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平板のまわりのオゼーン流れ（迎角の小さい場合）

~~定常問題~~

~~線型的~~
~~1. 迎角 0~~
~~2. 迎角 α~~

非線型的
3. $\alpha=0$　ブラジウスとの比較
4. $\alpha\neq 0$　Liftの非線形影響

振動問題（迎角0.）

1. 線型的　前縁
2. 〃　後縁　ピッチングヨーイング？
3. 非　一様流れとの干渉　　不安定性
4. 〃

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平均の 左右振動

$$\frac{1}{\sqrt{2\pi k}} \int_0^{x'} \frac{Y}{\sqrt{x'-x}} e^{-\frac{k y'^2}{2(x'-x)} - i\omega(x'-x)} dx$$

$$Y = \frac{\sqrt{\cancel{x'-x}}}{\sqrt{x}} \doteqdot \frac{1}{\sqrt{x}}$$

$$\eta = \sqrt{\frac{k}{2x'}}\, y'$$

$$\int_0^{x'} \frac{e^{-\frac{k y'^2}{2(x'-x)} - i\omega(x'-x)}}{\sqrt{x(x'-x)}} dx = 2\int_0^{\frac{\pi}{2}} e^{-\frac{\eta^2}{\cos^2\theta} - i\omega x' \cos^2\theta} d\theta$$

$$= 2\int_0^1 \frac{e^{-\frac{k y'^2}{2x' t^2} + i\omega x' t^2}}{\sqrt{1-t^2}} dt$$

$$\frac{\eta^2}{\cos^2\theta} = \eta^2 - \frac{2\eta^2 \sin\theta}{\cos^3\theta}\Delta\theta + 2\eta^2 \Delta\theta^2$$

$$\left(\frac{1}{\cos^2\theta}\right)' = +\frac{2\sin\theta}{\cos^3\theta}, \quad \left(\frac{1}{\cos^2\theta}\right)'' = +\frac{2}{\cos^2\theta} + \frac{6\sin^2\theta}{\cos^4\theta}$$

$$(1-\sin^2\theta)$$

$$= 2 e^{-\eta^2 - i\omega x'} \int_0^{\infty} e^{(-2\eta^2 + i\omega)\theta^2} d\theta = \frac{2\sqrt{\pi}\, e^{-\eta^2 - i\omega x'}}{2\sqrt{2\eta^2 - i\omega x'}}$$

$$\frac{1}{\sqrt{2}\,\eta}\frac{\sqrt{\pi}}{2}$$

$$\simeq \frac{\sqrt{\pi}}{\sqrt{2}\,\eta} e^{-\eta^2 - i\omega x'} \quad \sqrt{\frac{\pi x'}{k}} \frac{1}{y'} e^{-\frac{k y'^2}{2x'}}$$

$$-\frac{k y'}{x'}$$

$$u_H = \frac{e^{-\frac{k}{2x'}y'^2 - i\omega x'}}{\sqrt{x'}}$$

$$\sqrt{\frac{k\pi}{x'}}$$

$$\omega\delta^2 = 0.1 \qquad \omega = \frac{0.1}{\delta^2} \cdots$$

$$u(x',y') = \sqrt{\frac{2k}{\pi}} \int_0^{x'} \frac{X(x)\, e^{+i\omega(x-x') - \frac{k y'^2}{2(x'-x)}}}{\sqrt{x'-x}}\, dx$$

$$u(x',0) = \sqrt{\frac{2k}{\pi}} \int_0^{x'} \frac{X(x)}{\sqrt{x'-x}}\, e^{i\omega(x-x')}\, dx$$

$$X(x) = \frac{e^{-i\omega x}}{\sqrt{2\pi k x}} + i\sqrt{\frac{\omega}{k}}\left[C\left(\sqrt{\frac{2\omega x}{T}}\right) - i S \right]$$

$$u(x',0) = e^{-i\omega x'} \sqrt{\frac{2k}{\pi}} \int_0^{x'} \left[\frac{1}{\sqrt{2\pi k x}} + e^{i\omega x} i \sqrt{\frac{\omega}{k}} (C - iS) \right] \frac{dx}{\sqrt{x'-x}}$$

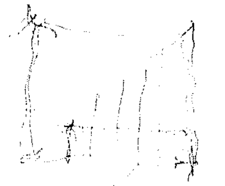
$$\int_0^{x'} \frac{dx}{\sqrt{x}\sqrt{x'-x}} = \int_0^1 \frac{dt}{\sqrt{t}\sqrt{1-t}} = 2\int_0^{\frac{\pi}{2}} d\theta = \pi \qquad t = \sin^2\theta \qquad \frac{1}{\pi}$$

$$u(x',0) = e^{-i\omega x'} + i\sqrt{\frac{2\omega}{\pi}} \int_0^{x'} (C - iS) \frac{e^{i\omega(x-x')}\, dx}{\sqrt{x'-x}} = 1$$

$$\sqrt{\frac{\omega}{2\pi}}$$

$$I = \frac{i\omega}{\pi} \int_0^{x'} \frac{e^{-i\omega x'}\, dx}{\sqrt{x'-x}} \cdot \int_0^{x} \frac{e^{i\omega t}\, dt}{\sqrt{x-t}} = \frac{\omega}{\pi} \int_0^{x'} \frac{e^{-i\omega(x'-x)}\, dx}{\sqrt{x'-x}} \cdot \int_0^{x} \frac{e^{i\omega(t-x)}\, dt}{\sqrt{x-t}}$$

$$= \frac{\omega}{\pi} \int_0^{x'} dt \int_t^{x'} \frac{e^{i\omega(t-x')}\, dx}{\sqrt{x'-x}\sqrt{x-t}} = \omega \int_0^{x'} e^{i\omega(t-x')}\, dt$$



$$I = \frac{1 - e^{-i\omega x'}}{i}$$

$$-x^2 + (x'+t)x - x't = -\left(x - \frac{x'+t}{2}\right)^2 - x't + \frac{(x'+t)^2}{4}$$

$$\frac{(x'-t)^2}{4}$$

$$\int_t^{x'} \frac{dx}{\sqrt{(x'-x)(x-t)}} = \int_t^{x'} \frac{dx}{\sqrt{\frac{(x'-t)^2}{4} - \left(x - \frac{x'+t}{2}\right)^2}}$$

$$x - \frac{x'+t}{2} = a \sin\theta$$

$$\frac{x'-t}{2} = a$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta = \pi$$

$$M = 2\int_0^1 X\,dx = \sqrt{\frac{2}{\pi k}}\int_0^1 \frac{e^{-i\omega x}}{\sqrt{x}}\,dx + 2i\sqrt{\frac{\omega}{k}}\int_0^1 dx\,\sqrt{\frac{\omega}{2\pi}}\int_0^x e^{-i\omega u}\frac{du}{\sqrt{u}}$$

$$\omega i\sqrt{\frac{2}{\pi k}}\left[x\int_0^x \frac{e^{-i\omega u}}{\sqrt{u}}du - \int_0^1 \sqrt{x}\,e^{-i\omega x}dx\right]$$

$$\int_0^1 dx\int_0^x \frac{e^{-i\omega u}}{\sqrt{u}}du = x\int_0^x \frac{e^{-i\omega u}}{\sqrt{u}}du - \int_0^1 \sqrt{x}\,e^{-i\omega x}dx$$

$$= \int_0^1 \frac{e^{-i\omega u}}{\sqrt{u}}du - \left[\frac{\sqrt{x}\,e^{-i\omega x}}{-i\omega} - \frac{1}{2i\omega}\int_0^1 \frac{e^{-i\omega x}}{\sqrt{x}}dx\right]$$

$$= \left(1 - \frac{i}{2\omega}\right)\int_0^1 \frac{e^{-i\omega u}}{\sqrt{u}}du - \frac{i}{\omega}e^{-i\omega}$$

$$M = \sqrt{\frac{2}{\pi k}}\left[\left(1 + \frac{1}{2} + i\omega\right)\underbrace{\int_0^1 \frac{e^{-i\omega u}}{\sqrt{u}}du}_{=\sqrt{\frac{2\pi}{\omega}}(C - iS)} + e^{-i\omega}\right]$$

$$M = \sqrt{\frac{2}{\pi k}}\left[\left(\frac{3}{2} + i\omega\right)\sqrt{\frac{2\pi}{\omega}}\left\{C\left(\sqrt{\frac{2\omega}{\pi}}\right) - iS\right\} + e^{-i\omega}\right]$$

平板の場合は $D_0 = \int_0^1 V\,dx = 2\sqrt{\frac{2}{\pi k}}$

$\omega \to 0$ のとき $\int_0^1 \frac{e^{-i\omega u}}{\sqrt{u}}du = \int_0^1 \frac{du}{\sqrt{u}} = 2\left[\sqrt{u}\right]_0^1 = 2$.

$\therefore\ M \longrightarrow \sqrt{\frac{2}{\pi k}}\left(\frac{3}{2}\times 2 + 1\right) = 4\sqrt{\frac{2}{\pi k}} = 2D_0$

added mass.

$x' - x = u$

$$u = u_1 + u_2$$

$$u_1 = \sqrt{\frac{2k}{\pi}} \frac{e^{-i\omega x'}}{\sqrt{2\pi k}} \int_0^{x'} \frac{e^{-\frac{k y'^2}{2(x'-x)}}}{\sqrt{x(x'-x)}} dx = \frac{e^{-i\omega x'}}{\pi} \int_0^{x'} \frac{e^{-\frac{k y'^2}{2u}}}{\sqrt{u(x'-u)}} du$$

$$= \frac{e^{-i\omega x'}}{\pi} \int_0^1 \frac{e^{-\frac{k y'^2}{2x' t}}}{\sqrt{t(1-t)}} dt$$

$\frac{k y'^2}{2x'} = \eta^2$ //

$$= \frac{e^{-i\omega x'}}{\pi} \int_0^1 \frac{e^{-\eta^2/t}\, dt}{\sqrt{t(1-t)}}$$

$\frac{\eta^2}{t} = u^2, \quad dt = -\frac{2\eta^2}{u^3} du$

$\frac{dt}{\sqrt{t(1-t)}} = -\frac{2\eta^2 du}{u^3 \frac{\eta}{u}\sqrt{1-\frac{\eta^2}{u^2}}} = \frac{-2\eta\, du}{u\sqrt{u^2-\eta^2}}$

$$= \frac{2\eta}{\pi} e^{-i\omega x'} \int_\eta^\infty \frac{e^{-u^2} du}{u\sqrt{u^2-\eta^2}} \quad //$$

$$= \frac{2}{\pi} e^{-i\omega x'} \int_0^{\frac{\pi}{2}} e^{-\eta^2 \sec^2\theta} d\theta$$

$d\theta = \frac{du}{\cosh u}$

$$= \frac{2}{\pi} e^{-i\omega x'} \int_0^\infty e^{-\eta^2 \cosh^2 u} \frac{du}{\cosh u} \quad //$$

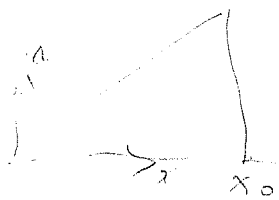
$$= e^{-i\omega x'} \operatorname{erfc}(\eta) \quad //$$

$$u_2 = i\sqrt{\frac{2\omega}{\pi}}\sqrt{\frac{\omega}{\pi}}\int_0^{x'}\left[C\sqrt{\frac{2\omega x}{\pi}} - iS\right]\frac{e^{i\omega(x-x') - \frac{f_1 y'^2}{2(x'-x)}}}{\sqrt{x'-x}}\,dx$$

$$\cancel{e}^{i\omega v}\Big[C - iS\Big] = \sqrt{\frac{\omega}{2\pi}}\int_0^{x}\frac{e^{i\omega \cancel{t}}\,d\cancel{t}}{\sqrt{x-t}}$$

$$u_2 = \frac{i\omega}{\pi}\int_0^{x'}\frac{e^{-i\omega x' - \frac{\eta^2 x'}{x'-x}}}{\sqrt{x'-x}}\,dx \cdot \int_0^{x}\frac{e^{i\omega t}}{\sqrt{x-t}}\,dt.$$

$$= \frac{i\omega}{\pi}\int_0^{x'}dt\int_t^{x'}\frac{e^{-\frac{\eta^2 x'}{x'-x} + i\omega(t-x')}\,dx}{\sqrt{(x'-x)(x-t)}}$$



$$x - \frac{x'+t}{2} = a\sin\theta, \qquad \frac{x'-t}{2} = a.$$

$$x'-x = x' - \left(\frac{x'+t}{2} + a\sin\theta\right) = a(1-\sin\theta)$$

$$= \frac{i\omega}{\pi}\int_0^{x'}dt\; e^{i\omega(t-x')}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-\frac{\eta^2 x'}{a(1-\sin\theta)}}\,d\theta.$$

$\theta = -\frac{\pi}{2} + \vartheta$

$\sin\theta = -\cos\vartheta.$

$$\int_0^{\pi} e^{-\frac{\eta^2 x'}{a(1+\cos\vartheta)}}\,d\vartheta$$

$\vartheta = 2u$

$1+\cos\vartheta = 2\cos^2 u$

$$2\int_0^{\frac{\pi}{2}} e^{-\frac{x'\eta^2}{2a\cos^2 u}}\,du = 2\int_0^{\infty} e^{-\frac{x'\eta^2}{2a}\operatorname{ch}^2 u}\,\frac{du}{\operatorname{ch} u}$$

$$u_2 = \frac{2i\omega}{\pi}\int_0^{x'}dt\; e^{i\omega(t-x')}\int_0^{\infty} e^{-\frac{\eta^2 x'}{\cancel{x'-t}}\operatorname{ch}^2 u}\,\frac{du}{\operatorname{ch} u}$$

$x'-t = \xi$

$$= \frac{2i\omega}{\pi}\int_0^{x'} e^{-\frac{\eta^2 x'}{\xi}\operatorname{ch}^2 u - i\omega\xi}\,d\xi\int_0^{\infty}\frac{du}{\operatorname{ch} u}.$$

$$= i\omega\int_0^{x'} e^{-i\omega\xi}\operatorname{erfc}\left(\eta\sqrt{\frac{x'}{\xi}}\right)d\xi$$

$$J(z) = \int_0^\infty e^{-z\,\mathrm{ch}^2 u} \frac{du}{\mathrm{ch}\,u}, \quad J(0) = \int_0^\infty \frac{du}{\mathrm{ch}\,u} = \frac{\pi}{2}$$

$$\mathrm{ch}^2 u = \frac{1}{4}(e^{2u} + e^{-2u} + 2) = \frac{1+\mathrm{ch}\,2u}{2}$$

$$\frac{\partial}{\partial z} J = -\int_0^\infty e^{-z\,\mathrm{ch}^2 u}\,\mathrm{ch}\,u\,du = -e$$

$$= -e^{-\frac{z}{2}} \int_0^\infty e^{-\frac{z}{2}\mathrm{ch}\,2u}\,\mathrm{ch}\,u\,du = -\frac{e^{-\frac{z}{2}}}{2} \int_0^\infty e^{-\frac{z}{2}\mathrm{ch}\,v}\,\mathrm{ch}\frac{v}{2}\,dv \quad (u = \tfrac{v}{2})$$

$$= -\frac{e^{-\frac{z}{2}}}{2} K_{\frac{1}{2}}\left(\frac{z}{2}\right) = -\frac{e^{-\frac{z}{2}}}{2} \times \frac{i\pi}{2} e^{\frac{i\pi}{4}} H^{(1)}_{\frac{1}{2}}\left(\frac{iz}{2}\right)$$

$$= -\frac{\pi}{4} e^{\frac{i\pi}{4} - \frac{z}{2}} \sqrt{\frac{2}{\pi \frac{iz}{2}}}\, e^{-\frac{z}{2}} = -\frac{\sqrt{\pi}}{2} \frac{e^{-z}}{\sqrt{z}}$$

$$J(z) - J(0) = -\frac{\sqrt{\pi}}{2} \int_0^z \frac{e^{-z}}{\sqrt{z}}\,dz = -\sqrt{\pi} \int_0^t e^{-t^2}\,dt$$

$$\frac{2}{\pi} J(z) = 1 - \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2}\,dt = \mathrm{erfc}(\sqrt{z}) \quad //$$

$$\frac{\partial}{\partial \eta} \mathrm{erf}(\eta) = \frac{2}{\sqrt{\pi}} e^{-\eta^2}$$

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$$-\zeta_1 = \frac{\partial u_1}{\partial y} = \frac{\partial}{\partial y'} e^{-i\omega x'} \operatorname{erf}\eta = \sqrt{\frac{2R}{2\pi x'}}\, e^{-i\omega x' - \eta^2}, \quad \eta = \sqrt{\frac{R}{2x'}}\, y'$$

$$-\zeta_2 = \frac{\partial u_2}{\partial y} = i\omega\sqrt{\frac{R}{\pi}} \int_0^{x'} e^{-i\omega\xi - \eta^2\frac{x'}{\xi}} \frac{d\xi}{\sqrt{\xi}} = 2i\omega\sqrt{\frac{R}{\pi}} \int_0^{\sqrt{x'}} e^{-\frac{\eta^2 x'}{u^2} - i\omega u^2}\, du.$$

$$\frac{\eta^2 x'}{u^2} + i\omega u^2 - v = 0, \qquad u^2 = \frac{v \pm \sqrt{v^2 - 4i\omega x'\eta^2}}{2i\omega}.$$

$$2u\,du = \frac{1}{2i\omega}\left(1 \pm \frac{v}{\sqrt{v^2 - 4i\omega x'\eta^2}}\right) dv.$$

$= 2i\omega$

$$du = \frac{1}{4i\omega} \frac{\pm v + \sqrt{v^2}}{\sqrt{\frac{v \pm \sqrt{v^2}}{2i\omega}}} \cdots dv$$

$$= \pm \frac{\sqrt{v \pm \sqrt{v^2}}}{2\sqrt{2i\omega}} \cdots dv.$$

$$\frac{\partial u_2}{\partial y} = \frac{\sqrt{2i\omega}}{2}\sqrt{\frac{R}{\pi}} \int_{i\omega x'}^{\infty} e^{-v} \frac{}{\sqrt{v - \sqrt{v^2 - 4i\omega x'\eta^2}}}\, dv. \quad \longrightarrow 2\omega\sqrt{\frac{R}{\pi}\, x'} \; e^{i\omega x' \eta^2}$$

$$\sqrt{2i\omega}\, u \longrightarrow \sqrt{2i\omega x'}$$

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$$\int_0^1 (1+\omega x) e^{-i\omega x} \frac{dx}{\sqrt{x}} = i\omega e^{-i\omega} + \left(1+\frac{1}{2i}\right)\sqrt{\frac{2\pi}{\omega}}\,(C - iS)$$

$\omega x = \frac{\pi}{2} t^2$

$$\int_0^1 e^{-i\omega x} \frac{dx}{\sqrt{x}} = \sqrt{\frac{2\pi}{\omega}} \int_0^{\sqrt{\frac{2\omega}{\pi}}} e^{-\frac{\pi}{2} i t^2} dt = \sqrt{\frac{2\pi}{\omega}}\left[C\left(\sqrt{\frac{2\omega}{\pi}}\right) - iS\right]$$

$$\omega \int_0^1 \sqrt{x}\, e^{-i\omega x} dx = \frac{\omega}{-i\omega}\sqrt{x}\, e^{-i\omega x}\Big|_0^1 + \frac{1}{2i}\int_0^1 e^{-i\omega x}\frac{dx}{\sqrt{x}}$$

$$= i e^{-i\omega} + \frac{1}{2i}\sqrt{\frac{2\pi}{\omega}}\,(C - iS)$$

$$C(z) = \frac{1}{2} + f(z)\sin\left(\frac{\pi}{2}z^2\right) - g(z)\cos\left(\frac{\pi}{2}z^2\right) \qquad \frac{\pi}{2}z^2 = \omega$$

$$S = \frac{1}{2} - f\ \cos \quad - g\ \sin \qquad z = \sqrt{\frac{2\omega}{\pi}}$$

$$C - iS = \frac{1-i}{2} + if\, e^{-\frac{i\pi}{2}z^2} - g\, e^{-\frac{i\pi}{2}z^2} = \frac{1-i}{2} + (if - g)\, e^{-\frac{i}{2}\pi z^2}$$

$$= \frac{1-i}{2} + (if - g)\, e^{-i\omega}$$

$$f\left(\sqrt{\frac{2\omega}{\pi}}\right) \longrightarrow \frac{1}{\pi\sqrt{\frac{2\omega}{\pi}}} = \frac{1}{\sqrt{2\pi\omega}}$$

$$g(\quad) \longrightarrow O(\omega^{\frac{3}{2}})$$

$$\int_0^1 (1+\omega x) e^{-i\omega x} dx = \frac{(1+\omega x)}{-i\omega} e^{-i\omega x} + \frac{\omega}{i\omega}\int e^{-i\omega x} dx$$

$$= \frac{1+\omega}{-i\omega} e^{-i\omega} + \frac{1}{i\omega} + i\left(\frac{e^{-i\omega} - 1}{\omega}\right)$$

$$= -1 - \frac{i}{\omega} + i\left(1+\frac{1}{\omega}\right) e^{-i\omega}$$

$\xi = \xi(\xi, \eta)$

$$u_0' = \frac{+2\alpha}{\sqrt{\pi}}\eta e^{-\eta^2} = \alpha\sqrt{\frac{2k}{\pi x}}\, y\, e^{-\eta^2} \qquad \eta = \frac{y}{2\sqrt{x\nu}} = \frac{\sqrt{k}}{\sqrt{2x}}y$$

$$v_0 = \alpha\sqrt{\frac{\nu}{\pi x}}\left(1 - e^{-\eta^2}\right) = \alpha\sqrt{\frac{1}{2\pi x k}}\left(1 - \right.$$

$$\zeta_0 = -\frac{\alpha}{\sqrt{\pi (x\nu)}}e^{-\eta^2} = -\alpha\sqrt{\frac{2k}{\pi x}}\, e^{-\eta^2}$$

$$u' = y\sqrt{\frac{k}{\pi x}}\,(1+\omega x)\, e^{-\eta^2 - i\omega x} = \sqrt{\frac{2}{\pi}}\,\eta\,(1 + \dots$$

$$v' = \frac{1}{2k}\sqrt{\frac{k}{\pi x}}\,(1+\omega x)\left(1 - e^{-\eta^2}\right)e^{-\eta^2 - i\omega x}$$

$$\zeta' = -\sqrt{\frac{k}{\pi x}}\,(1+\omega x)\, e^{-\eta^2 - i\omega x}$$

$$\eta' = \frac{\sqrt{k}}{\sqrt{2(1-x)}}\,y$$

$$\tilde{u}' = -y\sqrt{\frac{k}{\pi(1-x)}}\left(1 + \omega\overline{1-x}\right)e^{-\eta^2 - i\omega(1-x)} = -\sqrt{\frac{2}{\pi}}\,\eta'(1 + \cdots)$$

$$\tilde{v}' = \frac{1}{2k}\sqrt{\frac{k}{\pi(1-x)}}\left(1 + \omega\overline{1-x}\right)\left(1 - e^{-\eta^2}\right)e^{-i\omega(1-x)}$$

$$\zeta_0 v = -\frac{\alpha}{2k}\sqrt{\frac{2k}{\pi x}}\sqrt{\frac{k}{\pi x}}\,(1+\omega x)\,e^{-\eta^2}\left(1 - e^{-\eta^2}\right)e^{-i\omega x}$$

$$\zeta_0\tilde{v} = -\frac{\alpha k}{2k\pi}\sqrt{\frac{2}{(1-x)x}}\left(1 + \omega\overline{1-x}\right)e^{-\eta^2}\left(1 - e^{-\eta^2}\right)e^{-i\omega(1-x)}$$

$$\zeta v_0 = -\frac{\alpha}{\sqrt{2}\,\pi x}\,(1+\omega x)\,e^{-\eta^2}\left(1 - e^{-\eta^2}\right)e^{-i\omega x} = \zeta_0 v$$

$$\zeta\tilde{v} = -\frac{(1+\omega x)\left(1 + \omega\overline{1-x}\right)}{2\pi\sqrt{x(1-x)}}\,e^{-\eta^2}\left(1 - e^{-\eta^2}\right)e^{-i\omega}$$

$$\zeta_0 u' = -\frac{2\alpha\sqrt{k}}{\pi\sqrt{x}}\,\eta\,(1+\omega x)\,e^{-2\eta^2 - i\omega x} = \zeta u_0'$$

$$\zeta u_0' = -\frac{2\alpha\eta\sqrt{k}}{\pi\sqrt{x}}\,(1+\omega x)\,e^{-2\eta^2 - i\omega x}$$

$$+M = +2\oint_C X\,ds = 4\rho \iint_{D/2} \left[\zeta_0 (\tilde{u}v - u\tilde{v}) + \zeta(\tilde{u}v_0 - u_0\tilde{v}) \right] dx\,dy$$

$$\tilde{u}(\zeta_0 v + \zeta v_0) - \tilde{v}(\zeta_0 u + u_0 \zeta)$$

$$u = -1 + u', \quad \tilde{u} = +1 + \tilde{u}'$$

$$= +(\zeta_0 v + \zeta v_0) + \tilde{v}(\zeta_0 + \zeta) + \left[\tilde{u}'(\zeta_0 v + \zeta v_0) - \tilde{v}(\zeta_0 u' + u_0' \zeta)\right]$$

$$\left[\zeta_0 v + \zeta v_0 + \zeta_0 \tilde{v} + \tilde{v}\zeta \right]$$

$$\zeta_0(v + \tilde{v}) + \zeta(v_0 + \tilde{v})$$

$$M = 2\rho \iint_{D/2} \left[\zeta_0(v + \tilde{v}) + \zeta(v_0 + \tilde{v}) + \left[\tilde{u}'(\zeta_0 v + \zeta v_0) - \tilde{v}(\zeta_0 u' + u_0' \zeta) \right] \right] dx\,dy$$

$$= 4\rho \iint_{\frac{D}{2}} \left[\zeta_0 v + \frac{1}{2}(\zeta_0 \tilde{v} + \zeta \tilde{v}) + \left(\tilde{u}'\zeta_0 v - \tilde{v}\zeta_0 u' \right) \right] dx\,dy$$

$$\tilde{u}'\zeta_0 v = \alpha \frac{\eta'}{\sqrt{\pi}\,\pi x} (1 + \omega x)(1 + \omega\overline{1-x})\, e^{-\eta^2 - \eta'^2} (1 - e^{-\eta^2})\, e^{i\omega}$$

$$\tilde{v}\zeta_0 u' = -\frac{\alpha\,\eta\,(1 + \omega x)(1 + \omega\overline{1-x})}{\pi^{\frac{3}{2}} \sqrt{x(1-x)}}\, e^{-2\eta^2} (1 - e^{-\eta'^2})\, e^{i\omega}$$

$\omega x = \frac{\pi}{2} t^2 \quad x \quad \frac{\pi}{2\omega} t^2 \quad \sqrt{x} = \sqrt{\frac{\pi}{2\omega}}\, t$

$\frac{dx}{2\sqrt{x}} = \sqrt{\frac{\pi}{2\omega}}\, dt$

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$$\int_0^1 (1+\omega x)\frac{e^{-i\omega x}}{\sqrt{x}}\, dx = \sqrt{\frac{2\pi}{\omega}} \int_0^{\sqrt{\frac{2\omega}{\pi}}} \left(1+\frac{\pi}{2}t^2\right) e^{-\frac{\pi}{2}i t^2}\, dt$$

$$= \sqrt{\frac{2\pi}{\omega}} \left(C - iS + \right.$$

$$4\rho \iint \zeta_0 v\, dx\, dy = -\frac{4\alpha}{\pi\sqrt{2}} \iint (1+\omega x) e^{-\eta^2}(1-e^{-\eta^2}) e^{-i\omega x}\left(\frac{1}{x}\sqrt{\frac{2x}{R}}\right) d\eta\, dx$$

$$= \frac{4\alpha}{\pi\sqrt{R}} \int_0^1 (1+\omega x) e^{-i\omega x} \frac{dx}{\sqrt{x}} \cdot \int_0^\infty e^{-\eta^2}(1-e^{-\eta^2})\, d\eta$$

$$\frac{\sqrt{\pi}}{2} - \frac{\sqrt{\pi}}{2\sqrt{2}} = \frac{\sqrt{\pi}}{2}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{2\alpha}{\sqrt{\pi R}}\left(1-\frac{1}{\sqrt{2}}\right)\left[i\omega e^{-i\omega} + \left(1+\frac{1}{2i}\right)\sqrt{\frac{2\pi}{\omega}}(C - iS)\right].$$

$2\left(1-\frac{1}{\sqrt{2}}\right) = .5858$

$$2\rho \iint \zeta_0 \tilde{v}\, dx\, dy = -\frac{\alpha}{\pi}\sqrt{2} \iint \frac{1+\omega\sqrt{1-x}}{\sqrt{x(1-x)}}\, e^{-i\omega(1-x) - \eta^2}\left(1-e^{-\eta'^2}\right) dx\, dy.$$

$$\int_0^\infty e^{-\eta^2} dy = \sqrt{\frac{2x}{R}} \int_0^\infty e^{-\eta^2} d\eta = \sqrt{\frac{\pi x}{2R}}$$

$$\int_0^\infty e^{-\eta^2 - \eta'^2} dy = \sqrt{\frac{2x}{R}} \int_0^\infty e^{-\eta^2\left(1+\frac{x}{1-x}\right)} d\eta = \sqrt{\frac{2x}{R}} \times \frac{\sqrt{\pi(1-x)}}{2} = \frac{\sqrt{\pi x(1-x)}}{\sqrt{2R}}$$

$$= -\frac{\alpha\sqrt{2}}{\pi} \times \frac{\sqrt{\pi}}{\sqrt{2R}} \int_0^1 \frac{(1+\omega\sqrt{1-x})}{\sqrt{1-x}}(1-\sqrt{1-x})\, e^{-i\omega(1-x)}\, dx$$

$$= -\frac{\alpha}{\sqrt{\pi R}} \int_0^1 (1+\omega x)\left(\frac{1}{\sqrt{x}} - 1\right) e^{-i\omega x}\, dx$$

$$= -\frac{\alpha}{\sqrt{\pi R}}\left[i\omega e^{-i\omega} + \left(1+\frac{1}{2i}\right)\sqrt{\frac{2\pi}{\omega}}(C - iS) + 1 + \frac{i}{\omega} - i\left(1+\frac{1}{\omega}\right)e^{-i\omega} \right]$$

$$= \frac{\alpha}{\sqrt{\pi R}}\left[-.4142\left\{ i\omega e^{-i\omega} + \left(1+\frac{1}{2i}\right)\sqrt{\frac{2\pi}{\omega}}(C - iS)\right\} - 1 - \frac{i}{\omega} + i\left(1+\frac{1}{\omega}\right)e^{-i\omega} \right]$$

//

No. 10

Date

$$\int_0^1 \frac{dx}{\sqrt{1-x}} = 2\int_0^1 du = 2$$

$$\int_0^1 x(1-x)\,dx = \frac{x^2}{2} - \frac{x^3}{3} = \frac{1}{6}, \quad \int_0^1 x\sqrt{1-x}\,dx = 2\int_0^1 (1-u^2)u^2\,du = 2\left(\frac{1}{3}-\frac{1}{5}\right) = \frac{4}{15}$$

$$2\nu\iint \zeta\,\tilde{v}\,dx\,dy = -\frac{e^{-i\omega}}{\pi}\iint \frac{(1+\omega+\omega^2 x(1-x))}{\sqrt{x(1-x)}}\, e^{-\eta^2}(1-e^{-\eta'^2})\,dx\,d\eta$$

$$= \frac{-e^{-i\omega}}{\sqrt{2\pi k}}\int_0^1 \frac{\{1+\omega+\omega^2 x(1-x)\}}{\sqrt{1-x}}(1-\sqrt{1-x})\,dx \qquad \frac{4}{15}-\frac{1}{6} = \frac{8-5}{30}$$

$$= -\frac{e^{-i\omega}}{\sqrt{2\pi k}}\left[2(1+\omega)+\frac{4}{15}\omega^2 - \left\{(1+\omega)+\frac{\omega^2}{6}\right\}\right]$$

$$= -\frac{e^{-i\omega}}{\sqrt{2\pi k}}\left(1+\omega+\frac{\omega^2}{10}\right)$$

$$4\nu\iint \tilde{u}'\zeta_0 v\,dx\,dy = \frac{4\alpha}{\pi^{\frac{3}{2}}}e^{-i\omega}\int\frac{(1+\omega+\omega^2 x(1-x))}{x}\int_0^\infty \eta' e^{-\eta^2-\eta'^2}(1-e^{-\eta'^2})\,d\eta\sqrt{\frac{2x}{k}}$$

$$1+\frac{x}{1-x} \qquad \frac{\eta}{\sqrt{1-x}} = \eta'$$

$$\int_0^\infty \eta' e^{-\eta^2-\eta'^2}\,d\eta = \sqrt{\frac{x}{1-x}}\int_0^\infty e^{-\frac{\eta^2}{1-x}}\eta\,d\eta = \sqrt{x(1-x)}\int_0^\infty e^{-\eta'^2}\eta' d\eta' = \frac{\sqrt{x(1-x)}}{2}$$

$$\int_0^\infty \eta' e^{-2\eta^2-\eta'^2}\,d\eta = \sqrt{\frac{x}{1-x}}\int_0^\infty e^{-\frac{2+x}{1-x}\eta^2}\eta\,d\eta = \sqrt{\frac{x}{1-x}}\,\frac{1-x}{2+x}\times\frac{1}{2} = \frac{\sqrt{x(1-x)}}{2(2+x)}$$

$$\frac{1+x}{2+x}$$

$$= \frac{2\alpha\sqrt{2}\,e^{-i\omega}}{\pi\sqrt{\pi k}}\int_0^1 \frac{(1+\omega+\omega^2 x(1-x))}{\cancel{\sqrt{x}}}\sqrt{1-x}\left(1-\frac{1}{2+x}\right)dx = $$

$$1-x = u^2$$

$$\int_0^1 \sqrt{1-x}\,\frac{1+x}{2+x}\,dx = 2\int_0^1 \frac{2-u^2}{3-u^2}u^2\,du = 2\int_0^1\left(1-\frac{1}{3-u^2}\right)u^2\,du$$

$$= \frac{2}{3} - 2\int\frac{u^2\,du}{3-u^2} = \frac{2}{3}+2-6\int_0^1\frac{du}{3-u^2} = \frac{8}{3}-\frac{6}{2\sqrt{3}}\log\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\int\frac{du}{3-u^2} = \frac{1}{2\sqrt{3}}\log\frac{\sqrt{3}+u}{\sqrt{3}-u}$$

$= \cancel{+.3584}$ //o.K.

$$\int_0^1 x(1-x)^{\frac{3}{2}}\frac{1+x}{2+x}\,dx = \int_0^1 (1-u^2)u^4\left(1-\frac{1}{3-u^2}\right)du = \frac{1}{5}-\frac{1}{7}-\int\frac{(1-u^2)u^4}{3-u^2}du$$

$$= \frac{2}{35} - \int_0^1\frac{(3-u^2-2)u^4}{3-u^2}du = -\frac{1}{7}+2\int\frac{u^2(u^2-3+3)}{3-u^2}du = -\frac{1}{7}-\frac{2}{3}+6\int\frac{u^2\,du}{3-u^2}$$

$$= -\frac{1}{7}-\frac{2}{3}-6+18\underbrace{\int_0^1\frac{du}{3-u^2}}_{6.8431} = .0336 \text{ /o.K}$$

$$4\rho\iint \alpha' \zeta_0 v \, dx\, dy = \frac{2\alpha}{\pi} e^{-i\omega} \sqrt{\frac{2}{\pi k}} \left[(1+\omega) \times .3855 + \omega^2 \times .0336 \right]$$

$$-4\rho\iint v' \zeta_0 u \, dx\, dy = \frac{\alpha e^{-i\omega}}{\pi^{3}} \iint \frac{(1+\omega+\omega^2 x\sqrt{1-x})}{\sqrt{x(1-x)}} \eta e^{-2\eta^2} (1 - e^{-\eta'^2}) \, dx\, d\eta \sqrt{\frac{2x}{k}}$$

$$= \frac{\alpha e^{-i\omega}\sqrt{2}}{\pi\sqrt{\pi k}} \int_0^1 \left(\frac{x}{\sqrt{1-x}}\right) dx \int_0^\infty e^{-2\eta^2}(1 - e^{-\eta'^2})\eta \, d\eta$$

$$= \frac{3\alpha\sqrt{2}}{2\pi\sqrt{\pi k}} e^{-i\omega} \int_0^1 \frac{(1+\omega+\omega^2 x\sqrt{1-x})}{\sqrt{1-x}(2+x)} x\, dx, \quad \frac{1}{2}\int_0^\infty e^{-\eta'^2}\eta' d\eta' - \int_0^\infty e^{-\frac{2+x}{1-x}\eta^2} \eta\, d\eta$$

$$\frac{1}{4} - \frac{1-x}{2(2+x)} = \frac{2+x-2(1-x)}{4(2+x)} = \frac{3x}{4(2+x)}$$

$$\int_0^1 \frac{x\,dx}{\sqrt{1-x}(2+x)} = \int_0^1 \frac{dx}{\sqrt{1-x}} - 2\int_0^1 \frac{dx}{\sqrt{1-x}(2+x)} = 2 - 4\int_0^1 \frac{du}{3-u^2} = 2 - \frac{2}{\sqrt{3}} \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

$= .4793$

$$\int_0^1 \frac{x^2\sqrt{1-x}}{2+x} dx = \int_0^1 x\sqrt{1-x}\, dx - 2\int \frac{(x+2-2)\sqrt{1-x}}{x+2} dx = 2\left(\frac{1}{3} - \frac{1}{5}\right) - \frac{4}{3} + 4\int_0^1 \frac{\sqrt{1-x}}{x+2} dx$$

$$= -\frac{2}{3} - \frac{2}{3} + 4\left[3\int_0^1 \frac{dx}{(x+2)\sqrt{1-x}} - \int_0^1 \frac{dx}{\sqrt{1-x}}\right] = .59087$$

$(1-x)=3-2-x$

$$= -8 - \frac{8}{15} + \frac{24}{4\sqrt{3}} \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

$\frac{\ln\frac{\sqrt{3}+1}{\sqrt{3}-1}}{\sqrt{3}} = .76035$

$= -8.5333 -$

$$\int_0^1 \frac{\sqrt{1-x}\,dx}{x+2} = 2\int_0^1 \frac{u^2\,du}{3-u^2} = 6\int_0^1 \frac{du}{3-u^2} - 2 = \sqrt{3}\ln\frac{\sqrt{3}+u}{\sqrt{3}-u} - 2 = .2810$$

$$= \frac{3\alpha}{4\pi}\sqrt{\frac{2}{\pi k}} e^{-i\omega} \left[(1+\omega)(0.4793)\times\frac{3}{8} + \omega^2(.59087) \right]$$

$$= \frac{2\alpha}{\pi}\sqrt{\frac{2}{\pi k}} e^{-i\omega} \left[.5652(1+\omega) + .2552\,\omega^2 \right]$$

$$\omega = \frac{-.5652 \pm \sqrt{(.5652)^2 - 4 \times .2552 \times .5652}}{2 \times .2552}$$

$M \fallingdotseq \frac{1}{\sqrt{R}}$ $\quad M = O\left(\frac{1}{\sqrt{R}}\right)$ で [illegible]

この Mode では安定ではないように見える。

$k = $ [illegible]

				$\sqrt{R}$	
[illegible]	.3	[illegible]	.06	20.44	[illegible]
	[illegible]	[illegible]	6.[illegible]	24.[illegible]	4.840
	[illegible]		6.[illegible]	31.62	9.903
	10	[illegible]	8.0[illegible]	(4.[illegible])	[illegible]
	.3	[illegible]	[illegible]	16.[illegible]	
	.[illegible]	12000		21.91	
	.1	5×10^{5}			

Re	$\theta \times 10^{-4}$	$1/\sqrt{R}$
400	360	.0500
800	80	.0354
1200	40	.0289
1600	25	.0250