X}{\partial n} \right] dS,$$

とすると 無限遠 (a) での 2 重積分と等しい。

where

$$(u, h) \approx \psi_h, \quad (u, s) = \psi_s \quad \text{左右 (2)}$$

$$\int \left(\psi_h^{(1)}(u, s) + \psi_s^{(1)} \right) \quad (1)$$

$$\left. \begin{aligned} \frac{p}{p} &= - (i\omega + \frac{\partial}{\partial x}) \phi_p \\ \mu \dot{\phi} &= - (i\omega + \frac{\partial}{\partial x}) \psi_H \end{aligned} \right\}$$

$\omega = 0$ " " " "

$$\left. \begin{aligned} \frac{p}{p} &= - \frac{\partial \phi_p}{\partial x} = - \frac{\partial \psi_p}{\partial y} \\ \mu \dot{\phi} &= + \psi_H = \frac{\partial \psi_H}{\partial x} \end{aligned} \right\}$$

$$\begin{aligned} \frac{\partial \psi_p}{\partial s} &= \frac{\partial \psi_p}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial \psi_p}{\partial y} \frac{\partial y}{\partial s} = - \frac{p}{p} \frac{\partial y}{\partial s} + \frac{\partial \psi_H}{\partial x} \frac{\partial x}{\partial s} - \mu \dot{\phi} \frac{\partial x}{\partial s} \\ &= \frac{\partial \psi_H}{\partial x} \frac{\partial x}{\partial s} - \frac{p}{p} \frac{\partial y}{\partial s} + \mu \dot{\phi} \frac{\partial y}{\partial s} \end{aligned}$$

$$\frac{\partial \psi_p}{\partial t} = \frac{\partial \psi_p}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial \psi_p}{\partial y} \frac{\partial y}{\partial t} = - \frac{p}{p} \frac{\partial y}{\partial t} - \mu \dot{\phi} \frac{\partial x}{\partial t} + \frac{\partial \psi_H}{\partial x} \frac{\partial x}{\partial t}$$

$$\frac{\partial \psi_H}{\partial s} = - \frac{\partial \psi_H}{\partial x} \frac{\partial y}{\partial s} + \frac{\partial \psi_H}{\partial y} \frac{\partial x}{\partial s} = \frac{\partial \psi_H}{\partial y} \frac{\partial x}{\partial s} + \frac{p}{p} \frac{\partial y}{\partial s} - \mu \dot{\phi} \frac{\partial y}{\partial s}$$

$$\frac{\partial \psi_H}{\partial t} = \frac{\partial \psi_H}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial \psi_H}{\partial y} \frac{\partial y}{\partial t} = \frac{\partial \psi_H}{\partial y} \frac{\partial y}{\partial t} + \frac{p}{p} \frac{\partial y}{\partial t} + \mu \dot{\phi} \frac{\partial y}{\partial t}$$

$$\psi_{Fs} \frac{\partial y}{\partial t} + \psi_{Ft} \frac{\partial y}{\partial t} = \frac{\partial y}{\partial t} + \frac{p}{p} //$$

$$\psi_{Fs} \frac{\partial y}{\partial t} - \psi_{Ft} \frac{\partial x}{\partial t} = - \mu \dot{\phi} //$$

Circle (-1)

$$\mathcal{I} \log(w-1) = \begin{cases} 0 \text{ or } \pi, & -\frac{\pi}{2} > \theta > -\frac{\pi}{2} \\ \theta - \frac{\pi}{2}, & \text{for } \pi > \theta > \frac{\pi}{2} \\ \theta + \frac{\pi}{2}, & \text{for } \frac{\pi}{2} > \theta > -\pi \end{cases}$$



$$\overline{H}(z) = \lim_{r \rightarrow 1} f(z) = \sum_{n=0}^{\infty} \frac{A_n}{z^n}$$

$$A_0 = -$$

$$\varphi(\theta) = \mathcal{I} \left\{ \overline{H} \right\} \Big|_{|z|=1} = \sum_{n=1}^{\infty} A_n \lambda^n n \theta$$

$$A_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \varphi(\theta) \lambda^{-n} n \theta d\theta$$

$$\sum_{n=1}^{\infty} \frac{\lambda^n n \theta}{z^n} = \frac{\frac{1}{z} \lambda \theta}{1 - \frac{2}{z} \cos \theta + \frac{1}{z^2}}$$

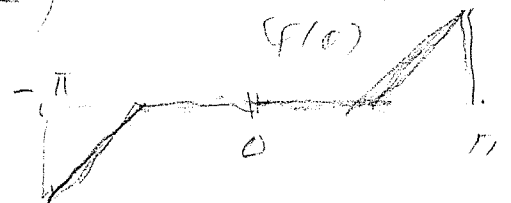
$$= \frac{\lambda \lambda \theta}{(z + \frac{1}{z}) - 2 \cos \theta}$$

$$\begin{aligned} \therefore \overline{H}(z) &= \frac{1}{\pi} \int_{-\pi}^{\pi} \varphi(\theta) d\theta \cdot \sum_{n=1}^{\infty} \frac{\lambda^n n \theta}{z^n} \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\varphi(\theta) \lambda \theta d\theta}{\frac{1}{2}(z + \frac{1}{z}) - \cos \theta} \end{aligned}$$

$$\int_0^{\frac{\pi}{2}} \frac{n \theta d\theta}{\cos u - \cos \theta} = \mathcal{I} (\cos u - \cos \theta) \Big|_{\theta=0}^{\frac{\pi}{2}} = \mathcal{I} \left(\frac{\cos u}{\cos u - 1} \right)$$

$$\int_{-\frac{\pi}{2}}^0 \frac{n \theta d\theta}{\cos u - \cos \theta} = \mathcal{I} \left(\frac{\cos u - 1}{\cos u} \right)$$

$$A_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \varphi(\theta) \lambda^n n \theta d\theta$$



$$\varphi(\theta) = \sum A_n \lambda^n n \theta$$

$$\begin{aligned} &= \frac{2}{\pi} \int_{\frac{\pi}{2}}^{\pi} (\theta - \frac{\pi}{2}) \lambda^n n \theta d\theta = \frac{2}{n\pi} \lambda^n \int_{\frac{\pi}{2}}^{\pi} \theta^2 d\theta \\ &= \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \theta \lambda^n \left(\frac{\pi}{2} - \theta \right) d\theta = \frac{2}{\pi} \left[\lambda^n \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \theta d\theta + \cos \frac{n\pi}{2} \int_0^{\frac{\pi}{2}} \theta \lambda^n \theta d\theta \right] \\ &= \frac{2}{\pi} \left[-\frac{(\theta - \frac{\pi}{2})}{n} \lambda^n n \theta \Big|_{\frac{\pi}{2}}^{\pi} + \frac{1}{n} \int_{\frac{\pi}{2}}^{\pi} \cos n \theta d\theta \right] \end{aligned}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8} = 1.234... = \frac{1}{.811}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad \log(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n 2^n} = -\log\left(1 + \frac{1}{2}\right)$$

$$S = -1 - \frac{1}{9} + \frac{1}{25} - \frac{1}{49} + \frac{1}{81} - \frac{1}{121} + \frac{1}{169} - \frac{1}{225} + \frac{1}{289} - \frac{1}{361} + \frac{1}{441} - \frac{1}{529} + \frac{1}{625} - \frac{1}{729} + \frac{1}{841} - \frac{1}{961} + \frac{1}{1089} - \frac{1}{1210} + \frac{1}{1369} - \frac{1}{1521} + \frac{1}{1681} - \frac{1}{1849} + \frac{1}{2025} - \frac{1}{2209} + \frac{1}{2401} - \frac{1}{2601} + \frac{1}{2809} - \frac{1}{3025} + \frac{1}{3249} - \frac{1}{3481} + \frac{1}{3721} - \frac{1}{3969} + \frac{1}{4225} - \frac{1}{4489} + \frac{1}{4761} - \frac{1}{5041} + \frac{1}{5329} - \frac{1}{5625} + \frac{1}{5929} - \frac{1}{6241} + \frac{1}{6561} - \frac{1}{6889} + \frac{1}{7225} - \frac{1}{7577} + \frac{1}{7937} - \frac{1}{8311} + \frac{1}{8699} - \frac{1}{9101} + \frac{1}{9517} - \frac{1}{9947} + \frac{1}{10391} - \frac{1}{10849} + \frac{1}{11321} - \frac{1}{11807} + \frac{1}{12307} - \frac{1}{12821} + \frac{1}{13349} - \frac{1}{13891} + \frac{1}{14447} - \frac{1}{15017} + \frac{1}{15601} - \frac{1}{16201} + \frac{1}{16817} - \frac{1}{17449} + \frac{1}{18097} - \frac{1}{18761} + \frac{1}{19441} - \frac{1}{20137} + \frac{1}{20849} - \frac{1}{21577} + \frac{1}{22321} - \frac{1}{23081} + \frac{1}{23857} - \frac{1}{24649} + \frac{1}{25457} - \frac{1}{26281} + \frac{1}{27121} - \frac{1}{27977} + \frac{1}{28849} - \frac{1}{29737} + \frac{1}{30641} - \frac{1}{31561} + \frac{1}{32497} - \frac{1}{33449} + \frac{1}{34417} - \frac{1}{35401} + \frac{1}{36401} - \frac{1}{37417} + \frac{1}{38449} - \frac{1}{39497} + \frac{1}{40561} - \frac{1}{41641} + \frac{1}{42737} - \frac{1}{43849} + \frac{1}{44977} - \frac{1}{46121} + \frac{1}{47281} - \frac{1}{48457} + \frac{1}{49649} - \frac{1}{50857} + \frac{1}{52081} - \frac{1}{53321} + \frac{1}{54577} - \frac{1}{55849} + \frac{1}{57137} - \frac{1}{58441} + \frac{1}{59761} - \frac{1}{61097} + \frac{1}{62449} - \frac{1}{63817} + \frac{1}{65201} - \frac{1}{66601} + \frac{1}{68017} - \frac{1}{69449} + \frac{1}{70897} - \frac{1}{72361} + \frac{1}{73841} - \frac{1}{75337} + \frac{1}{76849} - \frac{1}{78377} + \frac{1}{79921} - \frac{1}{81481} + \frac{1}{83057} - \frac{1}{84649} + \frac{1}{86257} - \frac{1}{87881} + \frac{1}{89521} - \frac{1}{91177} + \frac{1}{92849} - \frac{1}{94537} + \frac{1}{96241} - \frac{1}{97961} + \frac{1}{99697} - \frac{1}{101449} + \frac{1}{103217} - \frac{1}{105001} + \frac{1}{106797} - \frac{1}{108601} + \frac{1}{110421} - \frac{1}{112257} + \frac{1}{114101} - \frac{1}{115961} + \frac{1}{117837} - \frac{1}{119729} + \frac{1}{121637} - \frac{1}{123561} + \frac{1}{125501} - \frac{1}{127457} + \frac{1}{129421} - \frac{1}{131401} + \frac{1}{133397} - \frac{1}{135401} + \frac{1}{137421} - \frac{1}{139449} + \frac{1}{141481} - \frac{1}{143537} + \frac{1}{145601} - \frac{1}{147681} + \frac{1}{149777} - \frac{1}{151881} + \frac{1}{153901} - \frac{1}{155937} + \frac{1}{157981} - \frac{1}{159941} + \frac{1}{161917} - \frac{1}{163901} + \frac{1}{165897} - \frac{1}{167901} + \frac{1}{169921} - \frac{1}{171949} + \frac{1}{173981} - 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\frac{1}{5113701} + \frac{1}{5128921} - \frac{1}{5144161} + \frac{1}{5159421} - \frac{1}{5174701} + \frac{1}{5190001} - \frac{1}{5205321} + \frac{1}{5220661} - \frac{1}{5236021} + \frac{1}{5251401} - \frac{1}{5266801} + \frac{1}{5282221} - \frac{1}{5297661} + \frac{1}{5313121} - \frac{1}{5328601} + \frac{1}{5344101} - \frac{1}{5359621} + \frac{1}{5375161} - \frac{1}{5390721} + \frac{1}{5406301} - \frac{1}{5421901} + \frac{1}{5437521} - \frac{1}{5453161} + \frac{1}{5468821} - \frac{1}{5484501} + \frac{1}{5499201} - \frac{1}{5514921} + \frac{1}{5530661} - \frac{1}{5546421} + \frac{1}{5562201} - \frac{1}{5578001} + \frac{1}{5593821} - \frac{1}{5609661} + \frac{1}{5625521} - \frac{1}{5641401} + \frac{1}{5657301} - \frac{1}{5673221} + \frac{1}{5689161} - \frac{1}{5705121} + \frac{1}{5721101} - \frac{1}{5737101} + \frac{1}{5753121} - \frac{1}{5769161} + \frac{1}{5785221} - \frac{1}{5801301} + \frac{1}{5817401} - \frac{1}{5833521} + \frac{1}{5849661} - \frac{1}{5865821} + \frac{1}{5882001} - \frac{1}{5898201} + \frac{1}{5914421} - \frac{1}{5930661} + \frac{1}{5946921} - \frac{1}{5963201} + \frac{1}{5979501} - \frac{1}{5995821} + \frac{1}{6012161} - \frac{1}{6028521} + \frac{1}{6044901} - \frac{1}{6061301} + \frac{1}{6077721} - \frac{1}{6094161} + \frac{1}{6110621} - \frac{1}{6127101} + \frac{1}{6143601} - \frac{1}{6160121} + \frac{1}{6176661} - \frac{1}{6193221} + \frac{1}{6209801} - \frac{1}{6226401} + \frac{1}{6243021} - \frac{1}{6259661} + \frac{1}{6276321} - \frac{1}{6293001} + \frac{1}{6309701} - \frac{1}{6326421} + \frac{1}{6343161} - \frac{1}{6359921} + \frac{1}{6376701} - \frac{1}{6393501} + \frac{1}{6410321} - \frac{1}{6427161} + \frac{1}{6444021} - \frac{1}{6460901} + \frac{1}{6477801} - \frac{1}{6494721} + \frac{1}{6511661} - \frac{1}{6528621} + \frac{1}{6545601} - \frac{1}{6562601} + \frac{1}{6579621} - \frac{1}{6596661} + \frac{1}{6613721} - \frac{1}{6630801} + \frac{1}{6647901} - \frac{1}{6665021} + \frac{1}{6682161} - \frac{1}{6699321} + \frac{1}{6716501} - \frac{1}{6733701} + \frac{1}{6750921} - \frac{1}{6768161} + \frac{1}{6785421} - \frac{1}{6802701} + \frac{1}{6820001} - \frac{1}{6837321} + \frac{1}{6854661} - \frac{1}{6872021} + \frac{1}{6889401} - \frac{1}{6906801} + \frac{1}{6924221} - \frac{1}{6941661} + \frac{1}{6959121} - \frac{1$$

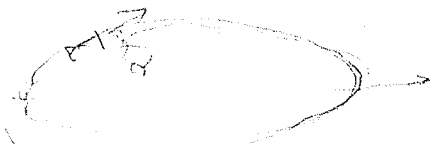
Ellipse

E-1

$$(u+1) - iV = \rho e^{-i\theta} = e^{f(z)}$$

$$f[f(z)] = -\alpha$$

$$dz = -d\theta [a \sin \theta - i b \cos \theta]$$



$$d = -\frac{d\theta}{2} [a \frac{e^{i\theta} - e^{-i\theta}}{i} - i b (e^{i\theta} + e^{-i\theta})]$$

$$z = a \cos \theta + i b \sin \theta$$

$$= \frac{i d\theta}{2} [(a+b) e^{i\theta} - (a-b) e^{-i\theta}] \quad z = \frac{a+b}{2} e^{i\theta} + \frac{a-b}{2} e^{-i\theta}$$

$$ds = \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta} d\theta$$

$$\xi = \frac{a+b}{2} \left(1 + \frac{e^{i\theta}}{2} \right)$$

$$e^{i\theta} = \frac{a-b}{a+b}$$

$$\frac{dz}{ds} = \frac{-a \sin \theta + i b \cos \theta}{\sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}} = e^{i(\pi + \alpha)}$$

$$\alpha = \frac{1}{2i} \log \left(\frac{a \sin \theta + i b \cos \theta}{a \cos \theta + i b \sin \theta} \right) = \frac{1}{2i} \log \left(\frac{(a+b) e^{i\theta} - (a-b) e^{-i\theta}}{(a-b) e^{i\theta} - (a+b) e^{-i\theta}} \right)$$

$$= \frac{1}{2i} \log \left(\frac{\xi - \frac{\xi^2}{\xi^2 - \frac{1}{\xi}}}{\xi^2 - \frac{1}{\xi}} \right) = \frac{1}{2i} \left[2 \log \xi + \log \left(\frac{1 - \xi^2}{1 - \xi^2 \xi^2} \right) \right]$$

$$\log \left(\frac{1 - \xi^2}{1 - \xi^2 \xi^2} \right) = - \sum_{n=1}^{\infty} \frac{\xi^{2n}}{n \xi^{2n}} + \sum_{n=1}^{\infty} \frac{(\xi^2 \xi^2)^n}{n \xi^{2n}}$$

$$= - \sum_{n=1}^{\infty} \frac{\xi^{2n}}{n} \left[e^{-2ni\theta} - e^{2ni\theta} \right]$$

$$= 2i \sum_{n=1}^{\infty} \frac{\xi^{2n}}{n} \sin 2n\theta$$

$$\alpha = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{\xi^{2n}}{n} \sin 2n\theta$$

$$= \dots + 2 \sum_{n=1}^{\infty} \frac{\xi^{4n+2}}{2n+1} \cos 4n\theta$$

$$f(x) = \sum_{n=1}^{\infty} \frac{A_n}{z^n}, \quad \mathcal{D}f = -\sum_{n=1}^{\infty} A_n n \theta$$

$$A_n = \frac{1}{\pi} \int_{-\pi}^{\pi} [\mathcal{D}f] n \theta d\theta = \frac{2}{\pi} \int_0^{\pi} [\mathcal{D}f] n \theta d\theta$$

$$= \frac{2}{\pi} \int_{\frac{\pi}{2}}^{\pi} [-\alpha] n \theta d\theta$$

$$-\alpha = \frac{\pi}{2} - \theta - \sum_{\nu=1}^{\infty} \frac{\varepsilon^{2\nu}}{\nu} n \nu \theta$$

$$\int_{\frac{\pi}{2}}^{\pi} (\frac{\pi}{2} - \theta) n \theta d\theta = -\frac{(\frac{\pi}{2} - \theta)}{n} \cos n\theta \Big|_{\frac{\pi}{2}}^{\pi} = \frac{1}{n} \int_{\frac{\pi}{2}}^{\pi} \cos n\theta d\theta$$

$$= + \frac{\pi \cos n\pi}{2n} + \frac{n' \frac{n}{2} \pi}{n^2}$$

$$\int_{\frac{\pi}{2}}^{\pi} n \nu \theta n \theta d\theta = \frac{1}{2} \int_{\frac{\pi}{2}}^{\pi} [\cos 2\nu - n \theta - \cos 2\nu + n \theta] d\theta$$

$$= \frac{1}{2} \left[\frac{-\cos(2\nu - n)\frac{\pi}{2}}{2\nu - n} - \frac{-\cos(2\nu + n)\frac{\pi}{2}}{2\nu + n} \right]$$

$$\cos n\pi n' \frac{n}{2} \pi$$

$$= \frac{\cos n\pi}{2} n' \frac{n}{2} \pi \left[\frac{1}{2\nu - n} + \frac{1}{2\nu + n} \right] = \frac{2\nu \cos n\pi}{4\nu^2 - n^2} n' \frac{n}{2} \pi$$

$$A_n = \frac{\cos n\pi}{n} + \frac{2 n' \frac{n}{2} \pi}{\pi n^2} - \frac{2}{\pi} \sum_{\nu=1}^{\infty} \frac{\varepsilon^{2\nu}}{\nu} \left[\frac{2\nu \cos n\pi}{4\nu^2 - n^2} n' \frac{n}{2} \pi \right]$$

$$= \frac{\cos n\pi}{n} + \frac{2}{\pi} n' \frac{n}{2} \pi \left[\frac{1}{n^2} - 2 \sum_{\nu=1}^{\infty} \frac{(-)^{\nu} \varepsilon^{2\nu}}{4\nu^2 - n^2} \right]$$

$$A_1 = -1 + \frac{2}{\pi} \left[1 - 2 \sum_{\nu=1}^{\infty} \frac{(-)^{\nu} \varepsilon^{2\nu}}{4\nu^2 - 1} \right]$$

$$2 - 1 + \frac{2}{\pi} \left[+ \frac{1}{2} \left(\varepsilon + \frac{1}{\varepsilon} \right) \delta \right], \quad \tan \delta = \varepsilon = \sqrt{\frac{a-b}{a+b}}$$

$$= \frac{2}{\pi} \left[\frac{1}{2} (1 + \varepsilon^2) \frac{\delta}{\varepsilon} - \frac{\pi}{2} \right] \xrightarrow{a \rightarrow b} \frac{2}{\pi} \left(\frac{\pi}{2} - \frac{\pi}{2} \right)$$

$$D = -4(2 - \pi) //$$

$$\delta = \varepsilon \rightarrow 0$$

2.3

$$\sum_{n=0}^{\infty} \frac{(-1)^n X^{2n}}{2n+1} = I = \frac{\log\left(\frac{1-iX}{1+iX}\right)}{2iX}$$

$$\frac{1}{2i} \left(\frac{1}{X-i} + \frac{1}{X+i} \right)$$

$$(XI)' = \sum_{n=0}^{\infty} (-1)^n X^{2n} = \frac{1}{1+X^2}, \quad I = \frac{1}{2} \int \frac{dx}{1+x^2} = \frac{\arctan(x)}{2}$$

$$\log(1-iX) = \sum_{n=1}^{\infty} \frac{(iX)^n}{n} = \frac{1}{2iX} \log\left(\frac{X-i}{X+i}\right)$$

$$\log(1+iX) = \sum_{n=1}^{\infty} \frac{(-1)^n (iX)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{(iX)^n (1-(-1)^n)}{n} = 2 \sum_{n=1}^{\infty} \frac{(iX)^{2n+1}}{2n+1} = 2iX \sum_{n=1}^{\infty} \frac{(-1)^n X^{2n}}{2n+1}$$

$$\frac{2}{4X^2-1} = \left(\frac{1}{2X-1} - \frac{1}{2X+1} \right)$$

$$\sum_{n=1}^{\infty} \frac{2}{4X^2-1} (-1)^n X^{2n} = \sum_{n=1}^{\infty} \left(\frac{1}{2X-1} - \frac{1}{2X+1} \right) (-1)^n X^{2n}$$

$$= -\varepsilon^2 I - [I - 1] = 1 - (1+\varepsilon^2)I$$

$$= 1 - \frac{(1+\varepsilon^2)}{2i\varepsilon} \log\left(\frac{1-i\varepsilon}{1+i\varepsilon}\right)$$

$$= 1 + \frac{\pi(1+\varepsilon^2)}{\varepsilon} \delta$$

$$\xrightarrow{\varepsilon \rightarrow 0} 1 + \frac{\pi}{2}$$

$$\varepsilon = \gamma \delta$$

$$\varepsilon = \frac{\sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2}}$$

$$\frac{b}{a} \rightarrow 0$$

$$C_D = 4 \left[\pi - \frac{\pi(1+\varepsilon^2)}{\varepsilon} \right]$$

$$\tan \delta = \varepsilon$$

$$\varepsilon = \frac{\sqrt{1 - \frac{b^2}{a^2}}}{\sqrt{1 + \frac{b^2}{a^2}}}$$

$\frac{b}{a}$				
ε				
δ				
C_D				

$$D = 2\pi A_1 = 4 \left(\frac{\pi}{2} - 1 \right) = 2\pi - 4 = 2.28$$

$$A_1 = \frac{2}{\pi} \left(\frac{\pi}{2} - 1 \right)$$

$$A_n = \frac{2}{\pi} \left[-\frac{\pi}{2n} \cos 2n\pi + \frac{\lambda^{2n}\pi - \lambda^{2n}\pi}{n^2} \right]$$

$$A_{2n+1} = \frac{2}{\pi} \left[+\frac{\pi}{2(2n+1)} + \frac{\cos n\pi}{(-n\pi)^2} \right]$$

$$A_{2n} = \frac{2}{\pi} \left[-\frac{\pi}{4n} \right] = -\frac{1}{2n}$$

$$A_{2n+1} = \frac{1}{2n+1} - \frac{2 \cos n\pi}{(2n+1)^2} = \frac{1}{2n+1} - \frac{2(-1)^n}{(2n+1)^2}$$

$$\therefore \bar{F}(z) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)z^{2n+1}} + \sum_{n=1}^{\infty} \frac{1}{2n z^{2n}} + \log\left(1 + \frac{1}{z}\right) - \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2 z^{2n+1}}$$

$$\frac{1}{1 + \frac{1}{z}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{z^n} \quad (|z| > 1)$$

$$\int \frac{dz}{z(1 + \frac{1}{z})} = \int \sum_{n=0}^{\infty} \frac{(-1)^n}{n z^{n+1}} dz = \log(z+1)$$

$$\bar{I} = \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2 z^{2n+1}}$$

$$\frac{1}{z} = t, \quad z = \frac{1}{t}$$

$$\bar{I}'' = \frac{2}{\pi} \sum \frac{dz}{1 + \frac{1}{z}} = -\frac{dt}{t^2(1+t)}$$

$$\bar{I}' = -\frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1) z^{2n+2}}, \quad \left(\frac{\bar{I}}{z} \right)' = \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{z^{2n+2}} \frac{2}{\pi} \log\left(1 + \frac{1}{z}\right)$$

$$\frac{z \bar{I}'}{\pi} = -\frac{2}{\pi} \int \frac{dz}{z^2(1 + \frac{1}{z})} = \frac{2}{\pi} \int \frac{dt}{1+t} = \frac{2}{\pi} \log\left(\frac{1+t}{1-t}\right)$$

$$= \frac{2}{\pi} \left[t - \frac{1}{2t} \left(\frac{1}{t-i} - \frac{1}{t+i} \right) \right] dt = \frac{2}{\pi} \left[t - \frac{1}{2i} \log \frac{t-i}{t+i} \right]$$

$$\bar{I}' = \frac{2}{\pi} z \left[\frac{1}{z} - \frac{1}{2iz} \log \frac{1-i/z}{1+i/z} \right] = \frac{2}{\pi} \left[\frac{1}{z} - \frac{1}{2iz} \log \left(\frac{z+i}{z-i} \right) \right]$$

~~2.1.2.1.3~~

E-5

err.

$$C_D = 4 \left[\frac{\pi}{2} - \frac{\pi}{2} (1 + \epsilon^2) \frac{\delta}{\epsilon} \right]$$

$$\delta = \left(\frac{\pi}{4} - \alpha \right)$$

$$\frac{h}{a} \rightarrow 0 \quad \epsilon \div 1 - \frac{h}{a}, \quad \tan \delta \div 1 - \frac{h}{a} = \frac{\cos(\frac{\pi}{4} - \alpha)}{\sin(\frac{\pi}{4} + \alpha)}$$

$$\delta \div \frac{\pi}{4} - \frac{h}{4a}$$

$$= \frac{\cos \alpha - \sin \alpha}{\cos \alpha + \sin \alpha}$$

$$1 + \epsilon^2 \frac{\delta}{\epsilon} = \frac{2 - \frac{h}{a}}{1 - \frac{h}{a}}$$

$$\div 1 - 2 \tan \alpha$$

$$\epsilon = \sqrt{\frac{1 - \frac{h}{a}}{1 + \frac{h}{a}}}$$

$$\alpha \div \frac{h}{2a}$$

$$\div \frac{2}{\pi} \left(1 - \frac{h}{2a} \right) \left(1 + \frac{h}{2a} \right) \left(\frac{\pi}{4} - \frac{h}{2a} \right) \div \frac{\pi}{2} \left(1 - \frac{2h}{\pi a} \right)$$

~~C₀ def.~~
~~err~~

$$C_D \div 4 \left[\frac{\pi}{2} \left(1 - \frac{2h}{\pi a} \right) \right] \div 2\pi \frac{2h}{\pi a} = \frac{4h}{a}$$

$\frac{1}{a}$

$\frac{h}{a}$	ϵ	δ	C_D	$C_D \frac{a}{h}$	$C_D = C_D \frac{a}{\sqrt{a^2 - h^2}}$
1	0	0	2.283	2.283	2.283
.5	.5774	.52359	1.4866	2.8932	4.0916
.25	.7746	.6591	.8378	3.3512	6.7024
.2	.8165	.6847	.6925	3.4625	
.1	.9045	.7353	.3711	3.711	
.05	.95119	.76039	.1925	3.8495	
0			0	4	
∞			∞	3.1416	
2	.5774	1.3171	3.2417	1.6209	.021 = $\frac{h}{a}$
3	.7746	1.7627	3.7903	1.2634	
5	.8165	2.2925	4.4114	.8823	
10	.9045	2.9932	5.0131	.5013	
∞	.95119		6.2832	0	
			0	3.1416	

err.

$$C_{DA} = \frac{C_D}{a + h}$$

$$\times \frac{1 + \frac{h}{a}}{2}$$



$$\frac{4}{2b} \left(1 + \frac{h}{a} \right)$$

$$a < b. \quad z = \frac{a+b}{2} \left(5 - \frac{\varepsilon^2}{\varepsilon} \right) \quad \varepsilon^2 = \frac{b-a}{a+b}$$

$$\alpha = \theta - \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n \varepsilon^{2n} \lambda'_{2n}}{2n}$$

$$A_n = \frac{\cos(n\theta)}{n} + \frac{2}{\pi n} \lambda'_{2n} \left[\frac{1}{2} - 2 \sum_{v=1}^{\infty} \frac{\varepsilon^{2v}}{4v^2 - 1} \right]$$

$$A_1 = -1 + \frac{2}{\pi} \left[1 - 2 \sum_{v=1}^{\infty} \frac{\varepsilon^{2v}}{4v^2 - 1} \right]$$

$$\sum_{v=1}^{\infty} \frac{2}{4v^2 - 1} \varepsilon^{2v} = \sum_{v=1}^{\infty} \left(\frac{1}{2v-1} - \frac{1}{2v+1} \right) \varepsilon^{2v} = \sum_{h=1}^{\infty} \frac{\varepsilon^{2h+2}}{2h+1} = \sum_{h=1}^{\infty} \frac{\varepsilon^{2h}}{2h+1}$$

$$\begin{aligned} \sum_{h=1}^{\infty} \frac{\varepsilon^{2h}}{2h+1} &= \sum_{h=1}^{\infty} \frac{x^{2h}}{2h} (1 - (-1)^h) = \sum_{h=1}^{\infty} \frac{x^{2h}}{2h} (1 - (-1)^h) \\ \ln \left(\frac{1-x}{1+x} \right) &= -2 \sum_{h=1}^{\infty} \frac{x^{2h}}{2h+1} \\ &= +1 + \frac{\ln(\varepsilon^2 + 1)}{2\varepsilon} \ln \left(\frac{1-\varepsilon}{1+\varepsilon} \right) \end{aligned}$$

$$A_1 = -1 + \frac{2}{\pi} \left[1 + \frac{\ln(\varepsilon^2 + 1)}{2\varepsilon} \ln \left(\frac{1-\varepsilon}{1+\varepsilon} \right) \right] \xrightarrow{\varepsilon \rightarrow 0} -1 + \frac{2}{\pi}$$

$$C_0 = -2\pi A_1 = 2 \left(\frac{\ln(\varepsilon^2 + 1)}{\varepsilon} \right) \ln \left(\frac{1-\varepsilon}{1+\varepsilon} \right) + 2\pi = 2\pi \left[1 - \frac{1-\varepsilon^2}{\pi \varepsilon} \ln \left(\frac{1+\varepsilon}{1-\varepsilon} \right) \right]$$

$$\xrightarrow{\varepsilon \rightarrow 1} 2\pi \left[1 - \frac{a}{\pi b} \ln \frac{2b}{a} \right] \xrightarrow{\varepsilon = \frac{b-a}{b+a}} 2\pi \left[1 - \frac{a}{\pi b} \ln \frac{2b}{a} \right]$$

$$\varepsilon = \frac{b-a}{b+a}, \quad \ln \frac{1+\varepsilon}{1-\varepsilon} = \ln \left(\frac{2 - \frac{a}{b}}{2 - \frac{a}{b}} \right) = \ln \left(\frac{2 - \frac{a}{b}}{2 - \frac{a}{b}} \right)$$

$$\frac{1 - \varepsilon^2}{\varepsilon} = \frac{2ab}{1 - \frac{a}{b}} \ln \left(\frac{2 - \frac{a}{b}}{2 - \frac{a}{b}} \right)$$

E-7

$$f(z) \doteq \frac{A_1}{z}$$

$$z = \frac{a+b}{2} \left(\xi \pm \frac{\varepsilon^2}{\xi} \right)$$

$$f(z) \doteq \frac{(a+b)A_1}{2z}$$

$$\xi^2 - \frac{2z}{a+b} \xi \pm \varepsilon^2 = 0$$

$$\xi = \frac{z}{a+b} \pm \sqrt{\left(\frac{z}{a+b}\right)^2 \mp \varepsilon^2}$$

$$\rightarrow \frac{z}{2(a+b)} \left[2 \mp \frac{4\varepsilon^2}{\left(\frac{z}{a+b}\right)^2} \right]$$

$$\doteq \frac{2z}{a+b}$$

$$D = \pi(a+b)A_1$$

$$C_{Da} = \pi \left(1 + \frac{b}{a}\right) A_1$$

$$C_{Db} = \pi \left(1 + \frac{a}{b}\right) A_1$$

(ii) ~~DA~~ b.

~~C_{Da}~~ a

$$\begin{cases} C_{Da} = \pi \left(1 + \frac{b}{a}\right) A_1 = 2 \left[\frac{\pi}{2} - \frac{(1+\varepsilon^2)}{\varepsilon} \delta \right] \left(1 + \frac{b}{a}\right) \\ C_{Db} = C_{Da} \times \frac{a}{b} \end{cases}$$

$$a < b$$

$$C_{Da} = \pi \left(1 + \frac{b}{a}\right) \left[1 - \frac{1-\varepsilon^2}{\pi \varepsilon} \ln \left(\frac{1+\varepsilon}{1-\varepsilon} \right) \right]$$