V

No. 1

Date

平面ポアゼイユ流れ

i) 平均流れ

$$U+\bar{u} = U(y), \quad \bar{v}=0, \quad \bar{\zeta} = -\frac{\partial U}{\partial y}, \qquad (1)$$

$$\left.\begin{aligned}&\frac{1}{\rho}\overline{G_x} - \nu\nabla^2 U - \frac{1}{4}(v\hat{\zeta}+\hat{v}\zeta)=0\\ &\frac{1}{\rho}\overline{G_y} + U\bar{\zeta} + \frac{1}{4}(u\hat{\zeta}+\hat{u}\zeta)=0\end{aligned}\right\} \qquad (2)$$

$$\frac{1}{\rho}\overline{G} = \frac{1}{\rho}\overline{p} + \frac{U^2}{2} + \frac{1}{4}(u\hat{u}+v\hat{v}), \qquad (3)$$

ここで、$v\hat{\zeta}$ 等は各固有関数についての和

$$v\hat{\zeta} = |X_n|^2 \sum_n v_n\hat{\zeta}_n \qquad (4)$$

を意味するものとする。X_n は ψ_n の振幅とする。

また、$$\psi_n = \phi_n(y)e^{-ik_n x}, \qquad (5)$$

とおくと (4) は y のみの関数となる。

そうすると (2) 第1式より

$$\overline{G_{xx}} = 0, \quad \overline{G} = \rho(Ax+B), \qquad (6)$$

(2) 第2式より A は y に依存しない定数で

$$B_y = +UU_y - \frac{1}{4}(u\hat{\zeta}+\hat{u}\zeta) \qquad (7)$$

第3式を y で微分すると

$$B_y = \frac{1}{\rho}\overline{p}_y + UU_y + \frac{1}{4}(u\hat{u}+v\hat{v})_y, \qquad (8)$$

両式から $$\frac{1}{\rho}\overline{p}_y = -\frac{1}{4}(u\hat{\zeta}+\hat{u}\zeta) - \frac{1}{4}(u\hat{u}+v\hat{v})_y. \qquad (9)$$

積分して

$$\frac{1}{\rho}\overline{p} = Ax - \frac{1}{4}(u\hat{u}+v\hat{v}) - \frac{1}{4}\int_1^y (u\hat{\zeta}+\hat{u}\zeta)\,dy, \qquad (10)$$

この定数Aは(10)の式より

$$A = \nu \nabla^2 U + \frac{1}{4}(\hat{v}\zeta + v\hat{\zeta}), \quad A = \nu U_{yy}|_{y=\pm 1}, \qquad (11)$$

として求まる。

上式をyで微分すれば偏差輸送方程式をうる。

$$\nu U_{yyy} + \frac{1}{4}(\hat{v}\zeta + v\hat{\zeta})_y = 0 , \qquad (12)$$

(5)のようにおくと

$$v_n = i k_n \phi_n(y) e^{ik_n x}, \quad \zeta_n = (k_n^2 \phi_n - \phi_{n\,yy}) e^{ik_n x},$$

となるから

$$\frac{1}{4}(\hat{v}\zeta + v\hat{\zeta}) = \frac{i}{4}\sum_n |X_n|^2 k_n \{\phi_n(k_n^2\hat{\phi}_n - \hat{\phi}_{n\,yy}) - \hat{\phi}_n(k_n^2\phi_n - \phi_{n\,yy})\}$$

$$= \frac{i}{4}\sum_n |X_n|^2 k_n (\phi_n \hat{\phi}_{n\,yy} - \hat{\phi}_n \phi_{n\,yy}) . \qquad (13)$$

それ故(12)を積分して (or (11))

$$\nu U_{yy}(y) - A y = -\frac{1}{4}[\hat{v}\zeta + v\hat{\zeta}] , \qquad (14)$$

Aは明らかに圧力勾配で (10)より

$$A = \frac{1}{\rho}\frac{\partial \bar{P}}{\partial x}\Big|_{y=\pm 1} , \qquad (14)$$

(14)をもう一度積分すると ($U_y(0)=0$)

$$U_y(y) - A y = -\frac{1}{4\nu}\int_0^y (\hat{v}\zeta + v\hat{\zeta})\,dy \qquad (15)$$

さらに

$$U(y) - \frac{A}{2}(y^2 - 1) = -\frac{1}{4\nu}\int_1^y dy \int_0^y (\hat{v}\zeta + v\hat{\zeta})\,dy , \qquad (16)$$

$$U(0) + \frac{A}{2} = +\frac{1}{4\nu}\int_0^1 dy \int_0^y (\hat{v}\zeta + v\hat{\zeta})\,dy , \qquad (17)$$

あるいは(16)を積分して、平均流速は

$$2U_m = \int_{-1}^{1} U(y)\,dy = -\frac{2}{3}A - \frac{1}{4\nu}\left[\int_{-1}^{1} dy\,(1+y)\int_0^y (\hat{v}\zeta + \hat{\bar{v}}\zeta)\,dy\right], \quad (18)$$

(16)より

$$U_y(1) = A - \frac{1}{4\nu}\int_0^1 (\hat{v}\zeta + \hat{\bar{v}}\zeta)\,dy \quad , \quad (19)$$

特に $v\zeta$ 等がなければ

$$\left.\begin{aligned} U(y) &= \frac{A}{2}(y^2-1) = \frac{3}{2}U_m(1-y^2), \\ U_y(y) &= -3U_m y, \quad U_{yy}(y) = -3U_m, \end{aligned}\right\} \quad (20)$$

従って一般にUはyの偶関数で

$$\left.\begin{aligned} U(y) &= \frac{A}{2}y^2 + \sum_{n=0} a_n \cos n\pi y \\ U(\pm 1) = 0 &= \frac{A}{2} + \sum_{n=0}^{\infty} (-)^n a_n \\ U_m &= \int_0^1 U(y)\,dy = \frac{A}{6} + a_0, \end{aligned}\right\} \quad (21)$$

こゝで(12)より

$$+\nu\sum_{n=1} (n\pi)^3 a_n \sin n\pi y = \frac{1}{4}\left(\hat{v}\zeta + \hat{\bar{v}}\zeta\right)_y \quad , \quad (22)$$

から $a_n\,(n \geq 1)$ は定まり、上式(下2式)より U_m が与えられれば

A, a_0 も定まる。

なお 摩擦応力 τ は

$$\left.\begin{aligned} \tau &= -\mu U_y(1), \quad C_F = \frac{\tau}{\frac{\rho}{2}U_m^2} = \frac{-2\nu U_y}{U_m^2} = \frac{2\gamma}{R} \\ \gamma &= -U_y(1)/U_m, \quad R = \frac{U_m}{\nu} \times \frac{B}{2} \end{aligned}\right\} \quad (23)$$

ii) 振動流れ

$$i\omega u + \frac{1}{\rho}G_x - \nu\nabla^2 u - v\bar{\zeta} = 0$$

$$i\omega v + \frac{1}{\rho}G_y - \nu\nabla^2 v + U\zeta + u\bar{\zeta} = 0 \qquad (1)$$

$$\frac{1}{\rho}G = \left[\frac{p}{\rho} + u \cdot (u\bar{u} + v\bar{v})\right] = \frac{p}{\rho} + uU(y), \qquad (2)$$

$$i\omega\zeta - \nu\nabla^2\zeta + (U\zeta + u\bar{\zeta})_x + (v\bar{\zeta})_y = 0$$

$$i\omega\zeta - \nu\nabla^2\zeta + U\zeta_x + v\bar{\zeta}_y = 0, \qquad (3)$$

$$\frac{1}{\rho}\nabla^2 G - (v\bar{\zeta})_x + (U\zeta + u\bar{\zeta})_y = 0, \qquad (4)$$

今 $\psi = \phi(y)e^{-ikx}\ [e^{i\omega t}]$, (5)

とおくと $u = \phi_y e^{-ikx}$, $v = +ik\phi e^{-ikx}$, $\zeta = (k^2\phi - \phi_{yy})e^{-ikx}$, (6)

(3)に代入して iで割ると

$$\omega(k^2\phi - \phi_{yy}) - kU(k^2\phi - \phi_{yy}) - kU_{yy}\phi = +i\nu\left(k^2 - \frac{d^2}{dy^2}\right)^2\phi, \ (7)$$

なる Orr-Sommerfeld eq. を得る。

まず両辺に $\bar{\phi}$ をかけて積分し

$$\frac{1}{2}\int_{-1}^{1} f\,dy = [f]$$

と記す事にしよう。

また以後固有値問題を取扱うので

$$\phi(\pm 1) = \phi_y(\pm 1) = 0, \ \cdots \qquad (8)$$

$$\omega\{k^2[\phi\hat{\phi}]+[\phi_y\hat{\phi}_y]\}-k\{[U\phi\hat{\phi}]+[U\phi_{yy}\hat{\phi}]\}-k[U_{yy}\phi\hat{\phi}]$$
$$=+i\nu\{k^4[\phi\hat{\phi}]+2k^2[\phi_y\hat{\phi}_y]+[\phi_{yy}\hat{\phi}_{yy}]\}, \quad (9)$$

これを実部と虚部にわけると。（$\omega=\omega_r+i\omega_i$ とすると）

$$\omega_r\{k^2[\phi\hat{\phi}]+[\phi_y\hat{\phi}_y]\}=k\{[U\phi\hat{\phi}]+[U_{yy}\phi\hat{\phi}]+\frac{1}{2}[U(\phi_{yy}\hat{\phi}+\phi\hat{\phi}_{yy})]\}, \quad (10)$$

$$\omega_i\{k^2[\phi\hat{\phi}]+[\phi_y\hat{\phi}_y]\}+\frac{ik}{2}[U(\phi_{yy}\hat{\phi}-\phi\hat{\phi}_{yy})]=+\nu\{k^4[\phi\hat{\phi}]+2k^2[\phi_y\hat{\phi}_y]+[\phi_{yy}\hat{\phi}_{yy}]\}, \quad (11)$$

ϕに適当な近似関数を与えれば(10)(11)からωの近似値が求められる。

さて小節の(11)にUをかけて積分すると

$$+\nu[U_y^2]=\frac{1}{4}[U(v\hat{\zeta}+\hat{v}\zeta)]=\frac{1}{4}\sum_n|X_n|^2k_n[U(\phi_n\hat{\phi}_{nyyy}-\hat{\phi}_n\phi_{nyyy})], \quad (12)$$
$\leftarrow -2\nu A U_m$

となるから (11) より

(13)

と変形。

(7) において $\omega = kc = kU_m\gamma$, $\gamma = c/U_m$, $R = \frac{U_m}{\nu}\times 1$, $U(y)/U_m = \eta(y)$ } (14)

とおき, kU_m で割ると

$$(\gamma-\eta)(k^2\phi-\phi_{yy}) - \eta''\phi - \frac{i}{kR}\left(k^2-\frac{\partial^2}{\partial y^2}\right)^2\phi = 0, \quad (7')$$

Adjoint sol. ϕ^* をかけて積分すると

$$\gamma\{k^2[\phi\phi^*]+[\phi_y\phi_y^*]\} - k^2[\eta\phi\phi^*] + [\eta\phi_{yy}\phi^*] - [\eta''\phi\phi^*]$$
$$- \frac{i}{kR}\{k^4[\phi\phi^*]+2k^2[\phi_y\phi_y^*]+[\phi_{yy}\phi_{yy}^*]\} = 0, \quad (15)$$

ϕ^* も境界条件 (8) を満たすものとして 第3項を部分積分すると

$$[\eta\phi_{yy}\phi^*] = [(\eta\phi^*)_{yy}\phi], \quad (16)$$

それ故 ϕ^* の満たす方程式は

$$\gamma(k^2\phi^*-\phi^*_{yy}) - k^2\eta\phi^* + \underbrace{(\eta\phi^*)_{yy}}_{2\eta_y\phi^*_y+\eta\phi^*_{yy}} - \eta''\phi^*$$
$$- \frac{i}{kR}\left(k^2-\frac{\partial^2}{\partial y^2}\right)^2\phi^* = 0, \quad (17)$$

さて今 $P_n(y)$, $Q_n(y)$ を (8) を満たす正規直交系 (P_n は even, Q_n は odd) としよう。

$$\frac{1}{2}\int_{-1}^{1} P_n P_m\,dy = \delta_{n,m}, \quad \frac{1}{2}\int_{-1}^{1} Q_n Q_m\,dy = \delta_{n,m}, \quad (18)$$

例えば
$$P_1 = \sqrt{\tfrac{2}{3}}(1+\cos\pi y), \quad P_2 = \frac{1}{\sqrt{12}}(1-2\cos\pi y-3\cos 2\pi y),$$
$$Q_1 = \sqrt{\tfrac{8}{5}}\left(\sin\pi y+\frac{\sin 2\pi y}{2}\right), \quad Q_2 = \sqrt{\tfrac{2}{3}}(\sin\pi y-\sin 2\pi y-\sin 3\pi y), \quad (19)$$

$$\phi = \sum_{n=1}\{a_n p_n(y) + b_n q_n(y)\}, \quad \phi^* = \sum_{n=1}(a_n^* p_n + b_n^* q_n), \qquad (20)$$

とおいて(7)に $p_m(y)$ をかけて積分すれば

$$\left(\gamma k^2 \cancel{+} \frac{ik^3}{R}\right)a_m + \sum_{n=1} a_n\Big\{\gamma[p_{ny}, p_{my}] - k^2[\eta p_n p_m] - [\eta'' p_n p_m] + [\eta p_{nyy} p_m] \cancel{+} \frac{i}{kR}\{2k^2[p_{ny} p_{my}] + [p_{nyy} p_{myy}]\}\Big\} = 0 \qquad (21)$$

b_n, q_n についても全く同型の式が成立つ。

(17)に p_m をかけて積分するとやはり

$$\left(\gamma k^2 \cancel{+} \frac{ik^3}{R}\right)a_m^* + \sum_{n} a_n^*\Big\{\gamma[p_{ny}, p_{my}] - k^2[\eta p_n p_m] - [\eta'' p_n p_m] + [(\eta p_n)_{yy} p_m] \cancel{+} \frac{i}{kR}\{2k^2[p_{ny} p_{my}] + [p_{nyy} p_{myy}]\}\Big\} = 0. \qquad (22)$$

部分で

$$[\eta p_{nyy} p_m] = [(\eta p_m)_{yy} p_n], \qquad (23)$$

であるから(22)のマトリックスは(21)の転置行列である。

つまり a_n^* は a_n の転置行列式の解となっている。

さて(21)は与えられた k に対して固有値 γ を与える式で、一般には無限箇の解を有する。また ϕ と ϕ^* の固有値は等しい。

また(16)の関係があるから(15)の関係も $\phi = \phi_n$, $\phi^* = \phi_m^*$ なる異なる固有関数の関係と見ると $\gamma_m \neq \gamma_n$, γ_m とおいて

~~も成り立つから、~~

$$(\gamma_n - \gamma_m^*)\{k^2[\phi_n\phi_m^*] + [\phi_{ny}\phi_{my}^*]\} = 0 , \qquad (24)$$

つまり、$\gamma_n \neq \gamma_m$ ならば

$$k^2[\phi_n\phi_m^*] + [\phi_{ny}\phi_{my}^*] = 0 , \qquad (25)$$

のような意味で固有関数は直交している。

よく知られているように k を実とすると γ は複素値をとる。さて我々は更に γ が実つまり中立安定のもののみを考えたい。そうすると (21) の固有値方程式は実部、虚部にわかれ、2式から γ を消去すると k^2(実) の多項式を得、それの実解が中立安定波数を与える事になる。

具体的に解いてみよう。

今 $\quad \eta = \frac{3}{2}(1-y^2) , \quad \eta'' = -3 \quad , \quad . \quad . \qquad (25)$

と設定し、また2項近似 (19) のようにおいてみよう。

(21) は

$$a_1\Big\{\gamma k^2 + 3 \mp \frac{ik^3}{R} + \gamma[\phi_{1y}\phi_{1y}] - k^2[\eta\phi_1\phi_1] + [\eta\phi_{1yy}\phi_1] \mp \frac{i}{kR}\big(2k^2[\phi_{1y}\phi_{1y}] + [\phi_{1yy}\phi_{1yy}]\big)\Big\}$$
$$+ a_2\Big(\cancel{\gamma k^2 + \frac{ik^3}{R}} + \gamma[\phi_{1y}\phi_{2y}] - k^2[\eta\phi_1\phi_2] + [\eta\phi_{2yy}\phi_1] \mp \frac{i}{kR}\{2k^2[\phi_{1y}\phi_{2y}] + [\phi_{1yy}\phi_{2yy}]\}\Big) = 0 \qquad (26)$$

$$a_1\{\gamma[P_{1y}P_{2y}]-k^2[\eta P_1P_2]+[\eta P_{1yy}P_2]\cancel{+}\frac{1}{kR}(2k^2[P_{1y}P_{2y}]+[P_{1yy}P_{2yy}])\}$$

$$+a_2\{\gamma k^2+3\cancel{+}\frac{ik^3}{R}+\gamma[P_{2y}P_{2y}]-k^2[\eta P_2P_2]+[\eta P_{2yy}P_2]\cancel{+}\frac{1}{kR}(2k^2[P_{2y}P_{2y}]+[P_{2yy}P_{2yy}])\}=0$$

$$[P_{1y}P_{1y}]=\frac{1}{2}\times\frac{2}{3}\pi^2\int_{-1}^{1}\sin^2\pi y\,dy=\frac{\pi^2}{3}=3.2899$$

$$[P_{1y}P_{2y}]=\frac{1}{2}\times\frac{\pi^2}{\sqrt{18}}\int_{-1}^{1}(\sin\pi y)(+2\sin\pi y+6\sin2\pi y)\,dy=-\frac{\pi^2}{2\sqrt{18}}=-1.1631$$

$$[P_{2y}P_{2y}]=\frac{\pi^2}{2\times12}\int_{-1}^{1}(2\sin\pi y+6\sin2\pi y)^2dy=\frac{\pi^2}{24}(4+36)=\frac{5\pi^2}{3}=16.4493$$

$$[g_{1y}g_{1y}]=\frac{8}{2\times5}\pi^2\int_{-1}^{1}(\cos\pi y+\cos2\pi y)^2dy=\frac{8}{5}\pi^2=15.7914$$

$$[g_{1y}g_{2y}]=\frac{1}{2}\sqrt{\frac{16}{15}}\pi^2\int_{-1}^{1}(\cos\pi y+\cos2\pi y)(\cos\pi y-2\cos2\pi y-3\cos3\pi y)\,dy$$

$$=-\frac{2\pi^2}{\sqrt{15}}=-5.0966$$

$$[g_{2y}g_{2y}]=\frac{\pi^2}{2}\times\frac{2}{3}\int_{-1}^{1}(\cos\pi y-2\cos2\pi y-3\cos3\pi y)^2dy=\frac{14\pi^2}{3}=46.0582$$

$$[P_{1yy}\cdot P_{1yy}]=\frac{1}{3}\times\pi^4\int_{-1}^{1}\cos^2\pi y\,dy=\frac{\pi^4}{3}=32.4697,\quad[P_{2yy}P_{2yy}]=\frac{\pi^4}{24}\int_{-1}^{1}(2\cos\pi y+12\cos2\pi y)^2dy$$

$$[P_{1yy}P_{2yy}]=\frac{-\pi^4}{2\times\sqrt{18}}\int_{-1}^{1}\cos\pi y\,(2\cos\pi y+12\cos2\pi y)\,dy=-\frac{\pi^4}{\sqrt{18}}=-22.9595$$

$$[P_{2yy}P_{2yy}]=\frac{37}{6}\pi^4=600.688$$

$$[g_{1yy}g_{1yy}]=\pi^4\frac{4}{5}\int_{-1}^{1}(\sin\pi y+2\sin2\pi y)^2dy=4\pi^4=389.636$$

$$[g_{1yy}g_{2yy}]=\frac{2\pi^4}{\sqrt{15}}\int_{-1}^{1}(\sin\pi y+2\sin2\pi y)(\sin\pi y-4\sin2\pi y-9\sin3\pi y)\,dy=-\frac{14}{\sqrt{15}}\pi^4=-352.113$$

$$[g_{2yy}g_{2yy}]=\frac{\pi^4}{3}\int(\sin\pi y-4\sin2\pi y-9\sin3\pi y)^2dy=\frac{98}{3}\pi^4=3182.030$$

$1+2\cos\pi y+\cos^2\pi y = \frac{3}{2} + 2\cos\pi y + \frac{\cos 2\pi y}{2}$.

$$[\eta P_1 P_1] = \frac{3}{4}\times\frac{2}{3}\int_{-1}^{1}(1-y^2)(1+\cos\pi y)^2 dy = \int_0^1 (1-y^2)\left(\frac{3}{2}+2\cos\pi y+\frac{\cos 2\pi y}{2}\right)dy$$
$$= 1+\frac{4}{\pi}-\frac{1}{4\pi} = 1+\frac{15}{4\pi} = 2.19366$$

$$[\eta P_1 P_2] = \frac{3}{4}\times\frac{1}{\sqrt{18}}\int_{-1}^{1}(1-y^2)(1+\cos\pi y)(1-2\cos\pi y-3\cos 2\pi y)dy$$
$$1+\cos\pi y-2\cos^2\pi y-3\cos 2\pi y-3\cos\pi y\cos 2\pi y$$
$$-(1+\cos 2\pi y) \quad -\frac{3}{2}(\cos\pi y+\cos 3\pi y)$$
$$-\frac{5}{2}\cos\pi y-4\cos 2\pi y-\frac{3}{2}\cos 3\pi y$$
$$= \frac{-1}{2\sqrt{2}}\int_0^1(1-y^2)\left(\frac{5}{2}\cos\pi y+4\cos 2\pi y+\frac{3}{2}\cos 3\pi y\right)dy$$
$$= \frac{-1}{\sqrt{2}}\left[\frac{5}{\pi}-\frac{2}{\pi}+\frac{1}{3\pi}\right] = -\frac{10}{\sqrt{2}\times 3\pi} = -.75026$$

$\frac{3}{\pi}$ $\frac{2}{3\pi}$

$$[\eta P_2 P_2] = \frac{1}{24}\times\frac{3}{2}\int_{-1}^{1}(1-y^2)(1-2\cos\pi y-3\cos 2\pi y)^2 dy.$$

$3(\cos\pi y + \cos 3\pi y)$

$$1+4\cos^2\pi y+9\cos^2 2\pi y-4\cos\pi y-6\cos 2\pi y+6\cos\pi y\cos 2\pi y$$
$$1+2+\frac{9}{2}+\cos\pi y-4\cos 2\pi y+3\cos 3\pi y+\frac{9}{2}\cos 4\pi y$$
$$= \frac{1}{8}\left[\frac{15}{2}\times\frac{2}{3}-\frac{2}{\pi}+\frac{2}{\pi}+\frac{2}{3\pi}-\frac{9}{2}\times\frac{1}{8\pi}\right] = \frac{1}{8}\left[5+\frac{5}{48\pi}\right] = .62914$$

$$\int_0^1(1-y^2)dy = \frac{2}{3}, \quad \int_0^1(1-y^2)\cos\pi y\, dy = \frac{\sin\pi y}{\pi}(1-y^2)\Big|_0^1 + \frac{2}{\pi}\int_0^1 y\sin\pi y\, dy$$
$$= -\frac{2}{\pi^2}y\cos\pi y\Big|_0^1 = \frac{2}{\pi}$$

$$\int_0^1(1-y^2)\cos n\pi y\, dy = +\frac{2}{n\pi}\int_0^1 y\sin n\pi y\, dy = -\frac{2}{(n\pi)^2}n\pi\cos n\pi = \frac{2(-)^{n+1}}{\pi n^2}$$

$n=2 \quad 3 \quad 4$

$$\left[\frac{-2}{4\pi}, \frac{2}{9\pi}, \frac{-2}{16\pi}\right]$$

$$[\eta P_1 P_{1yy}] = \frac{-\pi^2\, 3\cdot 2}{2\cdot 3}\int_0^1(1-y^2)(1+\cos\pi y)\cos\pi y\, dy = -\pi^2\int_0^1(1-y^2)\left(\cos\pi y+\frac{1+\cos 2\pi y}{2}\right)dy$$
$$= -\pi^2\left[\frac{1}{3}+\frac{2}{\pi}-\frac{1}{4\pi}\right] = -\pi^2\left[\frac{1}{3}+\frac{7}{4\pi}\right] = -8.78765$$

$$[\eta P_1 P_{2yy}] = \frac{1}{4}\times\frac{\pi^2}{\sqrt{18}}\times\frac{3}{2}\int_0^1(1-y^2)(1+\cos\pi y)(2\cos\pi y+12\cos 2\pi y)dy$$
$$1+2\cos\pi y+12\cos 2\pi y+2\cos^2\pi y+12\cos\pi y\cos 2\pi y$$
$$1+\cos 2\pi y+6(\cos\pi y+\dots$$
$$(2+8\cos\pi y+13\cos 2\pi y+6\cos 3\pi y).$$
$$= \frac{\pi^2}{2\sqrt{2}}\left[\frac{4}{3}+\frac{16}{\pi}-\frac{13}{2\pi}+\frac{12}{9\pi}\right] = \frac{\pi^2}{2\sqrt{2}}\left[\frac{4}{3}+\frac{65}{6\pi}\right] = 16.685$$

$\frac{19}{2\pi}+\frac{4}{3\pi}$ $\quad \frac{57}{6}$

$$+\frac{31}{\pi}+\frac{10}{3\pi}-\frac{9}{4\pi}\quad+\frac{18+9-1}{18\pi}$$

$$=\frac{3(\times 12+40-27}{12\pi}\quad 385$$

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$$+\cos\pi y \;\; (1+\cos 2\pi y)-\tfrac{3}{2}(\cos\pi y+\cos 3\pi y)$$

$$[\zeta\, P_{yy}\, P_2] = -\frac{3}{2}\sqrt{\frac{2}{3}}\,\pi^2\times\frac{1}{\sqrt{12}}\int_0^1(1-y^2)(+\cos\pi y)\,(1-2\cos\pi y-3\cos 2\pi y)\,dy$$

$$-1-\tfrac{1}{2}\cos\pi y-\cos 2\pi y-\tfrac{3}{2}\cos 3\pi y$$

$$=+\sqrt{\frac{1}{18}}\,\pi^2\left[\frac{2}{3}+\frac{4}{\pi}+\frac{1}{2\pi}-\frac{1}{18\pi}\right]=\frac{\pi^2}{\sqrt{18}}\left[\frac{1}{3}+\frac{13}{4\pi}\right]\;\approx 1.8950$$

$$[\zeta\, P_{2yy}\, P_2] = \frac{3}{2}\times\frac{\pi^2}{12}\int_0^1(1-y^2)(2\cos\pi y+12\cos 2\pi y)\,(1-2\cos\pi y-3\cos 2\pi y)\,dy$$

$$-2(1+\cos 2\pi y)-12(\cos\pi y+\cos 3\pi y)$$

$$-3(\cos\pi y+\cos 3\pi y)-18(1+\cos 4\pi y)$$

$$=[-20-13\cos\pi y+10\cos 2\pi y-15\cos 3\pi y-18\cos 4\pi y$$

$$=\frac{\pi^2}{8}\left[-\frac{40}{3}-\frac{26}{\pi}-\frac{5}{\pi}-\frac{30}{3\cdot 9\pi}+\frac{18}{8\pi}\right]=-\frac{\pi^2}{8}\left[\frac{40}{3}+\frac{385}{12\pi}\right]\approx -29.0484$$

$$[\zeta\, \hat{s}_1\, \hat{s}_1] = \frac{3}{2}\times\frac{8}{5}\int_0^1(1-y^2)\left(\sin\pi y+\frac{\sin 2\pi y}{2}\right)^2 dy$$

$$[\zeta\, \hat{s}_1\, \hat{s}_2] = \sqrt{\frac{16}{15}}\times\frac{3}{2}\int_0^1(1-y^2)\left(\sin\pi y+\frac{\sin 2\pi y}{2}\right)(\sin\pi y-\sin 2\pi y-\sin 3\pi y)\,dy$$

$$[\zeta\, \hat{s}_2\, \hat{s}_2] = \frac{2}{3}\times\frac{3}{2}\int_0^1(1-y^2)(\sin\pi y+\sin 2\pi y-\sin 3\pi y)^2\,dy$$

$[\zeta\, \hat{s}_1\, \hat{s}_{1yy}]$

$[\zeta\, \hat{s}_{2yy}\, \hat{s}_1]$

$[\zeta\, \hat{s}_2\, \hat{s}_{1yy}]$

$[\zeta\, \hat{s}_2\, \hat{s}_{2yy}]$

$$a_1\left[\gamma A_1 + B_1 - \frac{i}{kR}C_1\right] + a_2\left[\gamma A_2 + B_2 - \frac{i}{kR}C_2\right] = 0$$

$$a_1\left[\gamma A_3 + B_3 - \frac{i}{kR}C_3\right] + a_2\left[\gamma A_4 + B_4 - \frac{i}{kR}C_4\right] = 0$$

$$A_1 = k^2 + [P_{1y}P_{1y}] = k^2 + 3.2899, \quad B_1 = 3 + [\eta P_1 P_{1yy}] - k^2[\eta P_1 P_1] = -5.78777 - 2.1937k^2$$

$$C_1 = k^4 + 2k^2[P_{1y}P_{1y}] + [P_{1yy}P_{1yy}] = k^4 + 6.5789k^2 + 32.4697$$

$$A_2 = [P_{1y}P_{2y}] = -1.1631, \quad B_2 = [\eta P_1 P_{2yy}] - k^2[\eta P_1 P_2] = (16.685 + 0.7503k^2)$$

$$C_2 = 2k^2[P_{1y}P_{2y}] + [P_{1yy}P_{2yy}] = -2.3262k^2 - 22.9595$$

$$A_3 = [P_{1y}P_{2y}] = A_2, \quad B_3 = [\eta P_2 P_{1yy}] - k^2[\eta P_1 P_2] = (1.8450 + .7503k^2)$$

$$C_3 = 2k^2[P_{1y}P_{2y}] + [P_{1yy}P_{2yy}] = C_2$$

$$A_4 = k^2 + [P_{2y}P_{2y}] = k^2 + 16.4493, \quad B_4 = 3 + [\eta P_2 P_{2yy}] - k^2[\eta P_2 P_2]$$
$$= -26.0484 - .6291k^2$$

$$C_4 = k^4 + 2k^2[P_{2y}P_{2y}] + [P_{2yy}P_{2yy}] = k^4 + 32.8986k^2 + 600.6890.$$

$$(\gamma A_1 + B_1)(\gamma A_4 + B_4) - \frac{C_1C_4}{k^2R^2} - (\gamma A_2 + B_2)(\gamma A_3 + B_3) + \frac{C_2C_3}{k^2R^2}$$
$$- \frac{i}{kR}\left[C_1(\gamma A_4 + B_4) + C_4(\gamma A_1 + B_1) - C_2(\gamma A_3 + B_3) - C_3(\gamma A_2 + B_2)\right] = 0.$$

$$\gamma^2\underbrace{(A_1A_4 - A_2^2)}_{P} + \gamma(A_1B_4 + A_4B_1 - B_2A_3 - A_2B_3) + B_1B_4 - B_2B_3 + \frac{C_2^2 - C_1C_4}{k^2R^2}$$
$$- \frac{i}{kR}\left[\gamma(C_1A_4 + A_1C_4 - 2C_2A_2) + C_1B_4 + C_4B_1 - C_2B_3 - C_3B_2\right] = 0.$$

$$\gamma^2 P + \gamma Q + R = 0$$

$$\gamma = \frac{-Q \pm \sqrt{Q^2 - 4PR}}{2P}$$

$$P = A_1 A_4 - A_2^2 = (k^2 + 3.2899)(k^2 + 16.4493) - 1.3328$$
$$= k^4 + 19.7392 k^2 + 52.7638$$

$$Q = A_1 B_4 + A_4 B_1 - (B_2 + B_3) A_2 - \frac{i}{kR}\left(C_1 A_4 + A_1 C_4 - 2 C_2 A_2\right)$$
$$= (k^2 + 3.2899)(-26.0489 - k^2 \times .6291) - (k^2 + 16.4493)(5.7877 + 2.1937 k^2)$$
$$+ 1.1631(16.685 + .7503 k^2 + 1.8450 + .7503 k^2)$$
$$- \frac{i}{kR}\Big[(k^2 + 16.4493)(k^4 + 6.5789 k^2 + 32.4697)$$
$$+ (k^2 + 3.2899)(k^4 + 32.8986 k^2 + 600.6890)$$
$$- 2.3262(2.3262 k^2 + 22.9595)\Big]$$
$$= -k^4(.6291 + 2.19637) + k^2\left(\begin{array}{l}-26.0884 - .6291 \times 3.2899 - 5.7877 - 2.1937 \times 16.4493 \\ + .7503 \times 1.1631 \times 2\end{array}\right)$$
$$- 3.2899 \times 26.0489 - 16.4493 \times 5.7877$$
$$- \frac{i}{kR}\Big[2k^6 + k^4(16.4493 + 6.5789 + 3.2899 + 32.8986) \quad (-(2.3262)^2)$$
$$+ k^2(32.4697 + 6.5789 \times 16.4493 + 600.6890 + 3.2899 \times 32.8986)$$
$$+ 16.4493 \times 32.4697 + 3.2899 \times 600.6890 - 2.3262 \times 22.9595\Big]$$

$$Q = -2.8255 k^4 + k^2(-68.2409) - 180.9002$$
$$- \frac{i}{kR}\left[2k^6 + 59.2167 k^4 + 844.1989 k^2 + 2456.9022\right]$$

$$R = B_1 B_4 - B_2 B_3 + \frac{C_2^2 - C_1 C_4}{k^2 R^2} - \frac{i}{kR}\left[C_1 B_4 + C_4 B_1 - C_2 B_3 - C_3 B_2\right]$$
$$= (5.7877 + 2.1937 k^2)(26.0489 + .6291 k^2) - (16.6850 + .7503 k^2)$$
$$\times (1.8450 + .7503 k^2)$$
$$+ \frac{1}{k^2 R^2}\Big[(2.3262 k^2 + 22.9595)^2 - (k^4 + 6.5789 k^2 + 32.4697)$$
$$\times (k^4 + 32.8986 k^2 + 600.6890)\Big]$$

$$- \frac{i}{kR}\Big[(k^4 + 6.5789 k^2 + 32.4697)(26.0488 + .6291 k^2)$$
$$- (k^4 + 32.8986 k^2 + 600.6890)(5.7877 + 2.1937 k^2)$$
$$+ (2.3262 k^2 + 22.9595)\underbrace{(1.8450 + .7503 k^2 + 16.685 + .7503 k^2)}_{18.530 + 1.5006 k^2}\Big]$$

$$= k^4(2.1937 \times .6291 - (.7503)^2) + k^2(2.1937 \times 26.0484 + .6291 \times 5.7877 - .7503 \times 18.530)$$
$$+ 5.7877 \times 26.0489 - 1.8450 \times 16.6850$$

$$+\frac{1}{k^2R^2}\Big[-k^8 - k^6(6.5789+32.8986) + k^4(2.3262^2 - 600.6890 - 32.4697)$$
$$+ k^2(2\times 2.3262\times 22.9595 - 6.5789\times 600.6890 - 32.8986\times 32.4697)$$
$$+22.9595^2 - 32.4697\times 600.6890\Big]$$

$$-\frac{i}{kR}\Big[-k^6(.6291+2.1937) - k^4\big(6.5789\times .6291 + 26.0484 + 5.7877 + 2.1937\times 32.8986$$
$$- .7503\times 2.3262\times 2\big)$$
$$-k^2\big(26.0484\times 6.5789 + .6291\times 32.4697 + 2.1937\times 600.6890 + 32.8986\times 5.7877$$
$$- 2.3262\times \cancel{18.5300}\ 18.530 - .7503\times 22.9595\times 2\big)$$
$$- 32.4697\times 26.0484 - 600.6890\times 5.7877 + 22.9595\times 18.5300$$

$$R = .8171k^4 + 46.8804k^2 + 119.9765$$
$$+\frac{1}{k^2R^2}\Big[-k^8 - 39.4775k^6 - 87.7475k^4 + 4913.2638k^2 - 18,997.053\Big]$$
$$-\frac{i}{kR}\Big[-2.8228k^6 - 129.1074k^4 - 1622.3777k^2 - 389.9519\Big]$$

$k = 1, \quad R = 10^4$

$P = \overset{147.006}{73.503}, \qquad Q = -251.9666 - i\times .3362$

$R = 117.5580 + 0.2144i$

$$Q^2 - 4PR = 251.9666^2 - .3362^2 + 2\times .3362\times 251.9666\, i$$
$$- 4\times 73.503\,(117.5580 + .2144i)$$
$$= -33889.311 + 106.3862i = 33889.478\times e^{i(180-.1800)^{deg.}}$$

$$\sqrt{Q^2-4PR} = 184.091\times e^{89.91^\circ i} = .2892 + 184.09i$$

$$\gamma = \frac{-Q}{2P} \pm \frac{\sqrt{Q^2-4PR}}{2P} = 1.7140 + .0023i \pm (.0020 + 1.2523i)$$
$$= \begin{cases} 1.716 + 1.2546i \\ 1.712 - 1.2500i \end{cases}$$

$$\frac{a_{k+1}}{a_1} = \frac{-\gamma A_1 + B_1 - \frac{iC_1}{kR}}{\gamma A_2 + B_2 - \frac{iC_2}{kR}} \doteqdot -\frac{\gamma A_1 + B_1}{\gamma A_2 + B_2} = \frac{\cancel{4.8997 + .988}}{1.1638 - .1743}$$

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$$a_1\left[\gamma(k^2+A_1)+B_1k^2+C_1-\frac{i}{kR}\left(k^4+k^2D_1+E_1\right)\right]$$
$$+a_2\left[\gamma A_2+k^2B_2+C_2-\frac{i}{kR}\left(k^2D_2+E_2\right)\right]=0$$

$$a_1\left[\gamma A_3+k^2B_3+C_3-\frac{i}{kR}\left(k^2D_3+E_3\right)\right]$$
$$+a_2\left[\gamma(k^2+A_4)+k^2B_4+C_4-\frac{i}{kR}\left(k^4+k^2D_4+E_4\right)\right]=0$$

$$\left\{\gamma(k^2+A_1)+B_1k^2+C_1\right\}\left\{\gamma(k^2+A_4)+k^2B_4+C_4\right\}-\frac{1}{k^2R^2}(k^4+k^2D_1+E_1)\times(k^4+k^2D_4+E_4)$$
$$-\frac{i}{kR}\left[\left\{\gamma(k^2+A_4)+k^2B_4+C_4\right\}(k^4+k^2D_1+E_1)+(k^4+k^2D_4+E_4)\times\left\{\gamma(k^2+A_1)+B_1k^2+C_1\right\}\right]$$
$$=(\gamma A_2+k^2B_2+C_2)(\gamma A_3+k^2B_3+C_3)-\frac{1}{k^2R^2}(k^2D_2+E_2)(k^2D_3+E_3)$$
$$-\frac{i}{kR}\left[(\gamma A_2+k^2B_2+C_2)(k^2D_3+E_3)+(\gamma A_3+k^2B_3+C_3)(k^2D_2+E_2)\right]$$

$$\gamma^2\left\{(k^2+A_1)(k^2+A_4)-A_2A_3\right\}$$
$$+\gamma\left[(k^2+A_1)(k^2B_4+C_4)+(k^2+A_4)(k^2B_1+C_1)-A_2(k^2B_3+C_3)-A_3(k^2B_2+C_2)\right]$$
$$-\frac{i\gamma}{kR}\left[(k^2+A_1)(k^4+k^2D_4+E_4)+(k^2+A_4)(k^4+k^2D_1+E_1)-A_2(k^2D_3+E_3)-A_3(k^2D_2+E_2)\right]$$
$$+(k^2B_1+C_1)(k^2B_4+C_4)-\frac{1}{k^2R^2}(k^4+k^2D_1+E_1)(k^4+k^2D_4+E_4)$$
$$-(k^2B_2+C_2)(k^2B_3+C_3)+\frac{1}{k^2R^2}(k^2D_2+E_2)(k^2D_3+E_3)$$
$$-\frac{i}{kR}\left[(k^2B_4+C_4)(k^4+k^2D_1+E_1)+(k^2B_1+C_1)(k^4+k^2D_4+E_4)-(k^2B_2+C_2)(k^2D_3+E_3)-(k^2B_3+C_3)(k^2D_2+E_2)\right]=0$$

$$A\gamma^2 + \gamma\left(B + \frac{iC}{kR}\right) + D + \frac{iE}{kR} - \frac{F}{k^2R^2} = 0$$

$$\gamma = \frac{1}{2A}\left[-B + \frac{iC}{kR} \pm \sqrt{\left(B - \frac{iC}{kR}\right)^2 - 4A\left(D - \frac{iE}{kR} - \frac{F}{k^2R^2}\right)}\right]$$

$$\left[B^2 - 4AD + \frac{1}{k^2R^2}\left(4AF - C^2\right) \quad \frac{i}{kR}\left(4AE - 2BC\right)\right]$$

(7)式において $\omega = kc = kU_m\gamma$, $\gamma = c/U_m$, $R = \frac{U_m \times \frac{h}{2}}{\nu}$ (14)

とおいて kU_mで割ると

$$(\gamma - \frac{U}{U_m})(k^2\phi - \phi_{yy}) - \frac{U_{yy}}{U_m}\phi = -\frac{i}{kR}(k^2 - \frac{\partial^2}{\partial y^2})^2\phi, \quad (7')$$

となる。

$U(y)$の一般形は前節(21)で与えられるが、先ず最初に従来解かれている前節(20)で与えられる場合を考えよう。

$$U/U_m = \frac{3}{2}(1-y^2), \quad U_{yy}/U_m = -3, \quad (15)$$

そして

$$\phi = \sum_n (A_n \cos n\pi y + B_n \sin n\pi y), \quad (16)$$

の形に展開出来るとすると(8)より

$$\sum_{n=0} (-)^n A_n = 0, \quad \sum_n n(-)^n B_n = 0, \quad (17)$$

でなければならない。

(16)を(7')に代入すると

$$\sum_n (A_n \cos n\pi y + B_n \sin n\pi y)\left[(\gamma - \frac{U}{U_m})(k^2 + n^2\pi^2) - \frac{U_{yy}}{U_m} + \frac{i}{kR}(k^2 + n^2\pi^2)^2\right] = 0 \quad (18)$$

これに $\cos m\pi y$, $\sin m\pi y$ をかけて積分すると

$$\frac{1}{2}\int_{-1}^{1} \cos n\pi y \cos m\pi y \, dy = \begin{cases} 1 & \text{for } n = m = 0 \\ \frac{1}{2} & \text{for } n = m \geq 1 \\ 0 & \text{for } n \neq m \end{cases}$$

$$\frac{1}{2}\int_{-1}^{1} \sin n\pi y \sin m\pi y \, dy = \begin{cases} \frac{1}{2} & \text{for } n = m \\ 0 & \text{for } n \neq m \end{cases}$$

(19)

$$\frac{3}{2}\int_0^1 \quad dy =$$

$$\frac{1}{2}\int_{-1}^{1}\frac{U}{U_m}\cos n\pi y\,dy = \frac{3}{\cancel{2}\pi}\int_0^{\pi}\left(1-\frac{y'^2}{\pi^2}\right)\cos n y'\,dy'$$

$$= \frac{3}{\cancel{2}\pi}\left[+\frac{1}{n\pi^2}\int_0^{\pi} y \sin n y\,dy\right] = \frac{3}{\cancel{2}n\pi^3}\left[-\frac{y\cos n y}{n}\Big|_0^{\pi} + \frac{1}{n}\int_0^{\pi}\cos n\pi y\,dy\right]$$

$$-\frac{\pi(-)^n}{n}$$

$$= \begin{cases} 1 & \text{for } n=0 \\ \dfrac{3}{\cancel{2}n\pi^3}\left[-\dfrac{\pi(-)^n}{n}\right] = \dfrac{3(-)^{n+1}}{2n^2\pi^2} \end{cases} \quad (20)$$

$$\frac{3}{4}\times\frac{1}{2}$$

$$\frac{1}{2}\int_{-1}^{1}\frac{U}{U_m}\cos n\pi y\cos m\pi y\,dy = \frac{3}{\cancel{4}}\times\int_0^1 (1-y^2)\{\cos \overline{n-m}\,\pi y + \cos \overline{n+m}\,\pi y\}\,dy$$

$$= \begin{cases} 1 & \text{for } n=m=0 \\ \dfrac{3}{2n^2\pi^2}(-)^{n+1} & \text{for } m=0,\ n\geq 1 \\ \dfrac{3}{\cancel{4}\pi^2}\left\{\dfrac{(-)^{n-m+1}}{(n-m)^2} + \dfrac{(-)^{n+m+1}}{(n+m)^2}\right\} & \text{for } n, m\geq 1 \\ \cancel{\dfrac{1}{2}} + \dfrac{3(-)^{n+m+1}}{\cancel{4}(n+m)^2\pi^2} & \text{for } n=m\geq 1 \end{cases} \quad (21)$$

$$\frac{1}{2}\int_{-1}^{1}\frac{U}{U_m}\sin n\pi y\sin m\pi y = \frac{3}{2\pi}\int_0^{\pi}\left(1-\frac{y'^2}{\pi^2}\right)\sin n y'\sin m y'\,dy'$$

$$= \frac{3}{4\pi}\int_0^{\pi}\left(1-\frac{y^2}{\pi^2}\right)\{\cos \overline{n-m}\,y - \cos\overline{n+m}\,y\}\,dy$$

$$= \begin{cases} \dfrac{1}{2} - \dfrac{3(-)^{n+m+1}}{4\,\overline{n+m}^2\,\pi^2} = \dfrac{1}{2} + \dfrac{3}{16\pi^2 n^2} & \text{for } n=m \\ \dfrac{3(-)^{n-m+1}}{\cancel{4}(n-m)^2\pi^2} - \dfrac{3(-)^{n+m+1}}{\cancel{4}(n+m)^2\pi^2}, & n\neq m \end{cases} \quad (22)$$

$$\frac{1}{2}\int \frac{U}{U_m}dy\left[1, \cos\pi y, \cos 2\pi y, \cos 3\pi y, \cos 4\pi y, \cos 5\pi y, \cos 6\pi y\right]$$
$$= \left[1, \frac{3}{2\pi^2}, -\frac{3}{8\pi^2}, \frac{3}{6\pi^2}, -\frac{3}{32\pi^2}, \frac{3}{50\pi^2}, \frac{-3}{72\pi^2}\right]$$

$$\frac{1}{2}\int \frac{U}{U_m}dy\left[\cos^2\pi y, \cos\pi y\cos 2\pi y, \cos^2 2\pi y\right]$$
$$= \left[\frac{1}{2}-\frac{3}{16\pi^2}, \frac{3}{2\pi^2}\left(\frac{1}{1}-\frac{1}{9}\right), \frac{1}{2}-\frac{3}{64\pi^2}\right]$$

$$\frac{1}{2}\int \frac{U}{U_m}dy\left[\sin^2\pi y, \sin\pi y\sin 2\pi y, \sin\pi y\sin 3\pi y, \sin^2 2\pi y, \sin\pi y\sin 3\pi y\right]$$
$$= \left[\frac{1}{2}+\frac{3}{16\pi^2}, \frac{1}{2\pi^2}, -\frac{3}{16\pi^2}\left(1-\frac{1}{4}\right), \frac{1}{2}+\frac{3}{64\pi^2}, \frac{3}{4\pi^2}\left(1-\frac{1}{25}\right), \frac{1}{2}+\frac{3}{144\pi^2}\right]$$
$$\left(-\frac{9}{64\pi^2}\right) \qquad \left(\frac{18}{25\pi^2}\right)$$

$$\frac{1}{2}\int_{-1}^{1}(1+\cos\pi y)^2 dy = 1+\frac{1}{2} = \frac{3}{2}$$

$$\frac{1}{2}\int(1+2\cos\pi y-3\cos 2\pi y)^2 dy$$
$$= 1+\frac{4}{2}+9 = 12$$

$\pi\cos\pi y + \pi\cos 2\pi y$

$$\frac{1}{2}\int_{-1}^{1}\left(\sin\pi y+\frac{1}{2}\sin 2\pi y\right)^2 = \frac{1}{2}\left[1+\frac{1}{4}\right] = \frac{5}{8}$$

$$A\sin\pi y+\frac{B}{2}\sin 2\pi y+\frac{C}{3}\sin 3\pi y$$

$$\frac{1}{2}\int(\sin\pi y-\sin 2\pi y-\sin 3\pi y)^2 dy = \frac{1}{2}+\frac{1}{2}+\frac{1}{2} = \frac{3}{2}$$

$-A+B-C=0, \quad C=B-A=-3A$

$\frac{A}{2}+\frac{B}{4}=0, \quad B=-2A$

$-1 \quad -2+3$

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$$A_0\left\{(\gamma-\frac{U}{U_m})k^2+3+\frac{i}{kR}k^4\right\}$$

$$+A_1\cos\pi y\left\{(\gamma-\frac{U}{U_m})(k^2+\pi^2)+3+\frac{i}{kR}(k^2+\pi^2)^2\right\}$$

$$+A_2\cos 2\pi y\left\{(\gamma-\frac{U}{U_m})(k^2+4\pi^2)+3+\frac{i}{kR}(k^2+4\pi^2)^2\right\}$$

$$+B_1\sin\pi y\left\{(\gamma-\frac{U}{U_m})(k^2+\pi^2)+3+\frac{i}{kR}(k^2+\pi^2)^2\right\}$$

$$+B_2\sin 2\pi y\left\{(\gamma-\frac{U}{U_m})(k^2+4\pi^2)+3+\frac{i}{kR}(k^2+4\pi^2)^2\right\}$$

$$+B_3\sin 3\pi y\left\{(\gamma-\frac{U}{U_m})(k^2+9\pi^2)+3+\frac{i}{kR}(k^2+9\pi^2)^2\right\}$$

$$+\cdots\cdots \qquad =0, \quad (83)$$

$$A_0\left\{k^2(\gamma-1)+3+\frac{ik^3}{R}\right\}+A_1\left\{\frac{3}{2\pi^2}(k^2+\pi^2)\right\}+A_2\left\{-\frac{3}{8\pi^2}(k^2+4\pi^2)\right\}=0.$$

$$(\tfrac{1}{2}-\cdots$$

$$A_0\left\{\frac{-3}{2\pi^2}k^2\right\}+A_1\left\{(\frac{\gamma}{2}-\frac{1}{2}+\frac{3}{16\pi^2})(k^2+\pi^2)+\frac{3}{2}+\frac{i}{2kR}(k^2+\pi^2)^2\right\}$$
$$+A_2\left\{-\frac{(k^2+4\pi^2)}{\pi^2}\right\}=0$$

$$A_0\left(k^2\frac{3}{8\pi^2}\right)+A_1\left\{-\frac{(k^2+\pi^2)}{\pi^2}\right\}$$
$$+A_2\left[\left\{\frac{\gamma}{2}-(\frac{1}{2}-\frac{3}{64\pi^2})\right\}(k^2+4\pi^2)+\frac{3}{2}+\frac{i}{2kR}(k^2+4\pi^2)^2\right]=0$$

$$B_1\left\{(\frac{\gamma}{2}-\frac{1}{2}-\frac{3}{16\pi^2})(k^2+\pi^2)+\frac{3}{2}+\frac{i}{2kR}(k^2+\pi^2)^2\right\}$$
$$+B_2\frac{(k^2+4\pi^2)}{2\pi^2}+\frac{B_3\,9}{64\pi^2}=0$$

$$\frac{i}{16\pi^2}-\frac{1}{\pi^2}$$

$$B_1\left\{\mp \frac{1}{2\pi^2}(k^2+\pi^2)\right\} + B_2\left\{\left\{\frac{\gamma}{2}-\left(\frac{1}{2}+\frac{3}{64\pi^2}\right)\right\}(k^2+4\pi^2)+\frac{3}{2}+\frac{i}{2kR}(k^2+4\pi^2)^2\right\}$$

$$+B_3\left\{-\frac{18}{25\pi^2}(k^2+9\pi^2)\right\}=0$$

$$B_1\left\{-\frac{9}{6\pi^2}(k^2+\pi^2)\right\}+B_2\left\{\frac{18}{25\pi^2}(k^2+4\pi^2)\right\}$$

$$+B_3\left\{\left(\frac{\gamma}{2}-\frac{1}{2}-\frac{1}{48\pi^2}\right)(k^2+9\pi^2)+\frac{3}{2}+\frac{i}{2kR}(k^2+9\pi^2)^2/2\right\}$$

$A_2 = A_1 - A_0$, $B_3 = \frac{2}{3}B_2 - \frac{1}{3}B_1$ を代入すると

$$A_0\left\{k^2(\gamma-1)+3+\frac{3}{8\pi^2}(k^2+4\pi^2)+\frac{ik^3}{R}\right\}+A_1\left\{\frac{9}{8\pi^2}k^2\right\}=0$$

$$A_0\left[\frac{k^2}{\pi^2}+4-\frac{3}{2\pi^2}k^2\right]+A_1\left\{\frac{\gamma-1}{2}(k^2+\pi^2)-\frac{13}{16\pi^2}k^2-\frac{37}{16}+\frac{i}{2kR}(k^2+\pi^2)^2\right\}=0$$

$$A_0\left\{(\gamma-1)k^2+\frac{17}{2}-\frac{k^2}{8\pi^2}+\frac{ik^3}{R}\right\}+A_1\left\{\frac{(\gamma-1)}{2}(k^2+\pi^2)-\frac{37}{16}+\frac{5k^2}{16\pi^2}+\frac{i(k^2+\pi^2)^2}{2kR}\right\}=0$$

$$A_0\left\{-\frac{1}{2}\left(\gamma-1+\frac{3}{32\pi^2}\right)(k^2+4\pi^2)-\frac{3}{2}+\frac{3k^2}{8\pi^2}+\frac{i}{2kR}(k^2+4\pi^2)^2\right\}$$

$$+A_1\left\{\frac{1}{2}\left(\gamma-1+\frac{3}{32\pi^2}\right)(k^2+4\pi^2)+\frac{1}{2}-\frac{k^2}{\pi^2}+\frac{i}{2kR}(k^2+4\pi^2)^2\right\}=0.$$

$$A_0\left\{-\frac{(\gamma-1)}{2}(k^2+4\pi^2)-\frac{27}{16}+\frac{21}{64\pi^2}k^2+\frac{i}{2kR}(k^2+4\pi^2)^2\right\}$$

$$+A_1\left\{\frac{(\gamma-1)}{2}(k^2+4\pi^2)+\frac{11}{16}-\frac{61}{64}\frac{k^2}{\pi^2}+\frac{i}{2kR}(k^2+4\pi^2)^2\right\}=0$$

$$A_0\left[(\gamma-1)\{k^2 \cdots\}+3-\frac{9}{8}+\frac{27\times3}{16}+\frac{3}{8\pi^2}k^2+\frac{k^2}{\pi^2}-\frac{21\times3}{64\pi^2}k^2+\frac{i}{kR}\{k^4-3(k^2+4\pi^2)^2\}\right]$$

$$+A_1\left[+(\gamma-1)\left\{-(k^2+\pi^2)-\frac{3}{2}(k^2+4\pi^2)\right\}+\frac{37}{8}-\frac{33}{16}+\frac{9}{8\pi^2}k^2+\frac{13}{8\pi^2}k^2+\frac{61\times3}{64\pi^2}k^2\right.$$

$$\left.+\frac{i}{kR}\left\{-(k^2+\pi^2)^2-\frac{3}{2}(k^2+4\pi^2)^2\right\}\right]=0$$

$$A_0\left\{(\gamma-1)k^2+\frac{17}{2}-\frac{k^2}{8\pi^2}+\frac{ik^3}{R}\right)+A_1\left\{\frac{\gamma-1}{2}(k^2+\pi^2)-\frac{37}{16}+\frac{5k^2}{16\pi^2}+\frac{i}{2kR}(k^2+\pi^2)^2\right\}=0.$$

$$A_0\left[(\gamma-1)\left(\frac{5}{2}k^2+6\pi^2\right)+\frac{25}{16}+\frac{25k^2}{64\pi^2}+\frac{i}{kR}\left\{k^4-3(k^2+4\pi^2)^2\right\}\right]$$

$$+A_1\left[-(\gamma-1)\left(\frac{5}{2}k^2+17\pi^2\right)+\frac{41}{16}+\frac{35}{64\pi^2}k^2-\frac{i}{kR}\left\{(k^2+\pi^2)^2+\frac{3}{2}(k^2+4\pi^2)^2\right\}\right]=0$$

$$A_0\left(\overline{\gamma-1}\,k^2+a_0+\frac{i}{R}b_0\right)+A_1\left(\frac{\overline{\gamma-1}(k^2+\pi^2)}{2}+a_1+\frac{ib_1}{R}\right)=0$$

$$A_0\left(\overline{\gamma-1}\left(\frac{5}{2}k^2+6\pi^2\right)+a_2+\frac{i}{R}b_2\right)+A_1\left(-(\gamma-1)\left(\frac{5}{2}k^2+7\pi^2\right)+a_3+\frac{ib_3}{R}\right)=0$$

$$-\left\{(\gamma-1)k^2+a_0\right\}\left\{(\gamma-1)\left(\frac{5}{2}k^2+7\pi^2\right)-a_3\right\}-\frac{b_0b_3}{R^2}$$

$$+\frac{i}{R}\left[b_0\left\{-(\gamma-1)\left(\frac{5}{2}k^2+7\pi^2\right)+a_3\right\}+b_3\left(\overline{\gamma-1}\,k^2+a_0\right)\right\}$$

$$-\left(\frac{\overline{\gamma-1}}{2}(k^2+\pi^2)+a_1\right)\left\{\overline{\gamma-1}\left(\frac{5}{2}k^2+6\pi^2\right)+a_2\right\}+\frac{b_1b_2}{R^2}.$$

$$-\frac{i}{R}\left[b_1\left(\overline{\gamma-1}\left(\frac{5}{2}k^2+6\pi^2\right)+a_2\right)+b_2\left(\frac{\overline{\gamma-1}(k^2+\pi^2)}{2}+a_1\right)\right]=0.$$

Im

$$b_0\left\{-(\gamma-1)\left(\frac{5}{2}k^2+7\pi^2\right)+a_3\right\}-b_1\left\{\overline{\gamma-1}\left(\frac{5}{2}k^2+6\pi^2\right)+a_2\right\}$$

$$+b_3\left(\overline{\gamma-1}\,k^2+a_0\right)-b_2\left\{\frac{\gamma-1}{2}(k^2+\pi^2)+a_1\right\}=0.$$

$$\overline{\gamma-1}\left\{-b_0\left(\frac{5}{2}k^2+7\pi^2\right)-b_1\left(\frac{5}{2}k^2+6\pi^2\right)+b_3k^2-\frac{b_2}{2}(k^2+\pi^2)\right\}$$

$$+b_0a_3-b_1a_2+a_0b_3-b_2a_1=0$$

$1-\gamma=\xi$

$$A_0\left(-\xi+a_0+\frac{i}{R}b_0\right)+A_1\left(\frac{\xi}{2}+a_1+\frac{i}{R}b_1\right)=0$$

$$A_0\left(\xi+c_0+\frac{i}{R}d_0\right)+A_1\left(\xi+c_1+\frac{i}{R}d_1\right)=0$$

$$(\zeta + a_0)(\zeta + c_1) - \frac{b_0 d_1}{R^2} - \left\{ (\zeta + a_1)(\zeta + c_0) - \frac{b_1 d_0}{R^2} \right\} = 0.$$

$$b_0(\zeta + c_1) + d_1(\zeta + a_0) = d_0(\zeta + a_1) + b_1(\zeta + c_0) \neq 0.$$

$$\begin{cases} \zeta(b_0 + d_1 + d_0 + b_1) = a_1 d_0 + b_1 c_0 - b_0 c_1 - a_0 d_1 \\ 2\zeta^2 + \zeta\{a_0 - c_1 + c_0 + a_1) + a_0 c_1 - a_1 c_0 - \frac{1}{R^2}(b_0 d_1 - b_1 d_0) = 0. \end{cases}$$

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$y = c/U_{max}$, $R = \frac{U_{max}}{\nu}$, $\alpha = kc = \ldots$

$(U-c)(\alpha^2\phi - \phi_{yy}) - \eta''\phi - \frac{i}{\alpha R}(\alpha^2 - \frac{\partial^2}{\partial y^2})^2\phi = 0$

$\eta = 1 - y^2$, $\eta'' = -2$, $\eta = 1 - \cos^2\theta = \sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\phi = \phi_y \Big|_{y=\pm 1} = 0$

$y = \cos\theta$, $\phi_y = -\sin\theta\, d\theta$

$\phi = \sum_{n=0}^{\infty} A_n \cos n\theta = \sum A_n P_n(\theta) = A P_1 + B P_2$

$P_1 = a_0 + a_2\cos 2\theta + a_4\cos 4\theta$, $a_0 + a_2 + a_4 = 0$

$\frac{1}{\sin\theta}\frac{d}{d\theta}P_1 = 4a_2\cos 2\theta + \frac{\ldots}{2\theta}\sin 4\theta$, $4a_2 + 16a_4 = 0$, $a_0 + \frac{3}{4}a_2 = 0$

$a_2 = -\frac{4}{3}a_0$

$a_4 = \frac{1}{3}a_0$

$P_1 = a_0(1 - \frac{4}{3}\cos 2\theta + \frac{1}{3}\cos 4\theta)$

$\frac{1}{\pi}\int_0^{\pi} P_1^2\, d\theta = a_0^2\left(1 + \frac{16}{9\times 2} + \frac{1}{2\cdot 9}\right) = \frac{35}{18}a_0^2$, $a_0 = \sqrt{\frac{18}{35}}$

$P_2 = a_0 + a_2\cos 2\theta + a_4\cos 4\theta + a_6\cos 6\theta$, $a_0 + a_2 + a_4 + a_6 = 0$

$= a_0[1 + \frac{3}{4}\cos 2\theta - 3\cos 4\theta + \frac{5}{4}\cos 6\theta]$

$P_{2y} = 4a_2\cos\theta + 4a_4\frac{\ldots}{\ldots} + 6\ldots$, $4a_2 + 16a_4 + 36a_6 = 0$

$\frac{1}{\pi}\int \frac{P_1}{a_0}P_2\, d\theta = 0$, $a_0 - \frac{2}{3}a_2 + \frac{a_4}{6} = 0$

$a_4 = 4a_2 - 6a_0$

$6a_0 + 17a_4 + 36a_6 = 0$

$9\times$ $10a_0 + 5a_4 + 4a_6 = 0$

~~$4a_0 - 12a_4 = 32a_6 = 0$~~

$\frac{1}{\pi}\int_0^{\pi} P_2^2 \ldots = a_0^2 + \frac{a_2^2}{2} + \frac{a_4^2}{2} + \frac{a_6^2}{2}$

$= a_0^2\left[1 + \frac{9}{16} + 9 + \frac{25}{16}\right] = 16.5625\, a_0^2$

$84a_0 + 28a_4 = 0$

$3a_0 + a_4 = 0$

$4a_2 = a_4 + 6a_0 = 3a_0$

$a_6 = -a_0 - \frac{3}{4}a_0 + 3a_0 = +\frac{5}{4}a_0$

$a_0 = .39036$

$a_2 = \frac{3}{4}a_0 = .29277$

$a_4 = -3a_0 = -1.02857$

$a_6 = +\frac{5}{4}a_0 = .48795$

$P_1 = \sqrt{\frac{15}{32}}\left(1 - \frac{4}{3}\cos 2\theta + \frac{1}{3} [illegible]\right)$

$P_{1y} = \sqrt{\frac{15}{32}}\left(-\frac{16}{3} [illegible] \frac{4\sin 4\theta}{3} [illegible]\right) = \sqrt{\frac{15}{32}}\left([illegible] + \frac{8}{3}\cos 3\theta [illegible]\right)$

[illegible]

$P_{1y} = \sqrt{\frac{15}{32}} \times \frac{8}{3}(-\cos\theta + \cos 3\theta)$

$P_{1yy} = \sqrt{\frac{15}{32}} \times \frac{8}{3}(-1 - 3 - 6\cos 2\theta) = -\frac{16}{3}\sqrt{\frac{15}{32}}(2 + 3\cos 2\theta)$

$(\cos 2\theta)_y = 4\cos\theta, \quad (\cos 4\theta)_y = 4\frac{\sin 4\theta}{\sin\theta} = 8(\cos 3\theta + \cos\theta)$

$(\cos 6\theta)_y = 6\frac{\sin 6\theta}{\sin\theta} = 12(\cos 5\theta + \cos 3\theta + \cos\theta)$

$(\cos 2\theta)_{yy} = 4, \quad (\cos 4\theta)_{yy} = 8 \times 3\frac{\sin 3\theta}{\sin\theta} + 8 = 32 + 48\cos 2\theta$

$(\cos 6\theta)_{yy} = 12 + 36\frac{\sin 3\theta}{\sin\theta} + 60\frac{\sin 5\theta}{\sin\theta} = 12 + 36(1 + 2\cos 2\theta) + 60(1 + 2\cos 2\theta + 2\cos 4\theta)$
$= 108 + 192\cos 2\theta + 120\cos 4\theta$

$P_2 = .8372(1 - .90790\cos 2\theta - .15873\cos 4\theta + .06579\cos 6\theta)$

~~$P_{2y} = .8372(-2 \times .90790\cos\theta - .15873(\cos 3\theta + \cos\theta) + .06579 [illegible])$~~
~~$= .8372(-2.88947\cos\theta - .40515\cos 3\theta + .78948\cos 5\theta)$~~

~~$P_{2yy} = .8372[-2 \times .90790 - .15873 \times (32 + 48\cos 2\theta) + .06579(108 + 192\cos 2\theta + 120\cos 4\theta)]$~~
~~$= .8372[.23896 + 5.05584\cos 2\theta + 7.8948\cos 4\theta]$~~

$\eta = \frac{1-\cos 2\theta}{2}$ $\qquad$ $\sqrt{\frac{2}{35}} = .23905$, $\frac{1}{\sqrt{105}} = .09759$

$$P_1 = \sqrt{\frac{18}{35}}\left(1 - \frac{4}{3}\cos 2\theta + \frac{1}{3}\cos 4\theta\right) = \sqrt{\frac{2}{35}}\left(3 - 4\cos 2\theta + \cos 4\theta\right)$$

$$P_2 = \frac{1}{\sqrt{6.5625}}\left(1 + \frac{3}{4}\cos 2\theta - 3\cos 4\theta + \frac{5}{4}\cos 6\theta\right)$$

$$= \frac{1}{\sqrt{105}}\left(4 + 3\cos 2\theta - 12\cos 4\theta + 5\cos 6\theta\right)$$

$$P_{1y} = \sqrt{\frac{2}{35}} \times 8\left(-\cos\theta + \cos 3\theta\right)$$

$$P_{1yy} = -16\sqrt{\frac{2}{35}}\left(1 + 3\cos 2\theta\right)$$

$$P_{1yyyy} = 192 \times \sqrt{\frac{2}{35}}$$

$$P_{2yy} = \frac{1}{\sqrt{105}}\left[3\times 4 - 12\times(32 + 48\cos 2\theta) + 5\left(108 + 192\cos 2\theta + 120\cos 4\theta\right)\right]$$

$$= \frac{1}{\sqrt{105}}\left[168 + 384\cos 2\theta + 600\cos 4\theta\right] = \frac{24}{\sqrt{105}}\left[7 + 16\cos 2\theta + 25\cos 4\theta\right]$$

$$P_{2yyyy} = \frac{1}{\sqrt{105}}\left[384\times 4 + 600(32 + 48\cos 2\theta)\right] = \frac{1}{\sqrt{105}}\left[20736 + 28800\cos 2\theta\right]$$

$$= \frac{20736}{\sqrt{105}}\left[1 + 1.388889\cos 2\theta\right]$$

$$P_1^2 = \frac{2}{35}\left[9 + 8(1 + \cos 4\theta) + \frac{1}{2}(1 + \cos 8\theta) - 24\cos 2\theta + 6\cos 4\theta - 4(\cos 2\theta + \cos 6\theta)\right]$$

$$= \frac{2}{35}\left[17.5 - 28\cos 2\theta + 14\cos 4\theta - 4\cos 6\theta + \frac{1}{2}\cos 8\theta\right]$$

$$= 1 - \frac{8}{5}\cos 2\theta + \frac{4}{5}\cos 4\theta - \frac{8}{35}\cos 6\theta + \frac{1}{35}\cos 8\theta.$$

$$P_1P_2 = \sqrt{\frac{2}{35\times 105}}\Big[12 + 9\cos 2\theta - 36\cos 4\theta + 15\cos 6\theta$$
$$- 4\cos 2\theta\left(4 + 3\cos 2\theta - 12\cos 4\theta + 5\cos 6\theta\right)$$
$$+ \cos 4\theta\left(4 + 3\cos 2\theta - 12\cos 4\theta + 5\cos 6\theta\right)\Big]$$

$$= \frac{1}{35}\sqrt{\frac{2}{3}}\Big[(9+16)\cos 2\theta - 32\cos 4\theta + 15\cos 6\theta - 6\cos 4\theta + 24(\cos 2\theta + \cos 6\theta)$$
$$- 10(\cos 4\theta + \cos 8\theta) + \frac{3}{2}(\cos 2\theta + \cos 6\theta) - 6\cos 8\theta + \frac{5}{2}(\cos 2\theta + \cos 10\theta)$$

$$= \frac{1}{35}\sqrt{\frac{2}{3}}\left[53\cos 2\theta - 48\cos 4\theta + 40.5\cos 6\theta - 16\cos 8\theta + 1.5\cos 10\theta\right]$$

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$$P_2^2 = \frac{1}{105}\Big[16 + \frac{9}{2}(1+\cos 4\theta) + 72(1+\cos 8\theta) + \frac{25}{2}(1+\cos 12\theta)$$
$$+24\cos 2\theta - 96\cos 4\theta + 40\cos 6\theta - 18(\cos 2\theta + \cos 6\theta)$$
$$- 30(\cos 2\theta + \cos 10\theta) + \frac{15}{2}(\cos 4\theta + \cos 8\theta)\Big]$$

$$= \frac{1}{105}\Big[105 + 24\cos 2\theta - 84\cos 4\theta + 22\cos 6\theta + 79.5\cos 8\theta - 30\cos 10\theta + 12.5\cos 12\theta\Big]$$

$$P_{1yy}P_1 = -\frac{32}{35}\Big[3 + 9\cos 2\theta - 4\cos 2\theta - 6(1+\cos 4\theta) + \frac{3}{2}(\cos 2\theta + \cos 6\theta)\Big]$$

$$= +\frac{32}{35}\Big[+3 - 6.5\cos 2\theta + 6\cos 4\theta - \frac{3}{2}\cos 6\theta\Big]$$

$$P_{1yy}P_2 = -\frac{16}{35}\sqrt{\frac{2}{5}}\Big[4 + 3\cos 2\theta - 12\cos 4\theta + 5\cos 6\theta$$
$$+12\cos 2\theta + \frac{9}{2}(1+\cos 4\theta) - 18(\cos 2\theta + \cos 6\theta) + \frac{15}{2}(\cos 4\theta + \cos 8\theta)\Big]$$

$$= -\frac{16}{35}\sqrt{\frac{2}{5}}\Big[8.5 - 3\cos 2\theta - 13\cos 6\theta + \frac{15}{2}\cos 8\theta\Big]$$

$$P_{2yy}P_1 = \frac{24}{35}\sqrt{\frac{2}{5}}\Big[21 + 48\cos 2\theta + 75\cos 4\theta - 28\cos 2\theta - 32(1+\cos 4\theta)$$
$$- 50(\cos 2\theta + \cos 6\theta) + 7\cos 4\theta + 8(\cos 2\theta + \cos 6\theta) + \frac{25}{2}(1+\cos 8\theta)\Big]$$

$$= \frac{24}{35}\sqrt{\frac{2}{5}}\Big[1.5 - 22\cos 2\theta + 50\cos 4\theta - 42\cos 6\theta + \frac{25}{2}\cos 8\theta\Big]$$

$$P_{2yy}P_2 = \frac{24}{105}\Big[28 + 21\cos 2\theta - 84\cos 4\theta + 35\cos 6\theta + 64\cos 2\theta + 24(1+\cos 4\theta)$$
$$- 96(\cos 2\theta + \cos 6\theta) + 40(\cos 4\theta + \cos 8\theta) - 100\cos 4\theta + \frac{75}{2}(\cos 2\theta + \cos 6\theta)$$
$$- 150(1+\cos 8\theta) + \frac{125}{2}(\cos 2\theta + \cos 10\theta)\Big]$$

$$= \frac{24}{105}\Big[-98 + 89\cos 2\theta + 80\cos 4\theta - 23.5\cos 6\theta - 110\cos 8\theta + \frac{125}{2}\cos 10\theta\Big]$$

$[\ \ ] = \frac{1}{2}\int d\theta(\ \ )$

$[P_1^2] = [P_2^2] = 1, \quad [P_1P_2] = 0$

$[P_{1yy}P_1] = \frac{32\times3}{35} = 2.74286, \quad [P_{1yy}P_2] = -\frac{16}{35}\times8.5\times\sqrt{\frac{2}{3}} = -3.17267$

$[P_{2yy}P_1] = \frac{36}{35}\sqrt{\frac{2}{3}} = .83983 \qquad [P_{2yy}P_2] = -\frac{24\times98}{105} = -22.4$

$[P_{1y(4)}, P_1] = 192\times\frac{6}{35} = 32.91429, \quad [P_{1y(4)}, P_2] = 192\times\sqrt{\frac{2}{35}}\times\frac{4}{\sqrt{105}} = \frac{4\times192}{35}\sqrt{\frac{2}{3}}$

$= 17.91627$

$[P_{2y(4)}, P_1] = \frac{20736\times3}{\sqrt{105}}\sqrt{\frac{2}{35}} = \frac{3\times20736}{35}\sqrt{\frac{2}{3}} = 1451.2177$

$[P_{2y(4)}, P_2] = \frac{20736}{\sqrt{105}}\times\frac{4}{\sqrt{105}} = 789.94286$

$[\eta P_{1yy}P_1] = \frac{3\times32}{35\times2} + \frac{1}{4}\times\frac{32\times6.5}{35} = 2.85714$

$[\eta P_{1yy}P_2] = \frac{1}{2}\times\left(-\frac{16\times8.5}{35}\sqrt{\frac{2}{3}}\right) - \frac{1}{4}\times\frac{48}{35}\times\sqrt{\frac{2}{3}} = -\sqrt{\frac{2}{3}}\left(\frac{8\times8.5}{35} + 12\right) = -1.86628$

$[\eta P_{2yy}P_1] = \frac{1}{2}\times\frac{24\times1.5}{35}\sqrt{\frac{2}{3}} + \frac{1}{4}\times\frac{24\times2}{35}\sqrt{\frac{2}{3}} = \sqrt{\frac{2}{3}}\left(\frac{18+12}{35}\right) = 3.49927$

$[\eta P_{2yy}P_2] = \frac{1}{2}\times\frac{24}{105}(-98) = \frac{1}{4}\times\frac{24\times89}{105} = -16.28571$

$[\eta P_1^2] = \frac{1}{2} + \frac{1}{4}\times\frac{8}{5} = .9$

$[\eta P_1P_2] = -\frac{1}{4}\times\frac{53}{35}\sqrt{\frac{2}{3}} = -.30910$

$[\eta P_2^2] = \frac{1}{2} + \frac{1}{4}\times\frac{24}{105} = .55714$

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$$\gamma(k^2\phi - \phi_{yy}) + 2\phi + \eta(\phi_{yy} - k^2\phi) - \frac{i}{kR}\left[k^4\phi - 2k^2\phi_{yy} + \phi_{yyyy}\right] = 0.$$

$$A_1\left[\gamma(k^2P_1 - P_{1yy}) + 2P_1 + \eta(P_{1yy} - k^2P_1) - \frac{i}{kR}\left(k^4P_1 - 2k^2P_{1yy} + P_{1y(4)}\right)\right]$$

$$+ B_1\left[\gamma(k^2P_2 - P_{2yy}) + 2P_2 + \eta(P_{2yy} - k^2P_2) - \frac{i}{kR}\left(k^4P_2 - 2k^2P_{2yy} + P_{2y4}\right)\right] = 0.$$

$$A_1\left[\gamma(k^2 - 2.74286) + 2 + 2.85714 - 0.9k^2 - \frac{i}{kR}\left(k^4 - 5.48592k^2 + 32.91429\right)\right]$$

$$+ B_1\left[-.839837\gamma + 3.49927 + .30910k^2 - \frac{i}{kR}\left(-1.67966k^2 + 1451.2177\right)\right] = 0$$

$$A_1\left[1.86628\gamma - 1.86628 + .30910k^2 - \frac{i}{kR}\left(6.34534k^2 + 17.91627\right)\right]$$

$$+ B_1\left[\gamma(k^2 + 22.4) + 2 - 16.28571 + .55714k^2 - \frac{i}{kR}\left(k^4 + 44.8k^2 + 787.9428\right)\right]$$

$$\gamma^2(k^2 + 22.4)(k^2 - 2.74286) +$$

$$+ \gamma\Big[(k^2 - 2.74286)\left\{-14.28571 + .55714k^2 - \frac{i}{kR}(k^4 + 44.8k^2 + 787.9428)\right\}$$

$$+ (k^2 + 22.4)\left\{4.85714 - 0.9k^2 - \frac{i}{kR}(k^4 - 5.48592k^2 + 32.91429)\right\}\Big]$$

$$+ \{-14.$$

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$k=1,\quad R=10^4$

$$A_1\left[-1.74286\gamma+3.45714-\frac{i}{10^4}\times(28.42857)\right]$$
$$+B\left[-.83983\gamma+3.80837-\frac{i}{10^4}(1449.538)\right]=0$$
$$A_1\left[1.86628\gamma-1.55718-\frac{i}{10^4}(24.26161)\right]$$
$$+B\left[23.4\gamma-13.72857-\frac{i}{10^4}(835.74286)\right]=0.$$

$$-23.4\times1.74286\gamma^2+\gamma\left[23.4\left(3.45714-\frac{28.42857i}{10^4}\right)+1.74286\left(-13.72857-.083574i\right)\right]$$
$$+(3.45714-.0028428i)(-13.72857-.083574i)$$
$$=-1.86628\times.83983\gamma^2+\gamma\left[1.86628(3.80837-.14495i)-.83983(-1.55718-.0024261i)\right]$$
$$+(3.80837-.14495i)(-1.55718-.0024261i)$$

$$39.21557\gamma^2+(-106.5219-.34762i)\gamma+5.93067+.21647i=0.$$

$$\gamma^2-(2.71642+.008864i)\gamma-.15123+.005522i=0$$
$$b^2-4ac$$

$$\gamma=\frac{2.71642+.008864i\pm\sqrt{\ }}{2}$$

$$b^2-4ac=(2.71642+.008864i)^2-4(-.15123+.005522i)$$
$$=7.98378+.02608i=$$
$$\frac{\sqrt{b^2-4ac}}{4}=1.99595+.00652i=1.99596\times e^{.1872\deg i}$$

$$\gamma=1.35821+.00443i\pm(1.40569)e^{.0936\deg i}$$
$$(1.406+.0023i)$$
$$=2.7\ \ +$$
$$-.05\ \ +$$